# Time Irreversibility in Turbulent Dispersion

Jennifer Jucha, Haitao Xu, Alain Pumir, Eberhard Bodenschatz

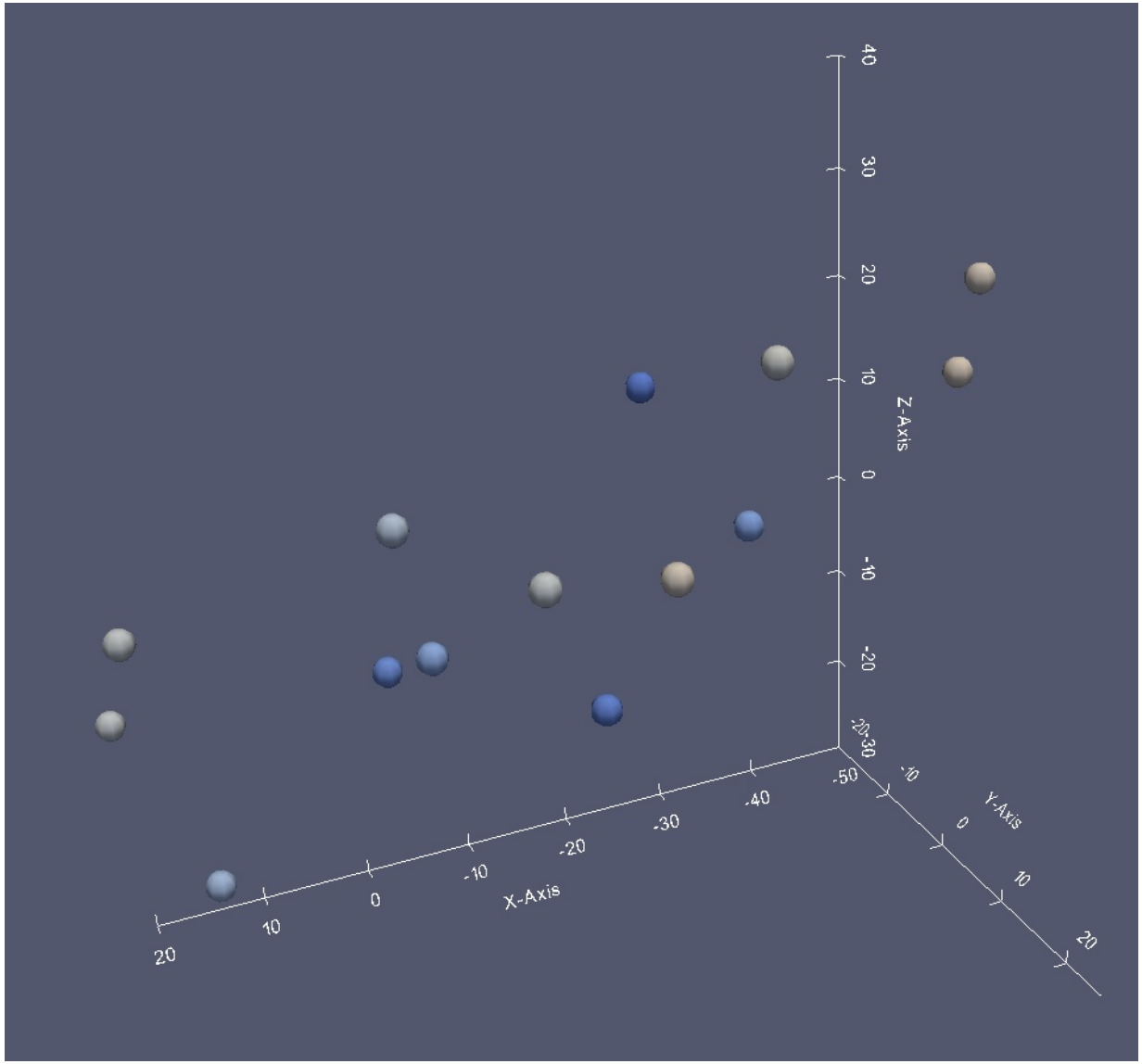
Réunion du GdR Turbulence

Juin 2, 2015

ENS DE LYON

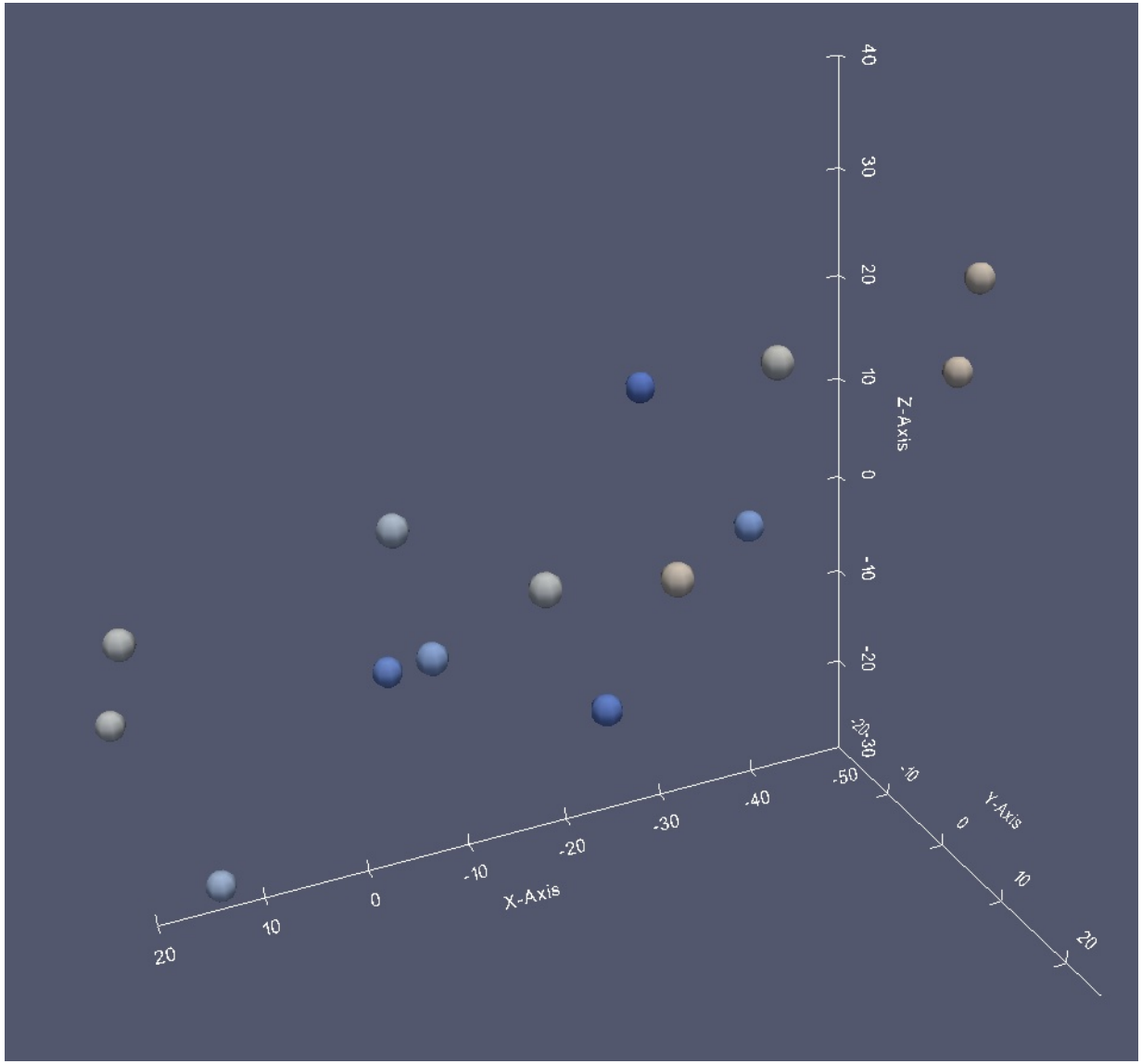
MAX-PLANCK-GESELLSCHAFT

# Introduction



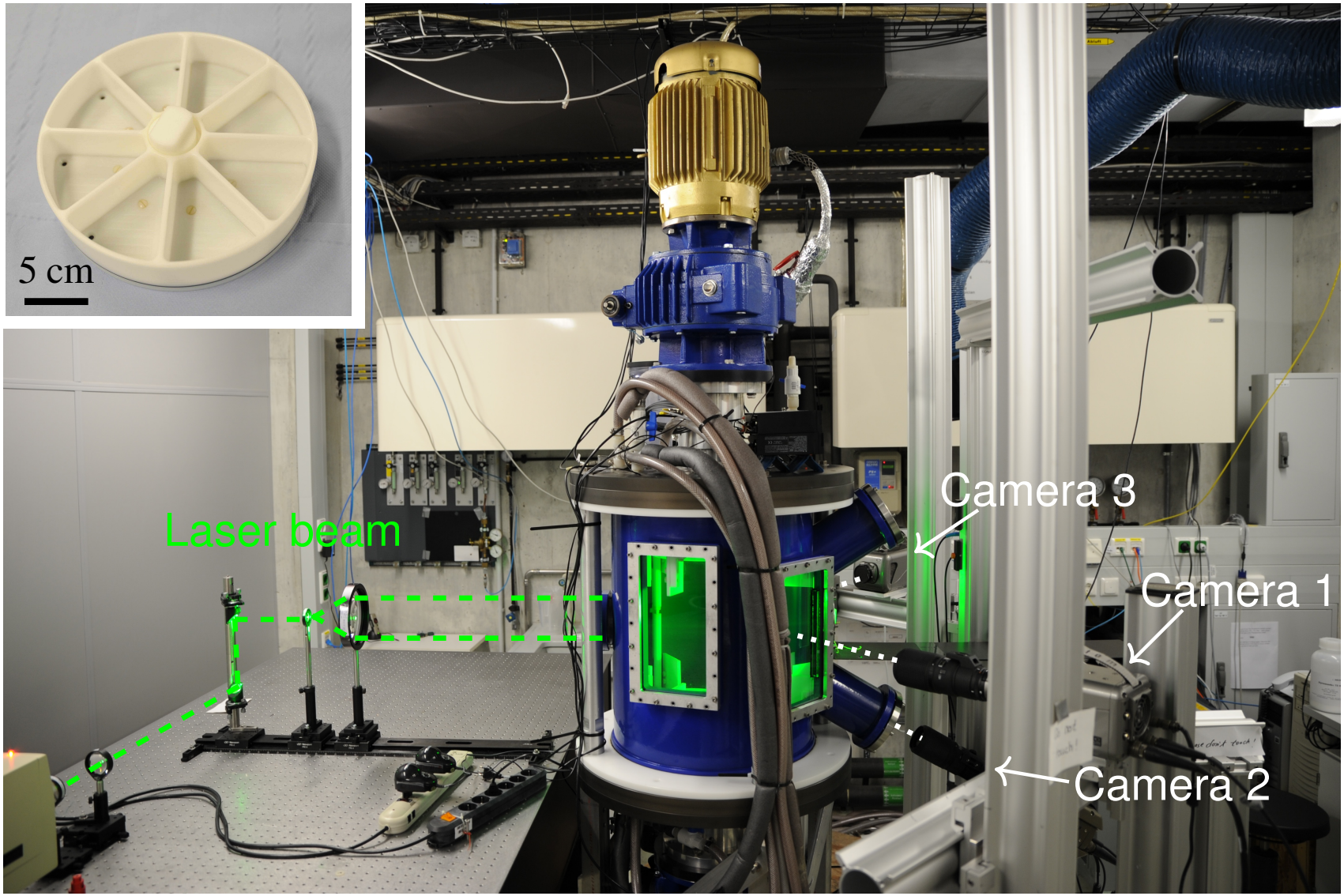

recorded at 7000 FPS, displayed at 140 FPS

# Introduction

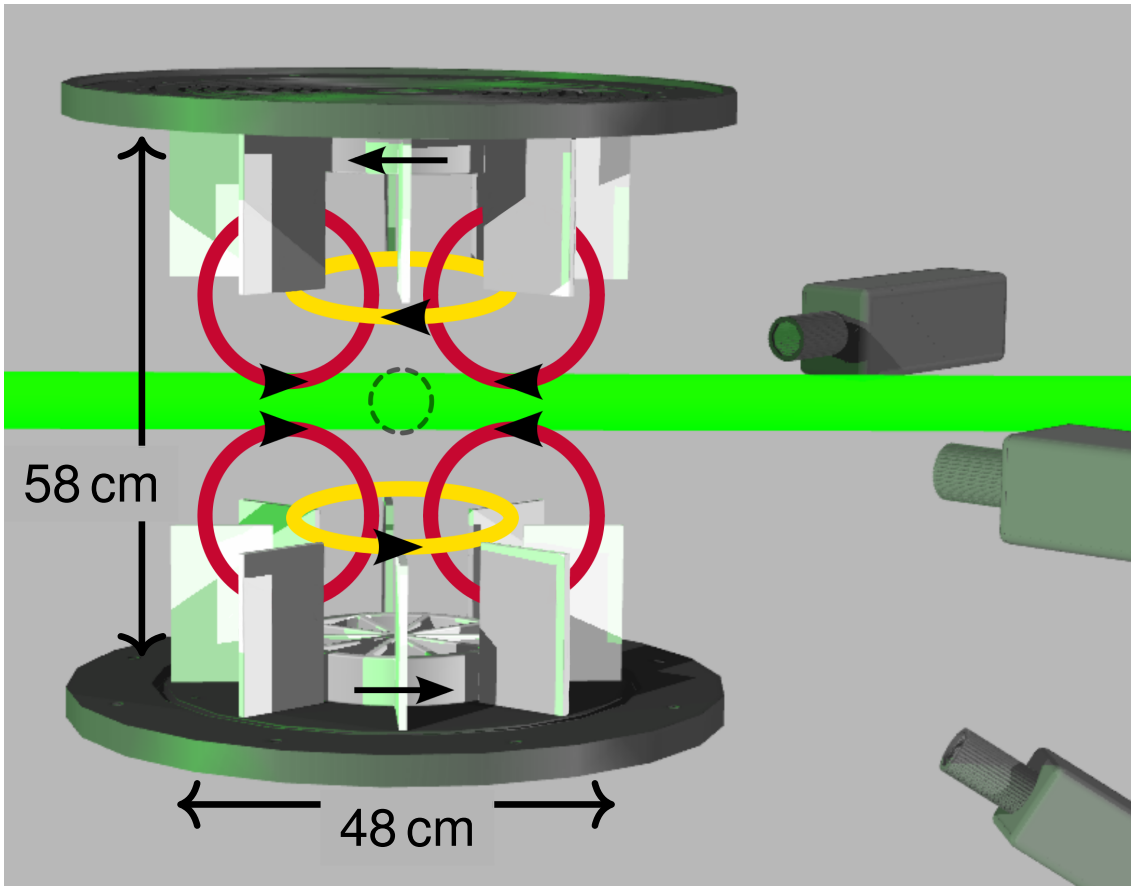



recorded at 7000 FPS, displayed at 140 FPS

# Experimental Setup



# Experimental Setup




Small Volume ($\varnothing = 27$ mm) :

$d_T = 45\ \mu$m, $\rho_T \approx \rho_{\text{water}}$

10 000 FPS

$640 \times 640$ pixels

Large Volume ($\varnothing = 80$ mm):

$d_T = 80\ \mu$m, $\rho_T \approx \rho_{\text{water}}$

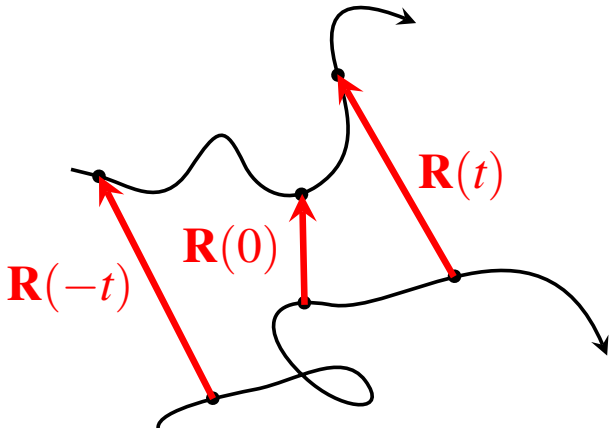
7 000 FPS

$640 \times 640$ pixels

Laser:

NdYag @ 50 W

# Pair Dispersion

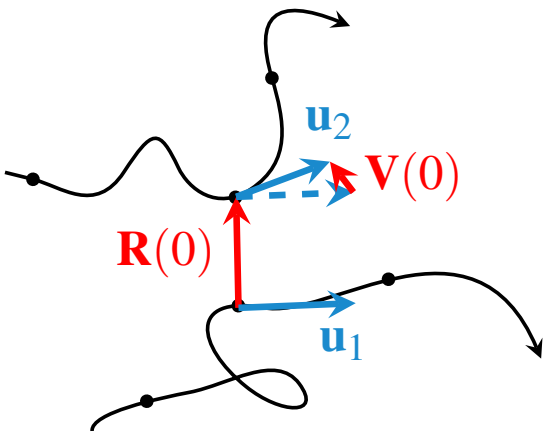



Mean squared change of separation:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle [\mathbf{R}(t) - \mathbf{R}(0)]^2 \right\rangle$$

$$|\mathbf{R}(0)| \stackrel{!}{=} R_0$$

