Continuous hybrid non-zonal RANS/LES simulations of turbulent flows using the PITM method

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OUTLINE

● Partial Integrated Transport Modeling (PITM) method: Hybrid RANS/LES simulations
  – Mathematical physics formalism developed in the spectral space
  – Generic subfilter dissipation-rate equation
  – Hypothesis of consistency with the RANS model at the zero cutoff limit

● Test of the generic dissipation-rate equation on a self-similar analytical flow example
  – Full statistical case where the cutoff wave number $\kappa_c$ reduces to zero
  – Subfilter turbulence case where the cutoff wave number $\kappa_c$ is non-zero

● The PITM in practice
  – Subfilter scale viscosity models
  – Subfilter scale stress models
  – Relaxation mechanism and convergence acceleration

● Some PITM simulation examples
  – Simulation of homogeneous turbulence
  – Simulation of non-homogeneous turbulence
MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- Cooperation between ONERA (Bruno Chaouat) and CNRS (Roland Schiestel)
  - Spectral partitioning \((m = \text{number of zones})\), definition of filtered and averaged quantities

\[
u_i = \langle u_i \rangle + \sum_{m=1}^{N} u_i^{(m)}; \quad u_i^{(m)}(\xi) = \int_{\kappa_{m-1}|\kappa|<\kappa_m} \hat{u}_i^{(m)}(\kappa) \exp(j\kappa\xi) d\kappa \quad (1)
\]

- Simulation LES \((m=2)\) : filtered velocity: \(\bar{u}_i = \langle u_i \rangle + u_i^{(1)}\)
  * Large-scale fluctuating velocity: \(u_i^\leq = u_i^{(1)}\)
  * Subgrid-scale fluctuating velocity: \(u_i^\geq = u_i^{(2)}\)
MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- Resulting equation in the spectral space by mean integrations over spherical shells,
\[ \varphi_{ij}(X, \kappa) = (R_{ij}(X, \xi))^\Delta \] (Jeandel and Cambon 1981, Schiestel, 1987, Chaouat and Schiestel, 2007)

\[ \frac{D\varphi_{ij}(X, \kappa)}{Dt} = P_{ij}(X, \kappa) + T_{ij}(X, \kappa) + \Pi_{ij}(X, \kappa) + J_{ij}(X, \kappa) - E_{ij}(X, \kappa) \] (2)

- Convection term \( D\varphi_{ij}/Dt \), Production term \( P_{ij} \), Transfer term \( T_{ij} \), Redistribution term \( \Pi_{ij} \), Diffusion term \( J_{ij} \), Dissipation term \( E_{ij} \)

- Temporal PITM (TPITM) developed in frequency space (Gatski et al., 2010)

- Equation for the partial turbulent stress \( \tau_{ij}^{(m)} \) by integration of equation (2) in \([\kappa_{m-1}, \kappa_m]\)

\[ \frac{\partial\tau_{ij}^{(m)}}{\partial t} = P_{ij}^{(m)} + F_{ij}^{(m-1)} - F_{ij}^{(m)} + \Psi_{ij}^{(m)} + J_{ij}^{(m)} - \varepsilon_{ij}^{(m)} \] (3)

with

\[ F_{ij}^{(m)} = F_{ij}^{(m)} - \varphi_{ij}(X, \kappa_m) \frac{\partial\kappa_m}{\partial t}, \] (4)

with

\[ F_{ij}^{(m)} = - \int_0^{\kappa_m} T_{ij}(X, \kappa) d\kappa \] (5)
MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

