GDR 2865 CNRS, Turbulence, Poitiers 15-17 octobre 2012

Bruit de mélange des jets subsoniques initialement turbulents

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- **Bruit de mélange des jets subsoniques initialement turbulents**

  - Motivations
  - Méthodologie pour la simulation des grandes échelles (LES-RF)
  - Influence du taux de turbulence initial sur un jet à Reynolds $\mathrm{Re}_D = 10^5$
  - Quelques remarques sur les contraintes de maillage
  - Jets trippés de $\mathrm{Re}_D = 25 \times 10^3$ à 200×10^3

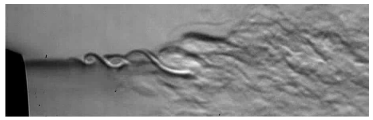
Bogey, Marsden & Bailly, 2012, *J. Fluid Mech.*, **701**
2011, *Phys. Fluids*, **23**, 035104 & **23**, 091702

- **Initial conditions at the nozzle exit**

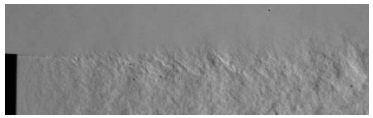
  (visualizations by T. Castelain & B. André, ECL)

$Re_D \sim 3 \times 10^7$

$Re_D \sim 3.3 \times 10^4$



$Re_D \sim 1.2 \times 10^5$



$Re_D \sim 8.7 \times 10^5$

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nominally laminar
$u'_e/u_j \sim 1\%$

nominally turbulent
$u'_e/u_j \sim 10\%$

fully laminar
$u'_e/u_j < 1\%$

transitional jets

fully turbulent

Re_D

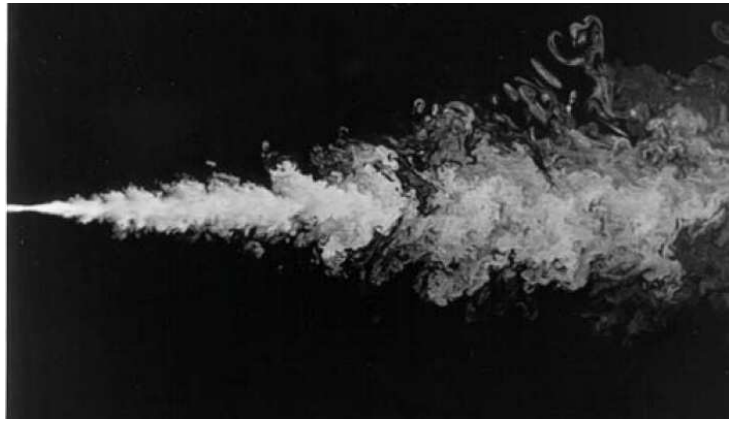
$Re_D \sim 10^5$
$(Re_{\delta_\theta} \simeq 300)$

$Re_D \sim 3 \times 10^5$

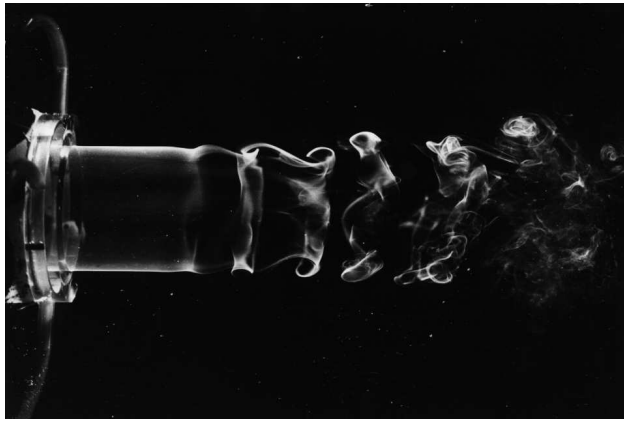
$$Re_D = u_j D/\nu \qquad Re_{\delta_\theta} = u_j \delta_\theta/\nu \qquad \sigma_{u_e} = u'_e/u_j$$

- Free shear flows : subsonic turbulent jets Reynolds number $Re_D = u_j D/\nu$

Prasad & Sreenivasan (1989)
$Re_D \simeq 4000$



Dimotakis *et al.* (1983)
$Re_D \simeq 10^4$



Kurima, Kasagi & Hirata (1983)
$Re_D \simeq 5.6 \times 10^3$

Ayrault, Balint & Schon (1981)
$Re_D \simeq 1.1 \times 10^4$

Mollo-Christensen (1963)
$Re_D = 4.6 \times 10^5$

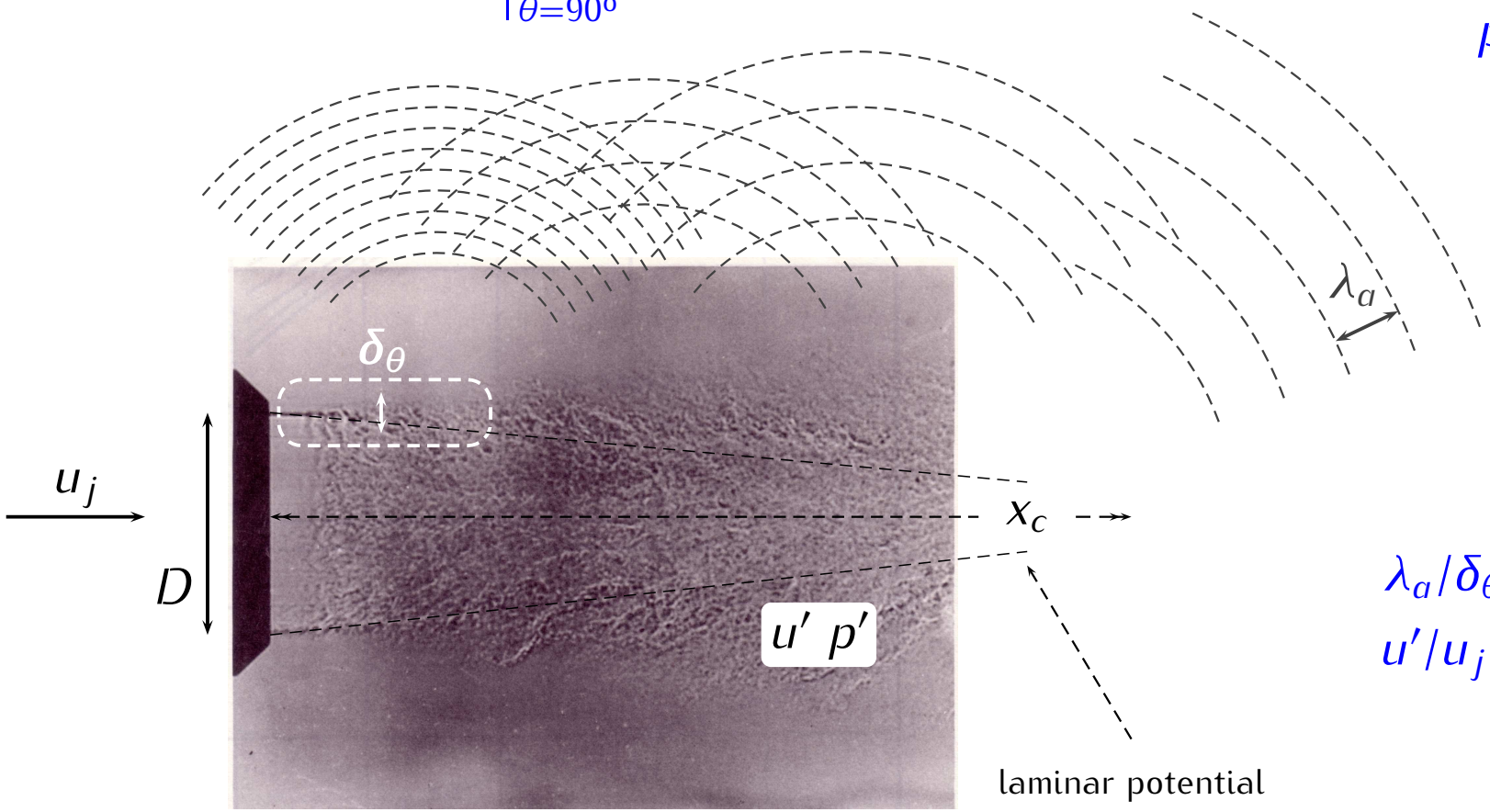
- **Disparity of scales**

Isothermal jet, $\lambda_a/\delta_\theta \simeq \sqrt{Re_D}/(M_a \times St)$ $(Re_D = 10^6, M_a = 0.9, r/D \sim 10)$

$$\overline{p_a'^2}\Big|_{\theta=90^o} \sim M_a^{7.5}$$

$u_a'/u' \sim 10^{-4}$

$p_a'/p' \sim 10^{-3}$



Mollo-Christensen (1963), $Re_D = 4.6 \times 10^5$

laminar potential core length x_c

$\lambda_a/\delta_\theta \simeq 10^3$

$u'/u_j \simeq 0.16$

- **Closure : subgrid-scale model** *e.g.* Sagaut (2006), Lesieur (2007)