# Pair Dispersion




Mean squared change of separation:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle [\mathbf{R}(t) - \mathbf{R}(0)]^2 \right\rangle$$

$$|\mathbf{R}(0)| \stackrel{!}{=} R_0$$

Short-time expansion:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle \mathbf{V}(0)^2 \right\rangle t^2 + \left\langle \mathbf{V}(0) \cdot \mathbf{A}(0) \right\rangle t^3 + \mathcal{O}(t^4)$$

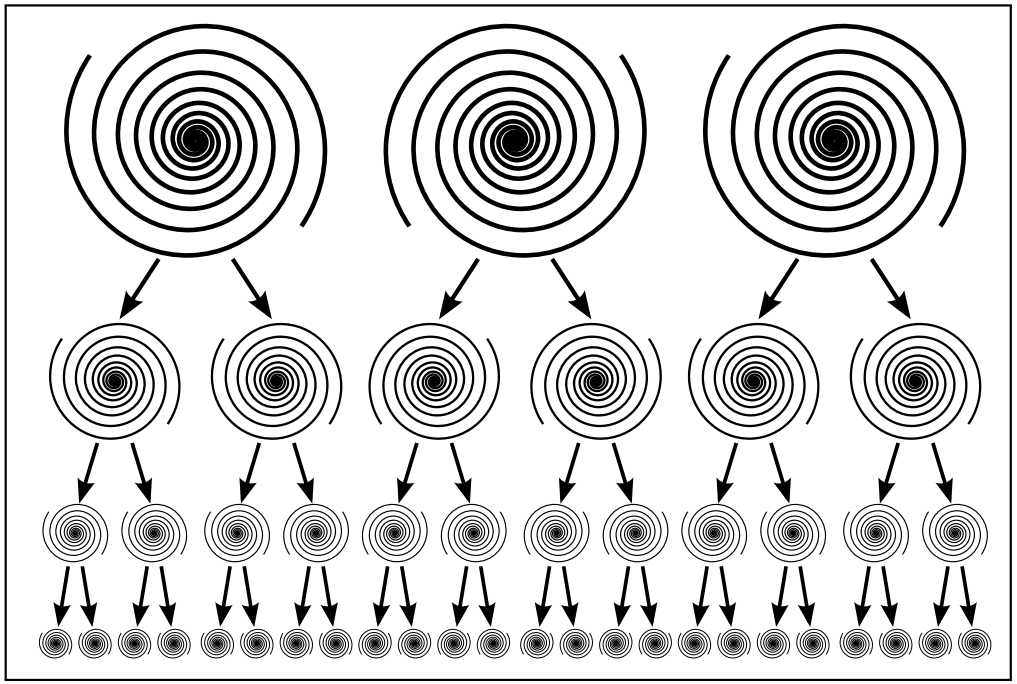
# The Energy Cascade

$$\left\langle \mathbf{V}(0) \cdot \mathbf{A}(0) \right\rangle = \left\langle \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{V}(t)^2 \Big|_{t=0} \right\rangle =$$

change of kinetic energy

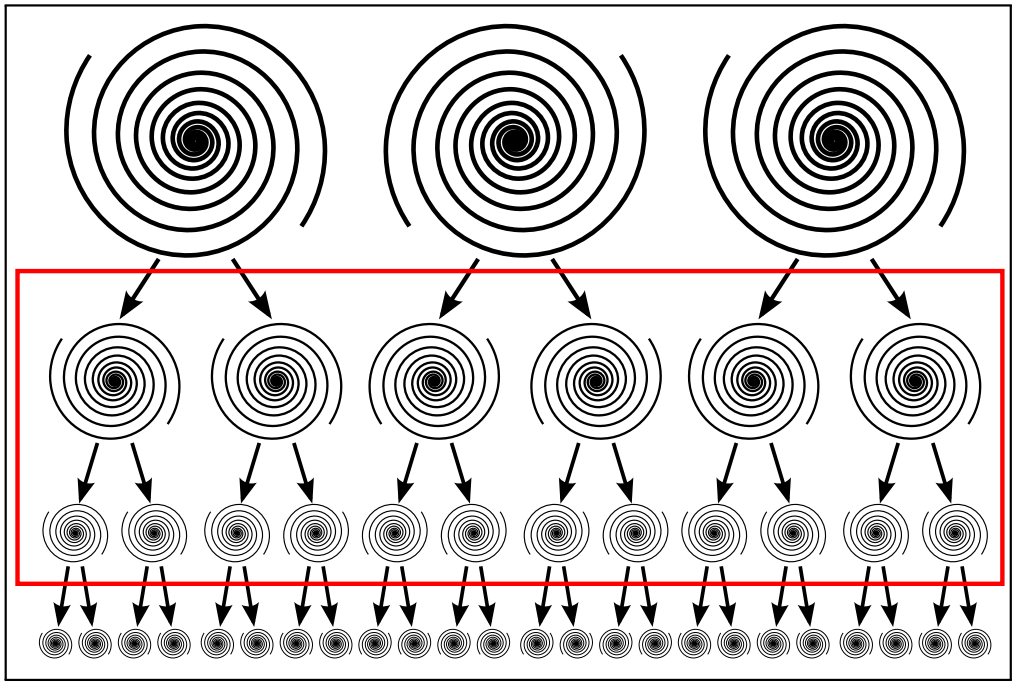
# The Energy Cascade

$$\left\langle \mathbf{V}(0)\cdot\mathbf{A}(0)\right\rangle = \left\langle \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V}(t)^2\Big|_{t=0}\right\rangle = \text{change of kinetic energy}$$



# The Energy Cascade

$$\left\langle \mathbf{V}(0)\cdot\mathbf{A}(0)\right\rangle = \left\langle \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V}(t)^2\Big|_{t=0}\right\rangle = \text{change of kinetic energy}$$