• Two scale models
  – Subfilter-scale turbulent stress
    \[
    \frac{\partial \tau_{ij}^{(1)}}{\partial t} = P_{ij}^{(1)} - F_{ij}^{(1)} + \Psi_{ij}^{(1)} + J_{ij}^{(1)} - \epsilon_{ij}^{(1)}
    \]  \hspace{1cm} (6)
    \[
    \frac{\partial \tau_{ij}^{(2)}}{\partial t} = P_{ij}^{(2)} + F_{ij}^{(1)} - F_{ij}^{(2)} + \Psi_{ij}^{(2)} + J_{ij}^{(2)} - \epsilon_{ij}^{(2)}
    \]  \hspace{1cm} (7)
    
    \[
    0 = F_{ij}^{(2)} - \epsilon_{ij}^{(3)}
    \]  \hspace{1cm} (8)
  – Subfilter-scale turbulent energy
    \[
    \frac{\partial k^{(1)}}{\partial t} = P^{(1)} - F^{(1)} + J^{(1)} - \epsilon^{(1)}
    \]  \hspace{1cm} (9)
    \[
    \frac{\partial k^{(2)}}{\partial t} = P^{(2)} + F^{(1)} - F^{(2)} + J^{(2)} - \epsilon^{(2)}
    \]  \hspace{1cm} (10)
    
    \[
    0 = F^{(2)} - \epsilon^{(3)}
    \]  \hspace{1cm} (11)
GENERIC SUBFILTER DISSIPATION RATE EQUATION

• Dissipation-rate equation

\[
\frac{\partial \langle \epsilon_{sfs} \rangle}{\partial t} = c_{sfs} \epsilon_1 \frac{\langle \epsilon_{sfs} \rangle}{\langle k_{sfs} \rangle} \langle P_{sfs} \rangle - c_{sfs} \epsilon_2 \frac{\langle \epsilon_{sfs} \rangle^2}{\langle k_{sfs} \rangle} \tag{12}
\]

where

\[
c_{sfs} \epsilon_1 = \frac{3}{2} \quad \text{and} \quad c_{sfs} \epsilon_2 = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{\kappa_d E(\kappa_d)} \left( \frac{F(\kappa_d) - F(\kappa_d)}{\langle \epsilon_{sfs} \rangle} \right) \tag{13}
\]

In a compact form, \( c_{sfs} \epsilon_2 \) can be written as

\[
c_{sfs} \epsilon_2 = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \tag{14}
\]

\[
\zeta(\kappa_d) = \frac{k}{\kappa_d E(\kappa_d)} \left( \frac{F(\kappa_d) - F(\kappa_d)}{\epsilon} \right) \tag{15}
\]

For \( \kappa_c = 0 \),

\[
\epsilon_2 = \frac{3}{2} - \zeta(\kappa_d) \tag{16}
\]

\[
c_{sfs} \epsilon_2 = \frac{3}{2} + \frac{\langle k_{sfs} \rangle}{k} \left( c_{\epsilon_2} - \frac{3}{2} \right) \tag{17}
\]
GENERIC SUBFILTER DISSIPATION-RATE EQUATION

• The value suggested for $c_{\epsilon_1} = c_{sf\epsilon_1} = 3/2$ seems restrictive if one remarks that this coefficient may take on different values in statistical RANS models according to its calibration made by different authors.

• Let us now consider the standard form of the statistical dissipation-rate equation written in homogeneous turbulence

$$\frac{\partial \epsilon}{\partial t} = c_{\epsilon_1} \frac{\epsilon}{k} P - c_{\epsilon_2} \frac{\epsilon^2}{k}$$

(18)

with given values of the coefficients $c_{\epsilon_1}$ and $c_{\epsilon_2}$.

• The issue to address is to compute the function $c_{sf\epsilon_2}$ when the coefficient $c_{sf\epsilon_1}$ differs from the value $3/2$ (Chaouat and Schiestel, Physics of Fluids, vol. 24, 2012)
GENERIC SUBFILTER DISSIPATION-RATE EQUATION

- After some algebra, one can obtain:

\[
\frac{\partial \langle \epsilon_{sfs} \rangle}{\partial t} = c_{\epsilon_1} \frac{\langle \epsilon_{sfs} \rangle}{\langle k_{sfs} \rangle} \langle P \rangle_{sfs} - \left[ \left( \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \right) - \left( \frac{3}{2} - c_{\epsilon_1} \right) \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} \right] \frac{\langle \epsilon_{sfs} \rangle^2}{\langle k_{sfs} \rangle}
\]

