Méthode de Galerkin discontinue pour la simulation des écoulements turbulents

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Outline

Context

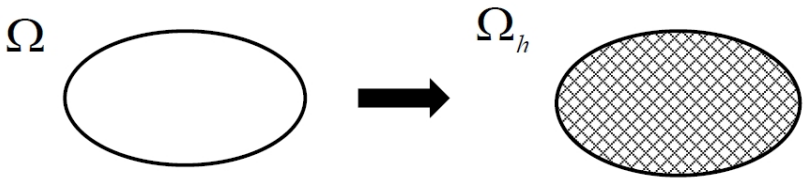
Discontinuous Galerkin (DG) methods

- Combine features of FEM and FVM
- Unstructured meshes (multi-elements, accurate representation of boundaries)
- High-order accuracy (polynomial approximation)
- Compact stencil, advantageous for parallelization based on MPI
- Natural framework for multiscale turbulence modelling

- 3D compressible Navier Stokes equations

$$\frac{\partial \mathbf{w}}{\partial t} + \nabla.\mathbf{F} = 0\ ,\ \ \mathbf{F} = \mathbf{F}^{conv}\left(\mathbf{w}\right) - \mathbf{F}^{visc}\left(\mathbf{w}, \nabla \mathbf{w}\right)$$

- Spatial discretization

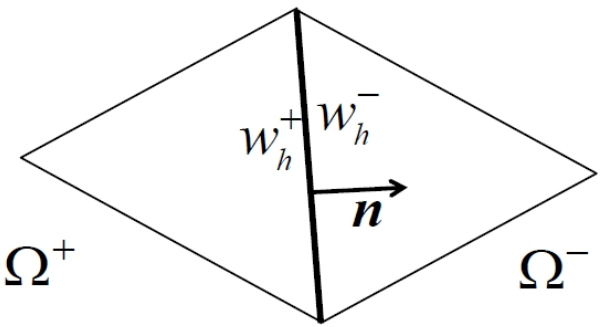


$$\Omega_h = \bigcup_{j=1}^{N} \kappa_j$$

- Polynomial expansion in the elements

$$\mathbf{w}_h\left(\mathbf{x}, t\right) = \sum_{k=1}^{N_p} \mathbf{W}_k\left(t\right)\phi_k\left(\mathbf{x}\right),\ \phi \in \mathcal{V}_h^p,\ dim\left(\mathcal{V}_h^p\right) = N_p$$

- Discontinuous solution across element's faces



- Numerical flux for convective terms, e.g. Local Lax-Friedrichs
- Numerical flux with lifting operators (BR2[1]) for viscous terms
- The BR2 scheme is **compact**
- RK3 scheme for the integration in time of the discrete system

[1] F. Bassi et al. *J. Comput. Phys.*, 218 (2006)