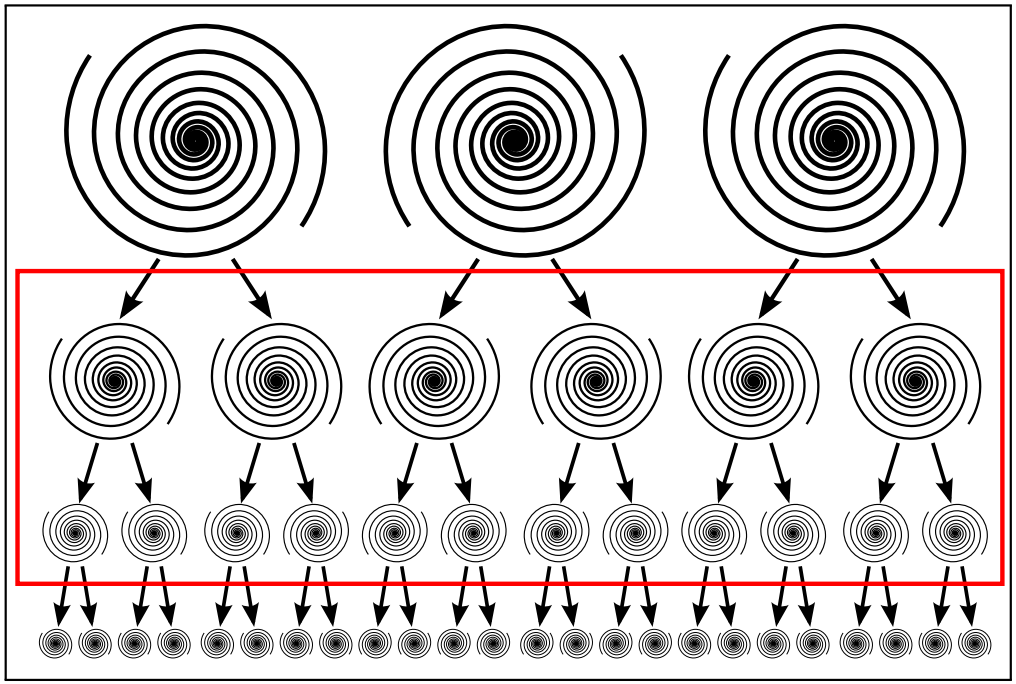



Kolmogorov, Dokl. Akad. Nauk SSSR **30**, 299 (1941)

# The Energy Cascade

$$\left\langle \mathbf{V}(0)\cdot\mathbf{A}(0)\right\rangle = \left\langle \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V}(t)^2\Big|_{t=0}\right\rangle = -2\varepsilon$$

Mann *et al.*, Tech. Rep. Risø-R-1036(EN) (1999)
Falkovich *et al.*, Rev. Mod. Phys. **73**, 913 (2001)
Pumir *et al.*, Europhys. Lett **56**,379 (2001)



Inertial Range

Forcing, $L$

$\varepsilon$

Dissipation, $\eta$

Kolmogorov, Dokl. Akad. Nauk SSSR **30**, 299 (1941)

# Time Asymmetry in Pair Dispersion

Short-time expansion:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle \mathbf{V}(0)^2 \right\rangle t^2 + \left\langle \mathbf{V}(0)\cdot\mathbf{A}(0) \right\rangle t^3 + \mathcal{O}(t^4)$$

# Time Asymmetry in Pair Dispersion

Short-time expansion:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle \mathbf{V}(0)^2 \right\rangle t^2 - 2\varepsilon\, t^3 + \mathcal{O}(t^4)$$

Pairs separate faster backwards in time!

# Time Asymmetry in Pair Dispersion

Short-time expansion:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle \mathbf{V}(0)^2 \right\rangle t^2 - 2\varepsilon\, t^3 + \mathcal{O}(t^4)$$

Difference between forward and backward evolution:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle - \left\langle \delta\mathbf{R}(-t)^2 \right\rangle = -4\,\varepsilon\, t^3 + \mathcal{O}(t^5)$$

# Time Asymmetry in Pair Dispersion

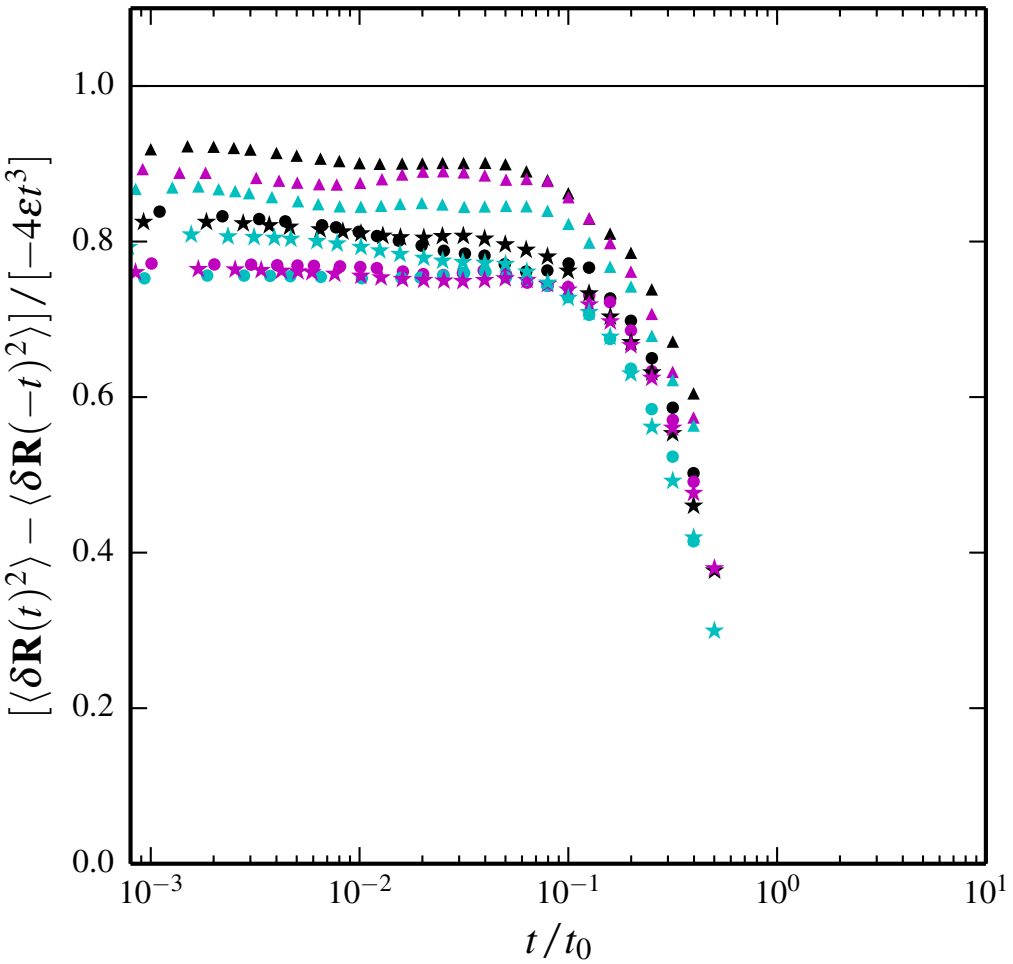
Short-time expansion:

$$\left\langle \delta\mathbf{R}(t)^2 \right\rangle = \left\langle \mathbf{V}(0)^2 \right\rangle t^2 - 2\varepsilon\, t^3 + \mathcal{O}(t^4)$$

Difference between forward and backward evolution:

$$\frac{\left\langle \delta\mathbf{R}(t)^2 \right\rangle - \left\langle \delta\mathbf{R}(-t)^2 \right\rangle}{-4\,\varepsilon\, t^3} = 1 + \mathcal{O}(t^2)$$

# Pair Dispersion



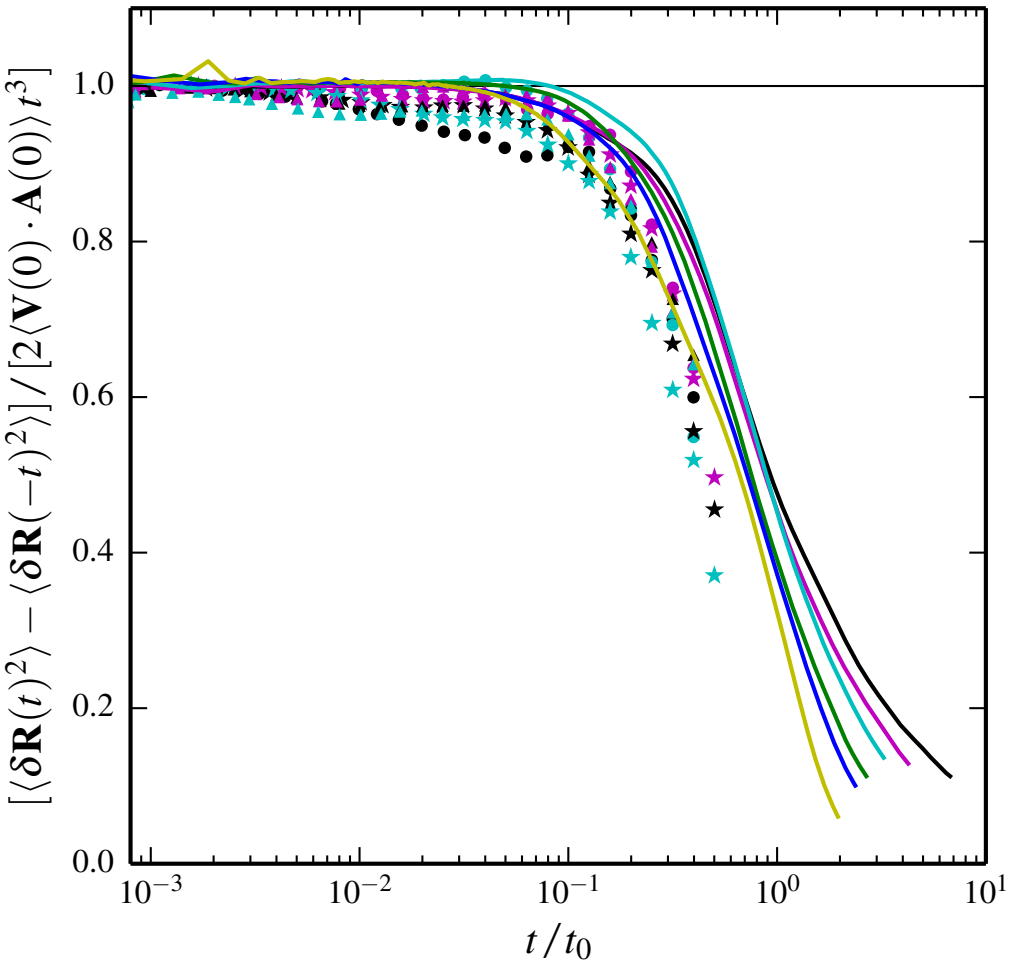

$R_\lambda = 390$
$R_0/\eta = 241, 276, 310$

$R_\lambda = 350$
$R_0/\eta = 212, 242, 273$

$R_\lambda = 270$
$R_0/\eta = 135, 154, 173$

$$t_0 = (R_0^2/\varepsilon)^{1/3}$$

# Pair Dispersion




$R_\lambda = 390$
$R_0/\eta = 241, 276, 310$

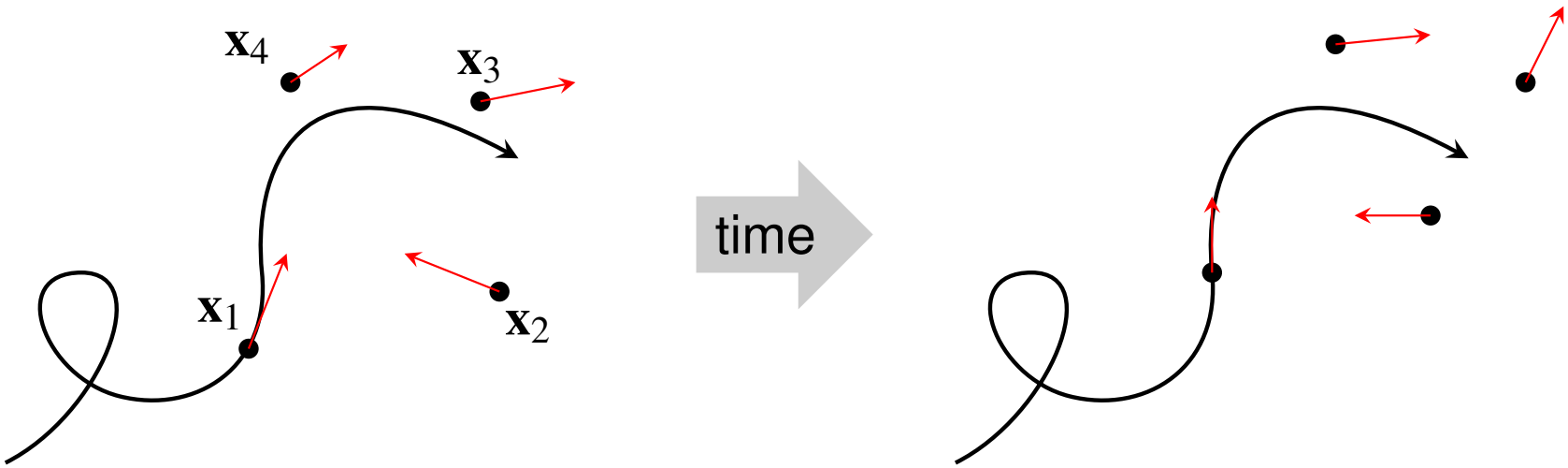
$R_\lambda = 350$
$R_0/\eta = 212, 242, 273$

