Effets stabilisants ou déstabilisants de la compressibilité en turbulence

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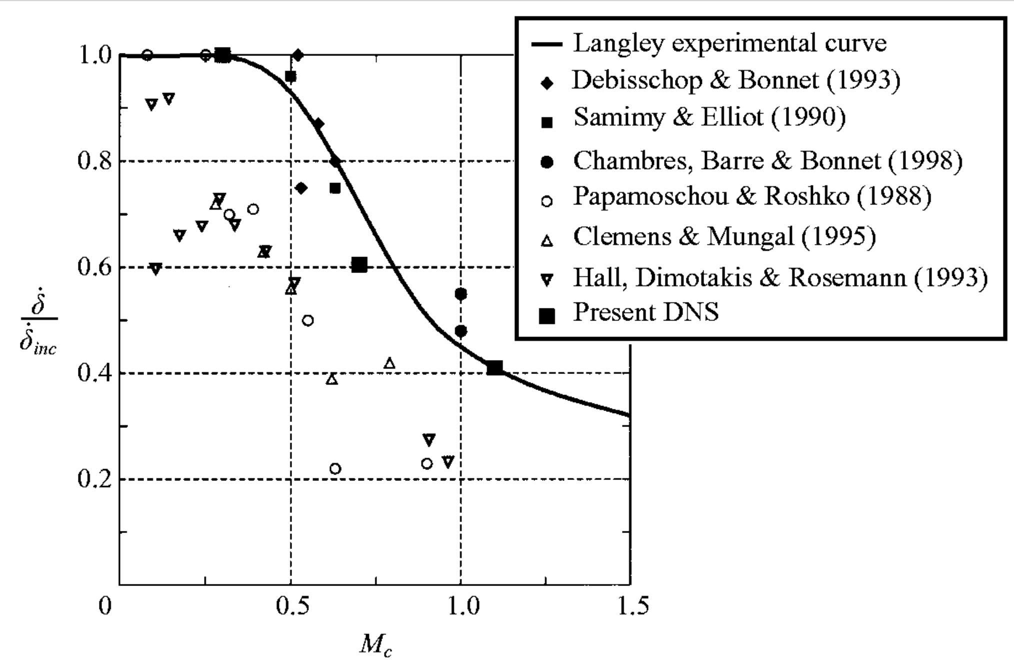
Reduction of mixing



- Is homogeneous turbulence relevant ? MAY BE ...
- Is the compressibility always stabilizing in homogeneous turbulence ? NO !

Simplified problem and strategy

- An overall agreement: Role of pressure fluctuation, *mollification of pressure effects* with compressibility
- But opposite effects looking at *linear* ‘rapid’ and *nonlinear* ‘slow’ pressure-strain rate terms in RSM. Possible controversy ?
- Investigation of the linear response : a crucial difference between *irrotational* (e.g. axial compression) and *rotational* (e.g. plane shear) flows
  -) Go back to linear theory (SLT) in the incompressible case
  -) Introduce new couplings in isentropic SLT ... and explain

Helmholtz decomposition - incompressible case

$$\boldsymbol{V}(\boldsymbol{x},t) = \boldsymbol{V}^{(sol)} + \boldsymbol{V}^{(dil)} = \underbrace{\boldsymbol{V}^{(tor)} + \boldsymbol{V}^{(pol)}}_{V^{(sol)}} + \boldsymbol{V}^{(dil)}$$

Applied to *strictly incompressible* Navier-Stokes equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{V} + \boldsymbol{\nabla} p = 0, \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{V}^{(sol)} = 0, \qquad \leftarrow \qquad \frac{\partial \boldsymbol{\omega}}{\partial t} = \ldots$$

,

$$\boldsymbol{V}^{(dil)} + \boldsymbol{\nabla} p = 0, \qquad \leftarrow \qquad \nabla^2 p = ..$$

Implicitely solve Poisson, Biot-Savart, and related equations ?

