Intermittence en turbulence d’ondes MHD

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Weak magnetohydrodynamic turbulence and intermittency

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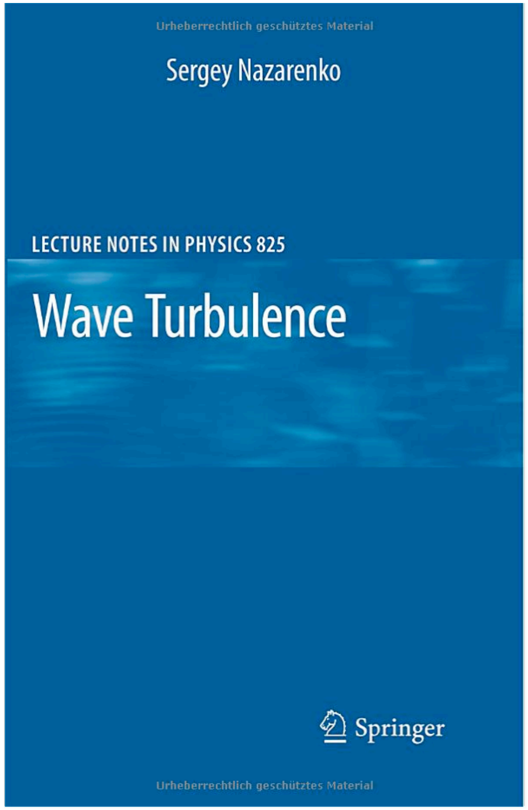
Front cover of JFM !

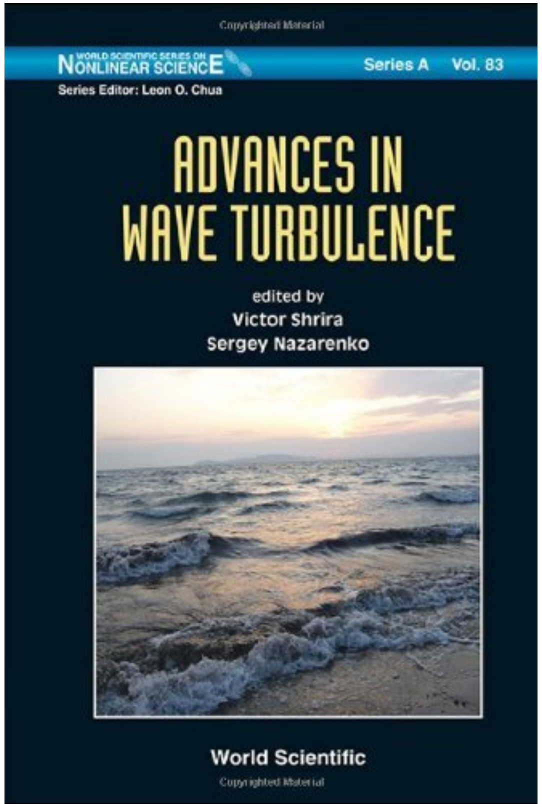
Weak wave turbulence

- ✓ Statistical theory of weakly nonlinear dispersive waves
- ✓ Exact solutions can be found *via* the Zakharov transform

2011



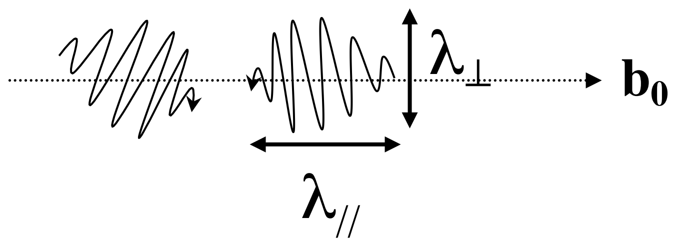
2013



Weak MHD turbulence

$$\frac{\partial \mathbf{z^s}}{\partial t} - s\mathbf{b}_0 \cdot \nabla \mathbf{z^s} = -\mathbf{z^{-s}} \cdot \nabla \mathbf{z^s} - \nabla P_*$$