$R_\lambda = 270$
$R_0/\eta = 135, 154, 173$

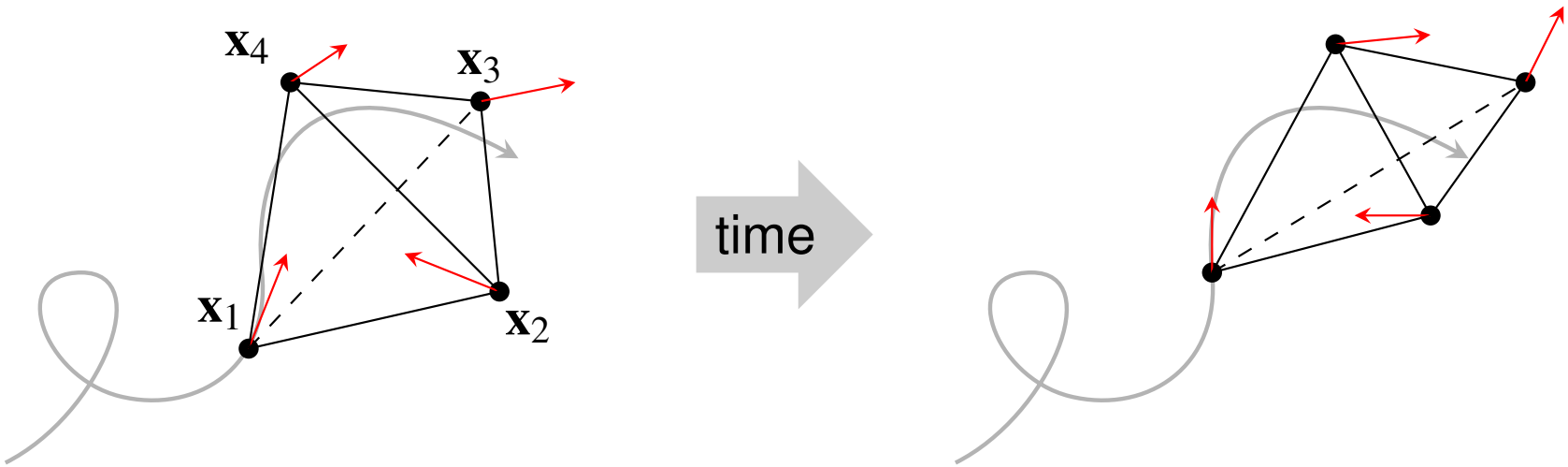
$R_\lambda = 300$ (DNS)
$R_0/\eta = 19, 38, 58,$
$77, 92, 123$

$$t_0 = (R_0^2/\varepsilon)^{1/3}$$

# Dispersion of Particle Clusters



# Dispersion of Particle Clusters

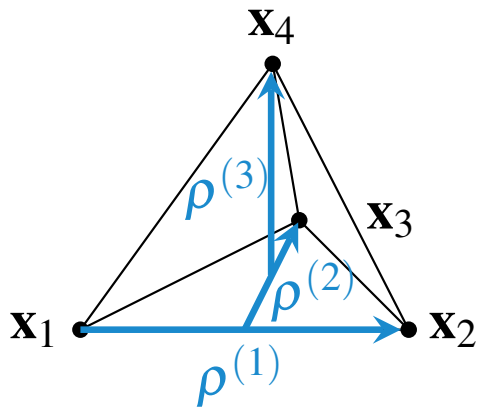


# Deformation of Tetrahedra

Shape Tensor: $G(t)_{ij} = \sum_a \rho_i^{(a)}(t)\rho_j^{(a)}(t)$

Size: radius of gyration $\mathbf{R}^2(t) = \mathrm{tr}[\mathbf{G}(t)]$

Shape: eigenvalues $g_i$



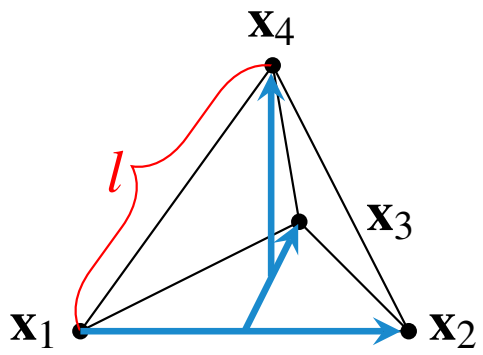


Chertkov *et al.*, Phys. Fluids **11**, 2394 (1999)
Pumir *et al.*, Phys. Rev. Lett. **85**, 5324 (2000)
Biferale *et al.*, Phys. Fluids **17**, 111701 (2005)
Lüthi *et al.*, J. Tubul. **8**, 45 (2007)
Xu *et al.*, New J. Phys. **10**, 013012 (2008)
Hackl *et al.*, Phys. Fluids **23**, 065103 (2011)

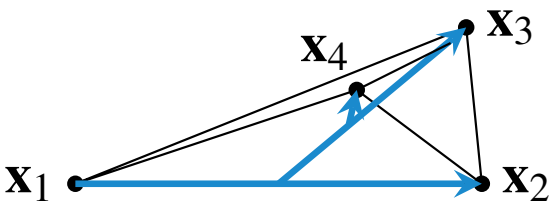
# Deformation of Tetrahedra

Initial Shape: Regular

Shape: $g_1(0) = g_2(0) = g_3(0) = \frac{l^2}{2}$

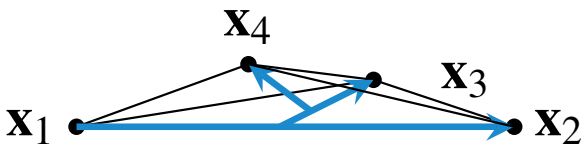



"Pancake"



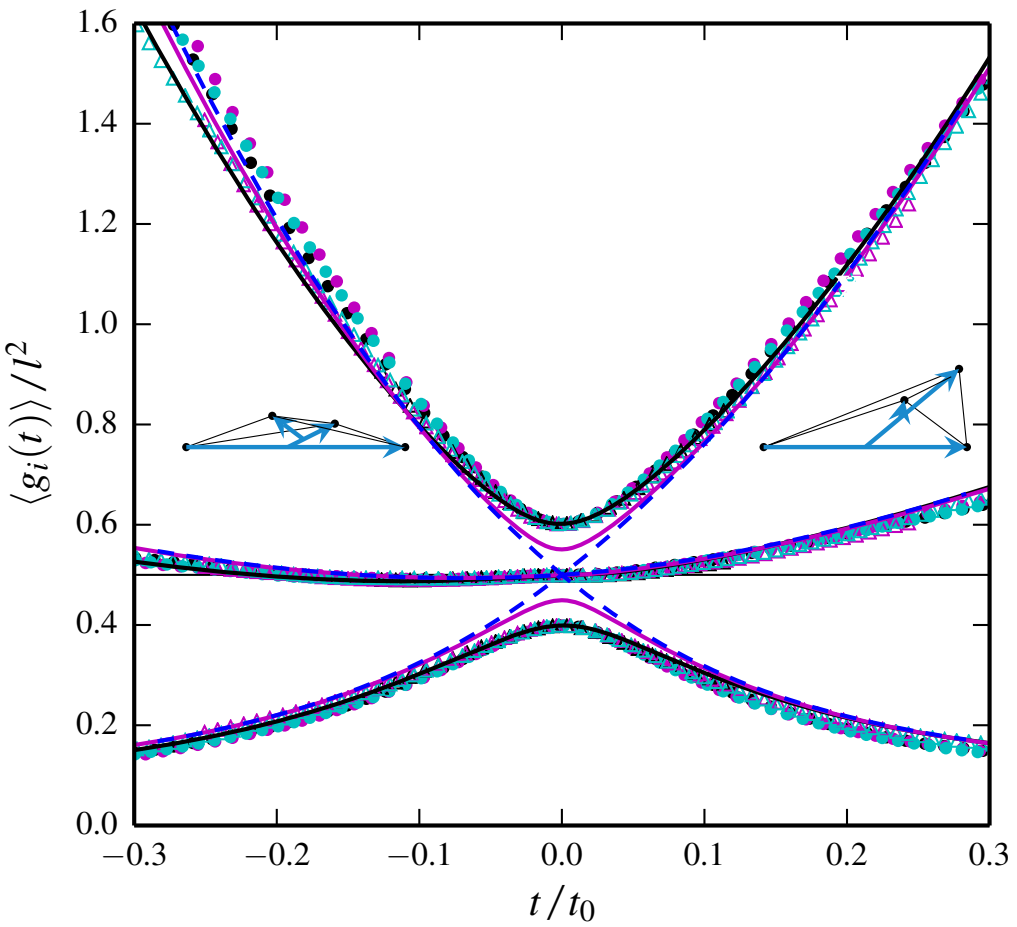

$g_1 \geq g_2 \gg g_3$

"Needle"




$g_1 \gg g_2 \geq g_3$

# Deformation of Tetrahedra




$R_\lambda = 690$
$l/\eta = 364, 424, 485$

$R_\lambda = 270$
$l/\eta = 135, 154, 173$

$R_\lambda = 300$ (DNS)
$l/\eta = 123$
$\Delta l/l = 10\%,\ 5\%$

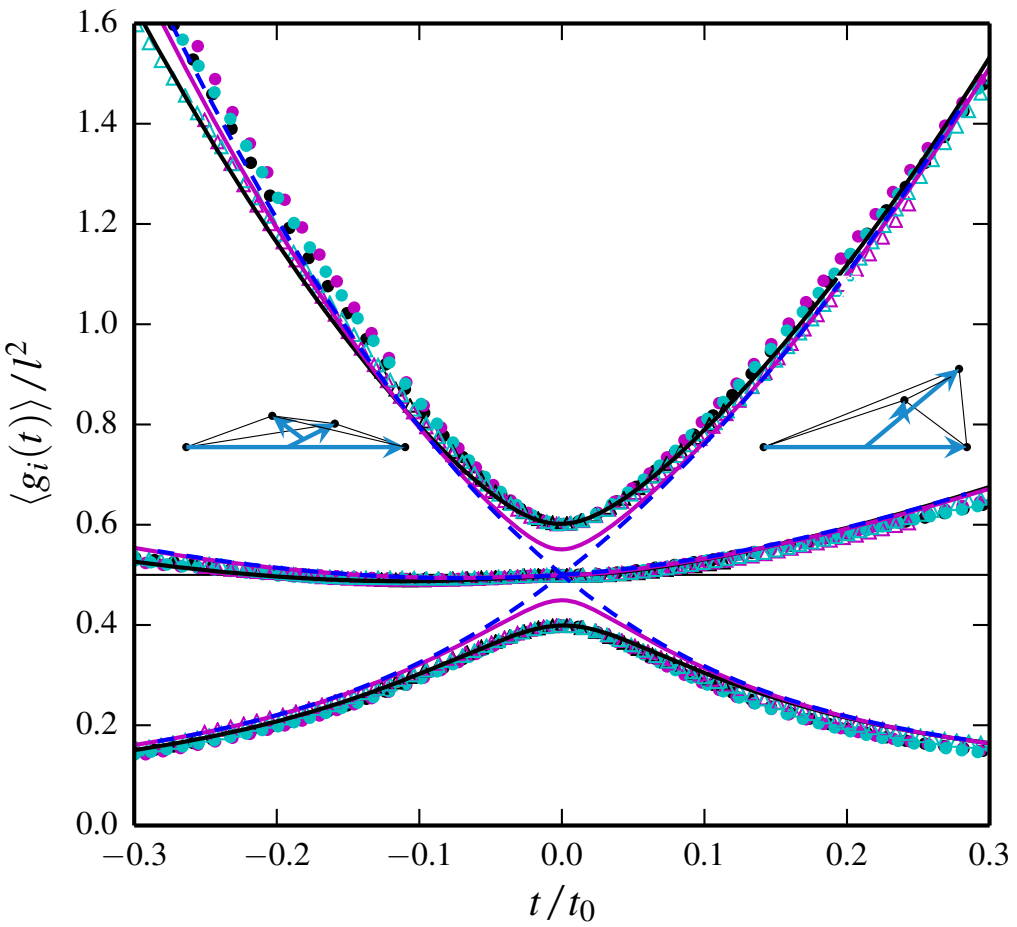
$R_\lambda = 430$ (DNS)(*)
$l/\eta = 167$
$\Delta l/l = 0\%$