(19)

and

\[
\frac{\partial \epsilon}{\partial t} = c_{\epsilon_1} \frac{\epsilon}{k} P - \left[ \left( \frac{3}{2} - \zeta(\kappa_d) \right) - \left( \frac{3}{2} - c_{\epsilon_1} \right) \frac{P}{\epsilon} \right] \frac{\epsilon^2}{k}
\]

(20)

showing that

\[
c_{\epsilon_2} = \left( \frac{3}{2} - \zeta(\kappa_d) \right) - \left( \frac{3}{2} - c_{\epsilon_1} \right) \frac{P}{\epsilon}
\]

(21)

and

\[
c_{sfs\epsilon_2} = \left( \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \right) - \left( \frac{3}{2} - c_{\epsilon_1} \right) \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle}
\]

(22)
Both equations (21) and (22) allow to determine the function $\zeta(\kappa_d)$ in two different forms as follows

$$\zeta(\kappa_d) = \left[ \frac{3}{2} - c \epsilon_2 + \left( c_\epsilon_1 - \frac{3}{2} \right) \frac{P}{\epsilon} \right]$$

(23)

and

$$\zeta(\kappa_d) = \left[ \frac{3}{2} - c_s f s \epsilon_2 + \left( c_\epsilon_1 - \frac{3}{2} \right) \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} \right] \frac{k}{\langle k_{sfs} \rangle}$$

(24)

$$c_{sfs} \epsilon_2 = c_\epsilon_2 + \left( \frac{3}{2} - c_\epsilon_2 \right) \left[ 1 - \frac{\langle k_{sfs} \rangle}{k} \right] + \left( c_\epsilon_1 - \frac{3}{2} \right) \left[ \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} - \frac{P}{\epsilon} \frac{\langle k_{sfs} \rangle}{k} \right]$$

(25)

If we assume that the ratio $\langle k_{sfs} \rangle / k$ of the subfilter energy to the total energy is constant or varies slowly with time

$$\frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} \approx 1 + \frac{\langle k_{sfs} \rangle}{k} \frac{P - \epsilon}{\epsilon}$$

(26)

Then, we obtain the final form of the coefficient $c_{sfs} \epsilon_2$ that reads

$$c_{sfs} \epsilon_2 = c_\epsilon_1 + \frac{\langle k_{sfs} \rangle}{k} (c_\epsilon_2 - c_\epsilon_1)$$

(27)

- Equation (27) established at high Reynolds number is the key equation
PHYSICAL MEANING OF \( C_s f_{s<1} \) IN THE SPECTRAL SPACE

- The Kovasznay hypothesis

\[
F(\kappa, t) = C_\kappa^{-3/2} E(\kappa, t)^{3/2} \kappa^{5/2}
\]  
(28)

Partial time derivative of both sides

\[
\frac{1}{F} \frac{\partial F(\kappa, t)}{\partial t} = \frac{1}{2E} \frac{\partial E(\kappa, t)}{\partial t}
\]  
(29)

\[
\frac{\partial E(\kappa, t)}{\partial t} = -\frac{\partial F(\kappa, t)}{\partial \kappa}
\]  
(30)

\[
\frac{\partial F(\kappa, t)}{\partial t} = -\frac{3 F \partial F(\kappa, t)}{2 E \partial \kappa}
\]  
(31)

Discretization in the spectral space

\[
\frac{\partial F^{(m)}}{\partial t} = \frac{3 F^{(m)} F^{(m-1)} - F^{(m)}}{2 E^{(m)} \Delta \kappa}
\]  
(32)
PHYSICAL MEANING OF $C_{Sf, \epsilon_1}$ IN THE SPECTRAL SPACE

- The term $E^{(m)} \Delta \kappa$ is in fact the partial turbulent energy within the spectral slice $[\kappa_{m-1}, \kappa_m]$

$$\frac{\partial F^{(m)}}{\partial t} = \frac{3}{2} \frac{F^{(m)} F^{(m-1)}}{k^{(m)}} - \frac{3}{2} \frac{(F^{(m)})^2}{k^{(m)}}$$  \hspace{1cm} (33)

