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Continuous hybrid non-zonal RANS/LES simulations of turbulent flows using the PITM method

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OUTLINE

- Partial Integrated Transport Modeling (PITM) method: Hybrid RANS/LES simulations
  - Mathematical physics formalism developed in the spectral space
  - Generic subfilter dissipation-rate equation
  - Hypothesis of consistency with the RANS model at the zero cutoff limit
- Test of the generic dissipation-rate equation on a self-similar analytical flow example
  - Full statistical case where the cutoff wave number κ_c reduces to zero
  - Subfilter turbulence case where the cutoff wave number κ_c is non-zero
- The PITM in practice
  - Subfilter scale viscosity models
  - Subfilter scale stress models
  - Relaxation mechanism and convergence acceleration
- Some PITM simulation examples
  - Simulation of homogeneous turbulence
  - Simulation of non-homogeneous turbulence

MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- Cooperation between ONERA (Bruno Chaouat) and CNRS (Roland Schiestel)
  - Spectral partitioning (m = number of zones), definition of filtered and averaged quantities

$$u_i = \langle u_i \rangle + \sum_{m=1}^{N} u_i^{\prime(m)} \;\; ; \;\; u_i^{\prime(m)}(\boldsymbol{\xi}) = \int_{\kappa_{m-1}<|\kappa|<\kappa_m} \widehat{u'}_i(\boldsymbol{\kappa}) \exp\left(j\boldsymbol{\kappa}\boldsymbol{\xi}\right) d\boldsymbol{\kappa} \qquad \textbf{(1)}$$

  - Simulation LES (m=2) : filtered velocity: $\bar{u}_i = \langle u_i \rangle + u_i^{\prime(1)}$
    - ∗ Large-scale fluctuating velocity: $u_i^{<} = u_i^{\prime(1)}$
    - ∗ subgrid-scale fluctuating velocity: $u_i^{>} = u_i^{\prime(2)}$

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MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Resulting equation in the spectral space by mean integrations over spherical shells, $\varphi_{ij}(\boldsymbol{X},\kappa) = (R_{ij}(\boldsymbol{X},\boldsymbol{\xi}))^{\Delta}$ (Jeandel and Cambon 1981, Schiestel, 1987, Chaouat and Schiestel, 2007)**

$$\frac{D\varphi_{ij}(\boldsymbol{X},\kappa)}{Dt} = \mathcal{P}_{ij}(\boldsymbol{X},\kappa) + \mathcal{T}_{ij}(\boldsymbol{X},\kappa) + \Pi_{ij}(\boldsymbol{X},\kappa) + \mathcal{J}_{ij}(\boldsymbol{X},\kappa) - \mathcal{E}_{ij}(\boldsymbol{X},\kappa) \qquad \textbf{(2)}$$

- **Convection term $D\varphi_{ij}/Dt$, Production term $\mathcal{P}_{ij}$, Transfer term $\mathcal{T}_{ij}$, Redistribution term Π_{ij}, Diffusion term $\mathcal{J}_{ij}$, Dissipation term $\mathcal{E}_{ij}$**
- **Temporal PITM (TPITM) developed in frequency space (Gatski et al., 2010)**
- **Equation for the partial turbulent stress $\tau_{ij}^{(m)}$ by integration of equation (2) in $[\kappa_{m-1},\kappa_m]$**

$$\frac{\partial\tau_{ij}^{(m)}}{\partial t} = P_{ij}^{(m)} + F_{ij}^{(m-1)} - F_{ij}^{(m)} + \Psi_{ij}^{(m)} + J_{ij}^{(m)} - \epsilon_{ij}^{(m)} \qquad \textbf{(3)}$$

with

$$F_{ij}^{(m)} = \mathcal{F}_{ij}^{(m)} - \varphi_{ij}(\boldsymbol{X},\kappa_m)\frac{\partial\kappa_m}{\partial t}, \qquad \textbf{(4)}$$

with

$$\mathcal{F}_{ij}^{(m)} = -\int_0^{\kappa_m} \mathcal{T}_{ij}(\boldsymbol{X},\kappa)d\kappa \qquad \textbf{(5)}$$

MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Two scale models**
  - **Subfilter-scale turbulent stress**

$$\frac{\partial \tau_{ij}^{(1)}}{\partial t} = P_{ij}^{(1)} - F_{ij}^{(1)} + \Psi_{ij}^{(1)} + J_{ij}^{(1)} - \epsilon_{ij}^{(1)} \qquad (6)$$

$$\frac{\partial \tau_{ij}^{(2)}}{\partial t} = P_{ij}^{(2)} + F_{ij}^{(1)} - F_{ij}^{(2)} + \Psi_{ij}^{(2)} + J_{ij}^{(2)} - \epsilon_{ij}^{(2)} \qquad (7)$$

$$0 = F_{ij}^{(2)} - \epsilon_{ij}^{(3)} \qquad (8)$$

  - **Subfilter-scale turbulent energy**

$$\frac{\partial k^{(1)}}{\partial t} = P^{(1)} - F^{(1)} + J^{(1)} - \epsilon^{(1)} \qquad (9)$$

$$\frac{\partial k^{(2)}}{\partial t} = P^{(2)} + F^{(1)} - F^{(2)} + J^{(2)} - \epsilon^{(2)} \qquad (10)$$

$$0 = F^{(2)} - \epsilon^{(3)} \qquad (11)$$

GENERIC SUBFILTER DISSIPATION-RATE EQUATION

- **Dissipation-rate equation**

$$\frac{\partial \langle \epsilon_{sfs} \rangle}{\partial t} = c_{sfs\epsilon_1} \frac{\langle \epsilon_{sfs} \rangle}{\langle k_{sfs} \rangle} \langle P_{sfs} \rangle - c_{sfs\epsilon_2} \frac{\langle \epsilon_{sfs} \rangle^2}{\langle k_{sfs} \rangle} \quad \textbf{(12)}$$

where

$$c_{sfs\epsilon_1} = 3/2 \;\; and \;\; c_{sfs\epsilon_2} = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\langle \epsilon_{sfs} \rangle} \right) \quad \textbf{(13)}$$