Projection modeling by a spacial convolution filtering $\bar{\boldsymbol{u}} = G * \boldsymbol{u}$

$$\partial_t \bar{\boldsymbol{u}} + \nabla \cdot (\bar{\boldsymbol{u}}\bar{\boldsymbol{u}}) + \nabla(\bar{p}/\rho) + \nu \nabla^2 \bar{\boldsymbol{u}} = \nabla \cdot \boldsymbol{\sigma}^{\mathrm{sgs}}$$

- structural approach, find a model for the sgs stress tensor
- functional approach, surrogate the mean action
  - turbulent kinetic energy balance is more important
  - turbulent kinetic energy cascade is dominant → dissipation

Two main classes of methods in physical space

- turbulent eddy viscosity models
- hyperviscosity ($\nabla \cdot \boldsymbol{\sigma}^{\mathrm{sgs}} \to \nu_h \nabla^{2n}$) or high-order explicit filtering

Pruett *et al.* – Domaradzki, Adams *et al.* – Visbal, Gaitonde, Rizzetta – ADM – ...

LES - Relaxation filtering $\nabla \cdot \boldsymbol{\sigma}^{\mathrm{sgs}} = -\chi G * \tilde{\boldsymbol{u}}$

Bogey & Bailly (2006), Berland *et al.*, *J. Comput. Phys.* (2008) & *JOT* (2008)

- Closure : LES based on relaxation filtering (LES-RF)

  - Spectrum model $E(k) = C_K(\epsilon L)^{2/3}(kL)^{-5/3} f_L(kL) f_\eta(k l_\eta)\, L$
    L large-scale length scale, l_η Kolmogorov length scale

  - Governing equation, $\partial_t \bar{\boldsymbol{u}} + \nabla \cdot (\bar{\boldsymbol{u}}\bar{\boldsymbol{u}}) + \nabla(\bar{p}/\rho) + \nu \nabla^2 \bar{\boldsymbol{u}} = -\chi G * \tilde{\boldsymbol{u}}$
    $\tilde{E}(k) = \hat{G}^2(\boldsymbol{k}) E(k)$, Lin's equation for $\tilde{E}$

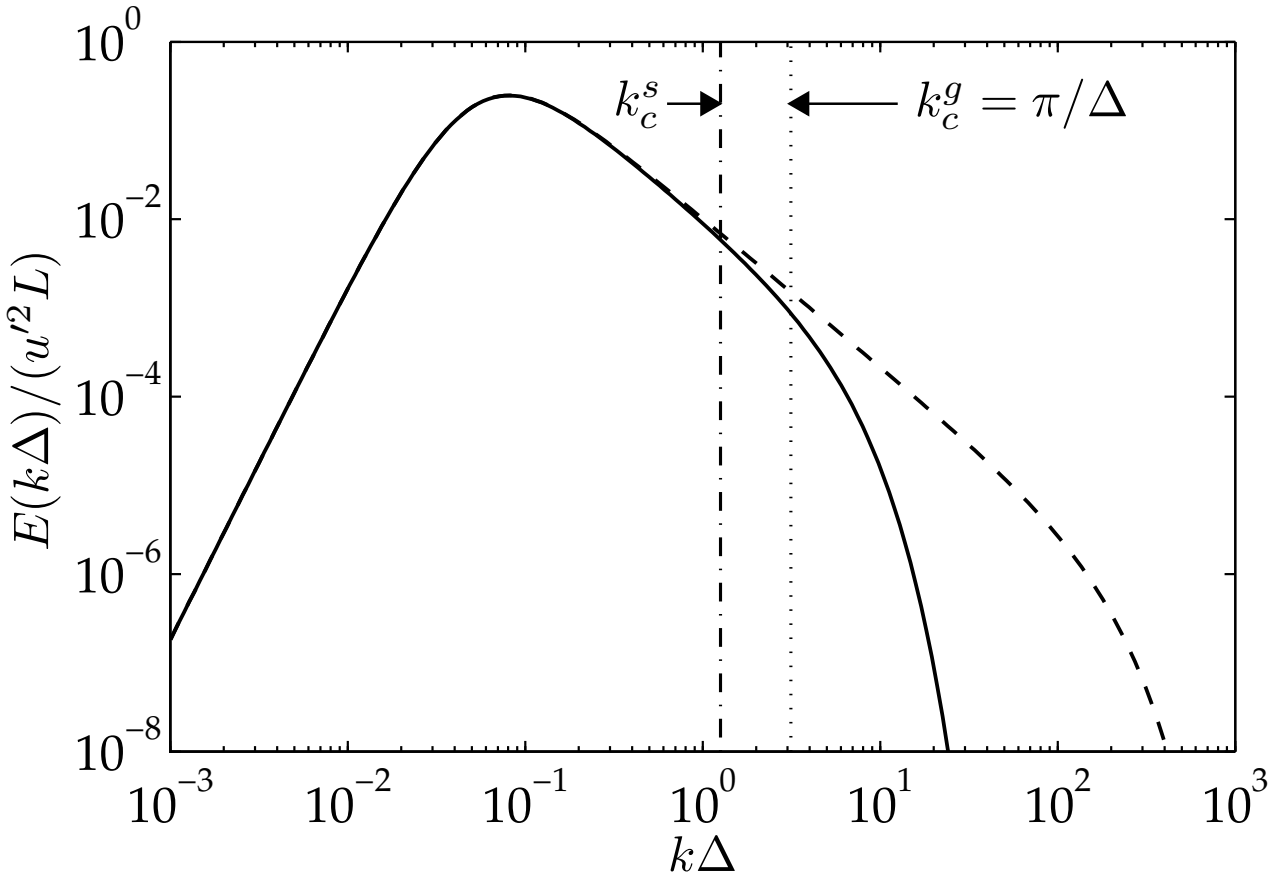
$$\partial_t \tilde{E}(k) = \tilde{T}(k) - 2\nu k^2 \tilde{E}(k) - 2\chi\big[1 - \hat{G}(k)\big]\tilde{E}(k)$$

    $D_\nu(k) = 2\nu k^2 \tilde{E}(k)$ viscous dissipation
    $D_f(k) = 2\chi\big[1 - \hat{G}(k)\big]\tilde{E}(k)$ dissipation induced by the explicit filtering

  - **Reynolds effects.** Given Mach number, $u' = \text{cst}$ and $k_t = \text{cst}$
    Reynolds number $\text{Re}_L = u'L/\nu \sim L$, and space discretization $L/\Delta = \text{cst} \Longrightarrow \Delta$

For a given wavenumber $k_\alpha \gg k_\eta$, $\dfrac{D_f(k_\alpha)}{u'^3} \simeq \text{cst}$ $\dfrac{D_\nu(k_\alpha)}{u'^3} \sim \text{Re}_L^{-1}$

- Closure : LES based on relaxation filtering (LES-RF)