- This equation looks like the flux equation in the multiple scale model for the spectral slice $(m)$

- Same structure as the standard dissipation-rate equation in which the coefficients are $c_{\epsilon_1} = c_{\epsilon_2} = 3/2$

- The constant value $3/2$ in the limiting case of a very thin spectral slice

- $c_{\epsilon_1} = 3/2$ is not a lucky haphazard!
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- Analytical solution (Reid and Harris (1959))

\[ E(\kappa, t) = H(t) G(\gamma) = \frac{b}{\sqrt{t}} G(\kappa \sqrt{at}) \]  \hspace{1cm} (34)

where \( H \) and \( G \) are similarity functions, \( \gamma(t) = \kappa \sqrt{at} \) is a similarity variable whereas \( a \) and \( b \) denote numerical constant coefficients.

Figure 1: Evolution of the density spectrum \( E \) corresponding to the self similar decay of turbulence.
On the FINITENESS of the \( c_{sf_1} e_2 \) at LARGE \( Re \)

Figure 2: Sketches of the spectral flux transfer \( \mathcal{F} \) and spectral transfer term \( \mathcal{T} \). Non-equilibrium flows. (a) High Reynolds number; (b) Infinite Reynolds number.
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- The dimensionless reduced variable involving the time is

\[ z(\kappa, t) = \epsilon(t)^{-1/3} \kappa^{-2/3} t^{-1} \quad (35) \]

- Spectrum and the transfer term are computed by Taylor series expansions of the dimensionless variable \( z = z(\kappa, t) \) where \( \kappa \) and \( t \) are independent variables as follows

\[ E(\kappa, t) = C_\kappa \epsilon(t)^{2/3} \kappa^{-5/3} \sum_{n=0}^{\infty} a_n z^n \quad (36) \]

The Kovasznay hypothesis for the transfer term

\[ \mathcal{F}(\kappa, t) = C_\kappa^{-3/2} E(\kappa, t)^{3/2} \kappa^{5/2} \quad (37) \]

- Spectral energy equation in time and wave number at large Reynolds number

\[ \frac{\partial E(\kappa, t)}{\partial t} = -\frac{\partial \mathcal{F}(\kappa, t)}{\partial \kappa} \quad (38) \]
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

The spectral flux transfer $\mathcal{F}$

$$\mathcal{F}(\kappa, t) = \epsilon(t) \left[ \sum_{n=0}^{\infty} a_n z^n \right]^{3/2}$$

(39)

The derivative of equation (36) reads

$$\frac{\partial E(\kappa, t)}{\partial t} = C_{\kappa} \kappa^{-5/3} \left[ \frac{2}{3} \epsilon^{-1/3} \frac{d\epsilon}{dt} \sum_{n=0}^{\infty} a_n z^n + \epsilon^{2/3} \sum_{n=0}^{\infty} n a_n z^{n-1} \frac{\partial z}{\partial t} \right]$$

(40)

The derivative $d\epsilon/\partial t$ is computed as

$$\frac{d\epsilon(t)}{dt} = -2 \epsilon^{4/3} \kappa^{2/3} z$$

(41)
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

The derivative $\frac{\partial z}{\partial t}$ is computed using the self preservation hypothesis

$$\frac{\partial z(\kappa, t)}{\partial t} = -\kappa^{-2/3} \left[ \frac{1}{3} \epsilon^{-4/3} \frac{d\epsilon}{dt} t^{-1} + \epsilon^{-1/3} t^{-2} \right] = -\frac{z}{3t} = -\frac{1}{3} \epsilon^{1/3} \kappa^{2/3} z^2$$  \hspace{1cm} (42)

So that equation (40) reads

$$\frac{\partial E(\kappa, t)}{\partial t} = \frac{2}{3} C_{\kappa} \epsilon \kappa^{-1} \left[ 2z + \frac{5}{2} a_1 z^2 + 3a_2 z^2 + \frac{7}{2} a_3 z^3 + O(z^4) \right]$$  \hspace{1cm} (43)