In a compact form, $c_{sfs\epsilon_2}$ can be written as

$$c_{sfs\epsilon_2} = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \quad \textbf{(14)}$$

$$\zeta(\kappa_d) = \frac{k}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\epsilon} \right) \quad \textbf{(15)}$$

For $\kappa_c = 0$,

$$c_{\epsilon_2} = \frac{3}{2} - \zeta(\kappa_d) \quad \textbf{(16)}$$

$$c_{\epsilon_{sfs2}} = \frac{3}{2} + \frac{\langle k_{sfs} \rangle}{k} \left(c_{\epsilon_2} - \frac{3}{2} \right) \quad \textbf{(17)}$$

GENERIC SUBFILTER DISSIPATION-RATE EQUATION

- **The value suggested for $c_{\epsilon_1} = c_{sfs\epsilon_1} = 3/2$ seems restrictive if one remarks that this coefficient may take on different values in statistical RANS models according to its calibration made by different authors**
- **Let us now consider the standard form of the statistical dissipation-rate equation written in homogeneous turbulence**

$$\frac{\partial \epsilon}{\partial t} = c_{\epsilon_1} \frac{\epsilon}{k} P - c_{\epsilon_2} \frac{\epsilon^2}{k} \qquad (18)$$

  with given values of the coefficients c_{ϵ_1} and c_{ϵ_2}.
- **The issue to address is to compute the function $c_{sfs\epsilon_2}$ when the coefficient $c_{sfs\epsilon_1}$ differs from the value $3/2$ (Chaouat and Schiestel, Physics of Fluids, vol. 24, 2012)**

GENERIC SUBFILTER DISSIPATION-RATE EQUATION

- After some algebra, one can obtain:

$$\frac{\partial \langle \epsilon_{sfs} \rangle}{\partial t} = c_{\epsilon 1} \frac{\langle \epsilon_{sfs} \rangle}{\langle k_{sfs} \rangle} \langle P \rangle_{sfs} - \left[\left(\frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \right) - \left(\frac{3}{2} - c_{\epsilon 1} \right) \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} \right] \frac{\langle \epsilon_{sfs} \rangle^2}{\langle k_{sfs} \rangle} \tag{19}$$

and

$$\frac{\partial \epsilon}{\partial t} = c_{\epsilon_1} \frac{\epsilon}{k} P - \left[\left(\frac{3}{2} - \zeta(\kappa_d) \right) - \left(\frac{3}{2} - c_{\epsilon_1} \right) \frac{P}{\epsilon} \right] \frac{\epsilon^2}{k} \tag{20}$$

showing that

$$c_{\epsilon_2} = \left(\frac{3}{2} - \zeta(\kappa_d) \right) - \left(\frac{3}{2} - c_{\epsilon_1} \right) \frac{P}{\epsilon} \tag{21}$$

and

$$c_{sfs\epsilon_2} = \left(\frac{3}{2} - \frac{\langle k_{sfs} \rangle}{k} \zeta(\kappa_d) \right) - \left(\frac{3}{2} - c_{\epsilon_1} \right) \frac{\langle P_{sfs} \rangle}{\langle \epsilon_{sfs} \rangle} \tag{22}$$

GENERIC SUBFILTER DISSIPATION-RATE EQUATION

Both equations (21) and (22) allow to determine the function $\zeta(\kappa_d)$ in two different forms as follows

$$\zeta(\kappa_d) = \left[\frac{3}{2} - c_{\epsilon_2} + \left(c_{\epsilon_1} - \frac{3}{2}\right)\frac{P}{\epsilon}\right] \quad \textbf{(23)}$$

and

$$\zeta(\kappa_d) = \left[\frac{3}{2} - c_{sfs\epsilon_2} + \left(c_{\epsilon_1} - \frac{3}{2}\right)\frac{\langle P_{sfs}\rangle}{\langle \epsilon_{sfs}\rangle}\right]\frac{k}{\langle k_{sfs}\rangle} \quad \textbf{(24)}$$

$$c_{sfs\epsilon_2} = c_{\epsilon_2} + \left(\frac{3}{2} - c_{\epsilon_2}\right)\left[1 - \frac{\langle k_{sfs}\rangle}{k}\right] + \left(c_{\epsilon_1} - \frac{3}{2}\right)\left[\frac{\langle P_{sfs}\rangle}{\langle \epsilon_{sfs}\rangle} - \frac{P}{\epsilon}\frac{\langle k_{sfs}\rangle}{k}\right] \quad \textbf{(25)}$$

If we assume that the ratio $\langle k_{sfs}\rangle / k$ of the subfilter energy to the total energy is constant or varies slowly with time

$$\frac{\langle P_{sfs}\rangle}{\langle \epsilon_{sfs}\rangle} \approx 1 + \frac{\langle k_{sfs}\rangle}{k}\frac{P-\epsilon}{\epsilon} \quad \textbf{(26)}$$