$\mathrm{Re}_L = 10^3$

$\mathrm{Re}_L = 10^5$

$E(k)$ energy spectrum

$k_c^g = \pi/\Delta$

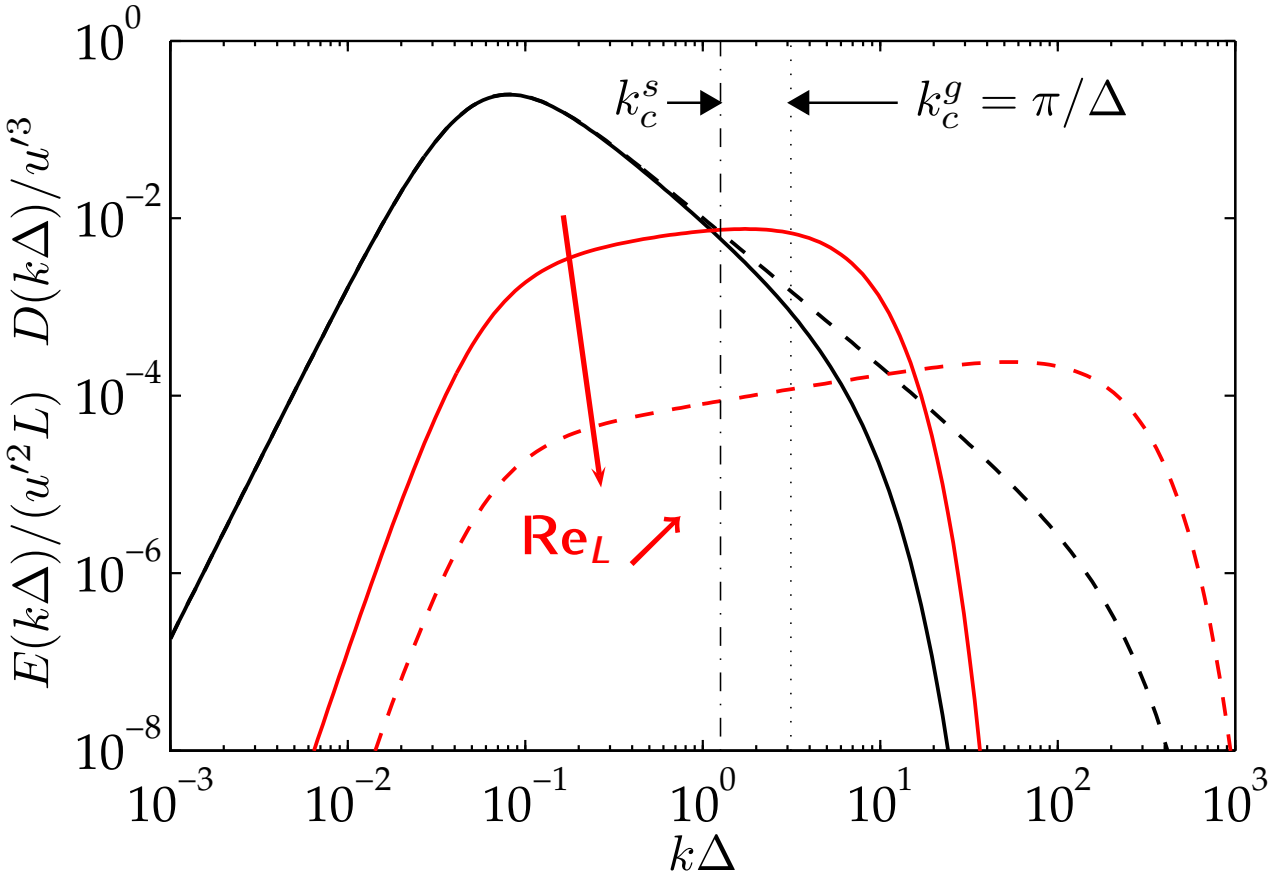
$k_c^s = 2\pi/(5\Delta)$

$L/\Delta = \mathcal{O}(10)$

von Kármán (1948) - Pao (1965)

$E(k) = C_K \epsilon^{2/3} k^{-5/3} f_L(kL) f_\eta(k l_\eta)$

- Closure : LES based on relaxation filtering (LES-RF)



$\mathrm{Re}_L = 10^3$

$\mathrm{Re}_L = 10^5$

$E(k)$ energy spectrum

$D_\nu(k)$ viscous dissipation

$k_c^g = \pi/\Delta$

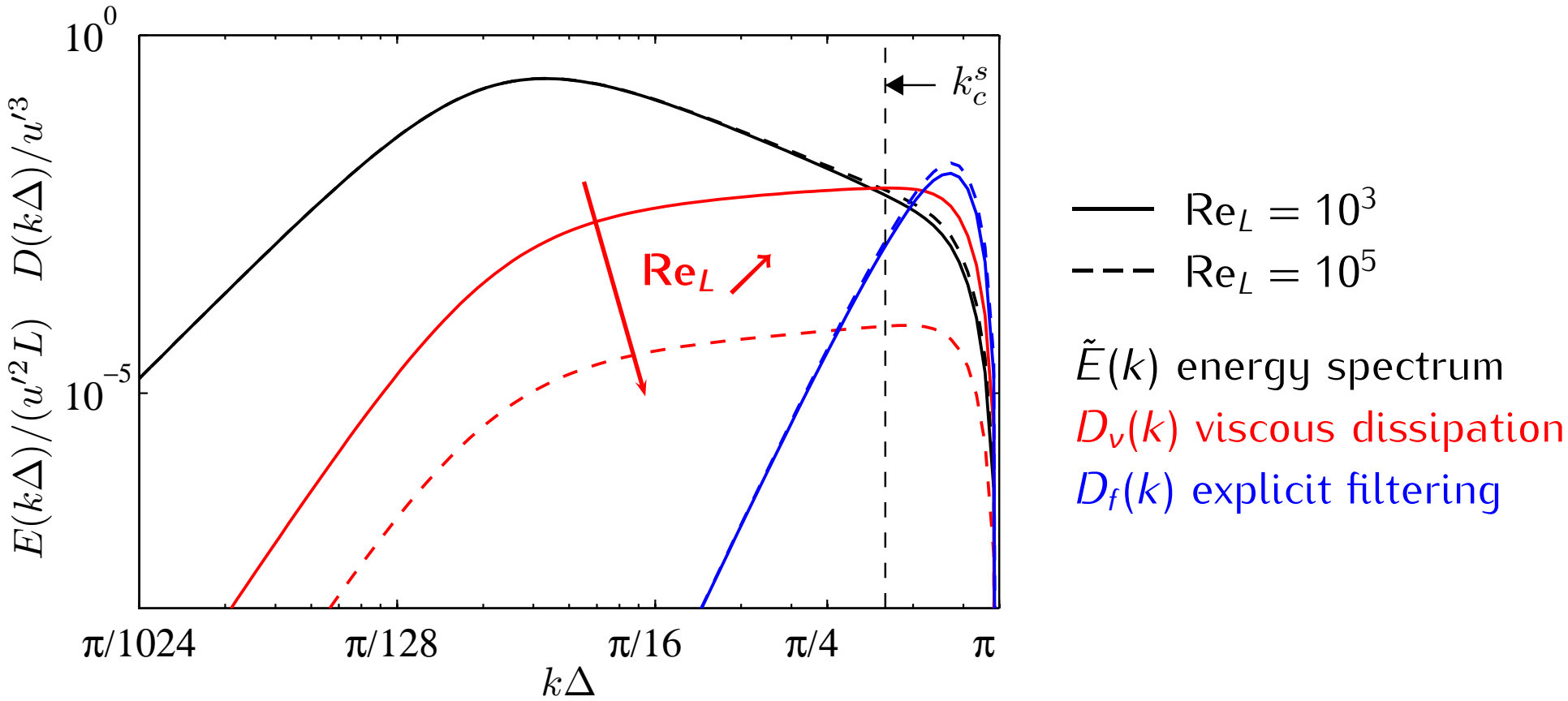
$k_c^s = 2\pi/(5\Delta)$

$$S(k) = -\int_0^k T(k')dk' \gtrsim 0 \quad \text{for} \quad k\lambda_g \gtrsim 1 \quad \Longrightarrow \quad k_c^s\lambda_g \sim 1$$