Solenoidal projection - linear and nonlinear terms

Projection operator in 3D Fourier space $P_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$, $\widehat{\boldsymbol{V}}^{[sol)} = \mathbf{P}\widehat{\boldsymbol{V}}$

$$\left(\frac{\partial}{\partial t} + \nu k^2\right)\hat{\boldsymbol{u}}(\boldsymbol{k},t) + \mathbf{P}\widehat{\boldsymbol{\omega} \times \boldsymbol{u}} = 0, \quad \boldsymbol{k}\cdot\hat{\boldsymbol{u}} = 0$$

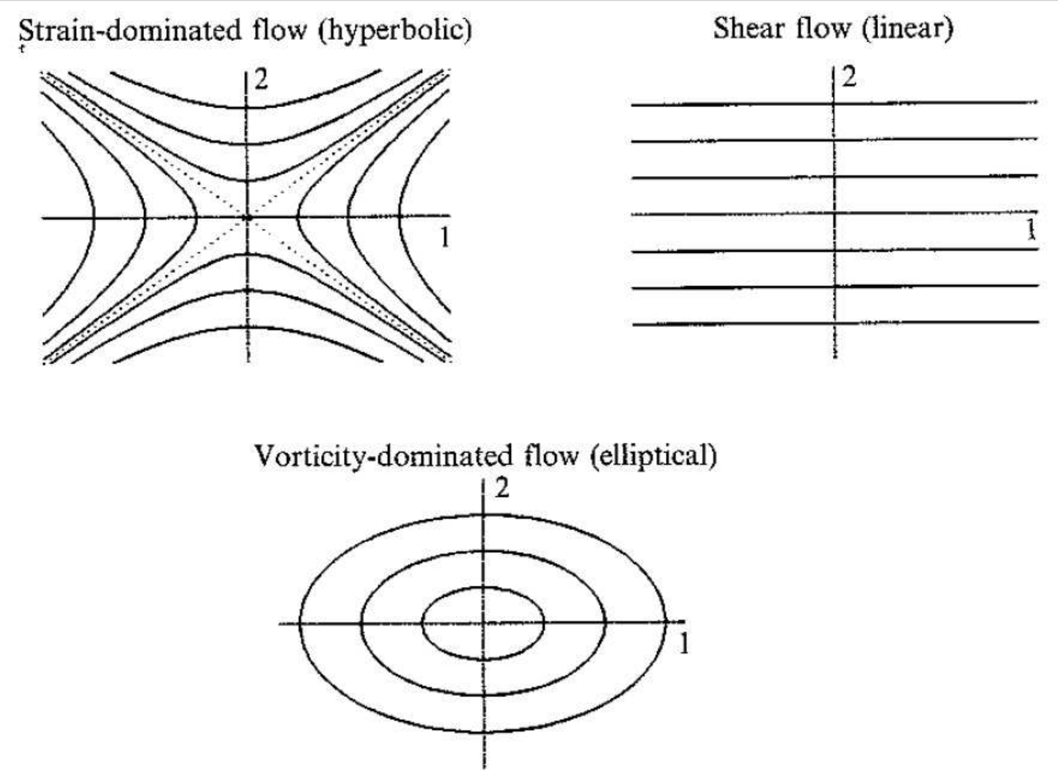
Add a mean flow in a rotating frame

$\boldsymbol{u} \rightarrow \underbrace{\boldsymbol{A}\boldsymbol{x}}_{\boldsymbol{U}} + \boldsymbol{u}$, $\boldsymbol{\omega} \rightarrow 2\boldsymbol{\Omega} + \boldsymbol{\omega}$, so that

$$\dot{\hat{\boldsymbol{u}}}(\boldsymbol{k}(t),t) + \nu k^2 \hat{\boldsymbol{u}} + \mathbf{M}\mathbf{A}\hat{\boldsymbol{u}} + \mathbf{P}(2\Omega \times \hat{\boldsymbol{u}}) = -\mathbf{P}\widehat{\boldsymbol{\omega} \times \boldsymbol{u}} = 0$$

Local — up to $\boldsymbol{k}(t)$ — and algebraic solution of the linear (left) part (SLT). Solenoidal projection (mediated by pressure fluctuation) for both linear and nonlinear terms via $k_i k_j / k^2$ (origin of rapid and slow contributions in subsequent statistical equations)

Basic mean flow



$\Omega^2 - S^2, < 0, 0, > 0$

Specific signature of linear response

- Conventional comparison between a 'linear' (inverse) time-scale S and a nonlinear one $\tau_{NL} = u'/\ell$? *no universality*
- Very different role of S in the basic linear operator, with or without production, and modulation by an angular term in Fourier space, as pure rotation 2Ω, specific $\pm\imath 2\Omega \cos\widehat{(\boldsymbol{k},\boldsymbol{\Omega})}$, very different from pure shear, due to solenoidal projection.
- Very different role in equations for statistical quantities, second-order single-point and two-point correlations