Then, we obtain the final form of the coefficient $c_{sfs\epsilon_2}$ that reads

$$c_{sfs\epsilon_2} = c_{\epsilon_1} + \frac{\langle k_{sfs}\rangle}{k}\left(c_{\epsilon_2} - c_{\epsilon_1}\right) \quad \textbf{(27)}$$

- **Equation (27) established at high Reynolds number is the key equation**

PHYSICAL MEANING OF $c_{sfs\epsilon_1}$ IN THE SPECTRAL SPACE

- **The Kovasznay hypothesis**

$$\mathcal{F}(\kappa,t) = C_\kappa^{-3/2}\, E(\kappa,t)^{3/2}\, \kappa^{5/2} \tag{28}$$

Partial time derivative of both sides

$$\frac{1}{\mathcal{F}}\frac{\partial \mathcal{F}(\kappa,t)}{\partial t} = \frac{3}{2}\frac{1}{E}\frac{\partial E(\kappa,t)}{\partial t} \tag{29}$$

$$\frac{\partial E(\kappa,t)}{\partial t} = -\frac{\partial \mathcal{F}(\kappa,t)}{\partial \kappa} \tag{30}$$

$$\frac{\partial \mathcal{F}(\kappa,t)}{\partial t} = -\frac{3}{2}\frac{\mathcal{F}}{E}\frac{\partial \mathcal{F}(\kappa,t)}{\partial \kappa} \tag{31}$$

Discretization in the spectral space

$$\frac{\partial \mathcal{F}^{(m)}}{\partial t} = \frac{3}{2}\frac{\mathcal{F}^{(m)}}{E^{(m)}}\frac{\mathcal{F}^{(m-1)} - \mathcal{F}^{(m)}}{\Delta\kappa} \tag{32}$$

PHYSICAL MEANING OF $c_{sfs\epsilon_1}$ IN THE SPECTRAL SPACE

- **The term $E^{(m)}\Delta\kappa$ is in fact the partial turbulent energy within the spectral slice $[\kappa_{m-1}, \kappa_m]$**

$$\frac{\partial \mathcal{F}^{(m)}}{\partial t} = \frac{3}{2}\frac{\mathcal{F}^{(m)}\mathcal{F}^{(m-1)}}{k^{(m)}} - \frac{3}{2}\frac{(\mathcal{F}^{(m)})^2}{k^{(m)}} \quad (33)$$

- **This equation looks like the flux equation in the multiple scale model for the spectral slice (m)**
- **Same structure as the standard dissipation-rate equation in which the coefficients are $c_{\epsilon_1} = c_{\epsilon_2} = 3/2$**
- **The constant value $3/2$ in the limiting case of a very thin spectral slice**
- **$c_{\epsilon_1} = 3/2$ is not a lucky haphazard !**

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- **Analytical solution (Reid and Harris (1959)**

$$E(\kappa,t) = H(t)\,G(\gamma) = \frac{b}{\sqrt{t}}\,G(\kappa\sqrt{at}) \quad (34)$$

where H and G are similarity functions, $\gamma(t) = \kappa\sqrt{at}$ is a similarity variable whereas a and b denote numerical constant coefficients

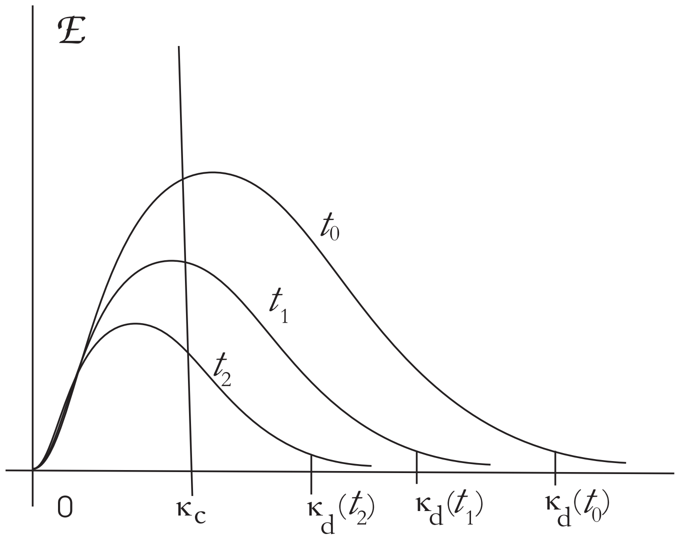

Figure 1: Evolution of the density spectrum E corresponding to the self similar decay of turbulence.

