Passive Scalar and Scalar Flux in Homogeneous Turbulence

Antoine BRIARD
Supervisor: Thomas GOMEZ
Collaboration: Claude Cambon and Vincent Mons

Paris, ∂'Alembert Institute

GDR Grenoble, 1-3 June 2015

Fluctuating fields

Velocity field

$$\left(\frac{\partial}{\partial t} - A_{ln}k_l\frac{\partial}{\partial k_n} + \nu k^2\right) u_i\hat{(\boldsymbol{k})} + M_{ij}(\boldsymbol{k})\hat{u}_j(\boldsymbol{k}) + iP_{imn}(\boldsymbol{k})\widehat{u_m u_n}(\boldsymbol{k}) = 0 \quad (1)$$

$M_{ij}(\boldsymbol{k}) = (\delta_{in} - 2\alpha_i\alpha_n)A_{n_j}$, A_{ij} mean velocity gradient matrix.

Scalar field

$$\left(\frac{\partial}{\partial t} - A_{jl}k_j\frac{\partial}{\partial k_l} + ak^2\right)\hat{\theta}(\boldsymbol{k}) + \lambda_j\hat{u}_j(\boldsymbol{k}) = -ik_j\widehat{\theta u_j}(\boldsymbol{k}) \quad (2)$$

λ_l scalar gradient

Kinetic Craya equation

$$\frac{d\hat{R}_{ij}}{dt} + 2\nu k^2 \hat{R}_{ij}(\boldsymbol{k}) + M_{in}(\boldsymbol{k})\hat{R_{nj}}(\boldsymbol{k}) + M_{jn}(\boldsymbol{k})\hat{R_{ni}}(\boldsymbol{k}) = T_{ij}^{NL}(\boldsymbol{k}) \qquad (3)$$

Spectral Reynolds tensor $\hat{R}_{ij}(\boldsymbol{k},t)\delta(\boldsymbol{k}-\boldsymbol{p}) =< \hat{u_i^*}(\boldsymbol{p},t)\hat{u}_j(\boldsymbol{k},t) >$.

Scalar Craya equation

$$\frac{d\mathcal{E}^T}{dt} + 2ak^2\mathcal{E}^T(\boldsymbol{k}) + 2\lambda_l F_l(\boldsymbol{k}) = T^{T,NL}(\boldsymbol{k}) \qquad (4)$$

Spectral scalar correlation $< \hat{\theta}^*(\boldsymbol{p})\hat{\theta}(\boldsymbol{k}) >= \mathcal{E}^T(\boldsymbol{k})\delta(\boldsymbol{k}-\boldsymbol{p})$.

Scalar Flux Craya equation

$$\frac{dF_i}{dt} + (\nu + a)k^2 F_i(\boldsymbol{k}) + M_{ij}(\boldsymbol{k})F_j(\boldsymbol{k}) + \lambda_j \hat{R}_{ij} = T_i^{F,NL}(\boldsymbol{k}) \qquad (5)$$

Spectral scalar flux correlation $< \hat{u_i^*}(\boldsymbol{p})\hat{\theta}(\boldsymbol{k}) >= F_i(\boldsymbol{k})\delta(\boldsymbol{k}-\boldsymbol{p})$.

Spherically-averaged Lin equations

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) = S^{L(iso)}(k,t) + S^{NL(iso)}(k,t) \quad (6)$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) H_{ij}^{(dir)}(k,t) = S_{ij}^{L(dir)}(k,t) + S_{ij}^{NL(dir)}(k,t) \quad (7)$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) H_{ij}^{(pol)}(k,t) = S_{ij}^{L(pol)}(k,t) + S_{ij}^{NL(pol)}(k,t) \quad (8)$$

$$\left(\frac{\partial}{\partial t} + 2a k^2\right) E_T(k,t) = S^{T,L(iso)}(k,t) + S^{T,NL(iso)}(k,t) \quad (9)$$

$$\left(\frac{\partial}{\partial t} + 2a k^2\right) E_T(k,t) H_{ij}^{(T)}(k,t) = S_{ij}^{T,L(dir)}(k,t) + S_{ij}^{T,NL(dir)}(k,t) \quad (10)$$

$$\left(\frac{\partial}{\partial t} + (a+\nu) k^2\right) E_F H_i^{(F)}(k,t) = S_i^{F,L}(k,t) + S_i^{F,NL}(k,t) \quad (11)$$

Spectra, energies, dissipation rates (1/2)