Very different domain of validity of the linear (SLT) solution, or nonlinear versus linear

- Isotropic compression-dilatation: The nonlinear term can be more rapid than the linear one ! No anisotropy, no significant explicit role of pressure.
- Pure rotation: Very delayed nonlinearity, inertial wave turbulence at $\Omega t \sim Ro^{-2}$
- Pure shear: Conventional (Hunt etal) view roughly valid: Validity of RDT $St \sim 1 - 10$, depending of initial 'shear rapidity' $S\tau_{NL}$ (inverse of a Rossby number)

Limitations of crude dimensional analysis, comparing S (spherical strain, anisotropic strain, shear), 2Ω, N (stable and unstable density stratification)

Irrotational vs. irrotational mean flow

From RDT (Batchelor & Proudman 1954) for irrotational strain to RDT (SLT) for rotational mean flow (Moffatt 1967, Sagaut and CC book 2008)

Helmholtz equation (inviscid) $\dot{\omega}_i = \frac{\partial u_i}{\partial x_j}\omega_j$ Integral form (Kelvin)

Linearization around $A_{ij} = S_{ij} + \epsilon_{inj} W_n$

$$\dot{\omega}_i = \underbrace{A_{ij}\omega_j}_{\smile} + \underbrace{\frac{\partial u_i}{\partial x_j} W_j}_{\frown}$$

Compressibility. Isentropic SLT

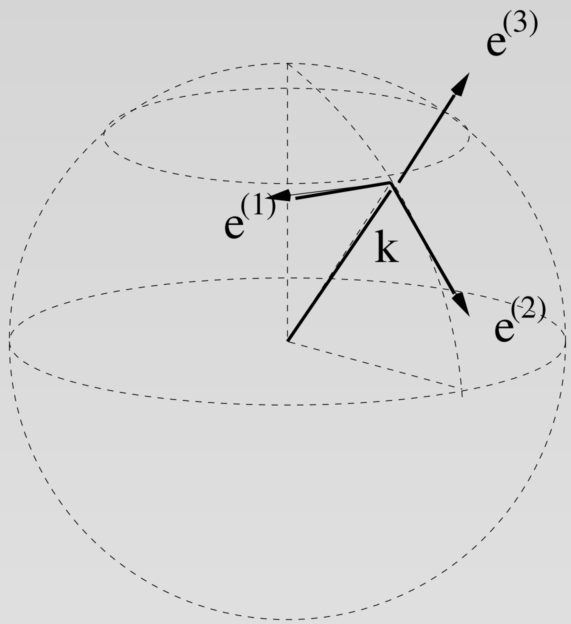
- Mean flow parameters $\boldsymbol{U} = \boldsymbol{A}\boldsymbol{x}$, $P(t)$, $\overline{\rho}(t)$,

$$a^2(t) = P/(\gamma\overline{\rho})$$

- Minimal set of dependent variables (fluctuating)
  -) Incompressible $\left(u_1, u_2, u_3, p\right) \rightarrow u^{(tor)}, u^{(pol)}$ ($u^{(dil)} = 0$, p slaved to them)
  -) Isentropic case $u^{(tor)}, u^{(pol)}, u^{(dil)}, u^{(p)} \sim p/(\overline{\rho}a)$

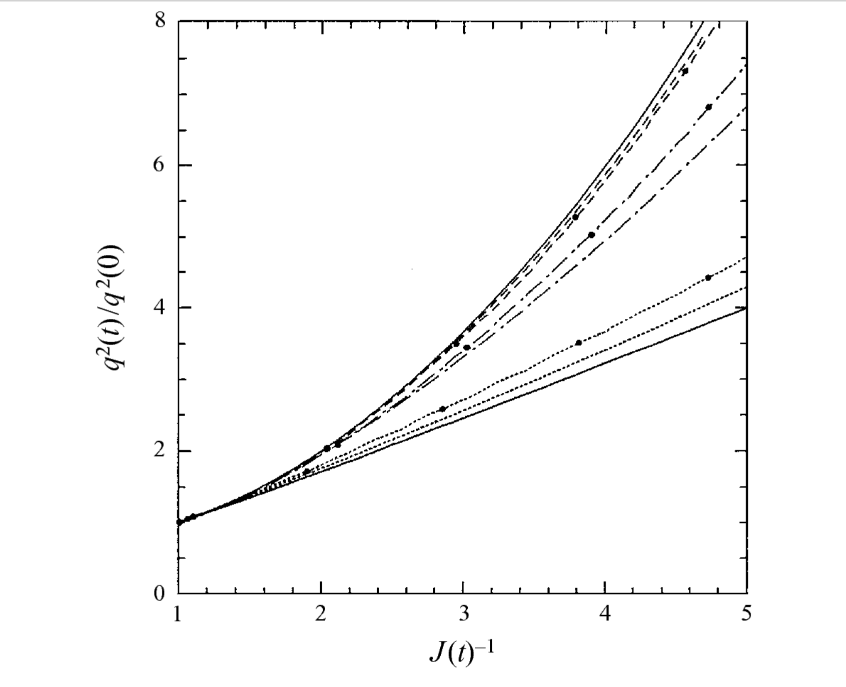
- Assumptions $\dot{s} = 0$, $p/P \ll 1$