for a well-resolved LES

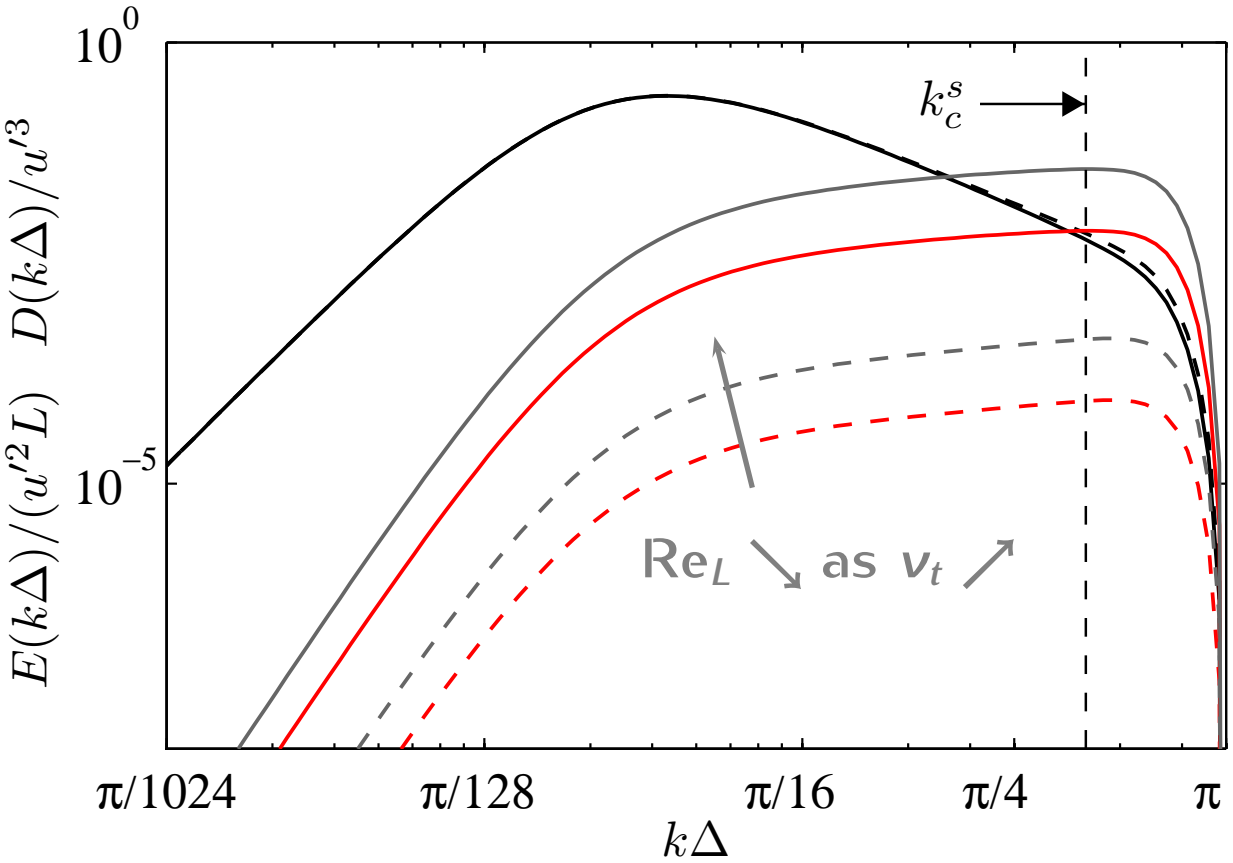
(Bailly & Comte-Bellot, 2003)

- Closure : LES based on relaxation filtering (LES-RF)



- In the well-resolved wavenumber range, we must have $D_f(k) < D_\nu(k)$
  $\rightsquigarrow$ constraint on the mesh size Δ
- Regularization term corresponding to the net drain associated with the turbulent kinetic energy cascade, larger scales mostly unaffected

- Closure : LES based on eddy viscosity concept



$\mathrm{Re}_L = 10^3$

$\mathrm{Re}_L = 10^5$

$E(k)$ energy spectrum

$D_\nu(k)$ viscous dissipation

$D_{\nu+\nu_t}(k)$ eddy-viscosity (with $\nu_t/\nu = 5$)

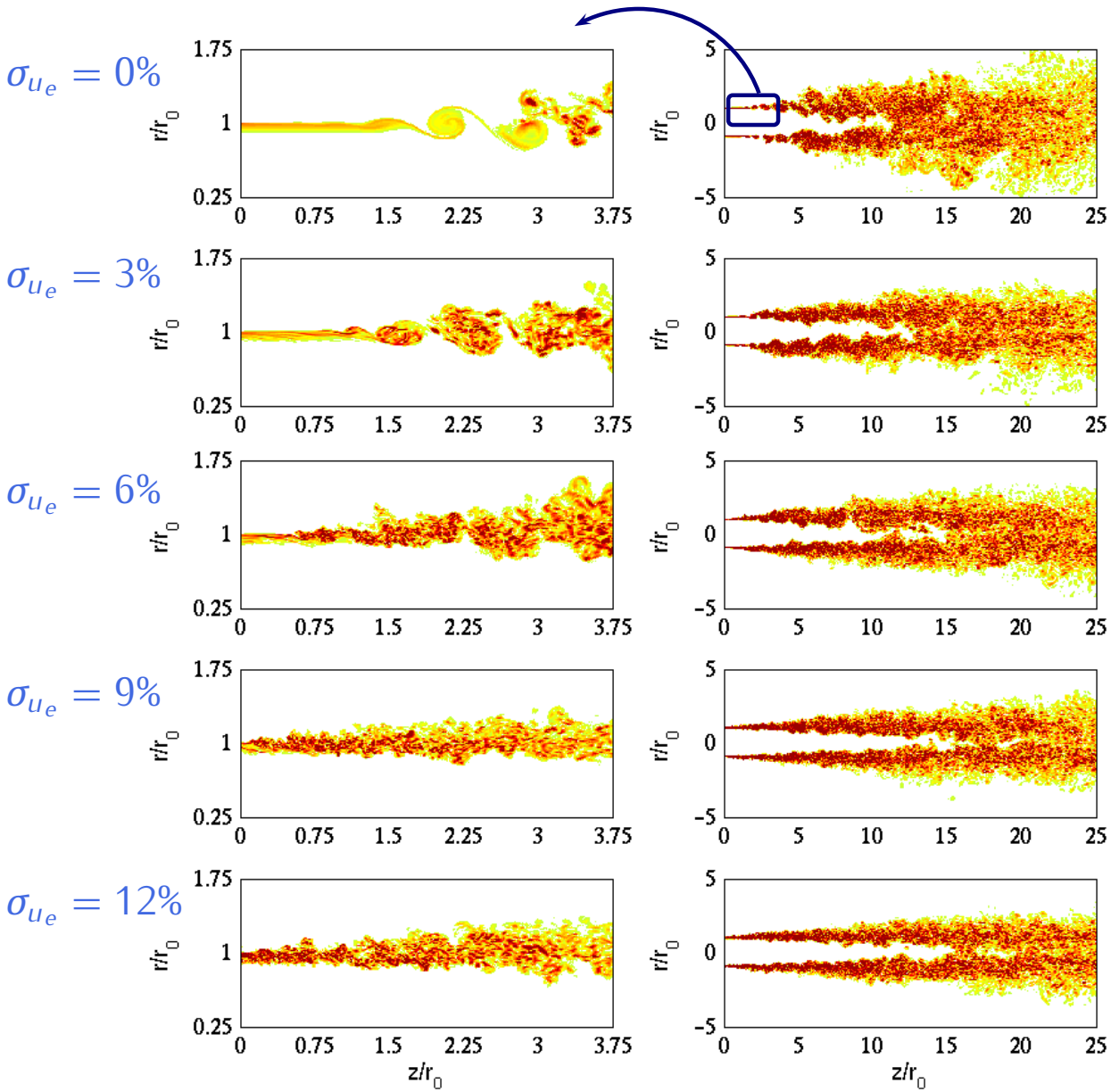
- In the well-resolved wavenumber range, $D_\nu(k)$ is replaced by $D_{\nu+\nu_t}(k)$, same functional form, $\rightsquigarrow$ overestimation of the dissipation for resolved scales
- However 'user-friendly' for unstructured grids

Status of large eddy simulation

- full-scale for laboratory jets (typically $D = 2$ cm, $M = 0.9$, $Re_D = 4 \times 10^5$) and nearly mature numerical tool *(basic statistics, turbulent kinetic energy budget, two-point space-time correlations, Reynolds effects)*
- advances in « alternative subgrid-scale models »
  e.g. removing energy at the smallest resolved scale by explicit filtering
- with $N \sim 10^8$ points,
  DNS at $\mathrm{Re}_D \sim 10^4$ and LES at $\mathrm{Re}_D \sim 10^5$, $\mathrm{St} \sim 10$.