*) Li *et al.*, J. Turbul. **9**, 31 (2008),

Yu *et al.*, J. Turbul. **13**, 12 (2012)

# Deformation of Tetrahedra

$$\frac{\langle g_2(t)\rangle - \langle g_2(-t)\rangle}{l^2} = 2\langle S_{0,22}\rangle t + \mathcal{O}(t^3)$$




$R_\lambda = 690$
$l/\eta = 364, 424, 485$

$R_\lambda = 270$
$l/\eta = 135, 154, 173$

$R_\lambda = 300$ (DNS)
$l/\eta = 123$
$\Delta l/l = 10\%,\ 5\%$

$R_\lambda = 430$ (DNS)(*)
$l/\eta = 167$
$\Delta l/l = 0\%$

*) Li *et al.*, J. Turbul. **9**, 31 (2008),

Yu *et al.*, J. Turbul. **13**, 12 (2012)

# Conclusion

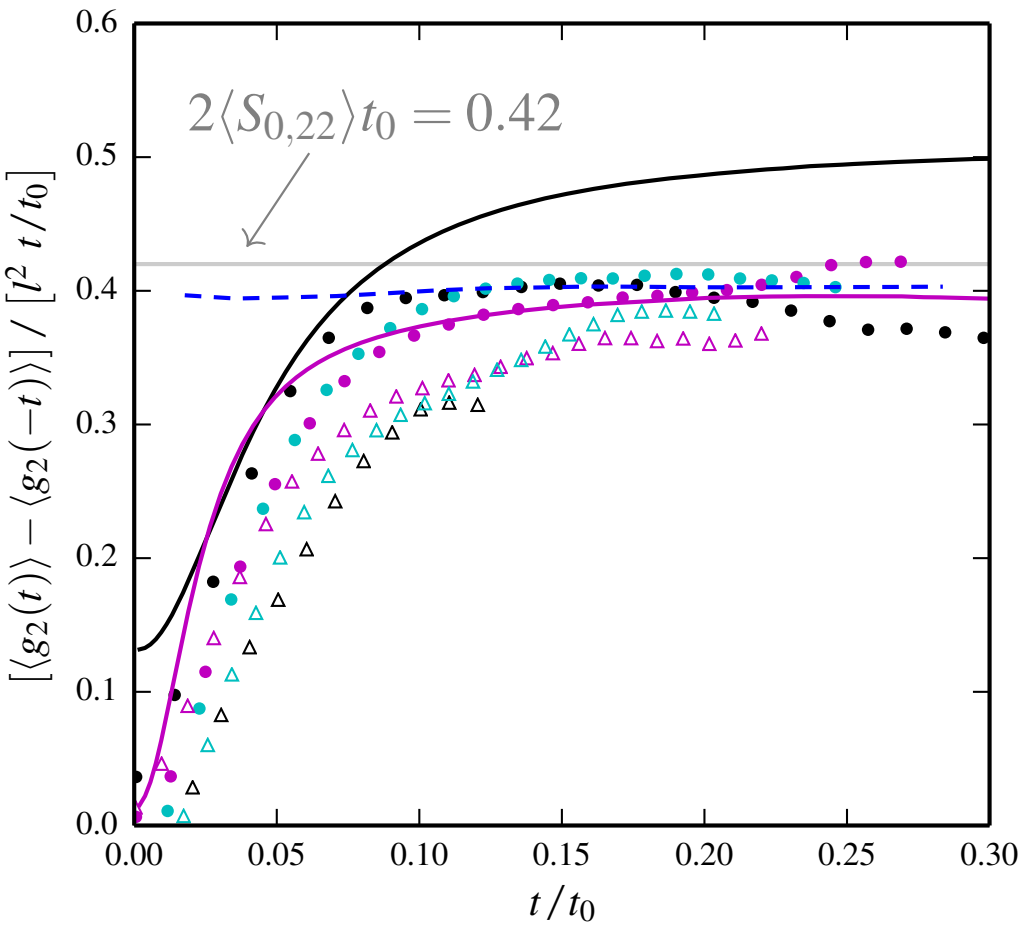
- Multi-particle dispersion is a sensitive measure to reveal time asymmetry.
- Time asymmetry can be observed already at very short times and can be traced back to the turbulence energy cascade.
- Pairs separate faster backwards in time.
- Tetrahedra become stronger elongated backwards in time.

Thank You!

# Appendix

- Lagrangian Particle Tracking
- Search for particle clusters
- Finite measurement volume
- Measurement Uncertainties

# Deformation of Tetrahedra




$R_\lambda = 690$
$l/\eta = 364, 424, 485$

$R_\lambda = 270$
$l/\eta = 135, 154, 173$

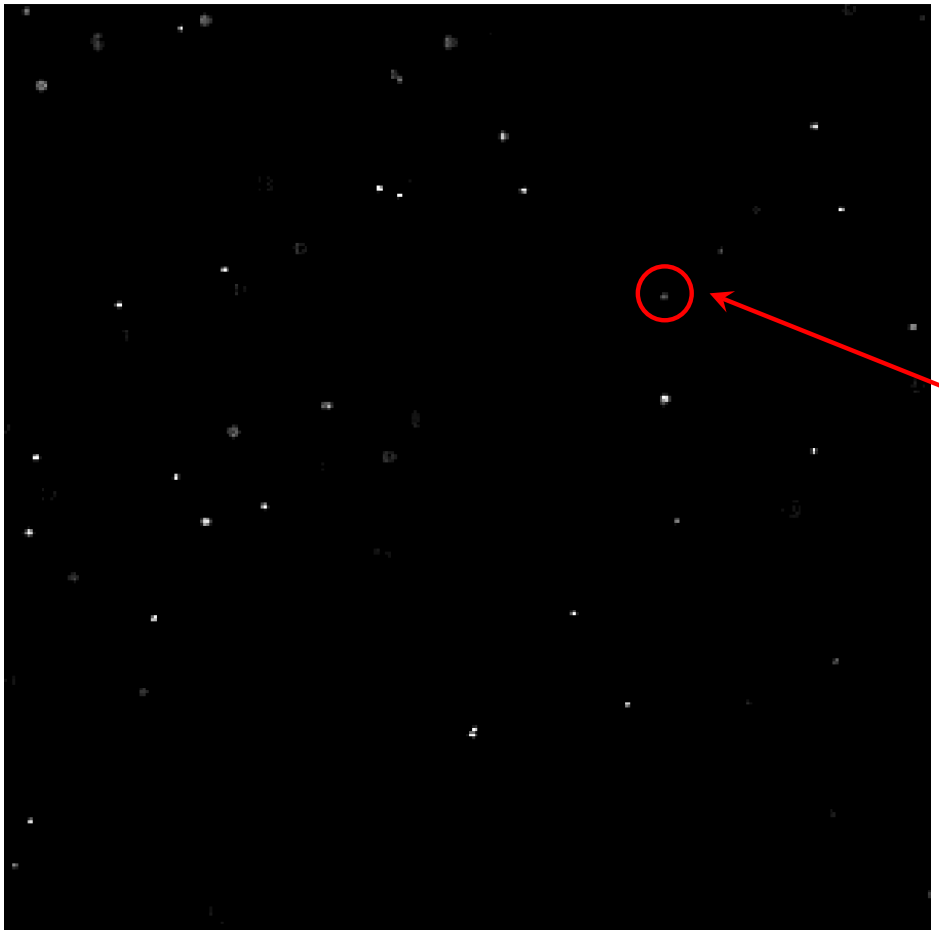
$R_\lambda = 300$ (DNS)
$l/\eta = 123$
$\Delta l/l = 10\%,\ 5\%$

$R_\lambda = 430$ (DNS)(*)
$l/\eta = 167$
$\Delta l/l = 0\%$

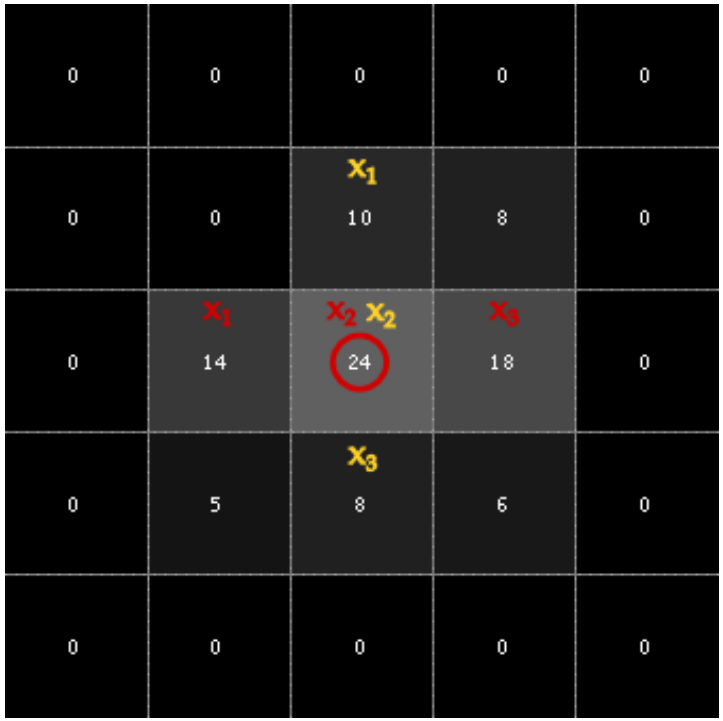
*) Li *et al.*, J. Turbul. **9**, 31 (2008),

Yu *et al.*, J. Turbul. **13**, 12 (2012)