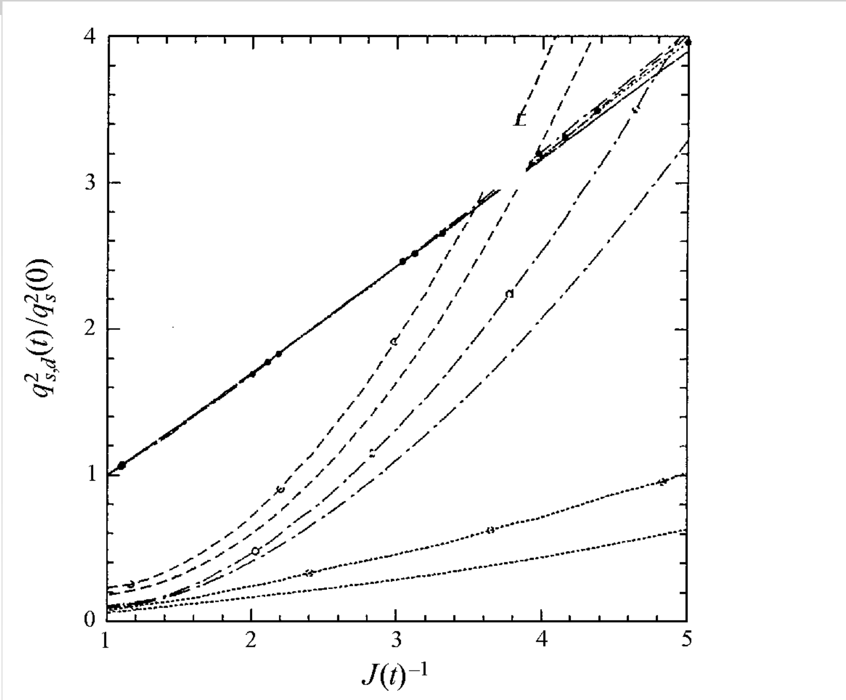
Toroidal-poloidal-dilatational decomposition



$$\hat{\boldsymbol{u}}(\boldsymbol{k}) = \underbrace{u^{(1)}\boldsymbol{e}^{(1)} + u^{(2)}\boldsymbol{e}^{(2)}}_{sol.} + \underbrace{u^{(3)}\boldsymbol{e}^{(3)}}_{dil.}$$

Symmetric mean strain : effect of rapid axial compression





Two relevant limits, isentropic homogeneous RDT + (DNS), monotonic *Destabilization*

- Pressure-released RDT $u_i(\boldsymbol{x},t) = F^{-1}_{ji}(\boldsymbol{X},t,t_0)u_j(\boldsymbol{X},t_0)$ (Debiève as well)
- Solenoidal RDT $u_i(\boldsymbol{x},t) = \left(F^{-1}_{ji}(\boldsymbol{X},t,t_0)u_j(\boldsymbol{X},t_0)\right)^{(sol)}$ Helmholtz decomposition. Pure addition of solenoidal and dilatational response.