$s = \pm \qquad \mathbf{z}^{\pm} \equiv \mathbf{u} \pm \mathbf{b}$



$$z_j^s(\mathbf{x},t) \equiv \iiint \hat{z}_j^s(\mathbf{k},t)\, e^{i\mathbf{k}\cdot\mathbf{x}}\, d\mathbf{k} = \iiint \left[\epsilon a_j^s(\mathbf{k},t) e^{is\omega_k t}\right]\, e^{i\mathbf{k}\cdot\mathbf{x}}\, d\mathbf{k}\,,$$

$$\Downarrow$$

$$\boxed{\frac{\partial a_j^s(\mathbf{k})}{\partial t} = -i\epsilon k_m P_{jn} \int_{\mathbf{R}^6} a_m^{-s}(\mathbf{q})\, a_n^s(\mathbf{p})\, e^{-is(\omega_k - \omega_p + \omega_q)t} \delta(\mathbf{k}-\mathbf{p}-\mathbf{q})\, d\mathbf{p} d\mathbf{q}\,,}$$

where $P_{jn}(k) \equiv \delta_{jn} - k_j k_n / k^2$ is the projection operator

Resonance condition (3-wave interactions):

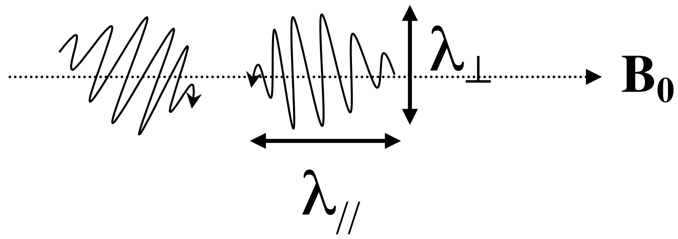
$$\begin{cases} \omega_k = \omega_p - \omega_q \\ \mathbf{k} = \mathbf{p} + \mathbf{q} \end{cases} \quad \Longrightarrow \quad \boxed{q_{\|} = 0}$$

No cascade is expected along b_0

[Shebalin et al., JPP, 1983]

Weak turbulence phenomenology

$z^+ \sim z^- \sim z$



balance turbulence

Same phenomenology as IK but with anisotropy

$$\tau_{tr} \sim \frac{\tau_{eddy}^2}{\tau_A} \sim \frac{(\ell_\perp / z_\ell)^2}{\ell_\parallel / b_0} \sim \frac{k_\parallel b_0}{k_\perp^2 z_\ell^2}$$

We thus obtain :

$$\varepsilon \sim \frac{z_\ell^2}{\tau_{tr}} \sim \frac{k_\perp^2 z_\ell^4}{k_\parallel b_0} \sim \frac{k_\perp^2 (E(k_\perp, k_\parallel) k_\perp k_\parallel)^2}{k_\parallel b_0} \sim \frac{k_\perp^4 k_\parallel E^2(k_\perp, k_\parallel)}{b_0} ,$$

hence the anisotropic (axisymmetric) spectrum :

$$\boxed{E^z(k_\perp, k_\parallel) \sim \sqrt{\varepsilon b_0}\, k_\perp^{-2} \cancel{k_\parallel^{-1/2}}}$$

[SG et al., JPP, 2000]

Weak MHD turbulence theory

• Asymptotic equation: (k$_\perp$ >> k$_{//}$ is assumed – transverse cascade)

$$\boxed{\frac{\partial E^\pm(k_\perp,k_\parallel)}{\partial t} = \frac{\pi\varepsilon^2}{B_0}\iint_\Delta \cos^2\phi\sin\theta\,\frac{k_\perp}{q_\perp}E^\mp(q_\perp,0)\,[k_\perp E^\pm(p_\perp,k_\parallel)-p_\perp E^\pm(k_\perp,k_\parallel)]\,dp_\perp dq_\perp}$$

[SG et al., JPP, 2000]

Resonance condition

• Spectral solutions:

$$E^\pm(k_\perp,k_\parallel) = E^\pm(k_\perp)f_\pm(k_\parallel)$$
$$E^\pm(k_\perp) \sim k_\perp^{n_\pm}$$