# Lagrangian Particle Tracking



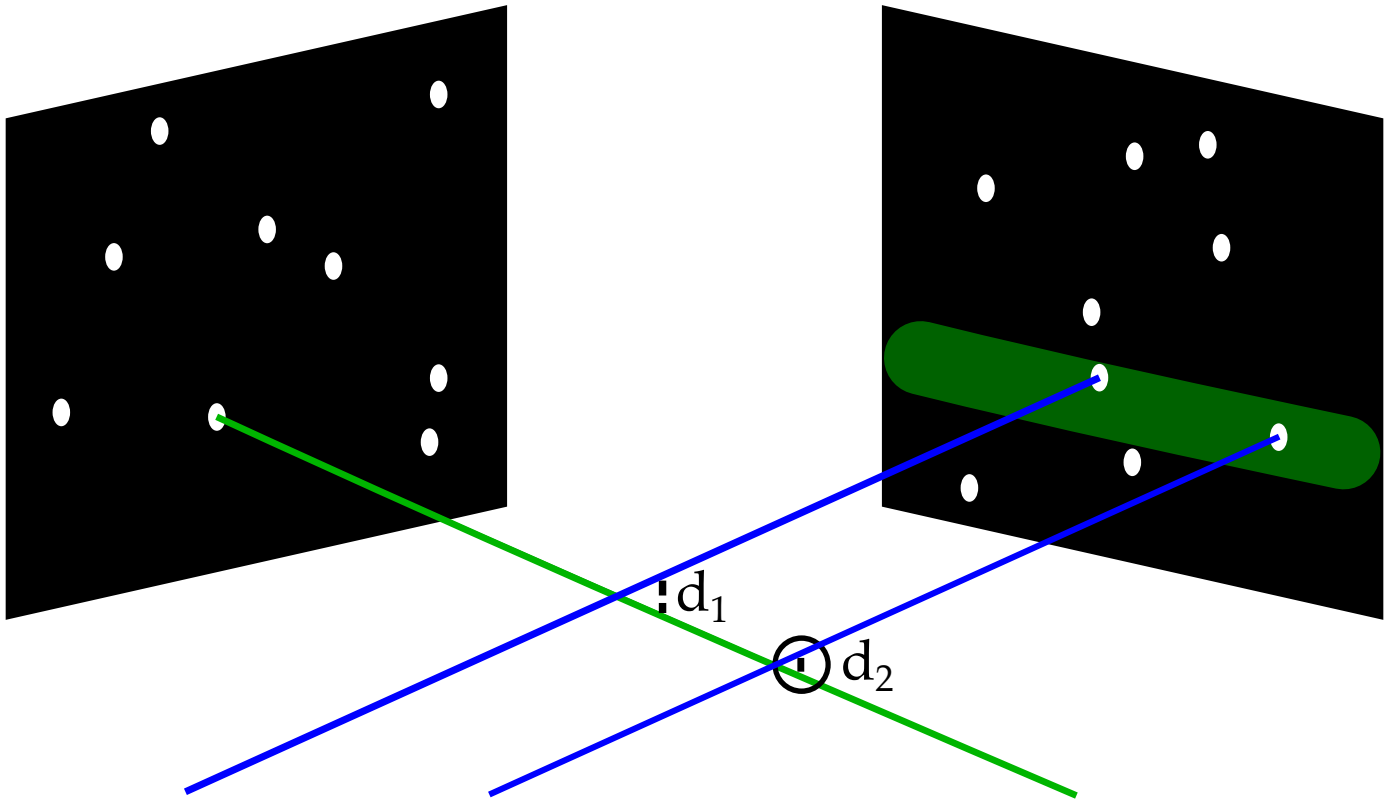
Quarter of a full camera image, contrast enhanced




Intensity profile of a single particle

Cowen *et al.*, Exp. Fluids **22**, 199 (1997)
Ouellette *et al.*, Exp. Fluids **40**, 301 (2006)

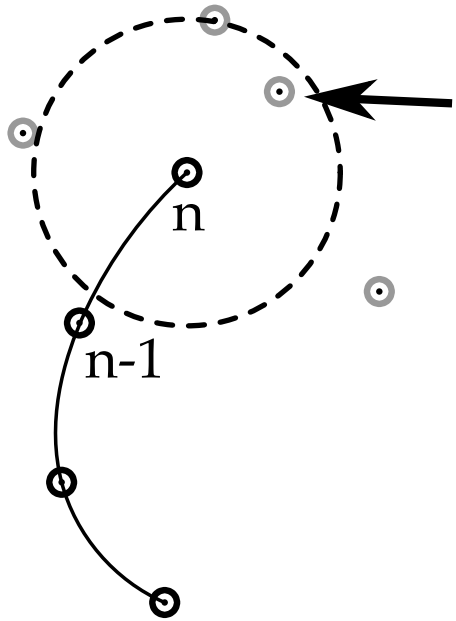
# Lagrangian Particle Tracking



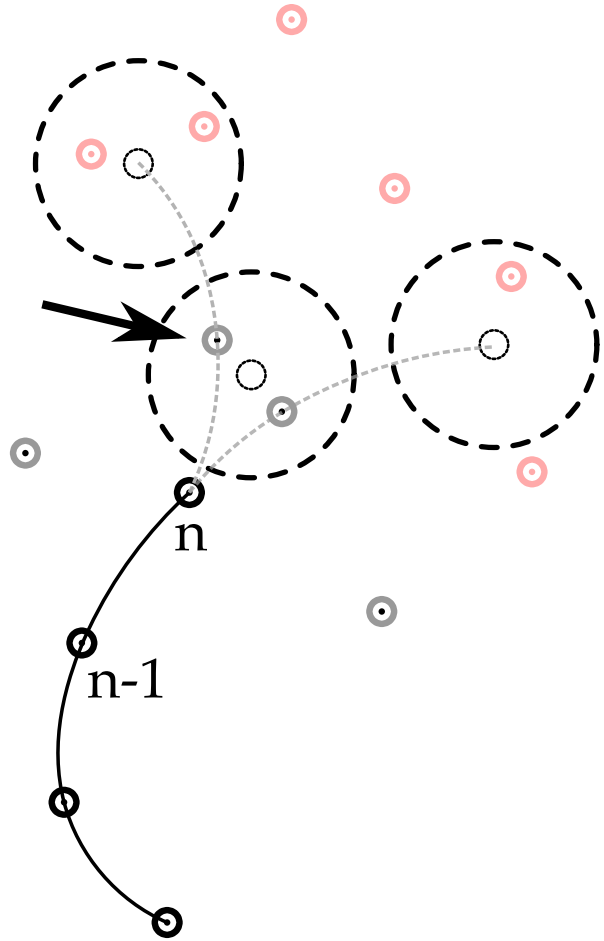

Tsai, IEEE J. Robotic. Atom. **3**, 323 (1987)

Ouellette *et al.*, Exp. Fluids **40**, 301 (2006)

# Lagrangian Particle Tracking



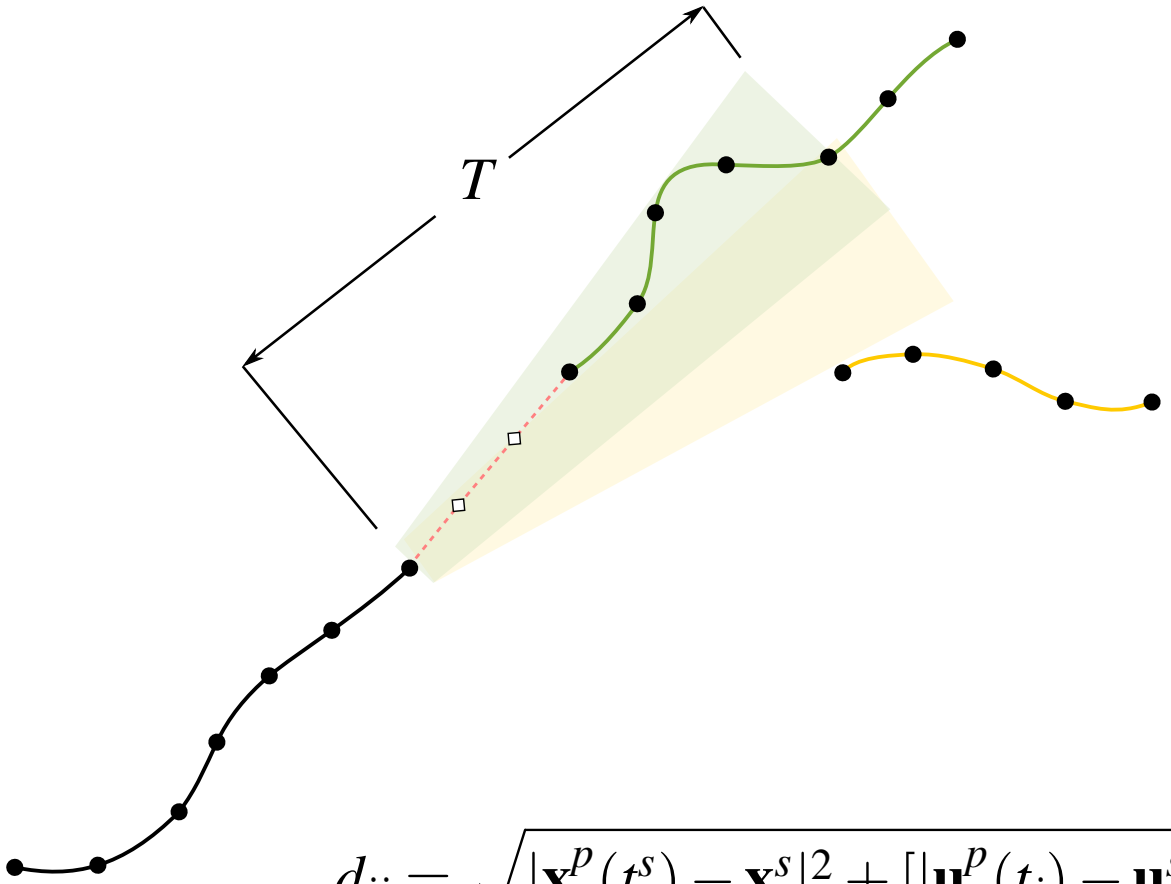

Nearest Neighbour




Four-Frame Best Estimate

Adapted from the PhD thesis of Nicholas Ouellette (Cornell University, 2006)