On the FINITENESS of the $c_{sfs\epsilon_2}$ at LARGE Re

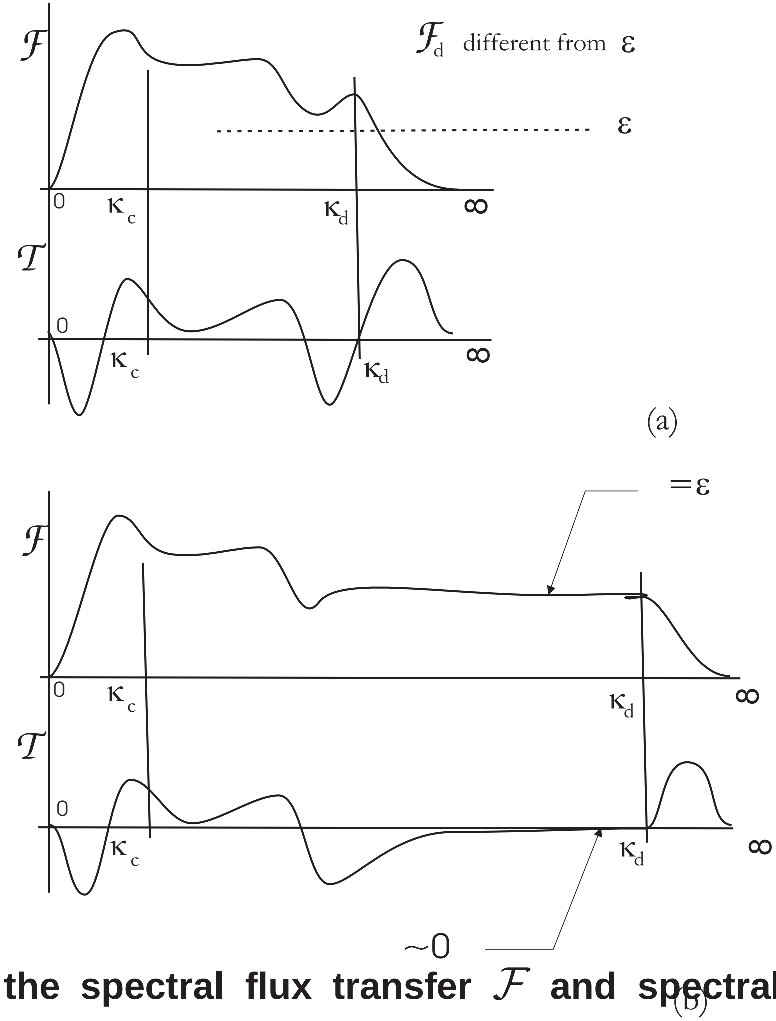

Figure 2: Sketches of the spectral flux transfer $\mathcal{F}$ and spectral transfer term $\mathcal{T}$. Non-equilibrium flows. (a) High Reynolds number; (b) Infinite Reynolds number.

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SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- The dimensionless reduced variable involving the time is

$$z(\kappa, t) = \epsilon(t)^{-1/3} \kappa^{-2/3} t^{-1} \quad (35)$$

- Spectrum and the transfer term are computed by Taylor series expansions of the dimensionless variable $z = z(\kappa, t)$ where κ and t are independent variables as follows

$$E(\kappa, t) = C_\kappa \, \epsilon(t)^{2/3} \kappa^{-5/3} \sum_{n=0}^{\infty} a_n \, z^n \quad (36)$$

The Kovasznay hypothesis for the transfer term

$$\mathcal{F}(\kappa, t) = C_\kappa^{-3/2} \, E(\kappa, t)^{3/2} \, \kappa^{5/2} \quad (37)$$

- Spectral energy equation in time and wave number at large Reynolds number

$$\frac{\partial E(\kappa, t)}{\partial t} = -\frac{\partial \mathcal{F}(\kappa, t)}{\partial \kappa} \quad (38)$$

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

The spectral flux transfer $\mathcal{F}$

$$\mathcal{F}(\kappa, t) = \epsilon(t) \left[\sum_{n=0}^{\infty} a_n \, z^n \right]^{3/2} \tag{39}$$

The derivative of equation (36) reads

$$\frac{\partial E(\kappa, t)}{\partial t} = C_\kappa \kappa^{-5/3} \left[\frac{2}{3} \epsilon^{-1/3} \frac{d\epsilon}{dt} \sum_{n=0}^{\infty} a_n \, z^n + \epsilon^{2/3} \sum_{n=0}^{\infty} n \, a_n \, z^{n-1} \frac{\partial z}{\partial t} \right] \tag{40}$$

The derivative $d\epsilon/dt$ is computed as

$$\frac{d\epsilon(t)}{dt} = -2\epsilon^{4/3} \kappa^{2/3} z \tag{41}$$

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SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

The derivative $\partial z / \partial t$ is computed using the self preservation hypothesis

$$\frac{\partial z(\kappa,t)}{\partial t} = -\kappa^{-2/3}\left[\frac{1}{3}\epsilon^{-4/3}\frac{d\epsilon}{dt}t^{-1} + \epsilon^{-1/3}t^{-2}\right] = -\frac{z}{3t} = -\frac{1}{3}\epsilon^{1/3}\kappa^{2/3}z^2 \qquad (42)$$

So that equation (40) reads

$$\frac{\partial E(\kappa,t)}{\partial t} = -\frac{2}{3}C_\kappa\,\epsilon\kappa^{-1}\left[2z + \frac{5}{2}a_1z^2 + 3a_2z^2 + \frac{7}{2}a_3z^3 + O\left(z^4\right)\right] \qquad (43)$$

One can then obtain

$$\mathcal{F}(\kappa,t) = \epsilon\left[1 + \frac{3}{2}a_1z + \frac{3}{2}\left(a_2 + \frac{a_1^2}{4}\right)z^2 + \frac{3}{2}\left(a_3 + \frac{1}{2}a_1a_2 - \frac{a_1^3}{24}\right)z^3 + O\left(z^4\right)\right] \qquad (44)$$

$$\frac{\partial \mathcal{F}(\kappa,t)}{\partial \kappa} = -\frac{2}{3}\epsilon\kappa^{-1}\left[\frac{3}{2}a_1z + 3\left(a_2 + \frac{a_1^2}{4}\right)z^2 + \frac{9}{2}\left(a_3 + \frac{a_1a_2}{2} - \frac{a_1^3}{24}\right)z^3 + O\left(z^4\right)\right] \qquad (45)$$