Zakharov $\Longrightarrow$ transformation

EXACT SOLUTION

$$\boxed{n_+ + n_- = -4}$$

$$-3 < n_\pm < -1$$

Condition of locality

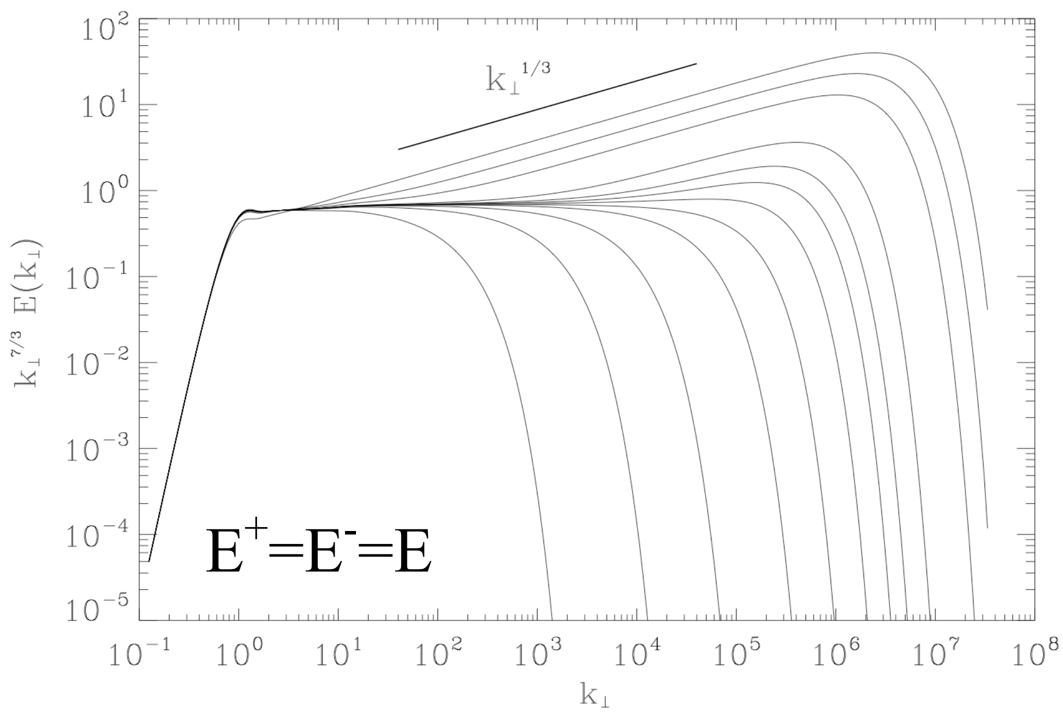
Direct $\perp$ cascade is proved

• Nature of the 2D modes; origin of intermittency ??

Simulations / observations

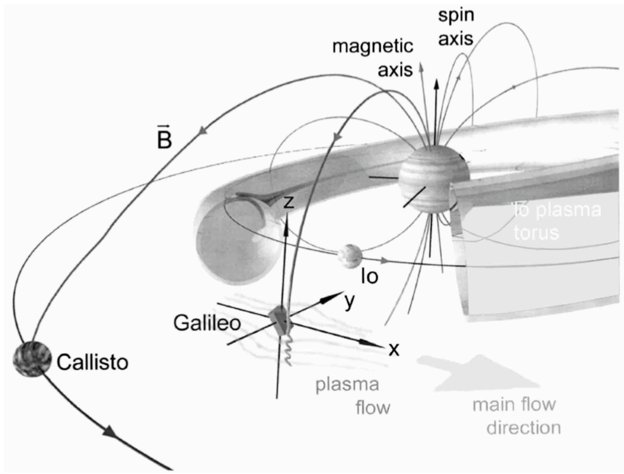
Zakharov solution is found but an anomalous scaling is obtained in the non-stationary phase

[Thalabard et al., will appear]



Indirect signature in the Jupiter’s magnetosphere

[Saur et al., A&A, 2002]



Direct numerical simulations

Parameters of the numerical experiences

✓ Case A: full equations $\partial_t z^{\pm} \mp b_0 \partial_{\|} z^{\pm} + z^{\mp} \cdot \nabla z^{\pm} = -\nabla P_* + \nu_3 \Delta^3 z^{\pm}$