Case description

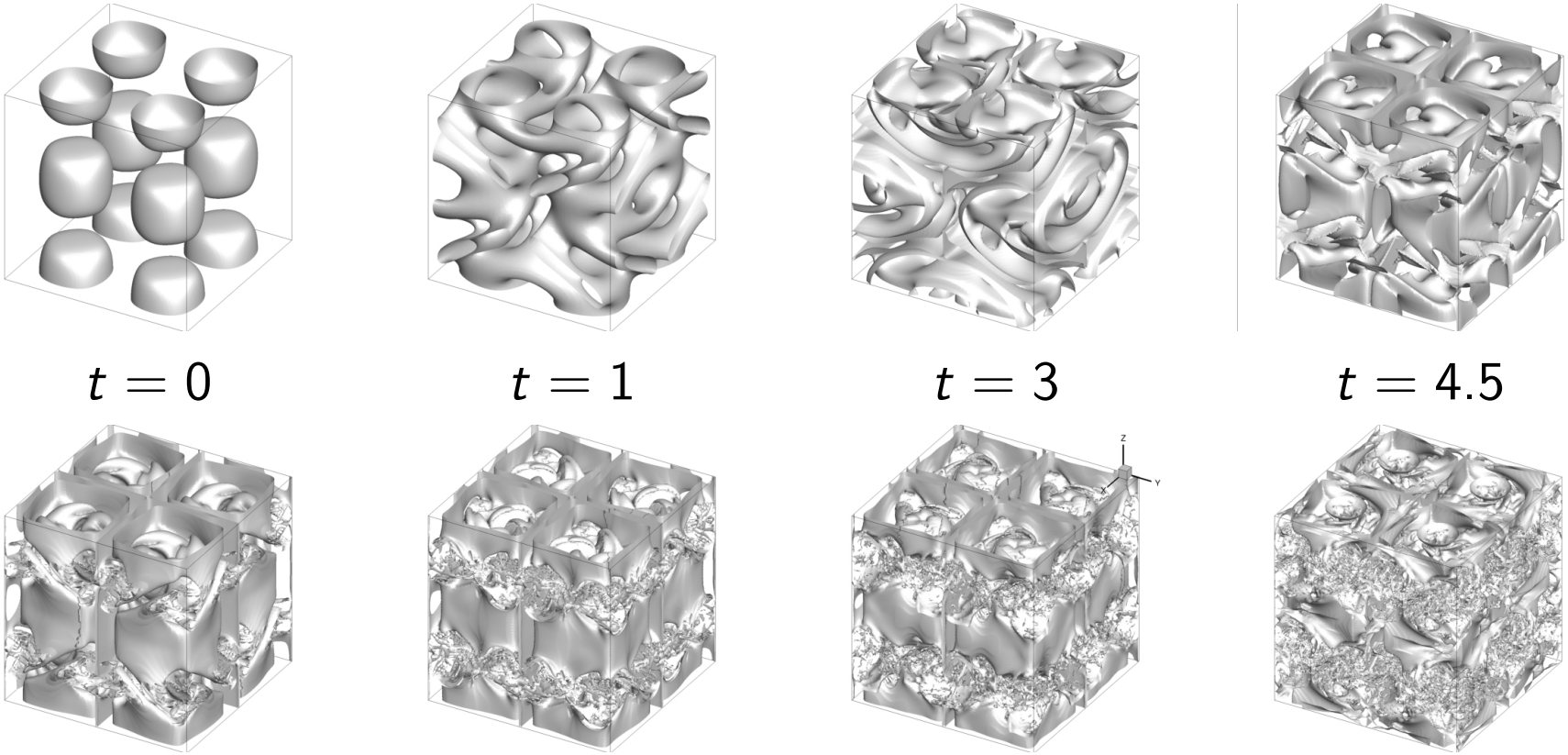
- Model for 3D incompressible freely decaying turbulence in a periodic box
- Analytical initialization in physical space

$$
\begin{aligned}
u_0(x,y,z) &= V_0 sin\left(\frac{x}{L}\right) cos\left(\frac{y}{L}\right) cos\left(\frac{z}{L}\right) \\
v_0(x,y,z) &= -V_0 cos\left(\frac{x}{L}\right) sin\left(\frac{y}{L}\right) cos\left(\frac{z}{L}\right) \\
w_0(x,y,z) &= 0 \\
p_0(x,y,z) &= p_\infty + \frac{\rho_0}{16}\left(cos(\frac{2x}{L}) + cos(\frac{2x}{L})\right)\left(cos(\frac{2z}{L}) + 2\right)
\end{aligned}
$$

- $Re = V_0 L/\nu$, $M = 0.1$ for all computations
- Monitoring the evolution of the mean energy and enstrophy

$$E_k = \frac{1}{V}\int_V \frac{\mathbf{u}.\mathbf{u}}{2} d\mathbf{x} \quad \xi = \frac{1}{V}\int_V \frac{\omega.\omega}{2} d\mathbf{x}$$