More general case with mean vorticity

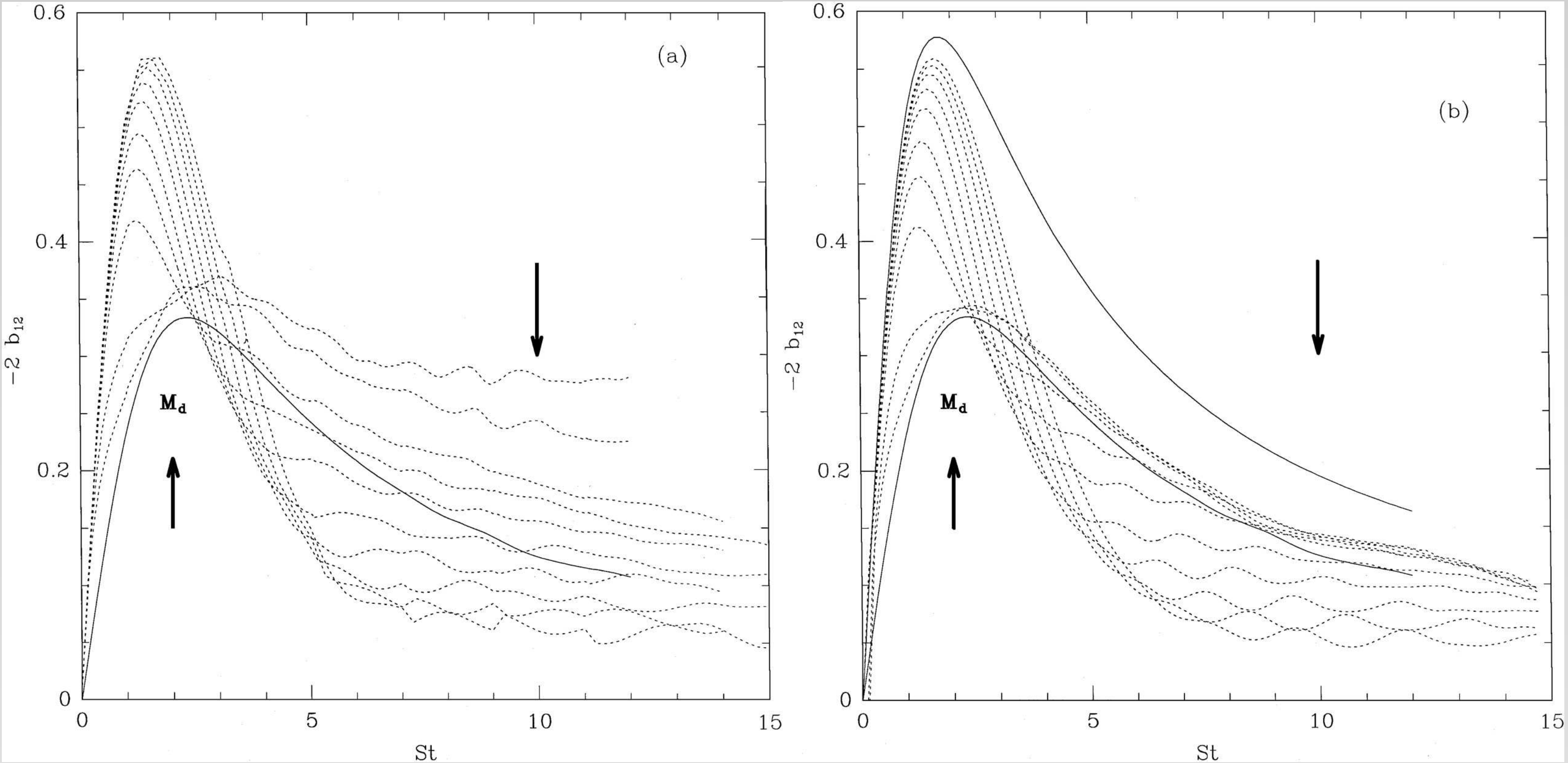
- In search for applications : compression ramps (suggested by Nagi Mansour, CTR, 1993), restarted collaboration with L. Jacquin, Pierre, Tom (here) ...
- Interpretation from RSM : the “rapid” pressure-strain rate inhibits the “production” $\longrightarrow$ mollification of pressure means *destabilization* in the linear (RDT) limit
- A more complex answer using the decomposition toroidal - poloidal - dilatational for the velocity. (Craya-Herring in Fourier space) + pressure + entropy.

Feedback from dilatational mode induced by the mean vorticity

A linear system of four equations for toroidal-poloidal-dilatational-pressure disturbance mode : A *tensorial* Green's function.

- quasi-isentropic (viscous terms for numerics) RDT
- Gradient Mach number : S and $ak \rightarrow M_d(k) = S/(ak)$, with $M_g = (SL)/c$
- Breaking of acoustical equilibrium $E^{(dil)} = E^{(pres)}$ by the mean shear effect
- *A linear feedback from dilatational to poloidal*, induced by *mean vorticity* (e.g. pure plane shear): a key effect for stabilizing ...
- ... even if explicit dilatational terms are marginally relevant ?