✓ Case B: $u(k_\perp, k_{//}) = b(k_\perp, k_{//}) = 0$ at each time step

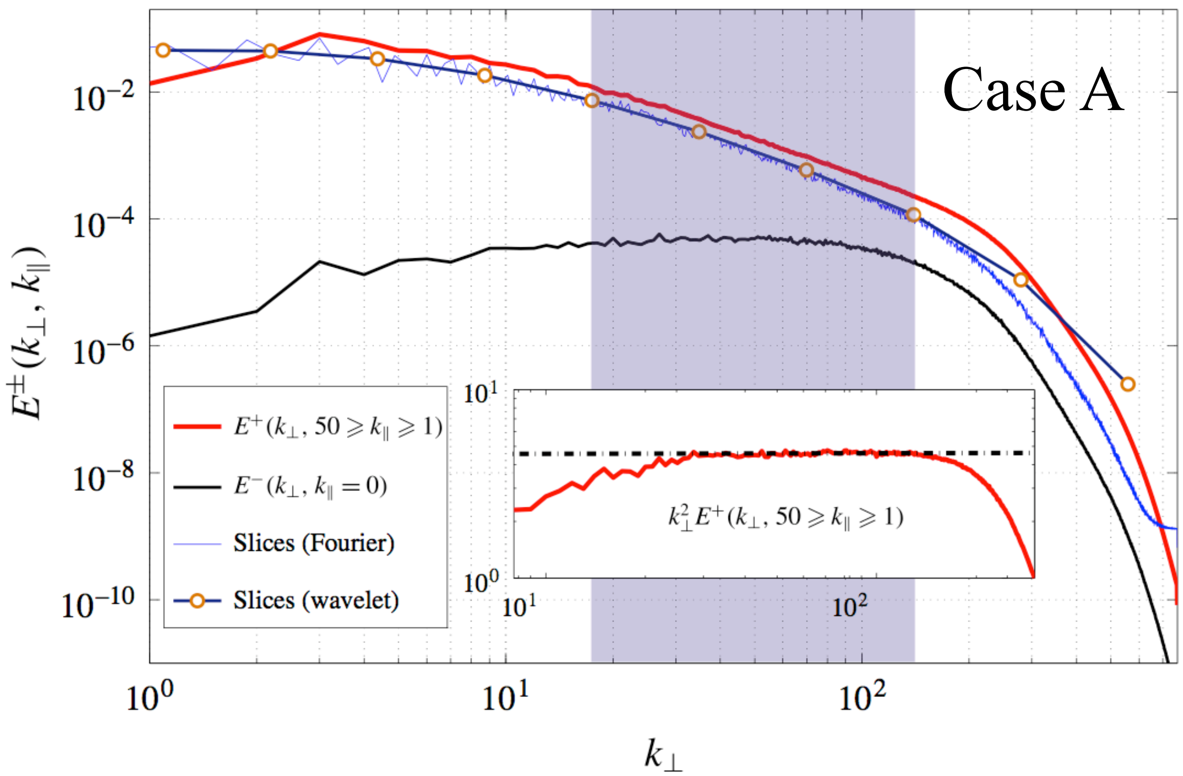
$nx \times ny \times nz$	$E^u_{t=0} = E^b_{t=0}$	$\lvert \mathbf{B}_0 \rvert$	$\int_V \mathbf{u} \cdot \mathbf{b} d\mathbf{x}$	ν_3
$1536 \times 1536 \times 128$	0.5	20.0	0	4.10^{-15}

✓ TURBO = solver for TURbulent flows with periodic BOundary conditions

Numerical results

$$\boxed{\chi^{\pm} = \frac{k_\perp z_\perp^{\pm}}{k_\parallel b_0} < 0.03 \quad \forall \quad (k_\perp, k_\parallel), \quad k_\parallel \neq 0}$$