Influence of initial turbulence level

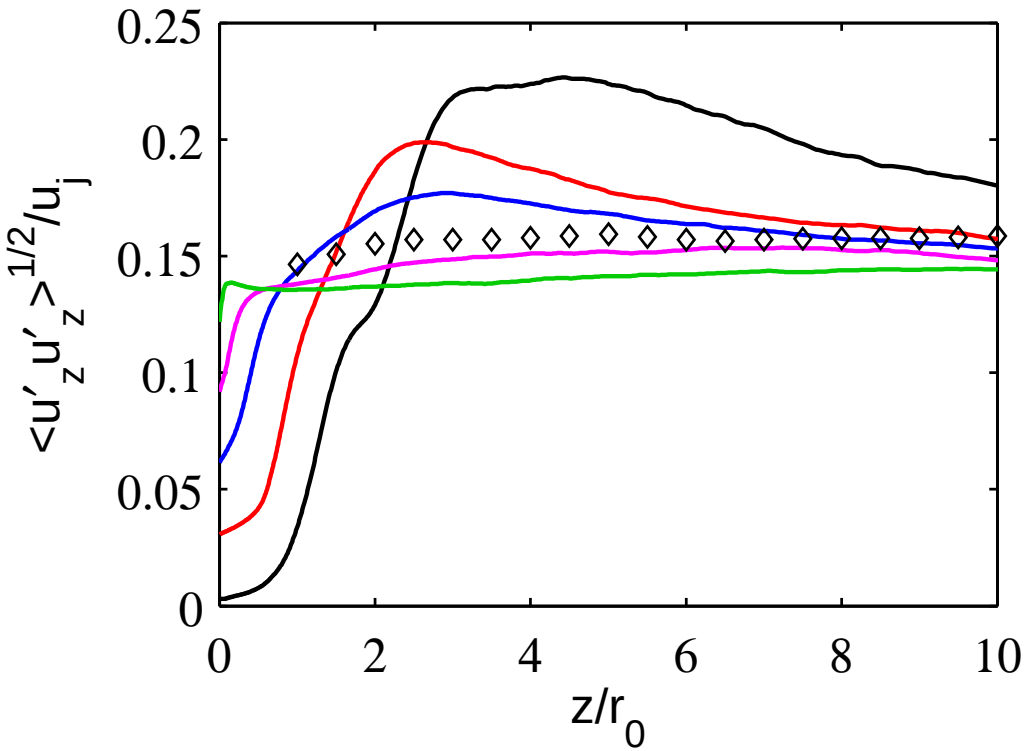
$M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$



$$n_r \times n_\theta = n_z = 256 \times 1024 \times 962 = 252 \text{ million pts}$$

- as the exit turbulence level increases, coherent structures (and consequently vortex rolling-ups and pairings) gradually disappear
- higher initial turbulence levels lead to longer potential cores

- Influence of initial turbulence level $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$



u'_z/u_j along $r = r_0$

$\sigma_{u_e} = 0\%$

$\sigma_{u_e} = 3\%$

$\sigma_{u_e} = 6\%$

$\sigma_{u_e} = 9\%$

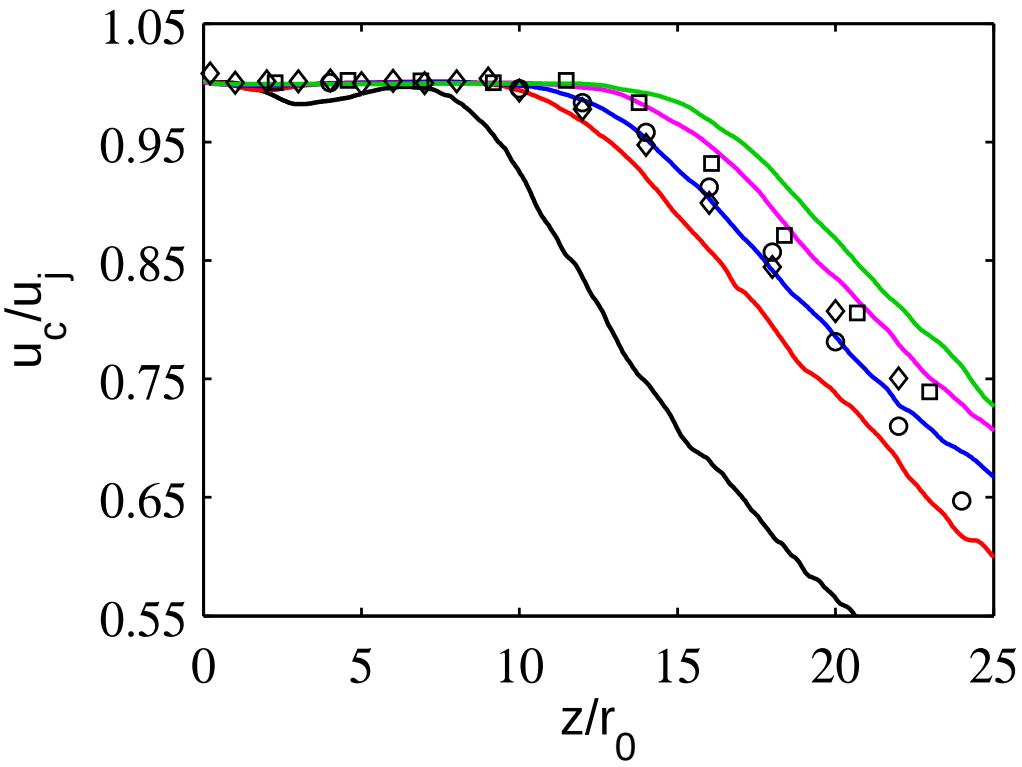
$\sigma_{u_e} = 12\%$

$M = 0.9$ & $Re_D = 7.7 \times 10^5$

◇ Fleury *et al.*, *AIAA Journal* (2008)

- As the initial turbulence level increases, the shear layers develop more slowly with lower rms velocity peaks (overshoot around the pairing position for $u'_e/u_j \leq 6\%$, but nearly monotonical growth for $u'_e/u_j \geq 9\%$)

- **Influence of initial turbulence level** $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$



Centerline mean axial velocity u_c/u_j

$\sigma_{u_e} = 0\%$

$\sigma_{u_e} = 3\%$

$\sigma_{u_e} = 6\%$

$\sigma_{u_e} = 9\%$

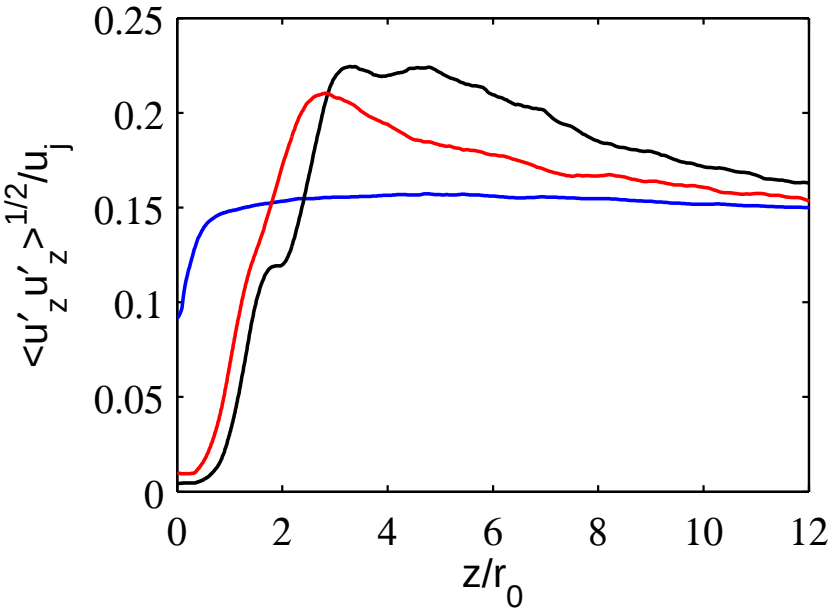
$\sigma_{u_e} = 12\%$

Experiments at Mach 0.9 jets at $\mathrm{Re}_D \geq 5 \times 10^5$

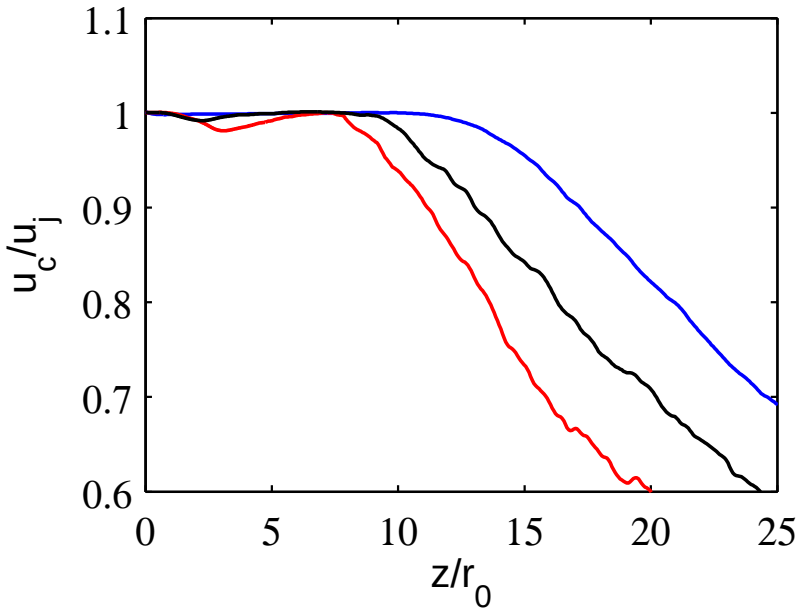
○ Lau *et al.* (1979), □ Arakeri *et al.* (2003), ◇ Fleury *et al.* (2008)