Homogeneous shear flow



- Full DNS / RDT : cross-over about $St = 4$ for destabilizing to stabilizing effect.
- The feed-back from dilatational to poloidal mode is essential for predicting the "stabilization" in the linear limit

Coupling in compressible RDT for pure shear: Details

$$\dot{u}^{(1)} + S\frac{K_3}{k(t)}u^{(2)} = S\frac{K_3 k_2(t)}{K_\perp}\frac{u^{(3)}}{k(t)} \tag{1}$$

$$\dot{\left(ku^{(2)}\right)} = -S\frac{K_1}{K_\perp}k(t)u^{(3)} \tag{2}$$

$$\dot{\left(\frac{u^{(3)}}{k(t)}\right)} = 2S\frac{K_1 K_\perp}{k^4(t)}k(t)u^{(2)} - a_0 u^{(4)} \tag{3}$$

$$\dot{u}^{(4)} = a_0 k(t) u^{(3)} \tag{4}$$

RSM modelling ?

Mollification of pressure yields

-) linear pressure-strain decreased $\longrightarrow$ production increased (e.g. basic deviatoric of production, LRR): M_t and M_g corrections ? Little hope to reproduce the complex SLT, which allows possible stabilization.

-) Nonlinear pressure-strain decreased $\longrightarrow$ production decreased (e.g. return-to-isotropy, feeding vertical component in pure shear): e.g. a simple factor (e.g. Heinz)

$$e^{-CM_g}$$

On tue le mauvais cochon ? en ‘tuant’ l’ effet de pression dans le terme ‘rapide’ (eg. Girimagi etal., Tacker etal.)

Discussion. Why stabilizing ?

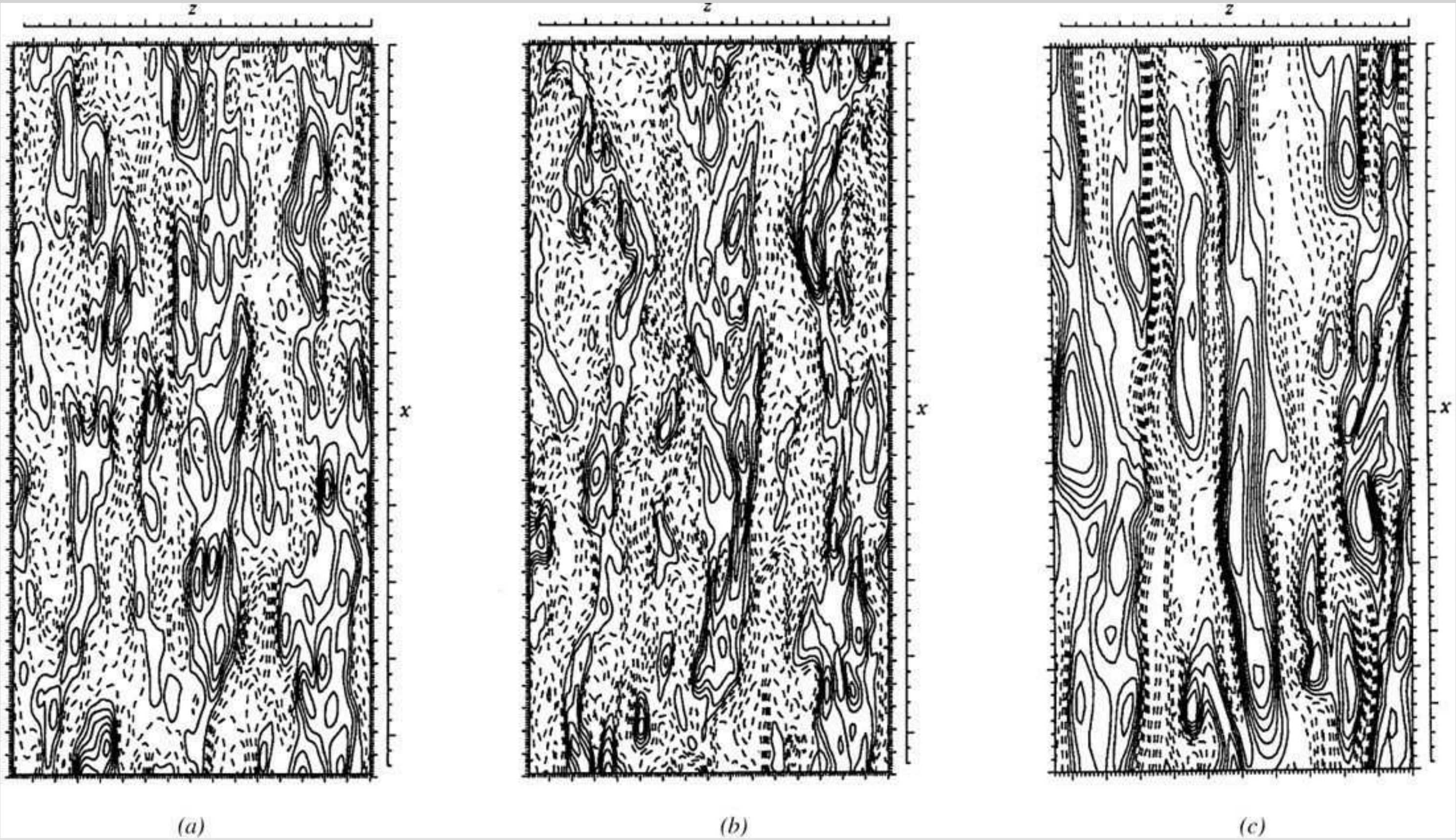
- Mollification of pressure-strain : opposite effects looking at “rapid = linear” and “slow = nonlinear” pressure-strain tensors *Depletion of nonlinearity* instead of compressibility effect ?
- The SCALAR Green’s function for fluctuating pressure is only a part (Thacker et al. 2006) *Role of the TENSORIAL Green’s function including both pressure effect and feedback from dilatational to poloidal (vertical)* (Simone et al. 1997).
- new progress in complete quasi-analytical linear solution. Initialization ? Invariant term combining poloidal and pressure component. See the context of accretion discs in astrophysics, *shearing box, non-modal growth* Chagelishvili etal.
- Relevance of the *pressure-less* limit (easy to obtain in RSM) ?