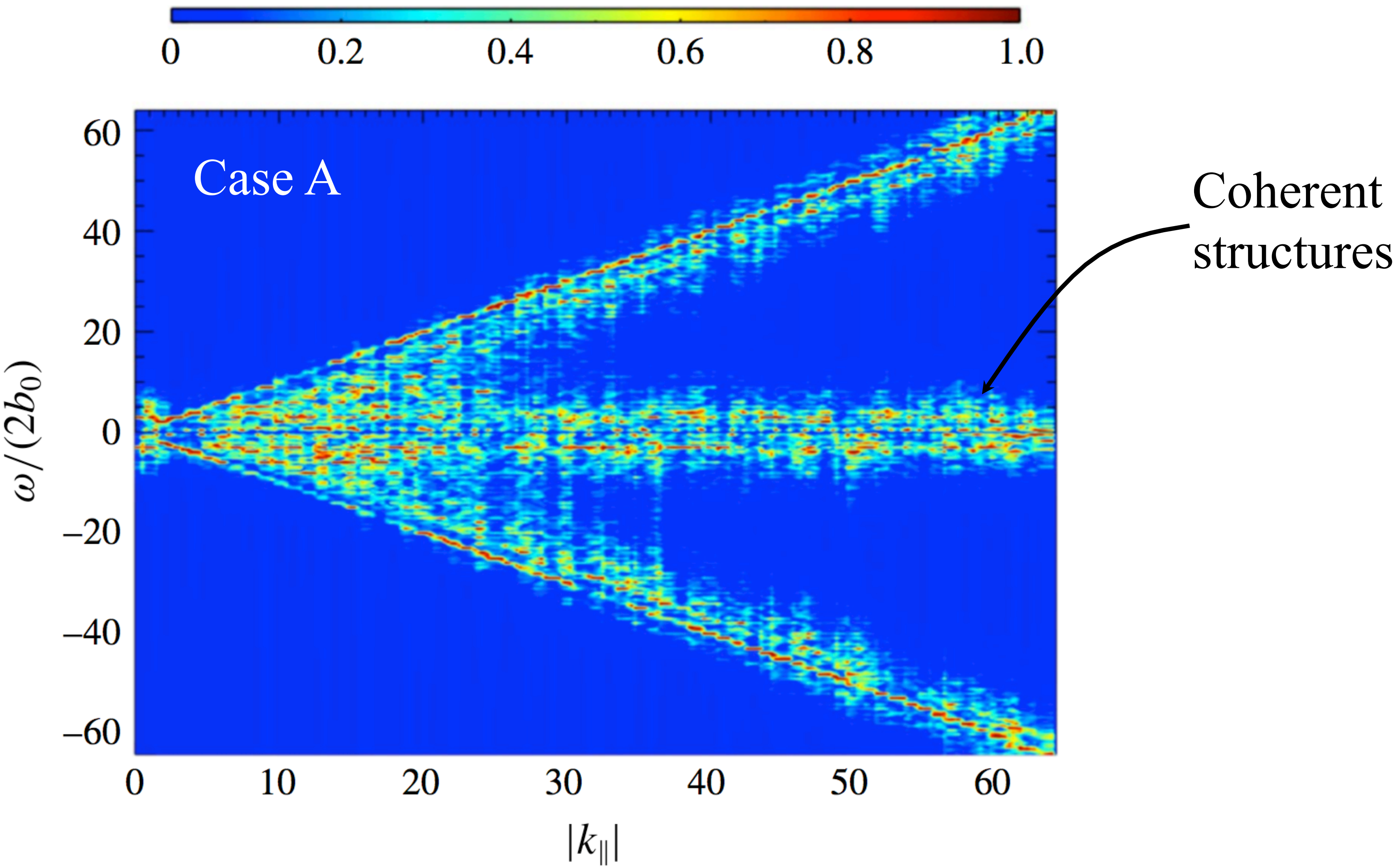
Condition for WT well satisfied



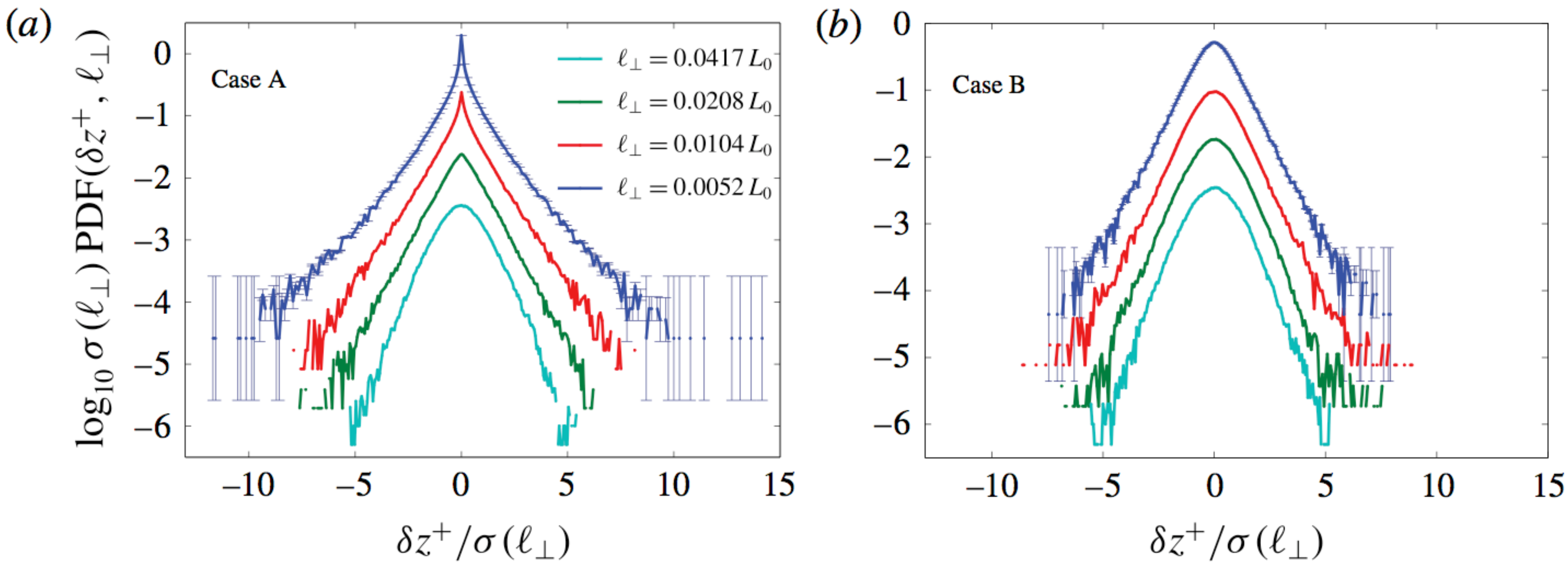
$$\int_1^{50} E^+(k_\perp, k_\parallel)\mathrm{d}k_\parallel \quad \text{(red)}$$