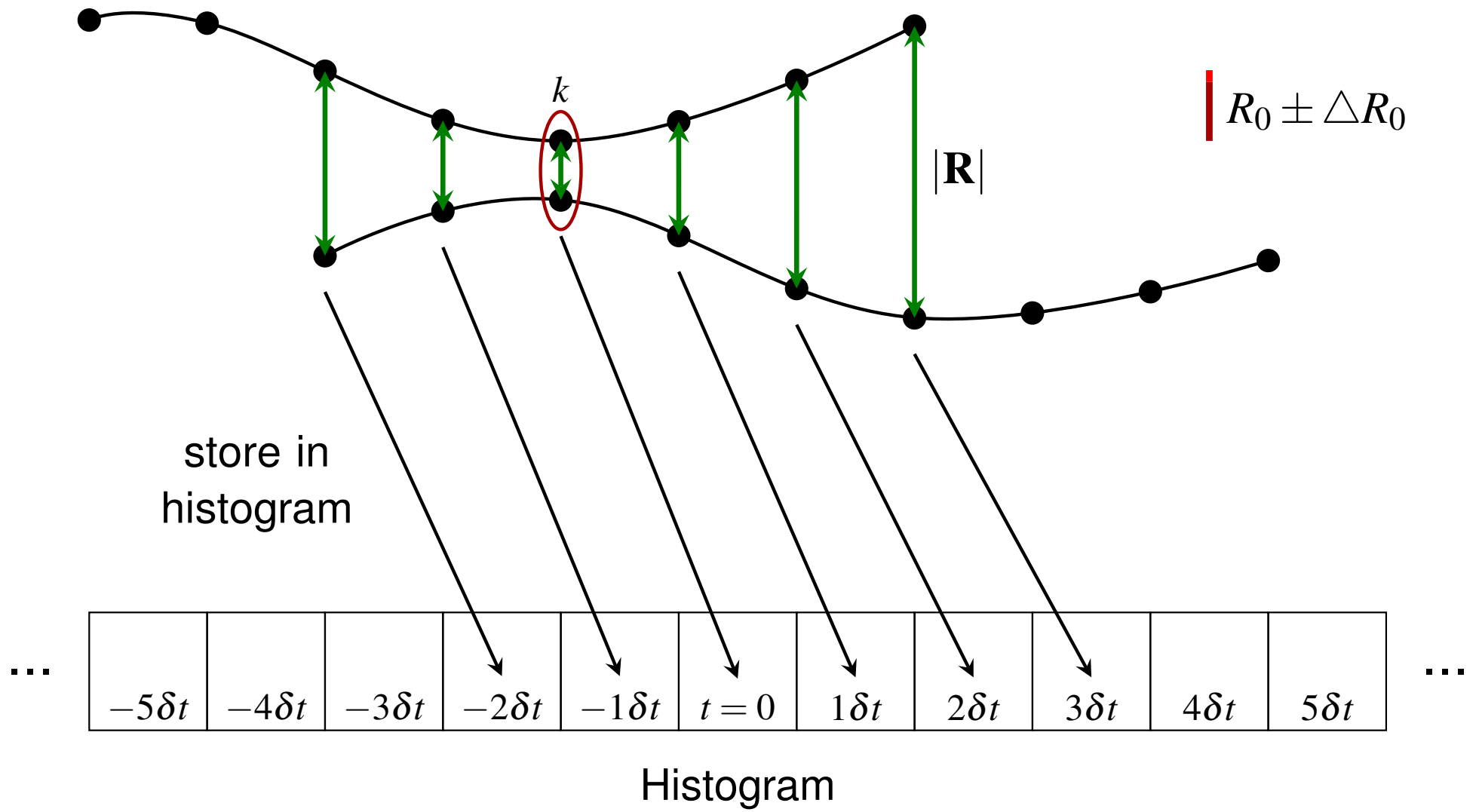
# Lagrangian Particle Tracking



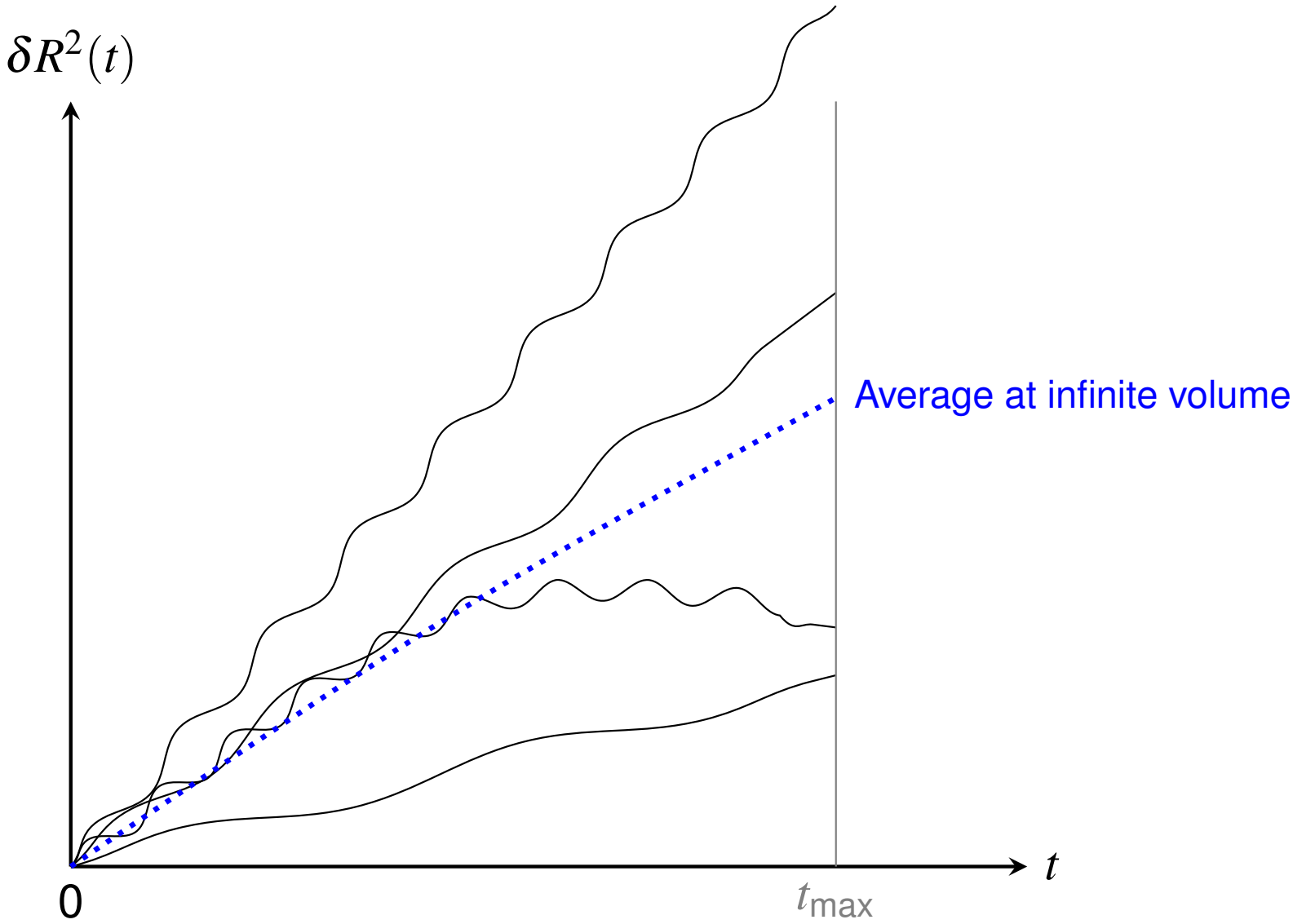

$$d_{ij} = \sqrt{|\mathbf{x}_i^p(t_j^s) - \mathbf{x}_j^s|^2 + [|\mathbf{u}_i^p(t_j) - \mathbf{u}_j^s|(t_j^s - t_i^e)]^2}$$

Xu, Meas. Sci. Technol. **19**, 075105 (2008)