One can then obtain

$$\mathcal{F}(\kappa, t) = \epsilon \left[ 1 + \frac{3}{2} a_1 z + \frac{3}{2} \left( a_2 + \frac{a_1^2}{4} \right) z^2 + \frac{3}{2} \left( a_3 + \frac{1}{2} a_1 a_2 - \frac{a_1^3}{24} \right) z^3 + O(z^4) \right]$$  \hspace{1cm} (44)

$$\frac{\partial \mathcal{F}(\kappa, t)}{\partial \kappa} = -\frac{2}{3} \epsilon \kappa^{-1} \left[ \frac{3}{2} a_1 z + 3 \left( a_2 + \frac{a_1^2}{4} \right) z^2 + \frac{9}{2} \left( a_3 + \frac{a_1 a_2}{2} - \frac{a_1^3}{24} \right) z^3 + O(z^4) \right]$$  \hspace{1cm} (45)
These analytical results will form the basis of a test of the generic model for the $\epsilon$ equation. (Chaouat and Schiestel, Physics of Fluids, vol. 24, 2012)

We will show that the coefficient $c_{sf \epsilon_2}$ introduced in the dissipation-rate equation takes a finite value.
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

Figure 3: Evolving of the density spectrum $E(\kappa, t)/E(\kappa, \infty)$ with respect to the dimensionless variable $\alpha = \kappa^{2/3} \epsilon^{1/3} t$

$$E(\kappa, t) = C_\kappa \kappa^{2/3} \epsilon^{-5/3} - \frac{4}{3} C_\kappa 2 \epsilon^{1/3} \kappa^{-7/3} \frac{1}{t} + \frac{2}{3} C_\kappa 3 \kappa^{-3} \frac{1}{t^2} \quad (49)$$
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

Figure 4: Evolving of the spectral flux transfer $\mathcal{F}(\kappa, t)/\epsilon$ with respect to the dimensionless variable $\alpha = \kappa^{2/3}\epsilon^{1/3}t$

$$
\mathcal{F}(\kappa, t) = \epsilon - 2C_\kappa \frac{\epsilon^{2/3}\kappa^{-2/3}}{t} + \frac{5}{3}C_\kappa^2 \frac{\epsilon^{1/3}\kappa^{-4/3}}{t^2}
$$

(50)
SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- Governing equations

\[
F(\kappa, t) = \epsilon - 2C_\kappa \frac{\epsilon^{2/3} \kappa^{-2/3}}{t} + \frac{5}{3} C_\kappa \frac{\epsilon^{1/3} \kappa^{-4/3}}{t^2}
\]  

\[
F(\kappa_d, t) = \epsilon + \frac{\partial}{\partial t} \int_{\kappa_d}^{\infty} E(\kappa, t) \, d\kappa
\]  

- Full statistical case where \( \kappa_c \) equals zero

\[
c_{\epsilon_2} = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{\kappa_d E(\kappa_d)} \left( \frac{F(\kappa_d) - F(\kappa_d)}{\epsilon} \right) = 2
\]

- Subfilter turbulence case where \( \kappa_c \) is non zero

\[
\frac{\langle k_{sfs} \rangle}{\kappa} = Q(\eta_c), \quad \eta_c = \kappa_c L
\]

\[
c_{sfs\epsilon_2} = \frac{3}{2} + \frac{1}{2} \frac{\langle k_{sfs} \rangle}{\kappa} \left[ 1 + \frac{3}{2} \eta_c \frac{d \ln Q(\eta_c)}{d\eta_c} \right]
\]

- \( c_{sfs\epsilon_2} \) always takes on a finite value whatever the domain variation of \( \kappa_d \)
PITM METHOD