Complements, to be discussed further ?

Modification of pressure Green's function: Not the whole story

- Example of incompressible turbulence in a rotating frame: -) Poisson equation for pressure with conventional time-independent Green's function $\nabla^2 p = f$
  ... but $f = 2\boldsymbol{\Omega}\cdot\boldsymbol{\omega}$ ...
  -) and eventually p satisfies a propagation equation (inertial waves !)
  $$\frac{\partial^2}{\partial t^2}\nabla^2 p + 4\Omega^2 \frac{\partial p}{\partial x_{\parallel}^2} = 0$$
- Isolating an operator with a 'frozen' right-hand-side can yield wrong results
- Incorporate all linear couplings via a full RDT tensorial Green's function is better

Qualitative effect of shear



History of RDT, cornerstones

see the cover of our SC book ... ! Babel tower, at least three communities

Kelvin mode $\leftrightarrow$ Rogallo space $\leftrightarrow$ Shear wave

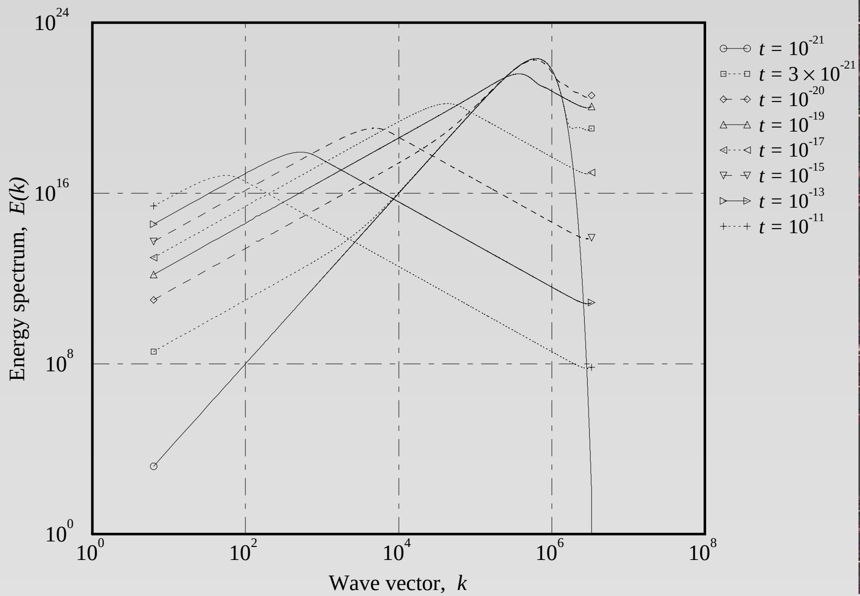
- Ideas more than hundred years ago (e.g. Kelvin), then Taylor, Prandtl and many others ...
- Batchelor & Proudman (1954), Townsend, Hunt, Moffatt (a Cambridge school ?)
- Extended by Rogallo towards pseudo-spectral DNS (Rogallo space !) 1981
- Rediscovered by a community in applied maths in 1986 (Bayly, Craik)
- WKB variants, Lifschitz & Hameiri 1991, ... B. Dubrulle
- Large community in astrophysics (Rogachevskii, Kleorin, Balbus, Chagelishvili ...)