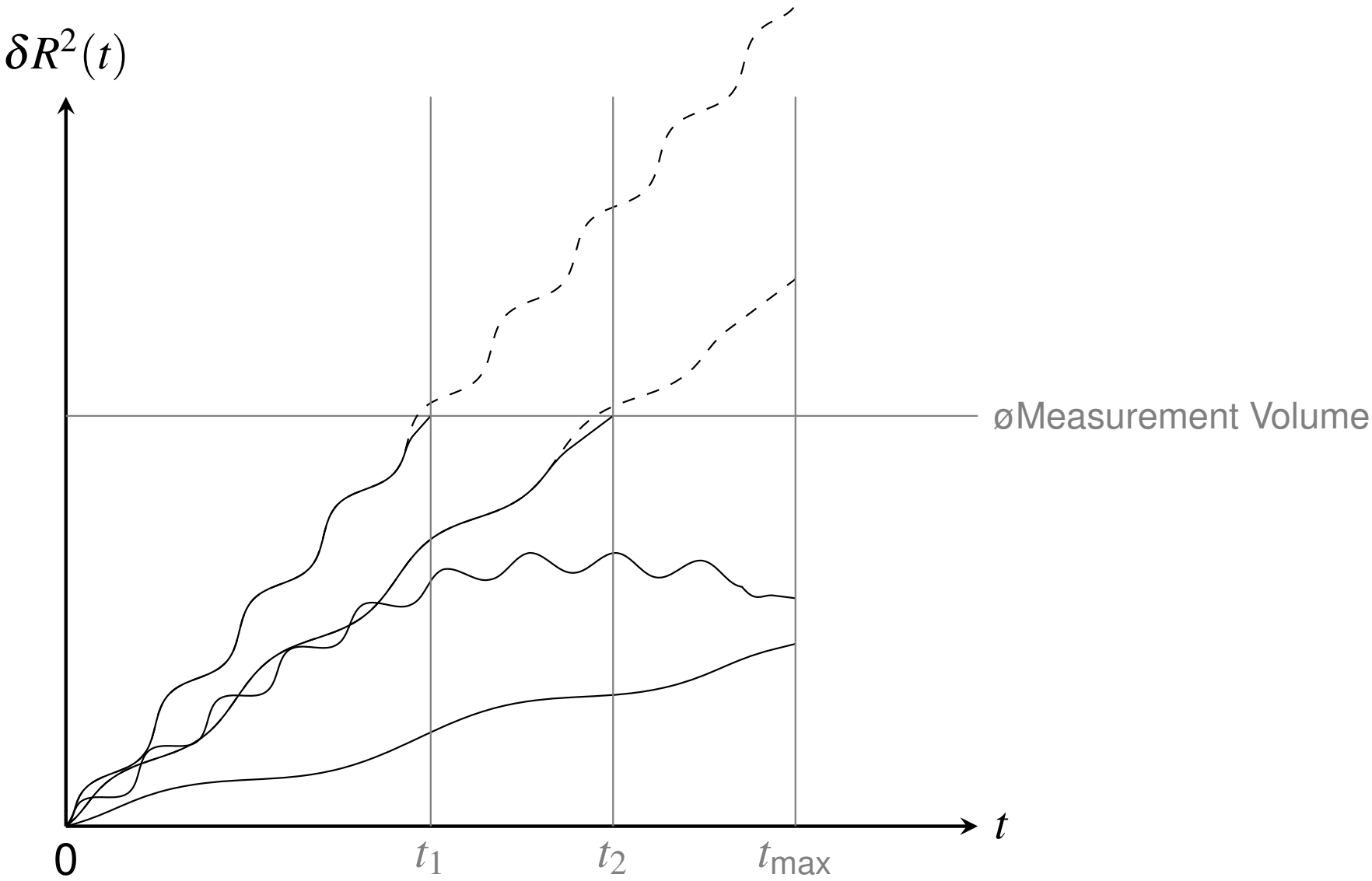
# Search for Particle Clusters



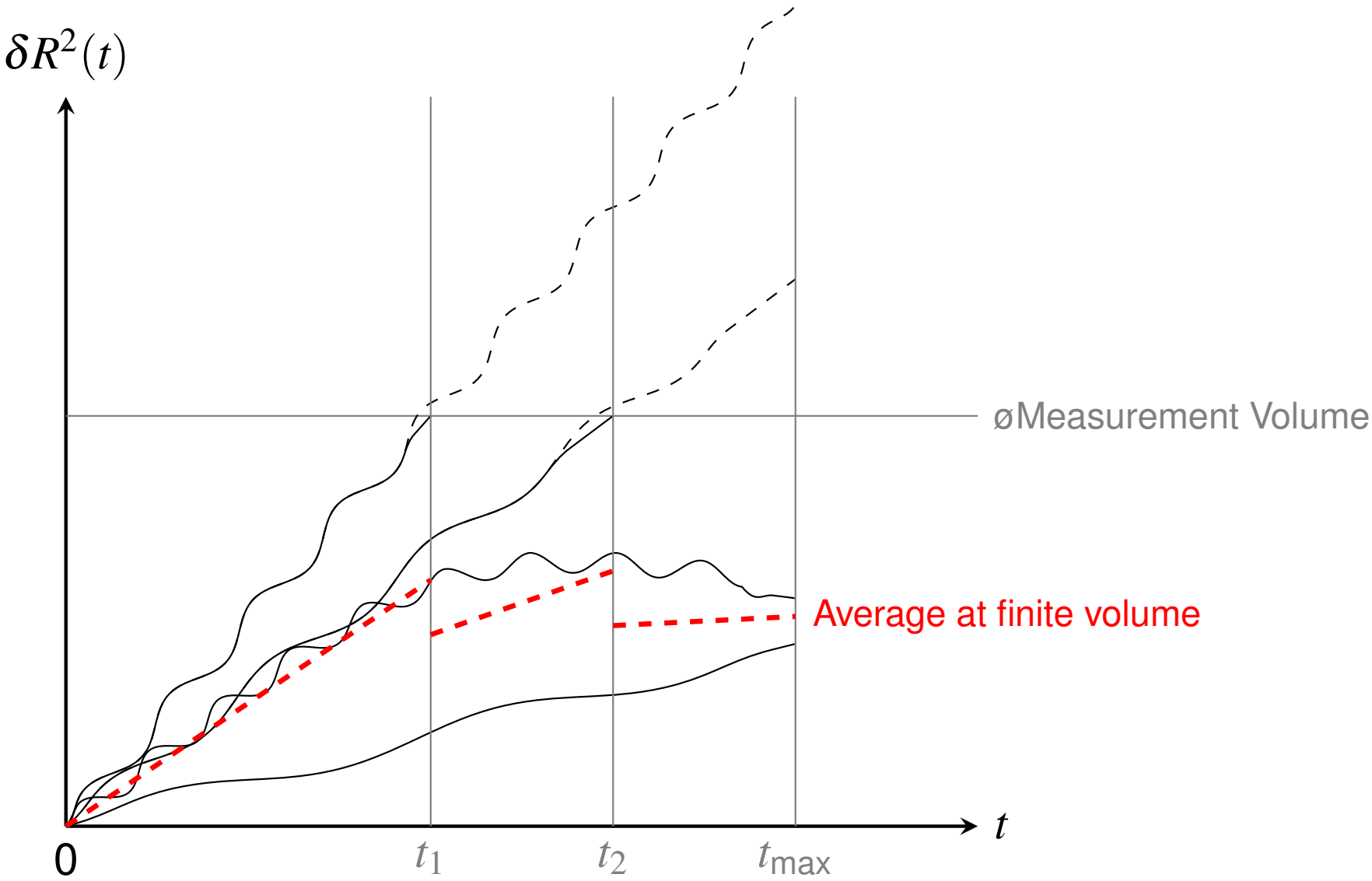
# A Finite Measurement Volume



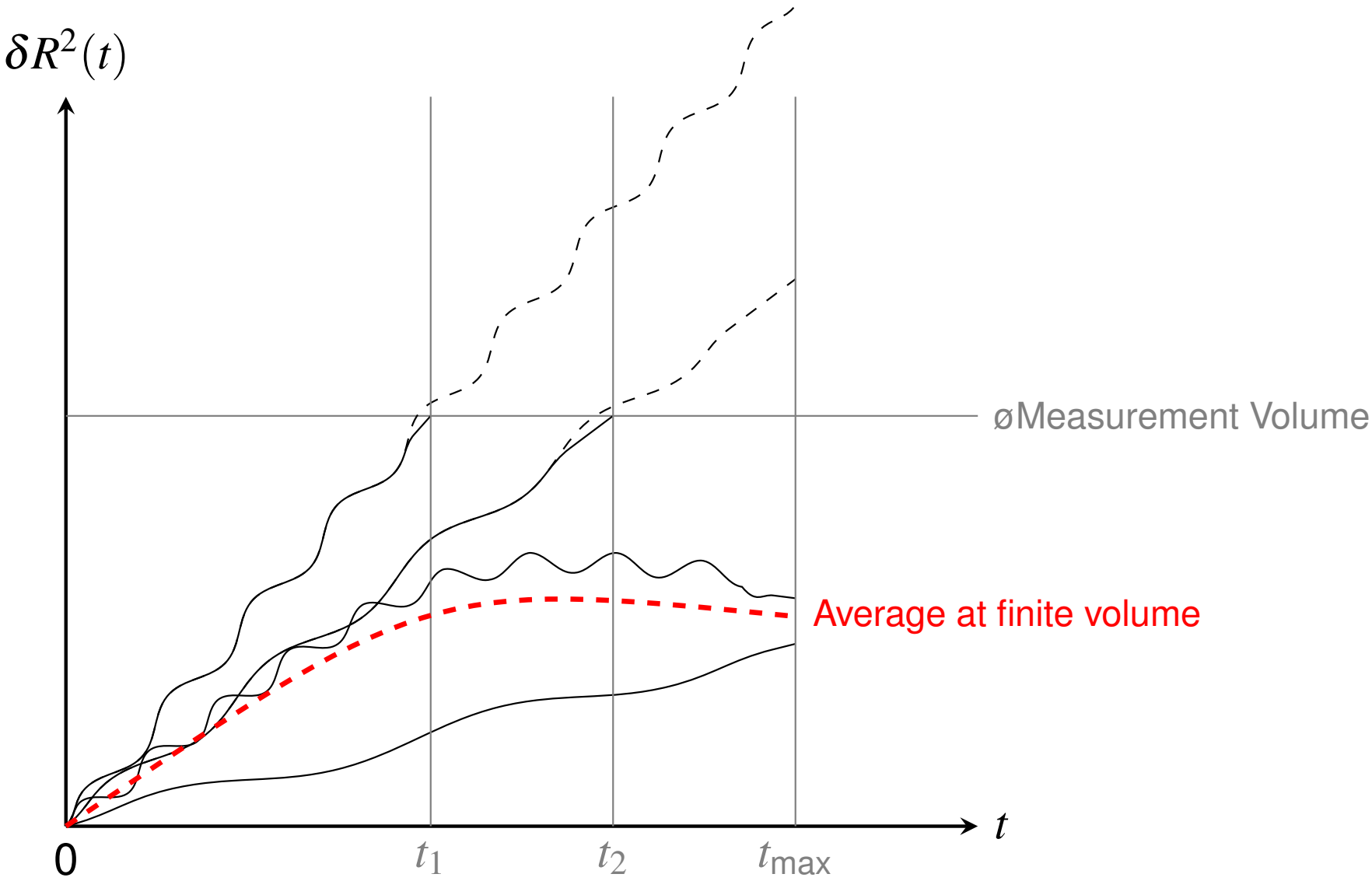
# A Finite Measurement Volume



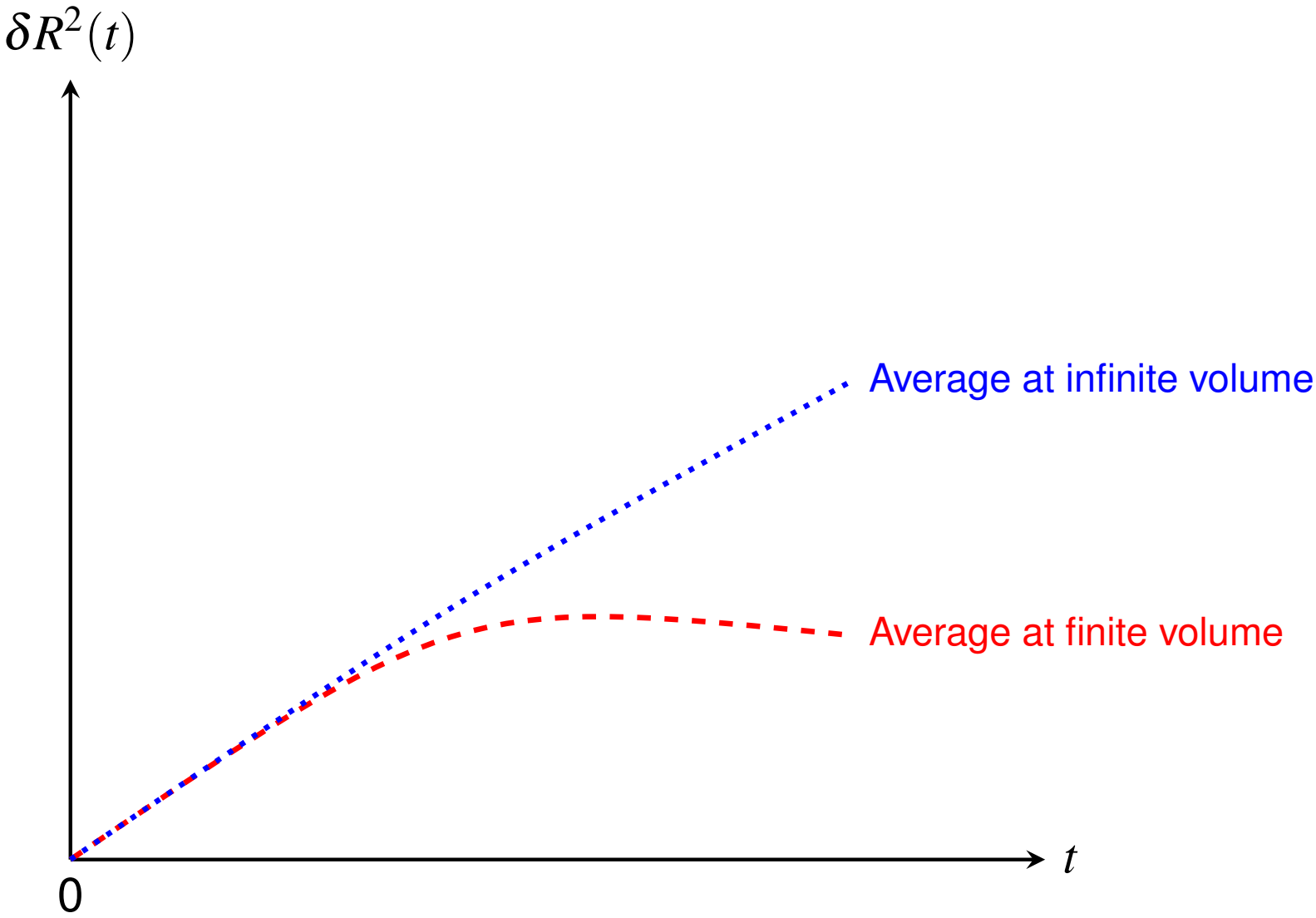
# A Finite Measurement Volume



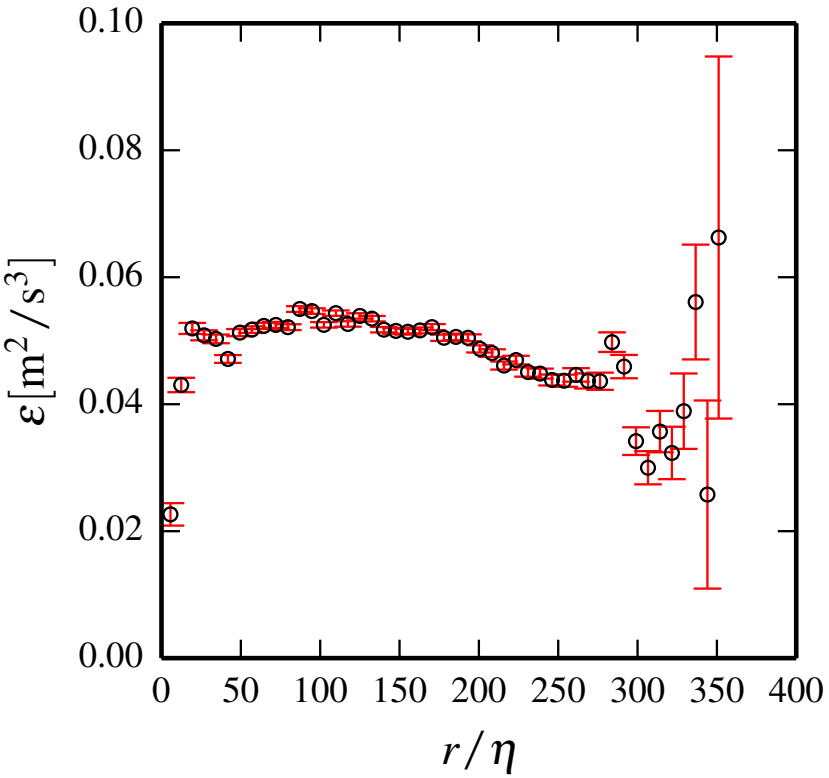
# A Finite Measurement Volume



# A Finite Measurement Volume

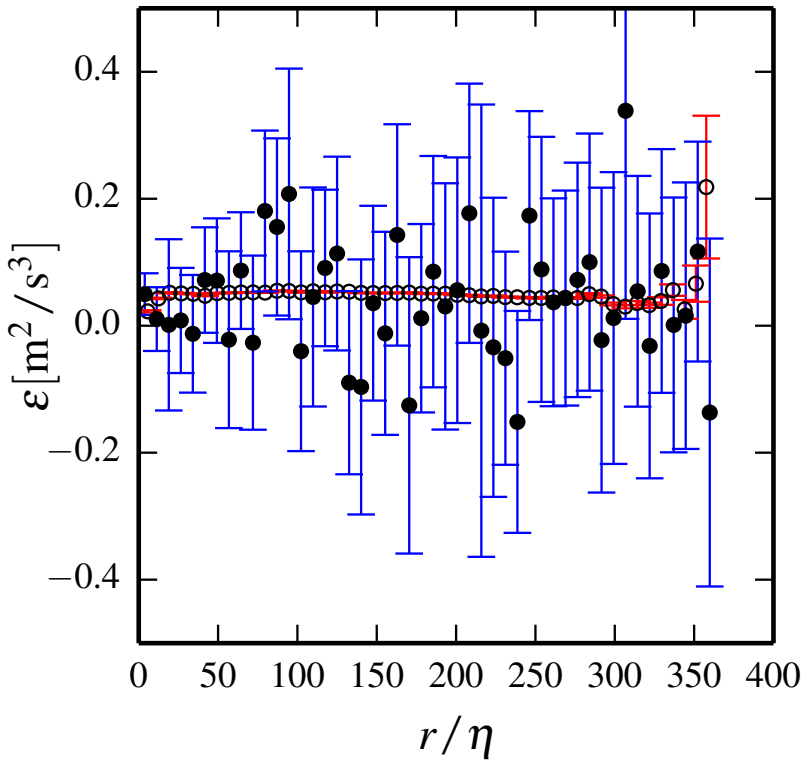


# Measurement Uncertainties ($R_\lambda = 350$)




Optimistic Estimate

"All trajectories are independent"




Conservative Estimate

"All videos are independent"