- Instantaneous transport equations and practical formulations (Schiestel and Dejoan, 2005, Chaouat and Schiestel, 2005-2012)

\[
\frac{Dk_{sfs}}{Dt} = P_{sfs} - \varepsilon_{sfs} + J_{sfs}
\] (56)

\[
\frac{D(\tau_{ij})_{sfs}}{Dt} = (P_{ij})_{sfs} - (\varepsilon_{ij})_{sfs} + (\Phi_{ij})_{sfs} + (J_{ij})_{sfs}
\] (57)

\[
\frac{D\varepsilon_{sfs}}{Dt} = c_{\varepsilon 1} \frac{\varepsilon_{sfs}}{k_{sfs}} \left( \frac{P_{mm}}{sfs} \right) \frac{2}{2} - c_{sfs \varepsilon 2} (\eta_c) \frac{\varepsilon_{sfs}^2}{k_{sfs}} + (J_{\varepsilon})_{sfs}
\] (58)

- “Exact” coefficient \( c_{\varepsilon 2} \) where \( \eta_c = (k^3/2K_c)/\varepsilon_{sfs} + \varepsilon^< \)

\[
c_{sfs \varepsilon 2} (\eta_c) = c_{\varepsilon 1} + \frac{c_{\varepsilon 2} - c_{\varepsilon 1}}{[1 + \beta \eta \eta_c^3]^{2/9}}
\] (59)
SOME PITM SIMULATIONS

• PITM challenges
  – The PITM has been especially developed for performing continuous hybrid non-zonal RANS-LES simulations on coarse grids
  – Flows which depart from spectral equilibrium

• Simulation of homogeneous turbulence
  – Decay of homogeneous turbulence (Chaouat and Schiestel, 2009)
  – Perturbed spectrum with a peak or defect of energy (Chaouat and Schiestel, 2009)

• Simulation of non-homogeneous turbulence
  – Pulsed channel flows (Schiestel and Dejoan, 2005)
  – Channel flow with mass injection (Chaouat and Schiestel, 2005)
  – Shearless mixing layer (Befeno and Schiestel, 2007)
  – Channel flows over periodic hills (Chaouat, 2010)
  – Turbulent rotating channel flows (Chaouat, 2012)
Figure 5. Mean velocity profile $\langle u_1 \rangle / u_m$ in global coordinate. (a) PITM1 $(24 \times 48 \times 64)$: ○; (b) PITM2 $(84 \times 64 \times 64)$: ○; (Chaouat, Physics of Fluids, 2012). Highly resolved LES (Lamballais et al., TCFD, 1998): —.
CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION

Figure 6. Mean velocity profile $\langle u_1 \rangle / u_m$ in global coordinate. (a) PITM1 $(24 \times 48 \times 64)$: ○; (c) PITM3 $(124 \times 84 \times 84)$: ○; (Chaouat, Physics of Fluids, 2012). Highly resolved LES (Lamballais et al., TCFD, 1998): —. $R_m = 14000$, $R_o = 1.50$. 
Figure 7. Isosurfaces of vorticity modulus $\omega = 3u_m/\delta = 8 \times 10^5$. $R_m = 14000$, $R_o = 0.17$.
(a) PITM1 ($24 \times 48 \times 64$); (b) PITM2 ($84 \times 64 \times 64$); (Chaouat, Physics of Fluids, 2012).
Figure 8. Isosurfaces of vorticity modulus $\omega = 3u_m/\delta = 12 \times 10^5$. $R_m = 14000$, $R_o = 1.50$. (a) PITM1 ($24 \times 48 \times 64$); (c) PITM3 ($124 \times 84 \times 84$). (Chaouat, Physics of Fluids, 2012).
CONCLUSION

• Partial integrated transport modeling (PITM) method
  – Mathematical physics formalism developed in the spectral space
  – Further insights into the physical interpretation of the PITM method, especially in its basic foundations

• PITM is a method and not a model itself!

• PITM can be applied to each RANS model to derive its corresponding subfilter model

• PITM allows one to perform continuous hybrid non-zonal RANS/LES simulations

• Drastic reductions of the computational cost by coarsening the meshes

• PITM is a new route for simulations of turbulent flows

• References can be found in Physics of Fluids and TCFD journals
References