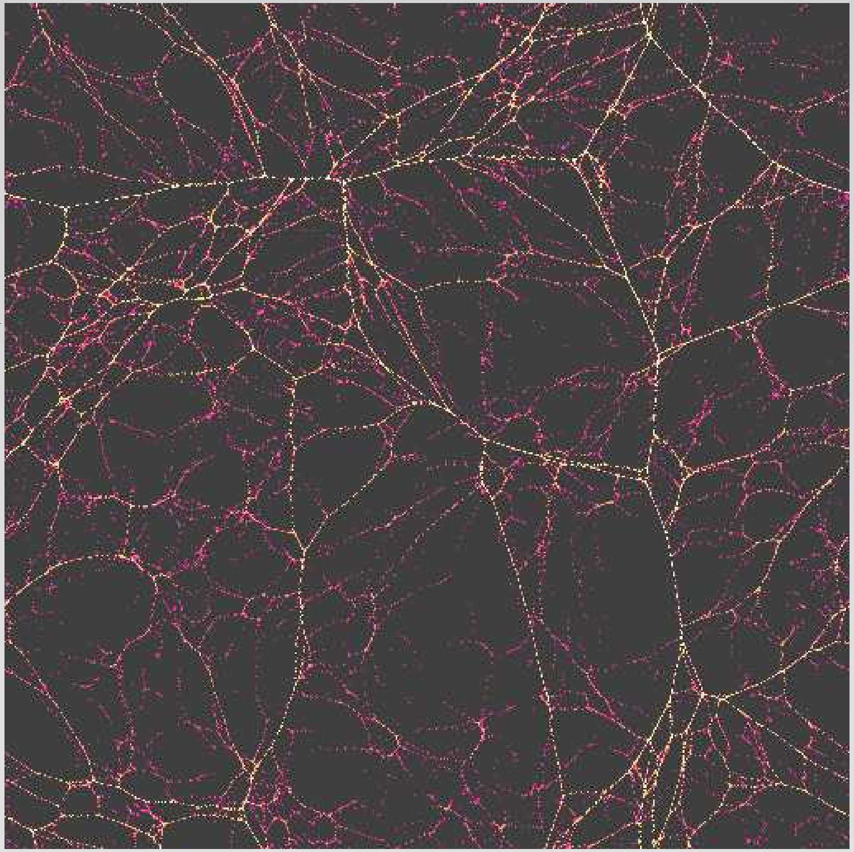
Some comments about RDT

- To identify the general *deterministic* Green's function is the best solution. (From Moffatt, 1967, extended from my thesis (1982), implicitely rediscovered in the stability community, Bayly, Craik, 1986).
  -) Further application to correlations of any order (e.g. 3 for cascade).
  -) Even applications to a known forcing (Astrophysicists) or to a closed nonlinear contribution (e.g. in anisotropic and/or multimodal EDQNM)

- The problem of *stabilizing or destabilizing* effect is often not correctly addressed.

Pressure-less Navier Stokes or multi-dimensional Burgers equation ?





Spectrum and snapshot in 2D Burgers turbulence, from Noullez et al., in SC book

pressure-less limit. Multidimensional Burgers equations

- A large material from Jérémie Bec (Nice), e.g. Bec & Khanin, Phys. Rep. , 2007 (and arxiv)
- Cosmological applications, 'reconstruction of the initial conditions of the universe ...' ! (Frisch et al., nature, 2002)
- What can be learn from that ?

  -) difference between 3D-Burgers with potential velocity and 'pressure-less' Navier-Stokes equations

  -) A maximum *internal* intermittency when the 'structures' are shocks.