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

$$E(\kappa,t) = C_\kappa\,\epsilon^{2/3}\kappa^{-5/3} - \frac{4}{3}{C_\kappa}^2\,\epsilon^{1/3}\frac{\kappa^{-7/3}}{t} + \frac{2}{3}{C_\kappa}^3\,\frac{\kappa^{-3}}{t^2} - \frac{8}{81}{C_\kappa}^4\,\epsilon^{-1/3}\frac{\kappa^{-11/3}}{t^3} + \epsilon^{2/3}\kappa^{-5/3}O\left(z^4\right) \quad (46)$$

$$\mathcal{F}(\kappa,t) = \epsilon - 2C_\kappa\,\frac{\epsilon^{2/3}\kappa^{-2/3}}{t} + \frac{5}{3}{C_\kappa}^2\,\frac{\epsilon^{1/3}\kappa^{-4/3}}{t^2} - \frac{2}{3}{C_\kappa}^3\,\frac{\kappa^{-2}}{t^3} + \epsilon O\left(z^4\right) \quad (47)$$

$$\frac{\partial\mathcal{F}}{\partial\kappa}(\kappa,t) = \frac{4}{3}C_\kappa\,\frac{\epsilon^{2/3}\kappa^{-5/3}}{t} - \frac{20}{9}{C_\kappa}^2\,\frac{\epsilon^{1/3}\kappa^{-7/3}}{t^2} + \frac{4}{3}{C_\kappa}^3\,\frac{\kappa^{-3}}{t^3} + \epsilon\kappa^{-1}O\left(z^4\right) \quad (48)$$

These analytical results will form the basis of a test of the generic model for the ϵ equation.

(Chaouat and Schiestel, Physics of Fluids, vol. 24, 2012)

We will show that the coefficient $c_{sfs\epsilon_2}$ introduced in the dissipation-rate equation takes a finite value.

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

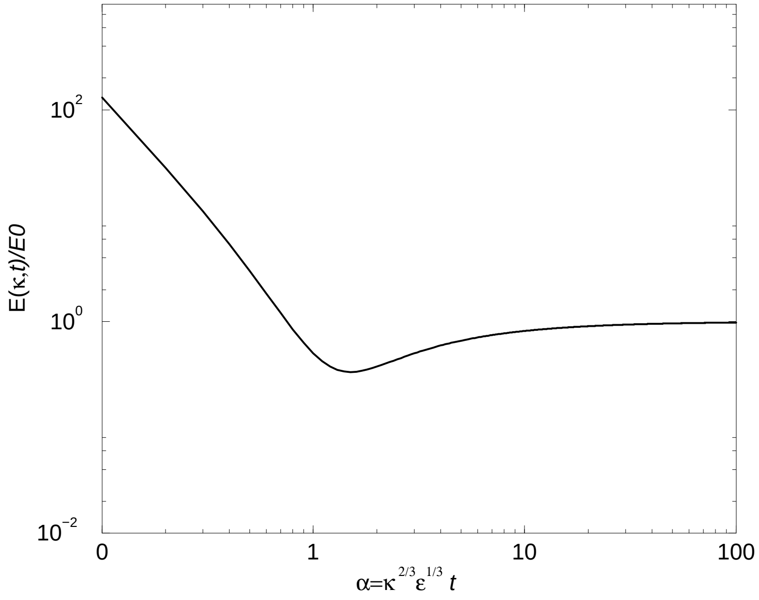

Figure 3: Evolving of the density spectrum $E(\kappa,t)/E(\kappa,\infty)$ with respect to the dimensionless variable $\alpha = \kappa^{2/3}\epsilon^{1/3}t$

$$E(\kappa,t) = C_\kappa\,\epsilon^{2/3}\kappa^{-5/3} - \frac{4}{3}C_\kappa{}^2\,\epsilon^{1/3}\frac{\kappa^{-7/3}}{t} + \frac{2}{3}C_\kappa{}^3\,\frac{\kappa^{-3}}{t^2} \qquad (49)$$

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

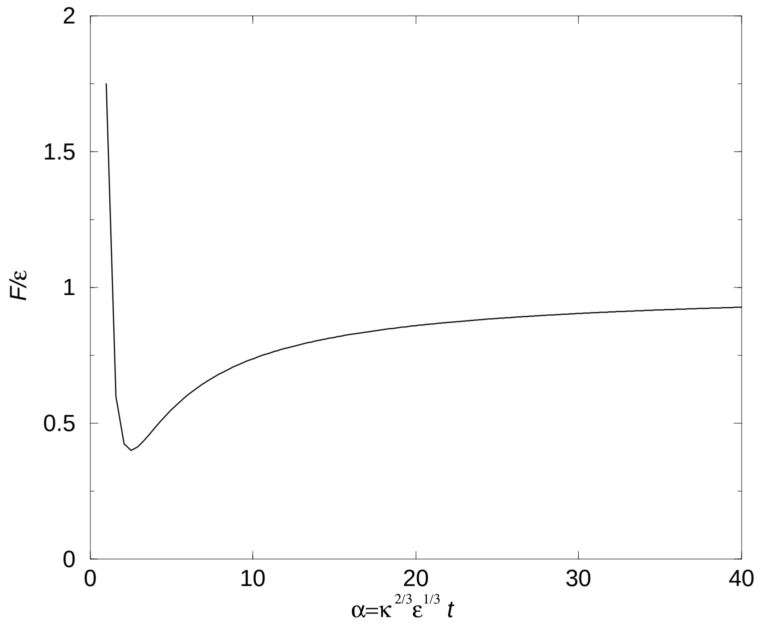

Figure 4: Evolving of the spectral flux transfer $\mathcal{F}(\kappa, t)/\epsilon$ with respect to the dimensionless variable $\alpha = \kappa^{2/3}\epsilon^{1/3}t$

$$\mathcal{F}(\kappa, t) = \epsilon - 2C_\kappa \frac{\epsilon^{2/3}\kappa^{-2/3}}{t} + \frac{5}{3}{C_\kappa}^2 \frac{\epsilon^{1/3}\kappa^{-4/3}}{t^2} \tag{50}$$