Illustration of the evolution of the flow[2]



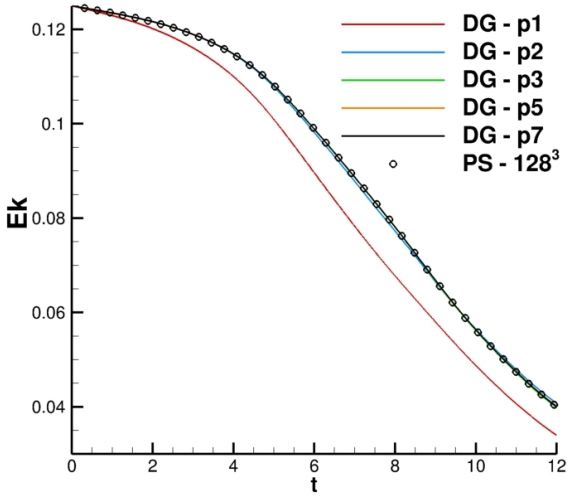
[2] Isosurfaces of $|\omega|$, Fourier spectral DNS computation (Code provided by R. Cant, *Cambridge Univ.)*

RE=500 : Details of the computations

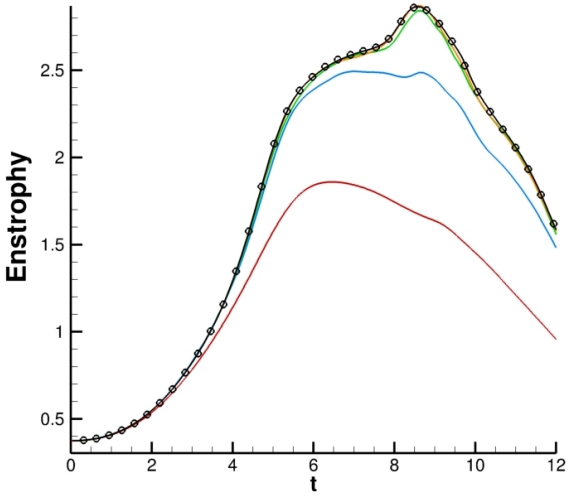
Computation	Method	Accuracy (p+1)	Nb. DOFs/Elements
DG 48p1	DG	2	96^3/ 48^3
DG 32p2	DG	3	96^3/ 32^3
DG 24p3	DG	4	96^3/ 24^3
DG 16p5	DG	6	96^3/ 16^3
DG 12p7	DG	8	96^3/ 12^3
PS 128	PS	-	128^3

- Key points
  - Evolution of the mean kinetic energy and enstrophy
  - Representation of the energy spectrum