$$E^-(k_\perp,\ k_\parallel = 0) \quad \text{(black)}$$

Spectrogram of the magnetic energy at $k_\perp = 64$



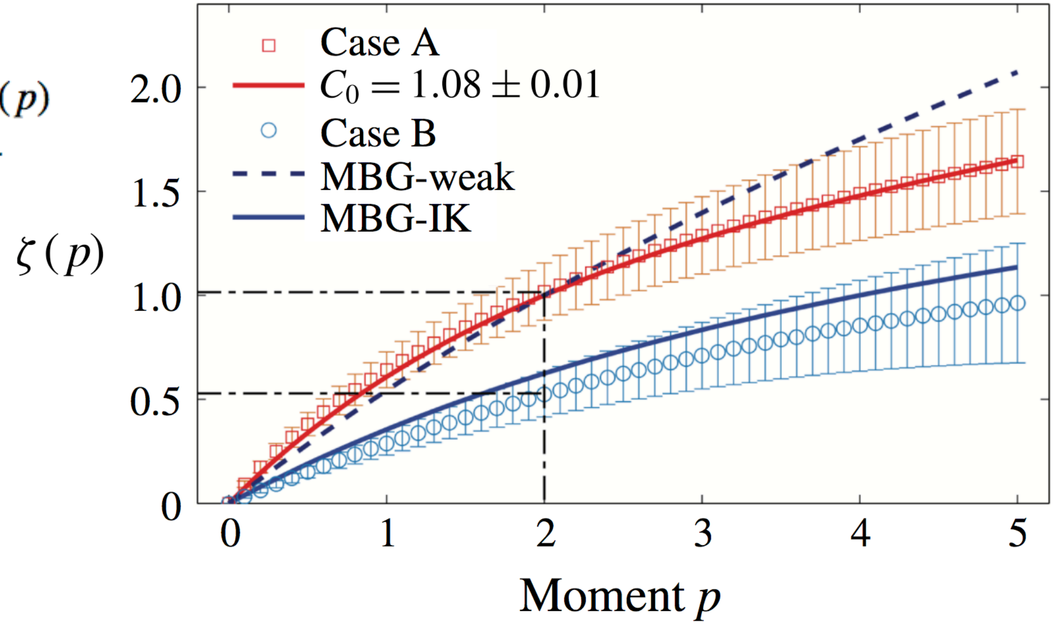
PDFs of the Elsässer field increments δz^+



Case A: strong intermittency is found

Case B: almost no intermittency

$$S_p = \langle(\delta z^+)^{p/2}\rangle\langle(\delta z^-)^{p/2}\rangle = C_p \ell_\perp^{\zeta(p)}$$