- Kinetic energy and scalar variance spectra

$$E(k,t) = \int_{S_k} \frac{\hat{R}_{ii}(\boldsymbol{k},t)}{2} d^2\boldsymbol{k}, \qquad E_T(k,t) = \int_{S_k} \mathcal{E}^T(\boldsymbol{k},t) d^2\boldsymbol{k} \quad (12)$$

- Kinetic and scalar energies and dissipation rates

$$K_{(T)}(t) = \int_0^\infty E_{(T)}(k,t)dk, \qquad \epsilon_{(T)}(t) = 2\nu_{(T)} \int_0^\infty k^2 E_{(T)}(k,t)dk \quad (13)$$

- Directional anisotropy

$$2E(k,t)H_{ij}^{(dir)}(k,t) = \int_{S_k} \hat{R}_{ij}^{(dir)}(\boldsymbol{k},t) d^2\boldsymbol{k} \quad (14)$$

- Polarization anisotropy

$$2E(k,t)H_{ij}^{(pol)}(k,t) = \int_{S_k} \hat{R}_{ij}^{(pol)}(\boldsymbol{k},t) d^2\boldsymbol{k} \quad (15)$$

Spectra, energies, dissipation rates (2/2)

- Scalar directional anisotropy

$$2E_T(k,t)H_{ij}^{(T)}(k,t) = \int_{S_k} \mathcal{E}^{(T,dir)} P_{ij}(\boldsymbol{k},t) d^2\boldsymbol{k} \qquad (16)$$

- Scalar flux anisotropy

$$E_F H_i^{(F)}(k,t) = \int_{S_k} F_i(\boldsymbol{k},t) d^2\boldsymbol{k}, \qquad F_i = \frac{3}{2}\mathcal{E}_0^F H_j^{(F)} P_{ij} \qquad (17)$$

- Cospectrum and streamwise flux

$$\mathcal{F}(k,t) = E_F H_3^{(F)}(k,t), \qquad \mathcal{F}_S(k,t) = E_F H_1^{(F)}(k,t) \qquad (18)$$

- Cospectrum and streamwise flux energies and dissipation rates

$$K_{\mathcal{F}}^{(S)}(t) = \int_0^\infty \mathcal{F}_{(S)}(k,t)dk, \qquad \epsilon_{\mathcal{F}}^{(S)}(t) = (\nu + a)\int_0^\infty k^2 \mathcal{F}_{(S)}(k,t)dk \qquad (19)$$

Homogeneous Isotropic Turbulence with Scalar Gradient

Cospectrum $\mathcal{F}$ with scalar gradient Λ

$$\mathcal{F}(k,t) = E_F H_3^{(F)}(k,t), \qquad \boldsymbol{\lambda} = (0,0,-\Lambda) \tag{20}$$

Spectral behavior : Lumley (1967), Bos (2005)

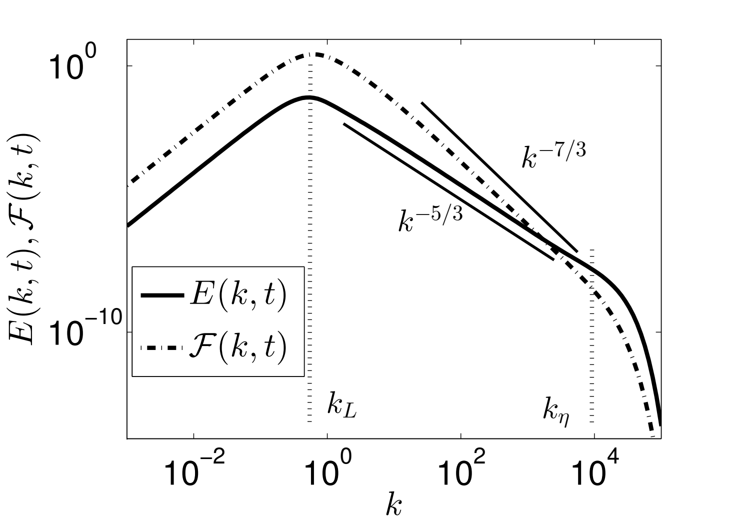
$$\mathcal{F}(k,t) = C_{\mathcal{F}} \Lambda \epsilon^{1/3} k^{-7/3} \tag{21}$$

Other scaling using $\epsilon_{\mathcal{F}}$

$$\mathcal{F}(k,t) = C_{\mathcal{F}} \epsilon^{-1/3} \epsilon_{\mathcal{F}} k^{-5/3} \tag{22}$$