SELF-SIMILAR ANALYTICAL FLOW EXAMPLE

- **Governing equations**

$$\mathcal{F}(\kappa,t) = \epsilon - 2C_\kappa \frac{\epsilon^{2/3}\kappa^{-2/3}}{t} + \frac{5}{3}{C_\kappa}^2 \frac{\epsilon^{1/3}\kappa^{-4/3}}{t^2} \quad (51)$$

$$F(\kappa_d,t) = \epsilon + \frac{\partial}{\partial t}\int_{\kappa_d}^{\infty} E(\kappa,t)\,d\kappa \quad (52)$$

- **Full statistical case where κ_c equals zero**

$$c_{\epsilon_2} = \frac{3}{2} - \frac{\langle k_{sfs}\rangle}{\kappa_d E(\kappa_d)}\left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\epsilon}\right) = 2 \quad (53)$$

- **Subfilter turbulence case where κ_c is non zero**

$$\frac{\langle k_{sfs}\rangle}{k} = Q(\eta_c) \quad , \quad \eta_c = \kappa_c L \quad (54)$$

$$c_{sfs\epsilon_2} = \frac{3}{2} + \frac{1}{2}\frac{\langle k_{sfs}\rangle}{k}\left[1 + \frac{3}{2}\eta_c \frac{d\ln Q(\eta_c)}{d\eta_c}\right] \quad (55)$$

- $c_{sfs\epsilon_2}$ **always takes on a finite value whatever the domain variation of** κ_d

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PITM METHOD

- **Instantaneous transport equations and practical formulations (Schiestel and Dejoan, 2005, Chaouat and Schiestel, 2005-2012)**

$$\frac{Dk_{sfs}}{Dt} = P_{sfs} - \epsilon_{sfs} + J_{sfs} \qquad \textbf{(56)}$$

$$\frac{D(\tau_{ij})_{sfs}}{Dt} = (P_{ij})_{sfs} - (\epsilon_{ij})_{sfs} + (\Phi_{ij})_{sfs} + (J_{ij})_{sfs} \qquad \textbf{(57)}$$

$$\frac{D\epsilon_{sfs}}{Dt} = c_{\epsilon_1} \frac{\epsilon_{sfs}}{k_{sfs}} \frac{(P_{mm})_{sfs}}{2} - c_{sfs\epsilon_2}(\eta_c) \frac{\epsilon^2_{sfs}}{k_{sfs}} + (J_\epsilon)_{sfs} \qquad \textbf{(58)}$$

  – **"Exact " coefficient** c_{ϵ_2} **where** $\eta_c = (k^{3/2}\kappa_c)/(\epsilon_{sfs} + \epsilon^<)$

$$c_{sfs\epsilon_2}(\eta_c) = c_{\epsilon_1} + \frac{c_{\epsilon_2} - c_{\epsilon_1}}{[1 + \beta_\eta \, \eta_c^3]^{2/9}} \qquad \textbf{(59)}$$

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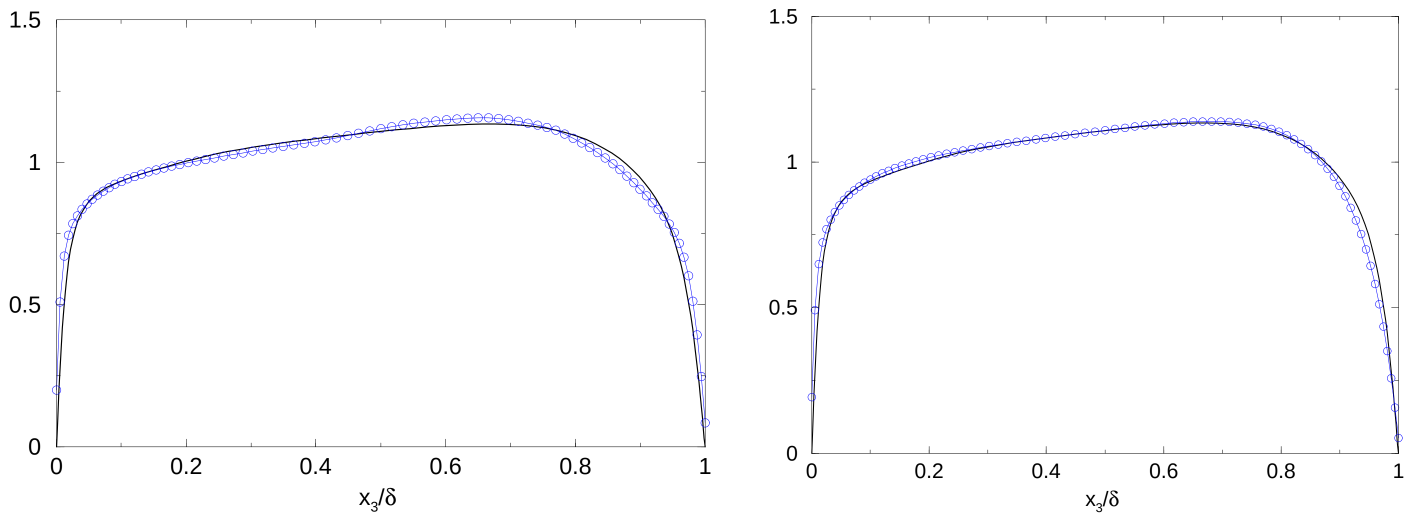
SOME PITM SIMULATIONS