- **Influence of initial turbulence level** $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$

peak rms velocity $[u'_z]_{rms}/u_j$

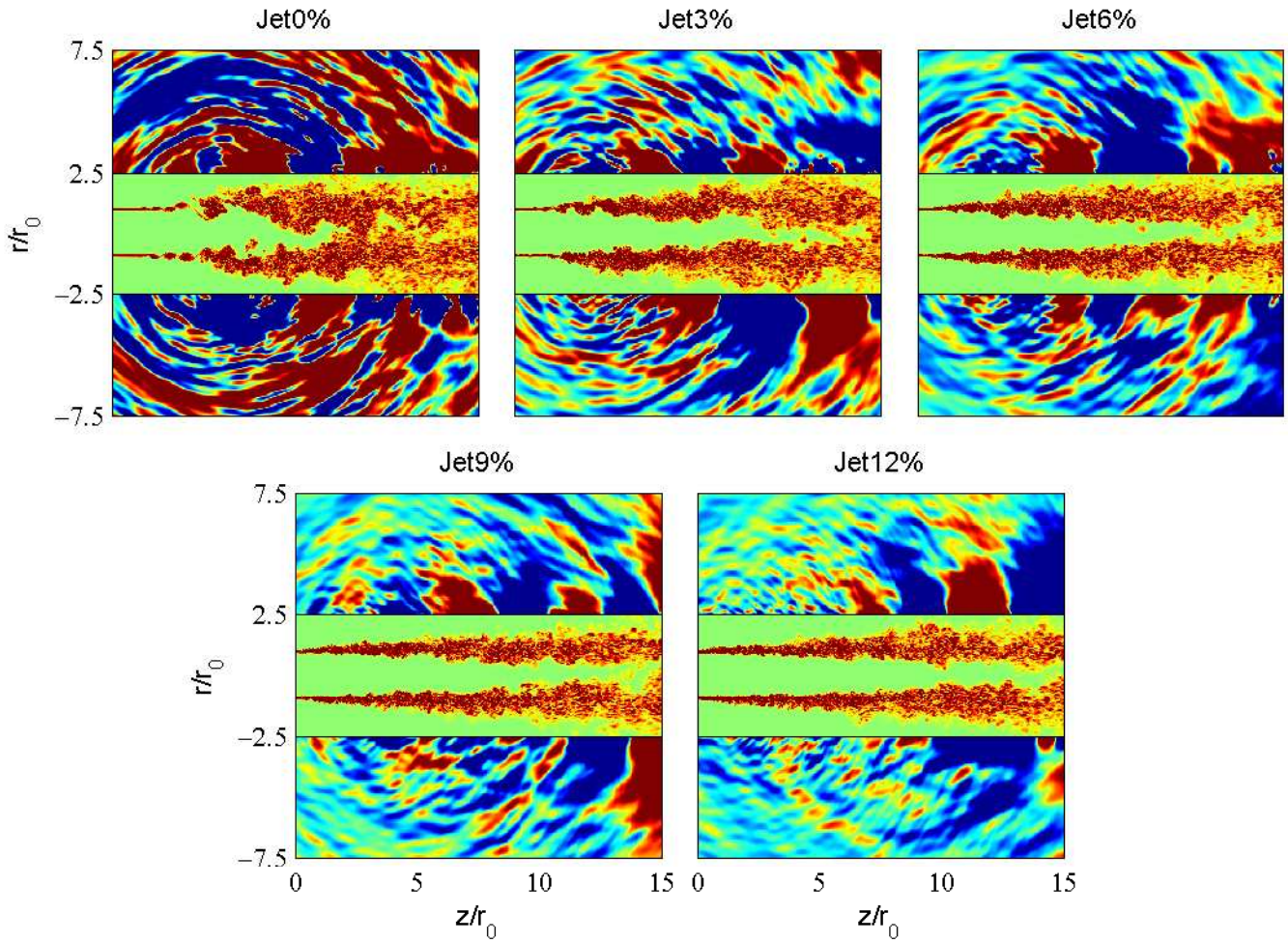


centerline mean velocity u_c/u_j

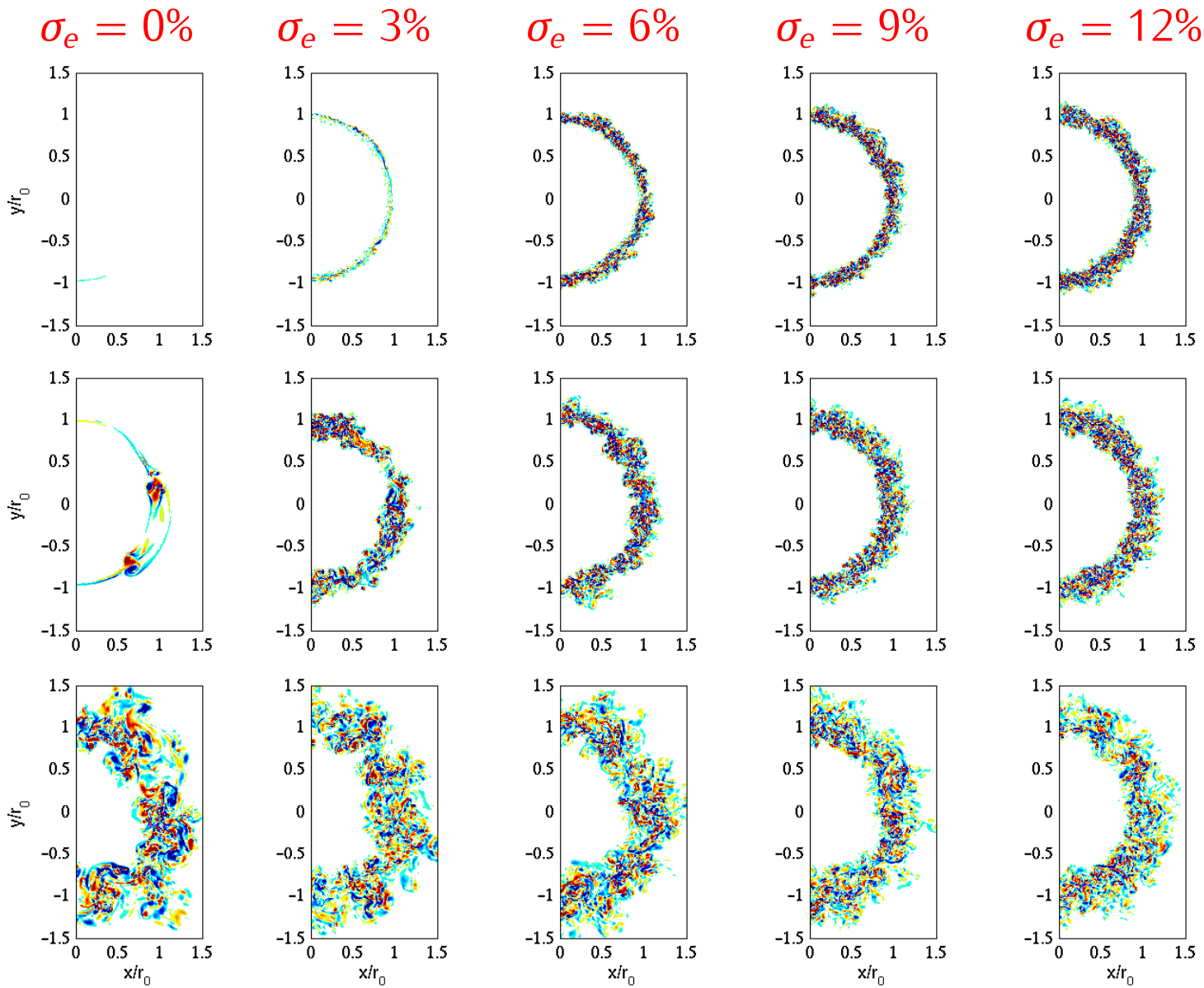


$\sigma_e < 1\%$ $\sigma_e = 1\%$ $\sigma_e = 9\%$

- Influence of initial turbulence level $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$



- Influence of initial turbulence level $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900$