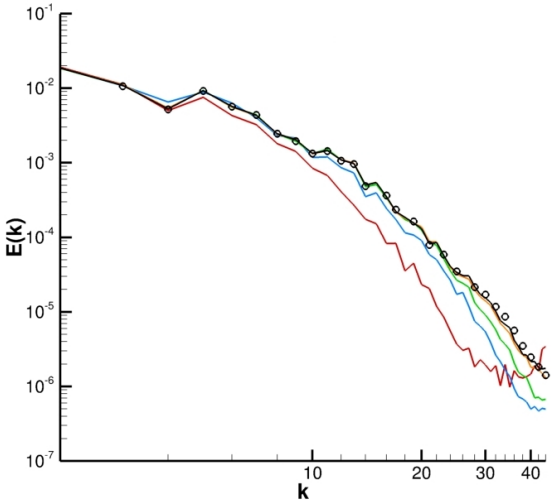
RE=500 : Statistics and spectra



Evolution of energy



Evolution of enstrophy



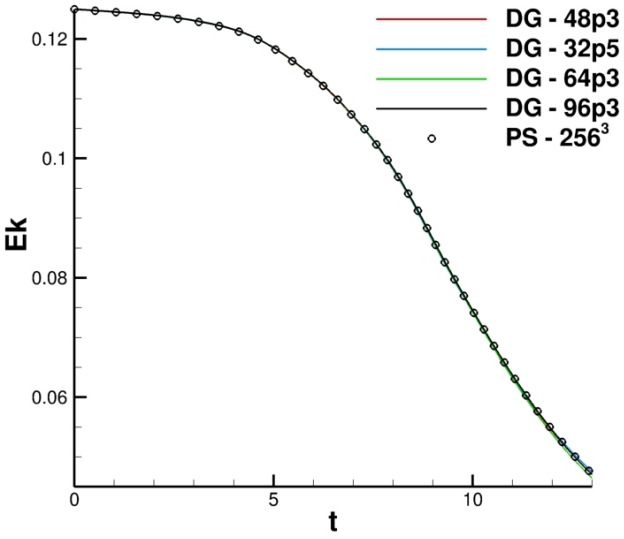
Energy spectra at t=9

RE=1600 : Details of the computations

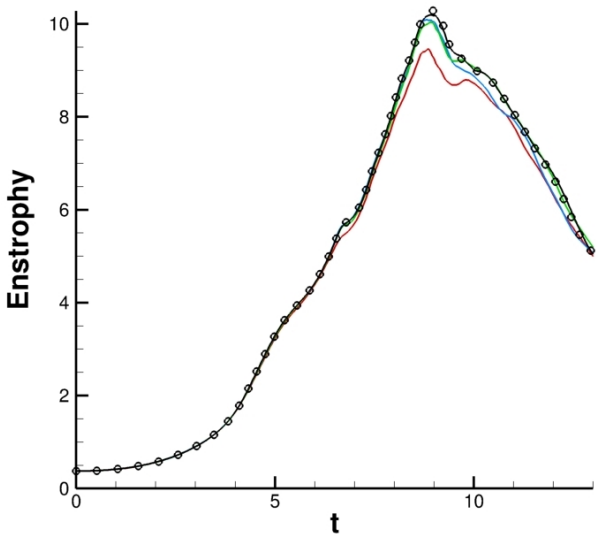
Computation	Method	Accuracy (p+1)	Nb. DOFs/Elements
DG 48p3	DG	4	192^3 / 48^3
DG 64p3	DG	4	256^3 / 64^3
DG 96p3	DG	4	384^3 / 96^3
DG 32p5	DG	6	192^3 / 32^3
PS 256	PS	-	256^3

- Key points
  - Physical representation of vortices
  - Behaviour of under-resolved DG computations