- PITM challenges
  - The PITM has been especially developed for performing continuous hybrid non-zonal RANS-LES simulations on coarse grids
  - Flows which depart from spectral equilibrium
- Simulation of homogeneous turbulence
  - Decay of homogeneous turbulence (Chaouat and Schiestel, 2009)
  - Perturbed spectrum with a peak or defect of energy (Chaouat and Schiestel, 2009)
- Simulation of non-homogeneous turbulence
  - Pulsed channel flows (Schiestel and Dejoan, 2005)
  - Channel flow with mass injection (Chaouat and Schiestel, 2005)
  - Shearless mixing layer (Befeno and Schiestel, 2007)
  - Channel flows over periodic hills (Chaouat, 2010)
  - Turbulent rotating channel flows (Chaouat, 2012)

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION



(a) **(b)**

Figure 5. Mean velocity profile $\langle u_1 \rangle / u_m$ in global coordinate. (a) PITM1 $(24 \times 48 \times 64)$: ○; (b) PITM2 $(84 \times 64 \times 64)$: ○; (Chaouat, Physics of Fuids, 2012). Highly resolved LES (Lamballais et al., TCFD, 1998): — .

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION

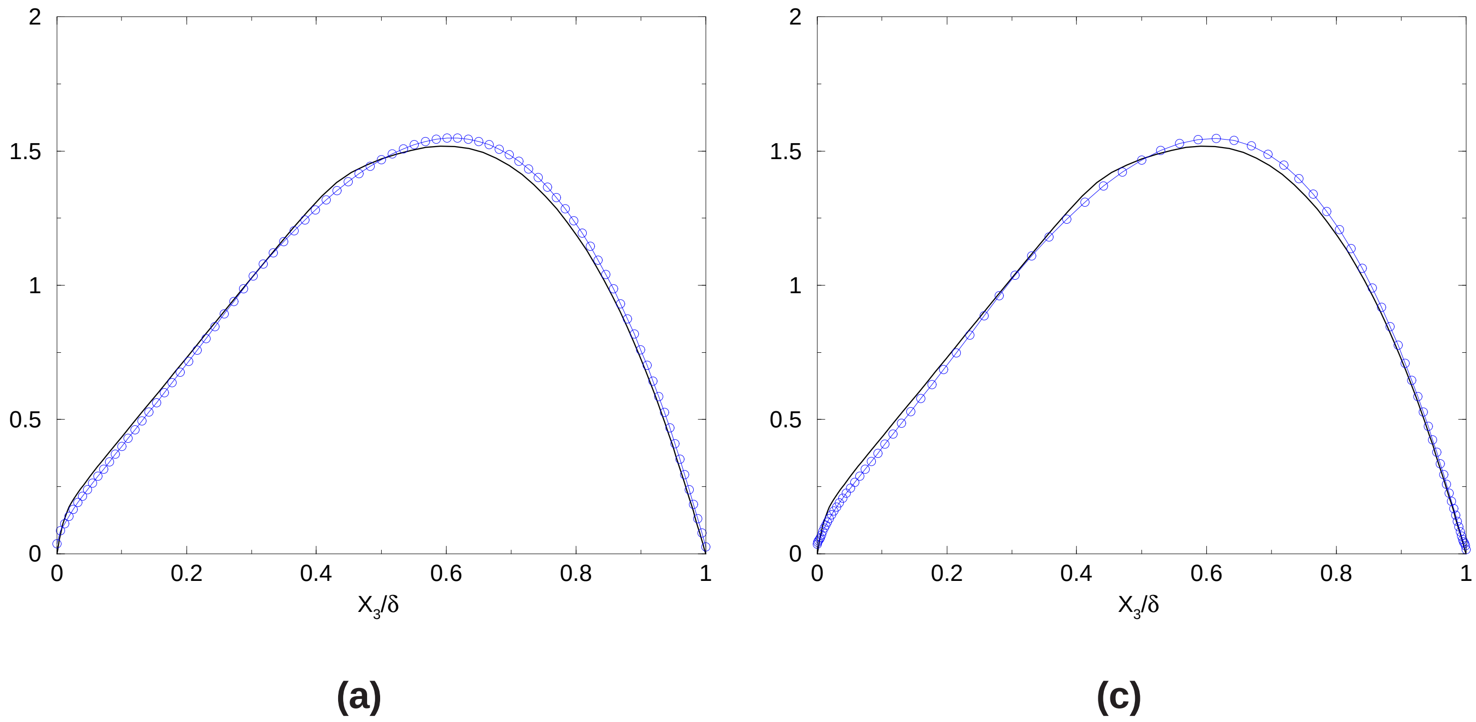
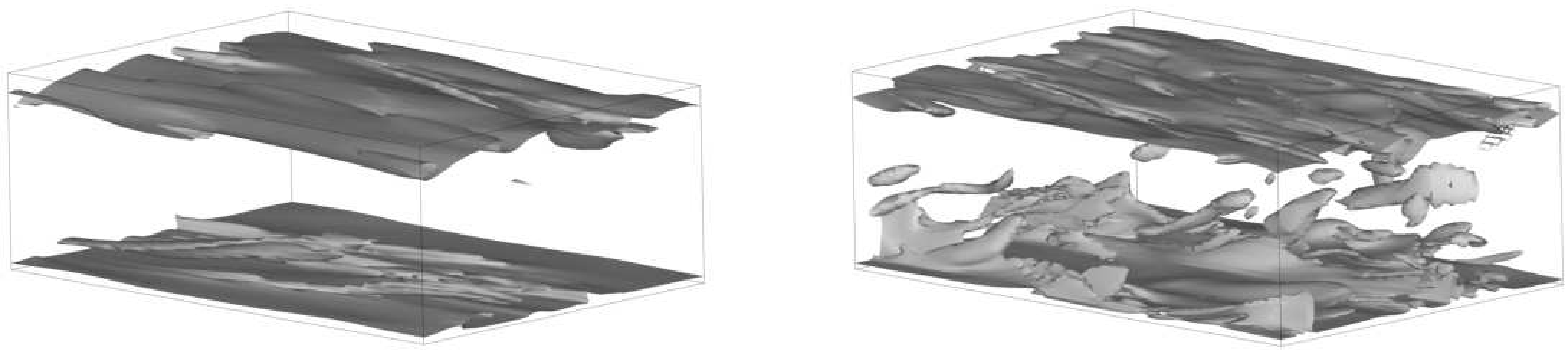