$z = r_0$

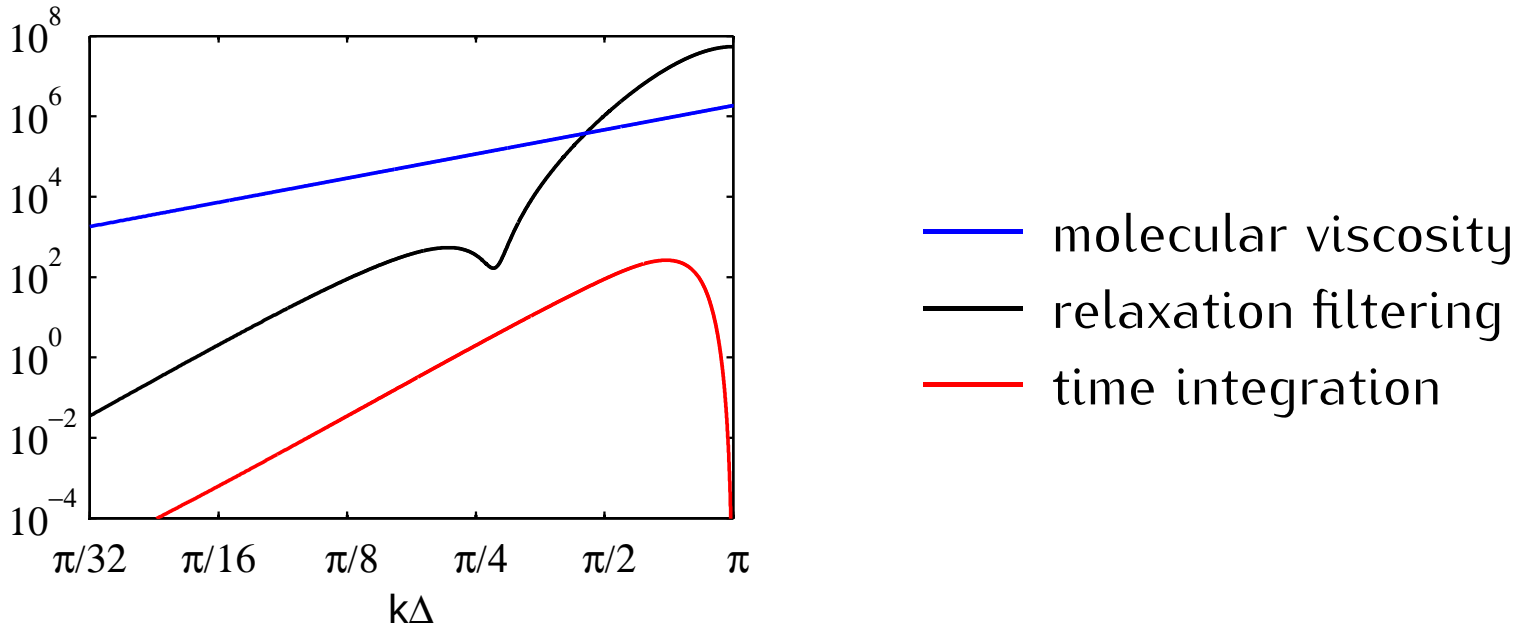
snapshots of vorticity $\omega_z(r, \theta)$

$z = 2r_0$

$z = 4r_0$

- **Dissipation functions versus normalized wave number $k\Delta$**

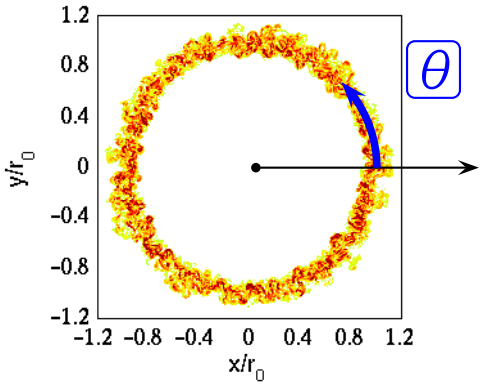
  Jetring1024drdz, $\Delta/r_0 = 0.36\%$ (Δr at $r = r_0$)



- negligible contribution of time integration
- damping from viscosity higher than that from the relaxation filtering for about $k\Delta < \pi/3 \Longrightarrow \lambda > 6\Delta$
- well discretized, *i.e.* numerical accuracy mainly affected by the molecular viscosity

Discretization quality

$M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900, \sigma_{u_e} = 9\%$



(snapshot of ω at $x = r_0$)

$$\mathcal{R}_{11}^{(\theta)}(\theta) = \frac{\langle u'_z(r,\theta_0,z)u'_z(r,\theta_0+\theta,z)\rangle}{\langle u'^2_z(r,\theta_0,z)\rangle^{1/2}\langle u'^2_z(r,\theta_0+\theta,z)\rangle^{1/2}}$$

$$= \sum_{n=0}^{\infty} a_n^{(\theta)}\cos(n\theta) \qquad k_\theta \equiv n/r_0$$

- Large scales, *i.e.* integral length scales $L_{ii}^{(\theta)}$, must be well discretized

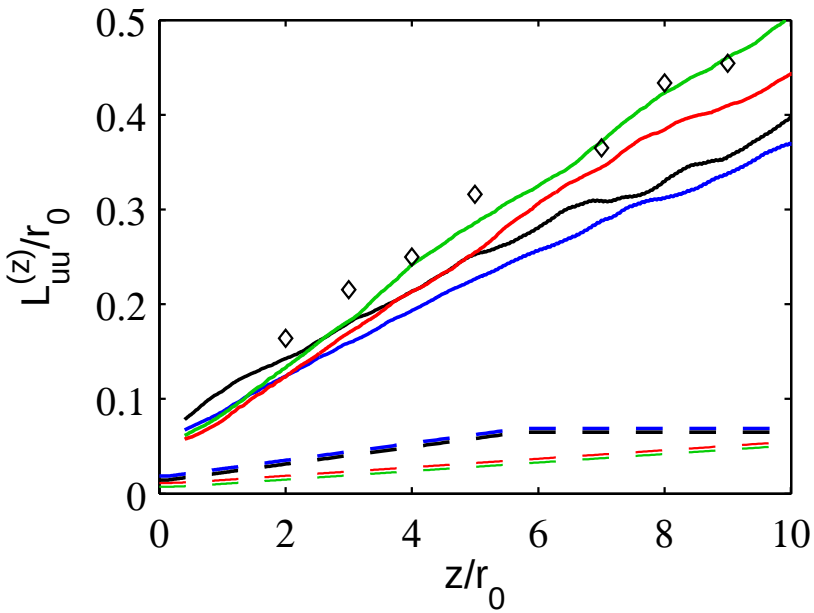
$$e.g. \qquad L_{11}^{(\theta)} = \frac{1}{r_0}\int_0^\pi \mathcal{R}_{11}^{(\theta)} d(r_0\theta)$$

$\rightsquigarrow$ mesh grid should be nearly isotropic near the nozzle exit

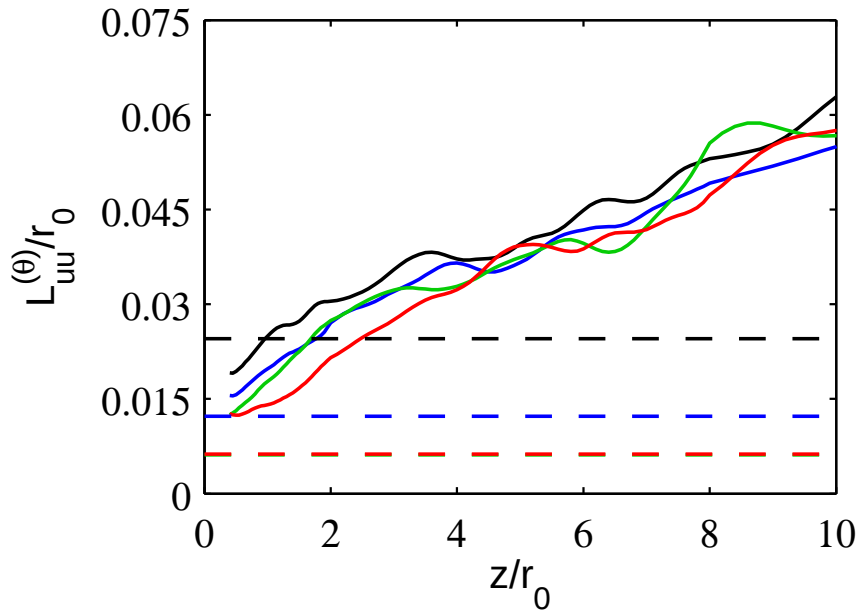
With $\Delta r < \delta_\theta/4$, $r_0\Delta\theta$ and $\Delta z < \delta_\theta/2$, practically grid-converged results

- Discretization quality $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900, \sigma_{u_e} = 9\%$

