Time Irreversibility in Turbulent Dispersion

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recorded at 7000 FPS, displayed at 140 FPS
recorded at 7000 FPS, displayed at 140 FPS
Experimental Setup

Laser beam

Camera 1

Camera 2

Camera 3

5 cm
Experimental Setup

Small Volume ($\varnothing = 27 \text{ mm}$) :
- $d_T = 45 \mu m$, $\rho_T \approx \rho_{\text{water}}$
- 10000 FPS
- $640 \times 640$ pixels

Large Volume ($\varnothing = 80 \text{ mm}$):
- $d_T = 80 \mu m$, $\rho_T \approx \rho_{\text{water}}$
- 7000 FPS
- $640 \times 640$ pixels

Laser:
- NdYag @ 50 W
Mean squared change of separation:

\[
\langle \delta R(t)^2 \rangle = \langle [R(t) - R(0)]^2 \rangle
\]

\[
|R(0)| \equiv R_0
\]
Pair Dispersion

Mean squared change of separation:

\[ \langle \delta \mathbf{R}(t)^2 \rangle = \langle (\mathbf{R}(t) - \mathbf{R}(0))^2 \rangle \]

\[ |\mathbf{R}(0)| \equiv R_0 \]

Short-time expansion:

\[ \langle \delta \mathbf{R}(t)^2 \rangle = \langle \mathbf{V}(0)^2 \rangle t^2 + \langle \mathbf{V}(0) \cdot \mathbf{A}(0) \rangle t^3 + O(t^4) \]
The Energy Cascade

\[ \langle V(0) \cdot A(0) \rangle = \left\langle \frac{1}{2} \frac{d}{dt} V(t)^2 \bigg|_{t=0} \right\rangle = \text{change of kinetic energy} \]
The Energy Cascade

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\langle V(0) \cdot A(0) \rangle = \left\langle \frac{1}{2} \frac{d}{dt} V(t)^2 \right|_{t=0} \rangle = \text{change of kinetic energy}
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The Energy Cascade

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Forcing, \( L \)

Inertial Range

Dissipation, \( \eta \)

The Energy Cascade

\[ \langle V(0) \cdot A(0) \rangle = \left\langle \frac{1}{2} \frac{d}{dt} V(t)^2 \bigg|_{t=0} \right \rangle = -2\varepsilon \]

Falkovich et al., Rev. Mod. Phys. 73, 913 (2001)
Pumir et al., Europhys. Lett 56,379 (2001)

Time Asymmetry in Pair Dispersion

Short-time expansion:

\[
\langle \delta R(t)^2 \rangle = \langle V(0)^2 \rangle t^2 + \langle V(0) \cdot A(0) \rangle t^3 + \mathcal{O}(t^4)
\]
Short-time expansion:

\[ \langle \delta R(t)^2 \rangle = \langle V(0)^2 \rangle t^2 - 2\varepsilon t^3 + O(t^4) \]

Pairs separate faster backwards in time!
Short-time expansion:

\[
\langle \delta R(t)^2 \rangle = \langle V(0)^2 \rangle t^2 - 2\varepsilon t^3 + O(t^4)
\]

Difference between forward and backward evolution:

\[
\langle \delta R(t)^2 \rangle - \langle \delta R(-t)^2 \rangle = -4\varepsilon t^3 + O(t^5)
\]
Short-time expansion:

\[ \langle \delta \mathbf{R}(t)^2 \rangle = \langle V(0)^2 \rangle t^2 - 2 \epsilon t^3 + \mathcal{O}(t^4) \]

Difference between forward and backward evolution:

\[ \frac{\langle \delta \mathbf{R}(t)^2 \rangle - \langle \delta \mathbf{R}(-t)^2 \rangle}{-4 \epsilon t^3} = 1 + \mathcal{O}(t^2) \]
Pair Dispersion

\[
\frac{[\langle (\delta R(t)^2) \rangle - \langle (\delta R(-t)^2) \rangle]}{[-4 \varepsilon t^3]} = \frac{R_\lambda}{R_0/\eta} = 390, 241, 276, 310
\]

\[
R_\lambda = 350
\quad R_0/\eta = 212, 242, 273
\]

\[
R_\lambda = 270
\quad R_0/\eta = 135, 154, 173
\]

\[
t_0 = (R_0^2/\varepsilon)^{1/3}
\]
Pair Dispersion

\[ \frac{\langle \delta R(t)^2 \rangle - \langle \delta R(-t)^2 \rangle}{2 \langle V(0) \cdot A(0) \rangle t^3} \]

\( R_\lambda = 390 \)
\( R_0/\eta = 241, 276, 310 \)

\( R_\lambda = 350 \)
\( R_0/\eta = 212, 242, 273 \)

\( R_\lambda = 270 \)
\( R_0/\eta = 135, 154, 173 \)

\( R_\lambda = 300 \) (DNS)
\( R_0/\eta = 19, 38, 58, 77, 92, 123 \)

\( t_0 = \left( \frac{R_0^2}{\varepsilon} \right)^{1/3} \)
Dispersion of Particle Clusters

\[ x_1, x_2, x_3, x_4 \]

Time Irreversibility
Dispersion of Particle Clusters

\[ \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4 \]

Time Irreversibility
Deformation of Tetrahedra

Shape Tensor: \( G(t)_{ij} = \sum_a \rho_i^{(a)}(t) \rho_j^{(a)}(t) \)