Figure 6. Mean velocity profile $\langle u_1 \rangle / u_m$ in global coordinate. (a) PITM1 $(24 \times 48 \times 64)$: ○; (c) PITM3 $(124 \times 84 \times 84)$: ○; (Chaouat, Physics of Fuids, 2012). Highly resolved LES (Lamballais et al., TCFD, 1998): — . $R_m = 14000$, $R_o = 1.50$.

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION



(a) **(c)**

Figure 7. Isosurfaces of vorticity modulus $\omega = 3u_m/\delta = 8.\,10^5$. $R_m = 14000$, $R_o = 0.17$.
(a) PITM1 $(24 \times 48 \times 64)$; **(b) PITM2** $(84 \times 64 \times 64)$; **(Chaouat, Physics of Fuids, 2012).**

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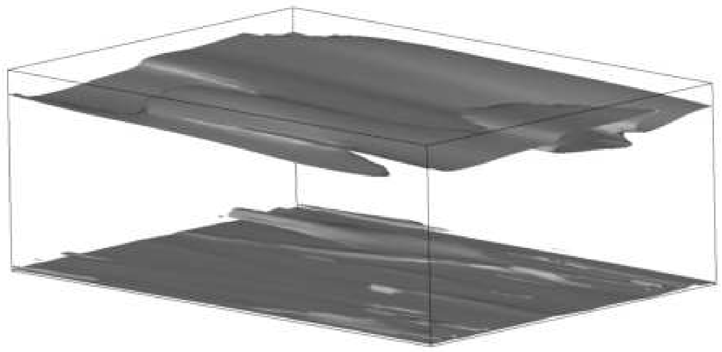
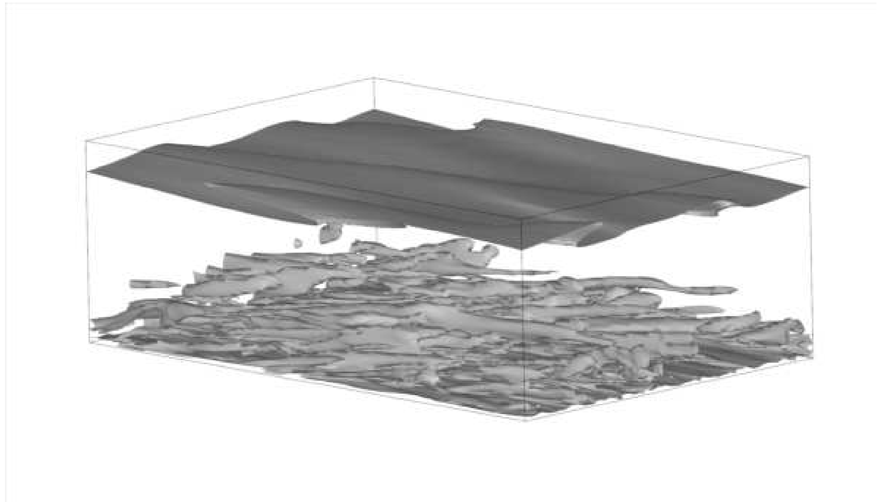
CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION



(a)



(c)

Figure 8. Isosurfaces of vorticity modulus $\omega = 3u_m/\delta = 12.\,10^5$. $R_m = 14000$, $R_o = 1.50$.
(a) PITM1 $(24 \times 48 \times 64)$; **(c) PITM3** $(124 \times 84 \times 84)$. **(Chaouat, Physics of Fuids, 2012).**

CONCLUSION

- Partial integrated transport modeling (PITM) method
  - Mathematical physics formalism developed in the spectral space
  - Further insights into the physical interpretation of the PITM method, especially in its basic foundations
- PITM is a method and not a model itself !
- PITM can be applied to each RANS model to derive its corresponding subfilter model
- PITM allows one to perform continuous hybrid non-zonal RANS/LES simulations
- Drastic reductions of the computational cost by coarsening the meshes
- PITM is a new route for simulations of turbulent flows
- References can be found in Physics of Fluids and TCFD journals

References

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