A Log-Poisson law is derived:

$$\zeta(p) = \frac{p}{8} + C_0 - C_0\left(1 - \frac{3}{4C_0}\right)^{p/2}$$

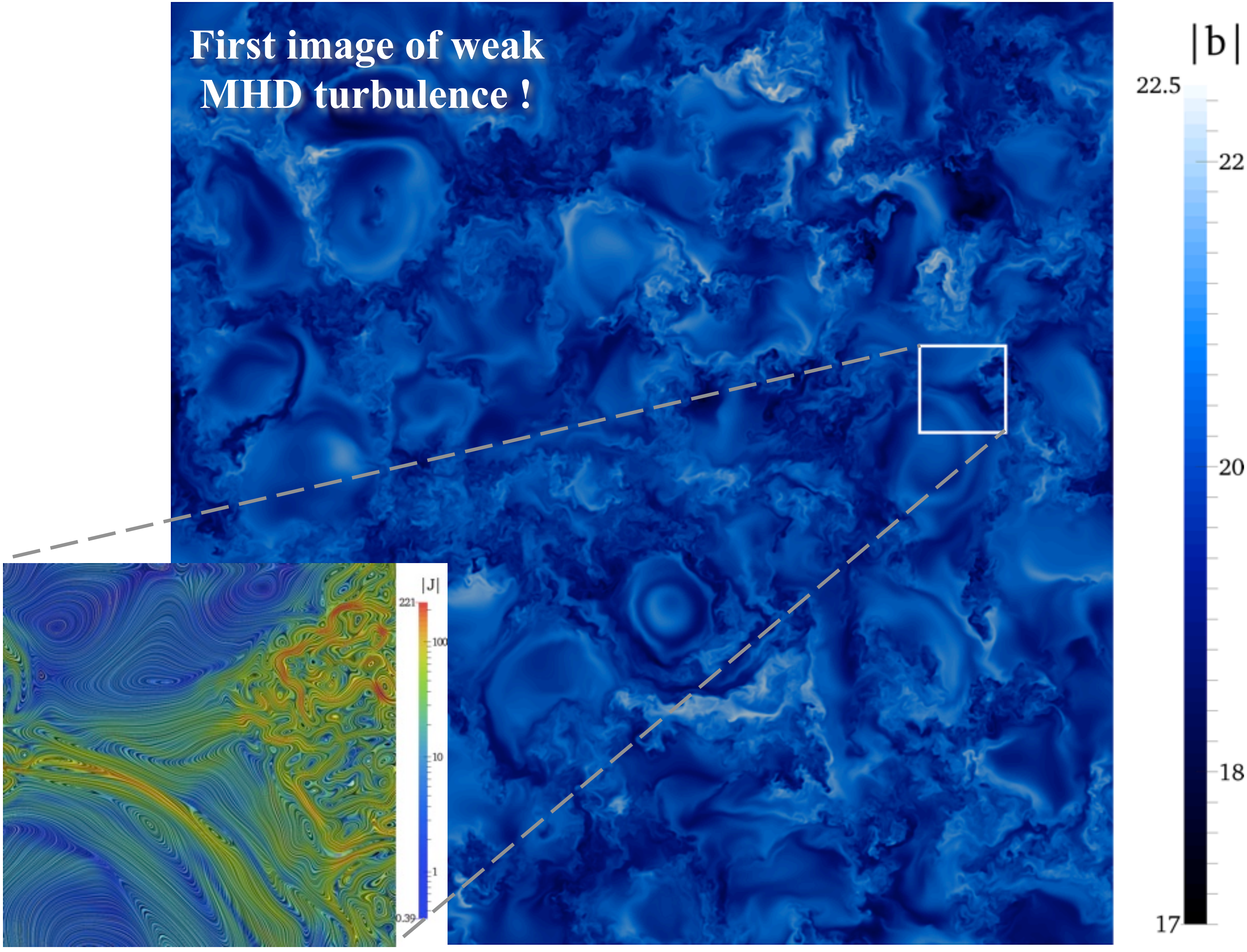
Co-dimension:

$C_0 = 1.08$

$\Rightarrow$ Current sheets

$$\Rightarrow \boxed{\zeta_p = p/8 + 1 - (1/4)^{p/2}}$$

(with C_0=1)



First image of weak
MHD turbulence !
|b|
22.5
22
20
18
17
|J|
221
100
10
1
0.39

Conclusion

[Meyrand et al., JFM-R 770, R1, 2015]

- ✓ Intermittency is found in weak MHD turbulence
- ✓ This intermittency can be modelled with a log-Poisson law
- ✓ The 2D modes play a central role *via* the dissipative structures
- ✓ Main application: solar/stellar magnetic turbulence