Axial scales $L_{11}^{(z)}/r_0$



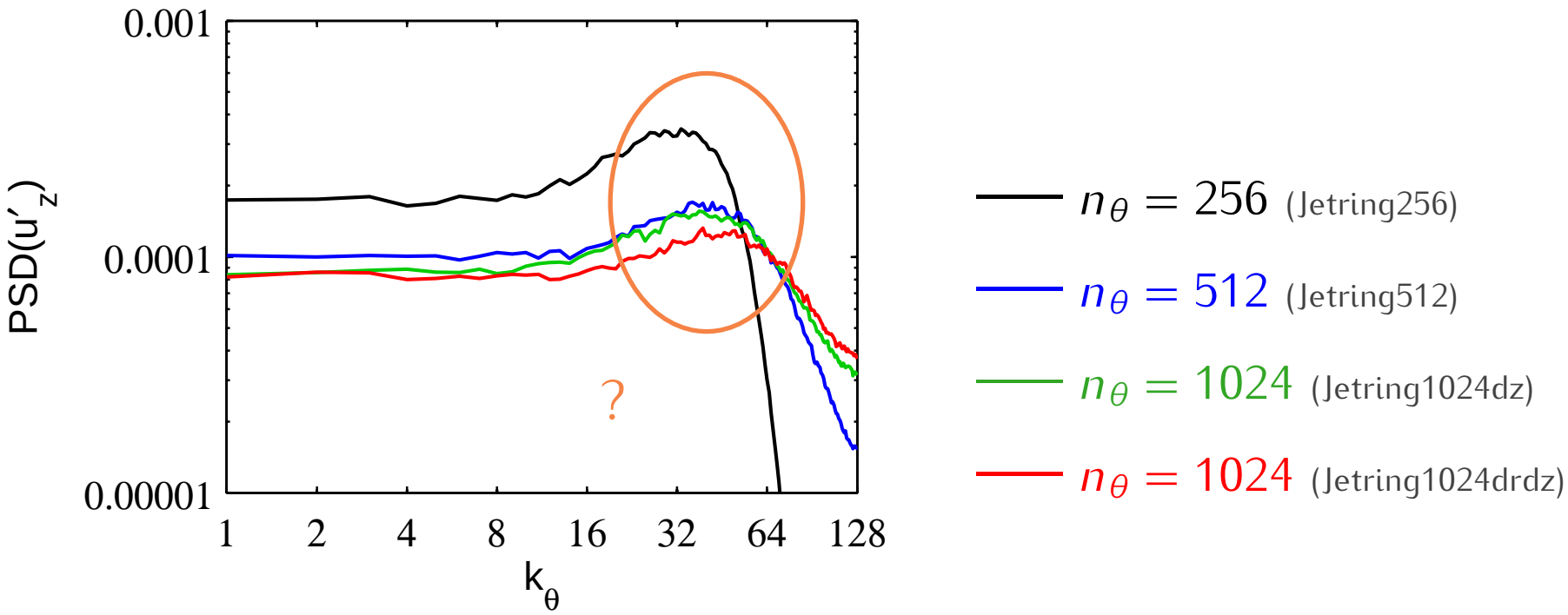
Azimuthal scales $L_{11}^{(\theta)}/r_0$



$n_\theta = 256$ (Jetring256) $n_\theta = 512$ (Jetring512)

$n_\theta = 1024$ (Jetring1024dz) $n_\theta = 1024$ (Jetring1024drdz)

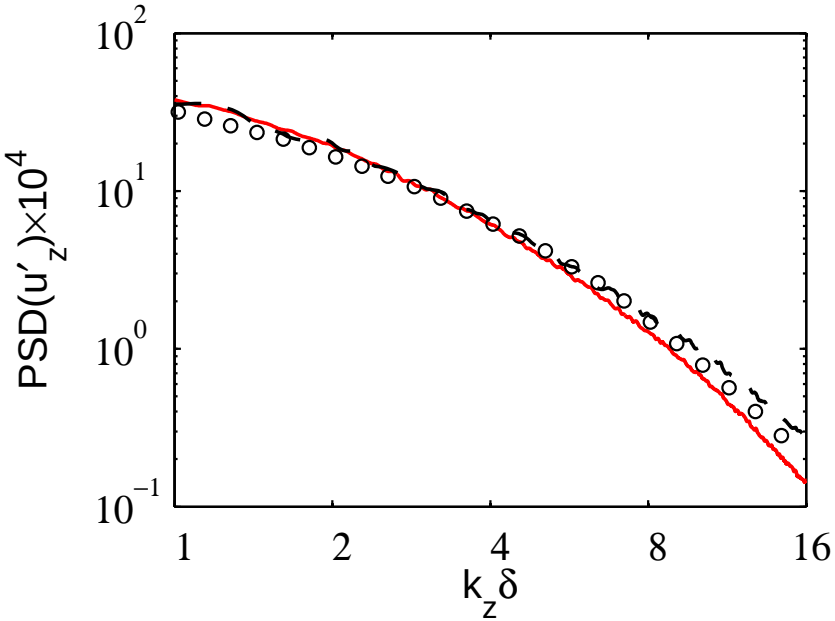
- Azimuthal spectra of u'_z $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900, \sigma_{u_e} = 9\%$



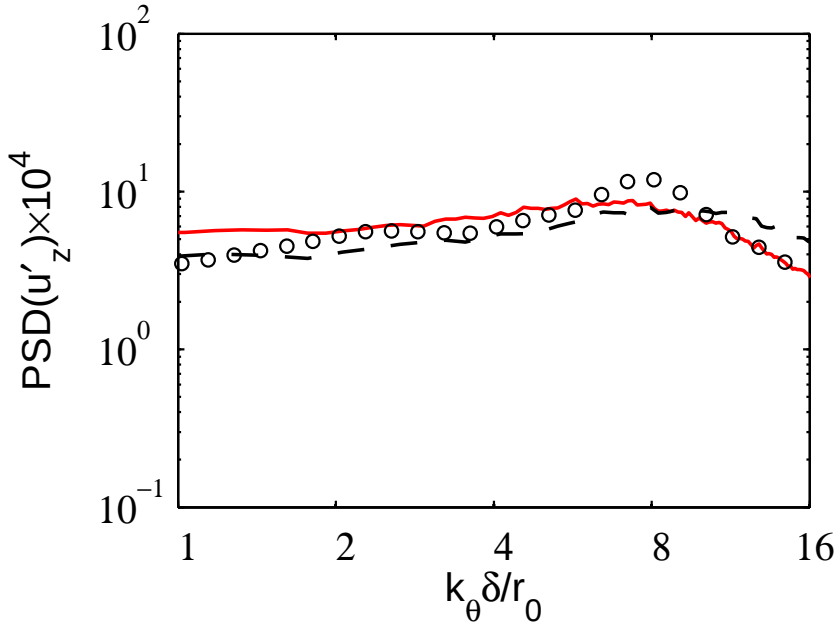
(at $r = r_0$ and $z = 0.4r_0$)

- Spectra of velocity u'_z $M = 0.9, Re_D = 10^5, \delta_\theta/r_0 = 1.8\%, Re_{\delta_\theta} = 900, \sigma_{u_e} = 9\%$

vs $k_z\delta$ (using Taylor hyp.)



vs $k_\theta\delta/r_0$



—— $n_\theta = 1024, \delta_\theta$ at $r = r_0$ and $z = 0.4r_0$ (Jetring1024drdz)

- - - $n_\theta = 1024, 2\delta_\theta$ at $r = r_0$ and $z = 0.8r_0$ (Jetring1024drdz2δ_θ)

∘ DNS turb. pipe flow, Eggels *et al.* (1994)

($\mathrm{Re}_{\delta_\theta} = 236$, $x_2^+ = 30$ or $x_2/\delta \simeq 0.17$)

Concluding remarks

- Spectra (vs axial and azimuthal wavenumbers) near the nozzle exit – similarity with boundary layer spectra ($0 \leq k_z\delta, k_\theta\delta/r_0 \leq 16$), peak at $k_\theta\delta/r_0 \simeq 7-8$

  Spalart (1988), Kim *et al.* (1987), Eggels *et al.* (1994), Tomkins & Adrian (2003)

- Tripped jets to deal with higher Reynolds number jets

  (many other things already discussed, Re_D at a given $\mathrm{Re}_{\delta_\theta}$, $\mathrm{Re}_{\delta_\theta}$ at a given Re_D, ...)

- LES is not a black box, and different configurations need to be clearly understood if the aim of numerics is to confidently extrapolate turbulence results at high Reynolds number.

Acknowledgments

- « CAA team » Christophe Bogey & Olivier Marsden

  with the contributions of Xavier Gloerfelt, Sébastien Barré, Julien Berland, Thomas Emmert, Vincent Fleury, Damien Desvignes, Nicolas de Cacqueray

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