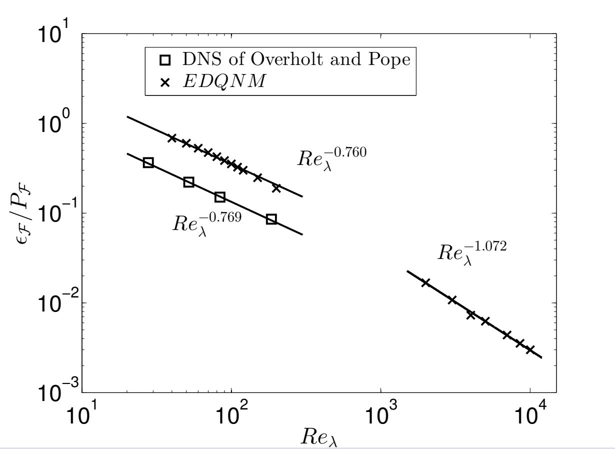
- $\epsilon_{\mathcal{F}}$ is not conserved
- Same scaling as for $E_T(k,t)$

Spectral behavior of $\mathcal{F}(k, t)$

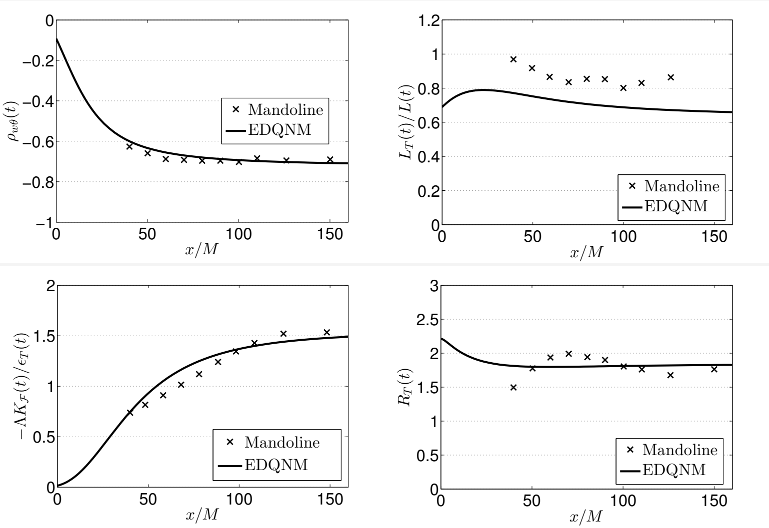


Production and dissipation - DNS Overholt & Pope (1996)

$$\epsilon_{\mathcal{F}}(t) = (\nu + a)\int_0^\infty k^2 \mathcal{F}(k,t)dk, \qquad P_{\mathcal{F}}(t) = -\frac{2}{3}\Lambda K(t) \qquad (23)$$



Sirivat & Warhaft (1983), $\beta = 1.78°C.m^{-1}$, $\Lambda = 0.152$



Decay of the cospectrum

- Power law decay

$$K_{\mathcal{F}}(t) \sim t^{\alpha_{\mathcal{F}}}$$

- High Reynolds regime : inertial range dominant

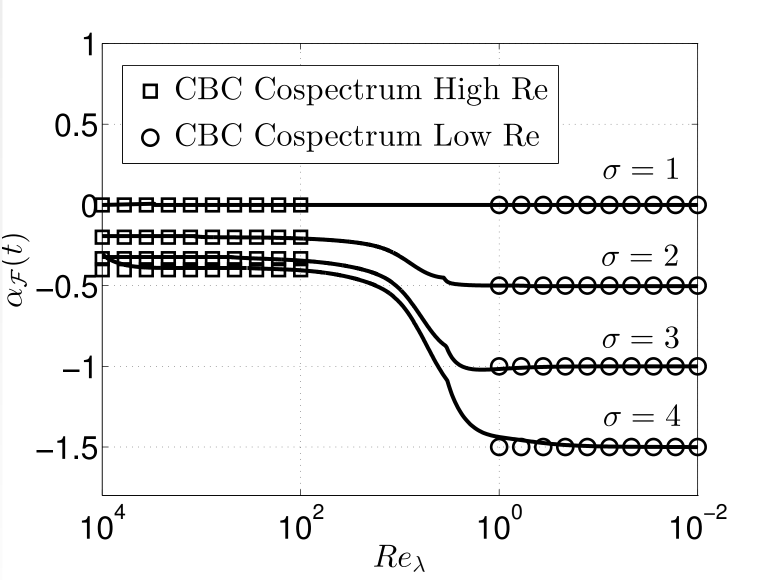
$$K_{\mathcal{F}}(t) = \int_{k_L}^{\infty} \mathcal{F}(k,t) \sim k_L^{-4/3} \epsilon^{1/3}$$

$$\alpha_{\mathcal{F}} = -\frac{\sigma - p_{\mathcal{F}} - 1}{\sigma - p + 3}, \qquad p_{\mathcal{F}} = \frac{1}{2}(p + p_T) = 0.4075$$