Size: radius of gyration \( R^2(t) = \text{tr}[G(t)] \)

Shape: eigenvalues \( g_i \)

Lüthi et al., J. Tubul. 8, 45 (2007)
Deformation of Tetrahedra

Initial Shape: Regular

Shape: \( g_1(0) = g_2(0) = g_3(0) = \frac{l^2}{2} \)

“Pancake”

\[ g_1 \geq g_2 \gg g_3 \]

“Needle”

\[ g_1 \gg g_2 \geq g_3 \]
Deformation of Tetrahedra

\[
\langle g_i(t) \rangle / l^2
\]

\[
R_\lambda = 690
\]
\[
l/\eta = 364, 424, 485
\]

\[
R_\lambda = 270
\]
\[
l/\eta = 135, 154, 173
\]

\[
R_\lambda = 300 \text{ (DNS)}
\]
\[
l/\eta = 123
\]
\[
\Delta l/l = 10\%, 5\%
\]

\[
R_\lambda = 430 \text{ (DNS)}(*)
\]
\[
l/\eta = 167
\]
\[
\Delta l/l = 0\%
\]

*) Li et al., J. Turbul. 9, 31 (2008),
Yu et al., J. Turbul. 13, 12 (2012)
Deformation of Tetrahedra

\[ \frac{\langle g_2(t) \rangle - \langle g_2(-t) \rangle}{l^2} = 2\langle S_{0,22} \rangle t + O(t^3) \]

- \( R_\lambda = 690 \)
  - \( l/\eta = 364, 424, 485 \)

- \( R_\lambda = 270 \)
  - \( l/\eta = 135, 154, 173 \)

- \( R_\lambda = 300 \) (DNS)
  - \( l/\eta = 123 \)
  - \( \Delta l/l = 10\%, 5\% \)

- \( R_\lambda = 430 \) (DNS)(*)
  - \( l/\eta = 167 \)
  - \( \Delta l/l = 0\% \)

*) Li et al., J. Turbul. 9, 31 (2008),

Yu et al., J. Turbul. 13, 12 (2012)
Conclusion

- Multi-particle dispersion is a sensitive measure to reveal time asymmetry.
- Time asymmetry can be observed already at very short times and can be traced back to the turbulence energy cascade.
- Pairs separate faster backwards in time.
- Tetrahedra become stronger elongated backwards in time.

Thank You!
Appendix

- Lagrangian Particle Tracking
- Search for particle clusters
- Finite measurement volume
- Measurement Uncertainties
Deformation of Tetrahedra

\[ \frac{2\langle S_{0,22} \rangle t_0}{l^2 t_0} = 0.42 \]

\[ \frac{\langle g_2(t) \rangle - \langle g_2(-t) \rangle}{l^2 t_0} \]

- \( R_\lambda = 690 \)
- \( l/\eta = 364, 424, 485 \)
- \( R_\lambda = 270 \)
- \( l/\eta = 135, 154, 173 \)
- \( R_\lambda = 300 \) (DNS)
- \( l/\eta = 123 \)
- \( \Delta l/l = 10\%, 5\% \)
- \( R_\lambda = 430 \) (DNS)(*)
- \( l/\eta = 167 \)
- \( \Delta l/l = 0\% \)

*) Li et al., J. Turbul. 9, 31 (2008), Yu et al., J. Turbul. 13, 12 (2012)
Lagrangian Particle Tracking

Quarter of a full camera image, contrast enhanced

Intensity profile of a single particle

Cowen et al., Exp. Fluids 22, 199 (1997)
Ouellette et al., Exp. Fluids 40, 301 (2006)
Lagrangian Particle Tracking

Ouellette et al., Exp. Fluids 40, 301 (2006)
Lagrangian Particle Tracking

Nearest Neighbour

Four-Frame Best Estimate

Adapted from the PhD thesis of Nicholas Ouellette (Cornell University, 2006)
Lagrangian Particle Tracking

\[ d_{ij} = \sqrt{|x_i^p(t_j^s) - x_j^s|^2 + [u_i^p(t_j) - u_j^s|(t_j^s - t_i^s)|]^2} \]

Search for Particle Clusters

\[ R_{0} \pm \Delta R_{0} \]

Histogram

store in histogram

|  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|
| \(-5\delta t\) | \(-4\delta t\) | \(-3\delta t\) | \(-2\delta t\) | \(-1\delta t\) | \(t = 0\) | \(1\delta t\) | \(2\delta t\) | \(3\delta t\) | \(4\delta t\) | \(5\delta t\) |
A Finite Measurement Volume

\[ \delta R^2(t) \]

Average at infinite volume
A Finite Measurement Volume

\[ \delta R^2(t) \]

\[ t_1 \quad t_2 \quad t_{\text{max}} \]

ØMeasurement Volume
A Finite Measurement Volume

\[ \delta R^2(t) \]

\[ \theta \text{Measurement Volume} \]

0 \[ t_1 \] \[ t_2 \] \[ t_{\text{max}} \] \[ t \] 

Average at finite volume
A Finite Measurement Volume

\[ \delta R^2(t) \]

Average at finite volume

J. Jucha

Time Irreversibility
A Finite Measurement Volume

$$\delta R^2(t)$$

- Average at infinite volume
- Average at finite volume
Measurement Uncertainties \((R_\lambda = 350)\)

Optimistic Estimate
“All trajectories are independent”

Conservative Estimate
“All videos are independent”