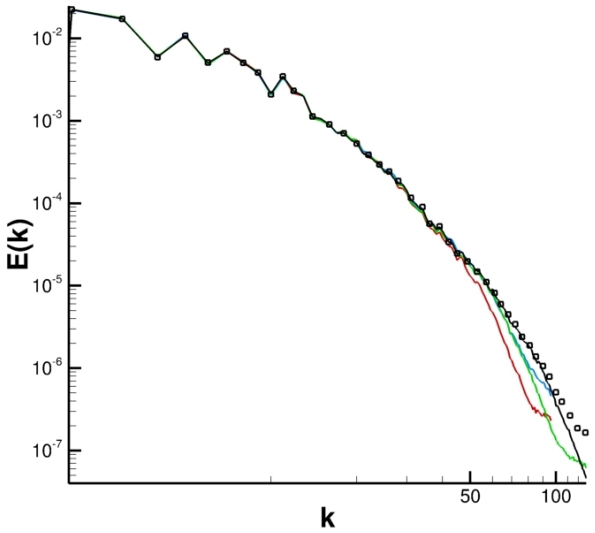
RE=1600 : Statistics and spectra



Evolution of energy



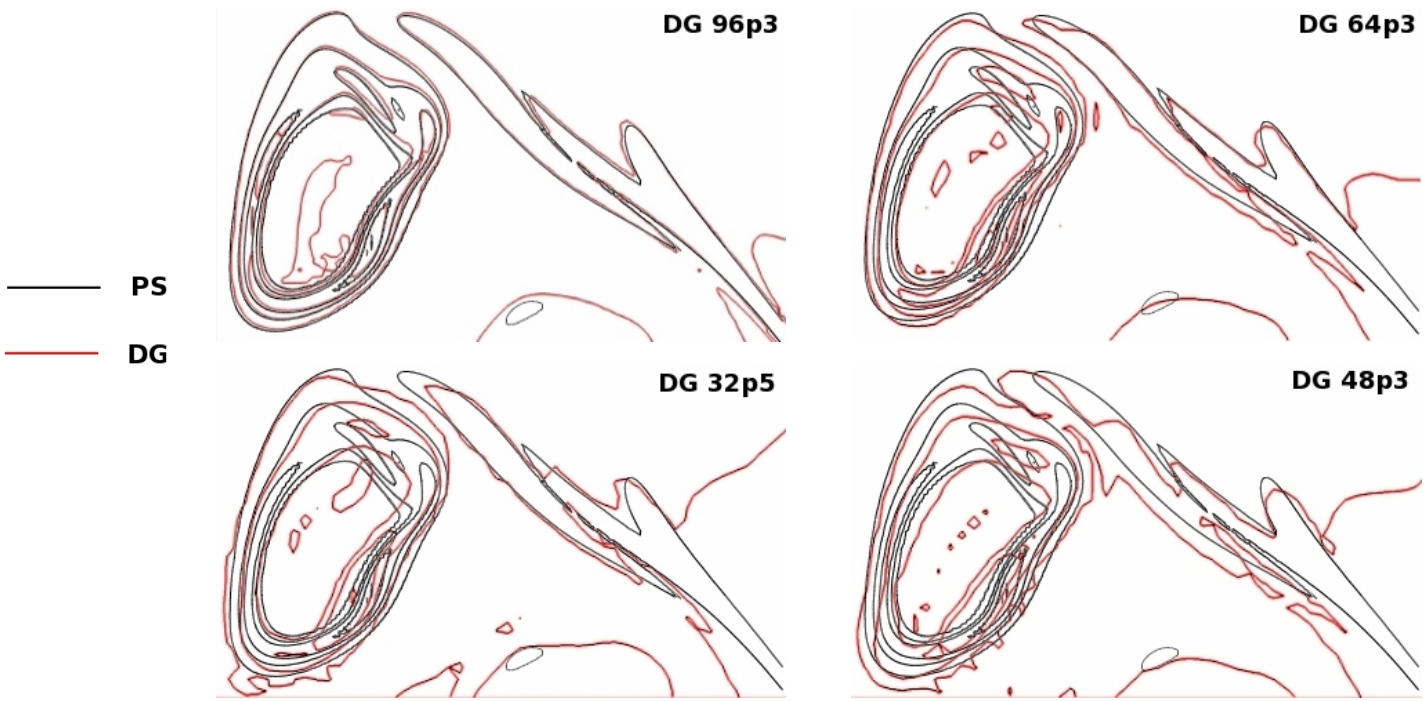
Evolution of enstrophy



Energy spectra at t=9

RE=1600 : Flow structure

- Modulus of vorticity on the (y, z) plane at $x = 0$



Taylor-Green : summary

- $Re = 500$
  - High-order DG simulations show similar results to PS computations at iso-number of DOFs
  - Representation of a richer spectral content as the polynomial order is increased
- $Re = 1600$
  - Accurate spectral and physical representation of turbulent structures for well-resolved DG computations
  - Acceptable results for under-resolved computations