- Low Reynolds regime : Production through scalar gradient dominant

$$\frac{dK_{\mathcal{F}}}{dt} = P_{\mathcal{F}} = \frac{2}{3}\Lambda K, \qquad \alpha_{\mathcal{F}} = -\frac{\sigma - 1}{2}$$

Cospectrum energy $K_{\mathcal{F}}$ and scalar dissipation ϵ_T



Growth of the passive scalar with the gradient Λ

- Power law growth

$$K_T(t) \sim t^{\alpha^{\Lambda}_{\mathcal{F}}}$$

- Production through the cospectrum dominant

$$\frac{dK_T}{dt} \sim \Lambda K_{\mathcal{F}}(t)$$

- High Reynolds regime

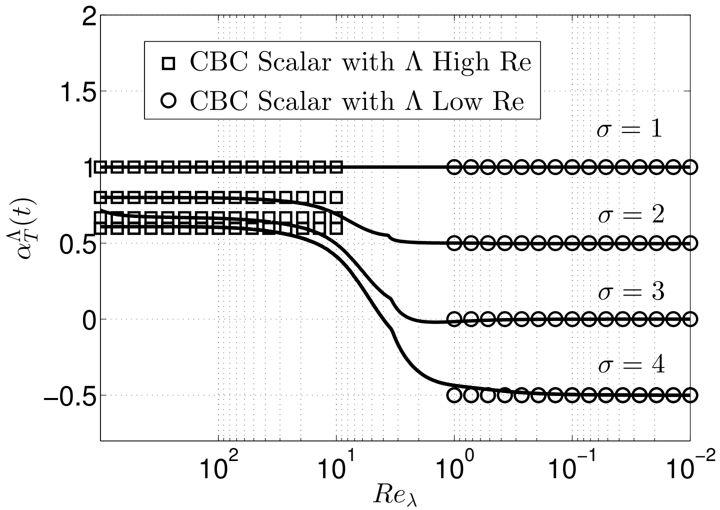
$$\alpha_T^{\Lambda} = \frac{1}{2}\frac{p_T - p + 8}{\sigma - p + 3}$$

Agreement with Chasnov (1995) for Saffman turbulence $\alpha_T^{\Lambda} = 4/5$.

- Low Reynolds regime

$$\alpha_T^{\Lambda} = -\frac{\sigma - 3}{2}$$

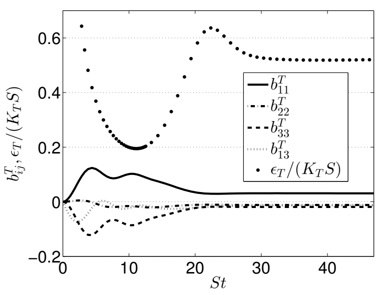
Scalar energy K_T

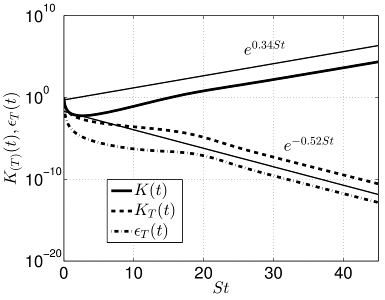


Passive scalar with an uniform shear S

Exponential decrease : Gonzalez (2000)

$$K_T(t) = K_T^\infty \exp(\gamma_T St), \qquad \epsilon_T(t) = \epsilon_T^\infty \exp(\gamma_T St), \qquad \gamma_T = -\frac{\epsilon_T}{SK_T}$$

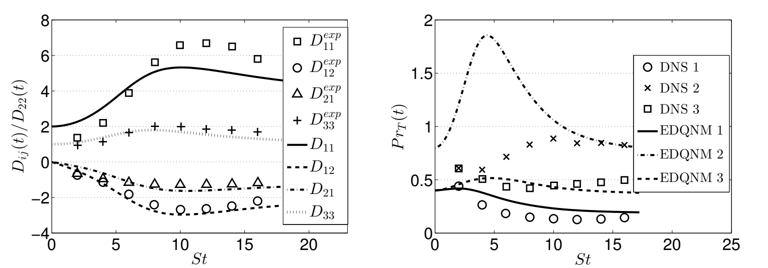




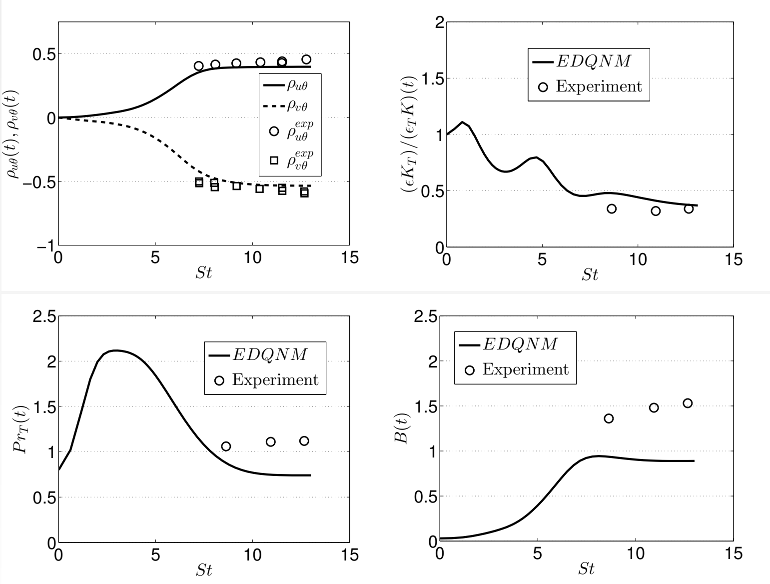
Rogers, Mansour & Reynolds (1989) : $S = 14.142$ and $\Lambda = 2.5$

Integrated quantities

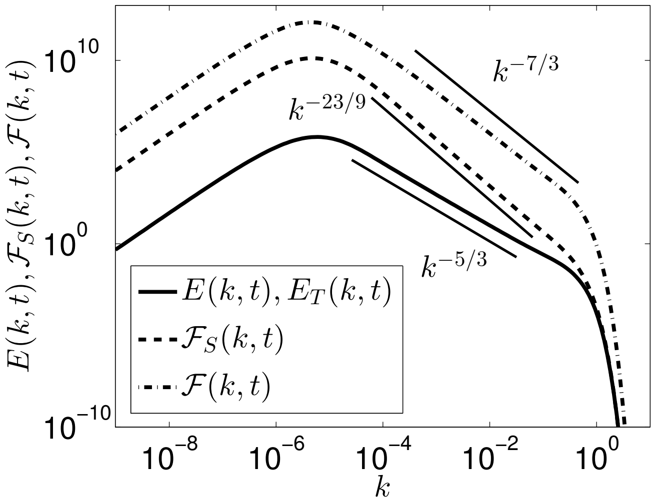
- Diffusivity tensor $D_{ij}(t) = - < \theta u_i > / \lambda_j$
- Turbulence Prandtl number $Pr_T(t) = -R_{12}(t)/(SD_{ii}(t))$
- Scalar flux correlation $\rho_{u_i\theta}(t) = \frac{<u_i\theta>}{\sqrt{<u_i^2><\theta^2>}} = \frac{K_{\mathcal{F}}}{\sqrt{2K_T R_{ii}}}$



Tavoularis & Corrsin (1981) : $S = 6.19$ and $\Lambda = 0.1823$



Spectral behavior of $\mathcal{F}_S(k,t)$



Bibliography

Bos, Touil, Bertoglio, *Physics of Fluids*, **Vol. 17** (2005)
Bos, Bertoglio, *Physics of Fluids*, **Vol. 19** (2007)
Briard, Gomez, Sagaut, Memari, *Journal of Fluid Mechanics*, (2015, review)
Briard, Gomez, Cambon, *Journal of Fluid Mechanics*, (2015, writing)
Chasnov, *Physics of Fluids*, **Vol. 7** (1995)
Corrsin, *Journal of Aero Science*, **Vol. 18** (1951)
Gonzalez, *Int. J. of Heat and Mass Transfer*, **Vol. 43** (2000)
Lumley, *Physics of Fluids*, **Vol. 10** (1967)
Mons, Cambon, Sagaut, *Journal of Fluid Mechanics*, (2015, review)
O'Gorman, Pullin, *Journal of Fluid Mechanics*, **Vol. 532** (2005)
Overholt, Pope, *Physics of Fluids*, **Vol. 8** (1996)
Rogers, Mansour, Reynolds *Journal of Fluid Mechanics*, **Vol. 203** (1989)
Sagaut, Cambon, *Cambridge University Press* (2008)
Sirivat, Warhaft, *Journal of Fluid Mechanics*, **Vol. 128** (1983)
Tavoularis, Corrsin, *Journal of Fluid Mechanics*, **Vol. 104** (1981)