Case description

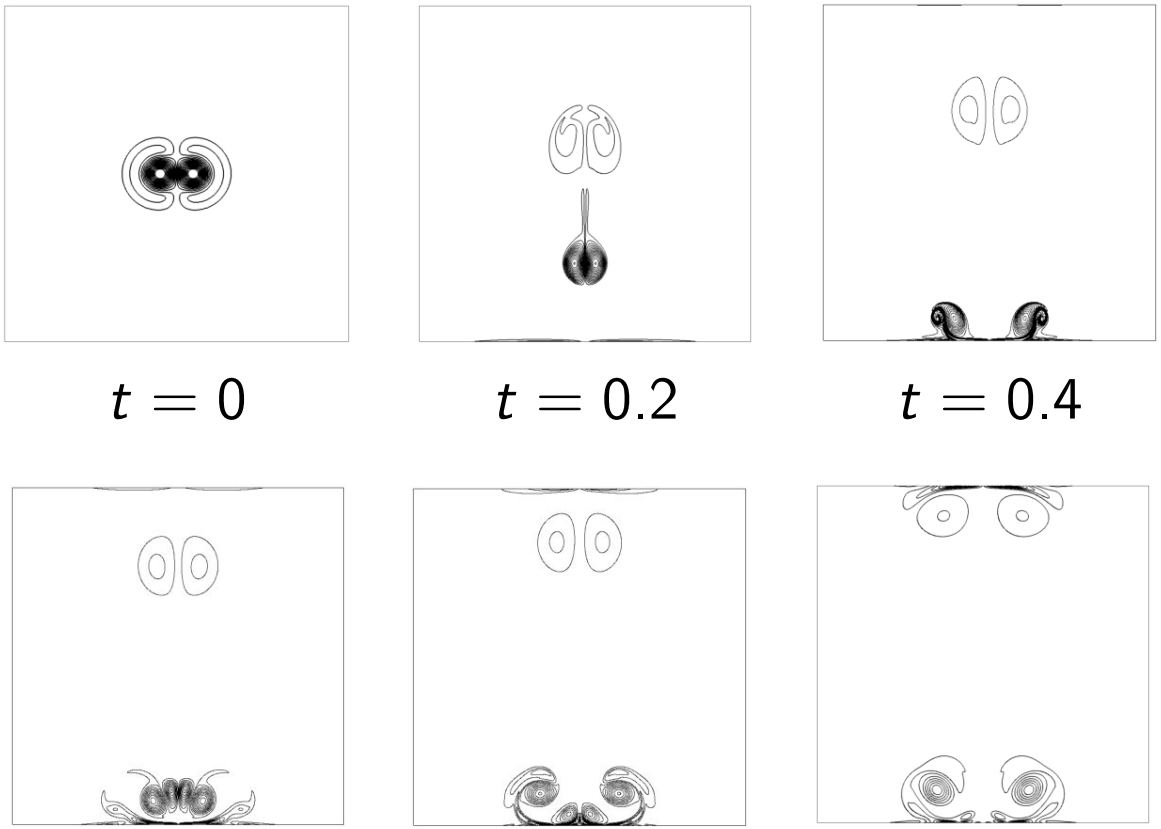
- 2D wall bounded flow involving complex wall-vortex interactions
- Analytical initialization of two counter rotating vortex monopoles in a square domain

$$\omega_i = \pm\omega_e \left(1 - (r_i/r_0)^2\right) e^{-\left(\frac{r_i}{r_0}\right)^2}, \; i = 1, 2$$

- $Re = h\sqrt{2E_0}/\nu = 1000$, where h is half of the domain length and E_0 the initial mean kinetic energy
- Monitoring the evolution of the mean energy and enstrophy

$$E_k = \frac{1}{V}\int_V \frac{\mathbf{u}.\mathbf{u}}{2} d\mathbf{x} \quad \xi = \frac{1}{V}\int_V \frac{\omega_z^2}{2} d\mathbf{x}$$

Illustration of the evolution of the flow[3]



$t = 0$ $t = 0.2$ $t = 0.4$

$t = 0.6$ $t = 0.8$ $t = 1.2$

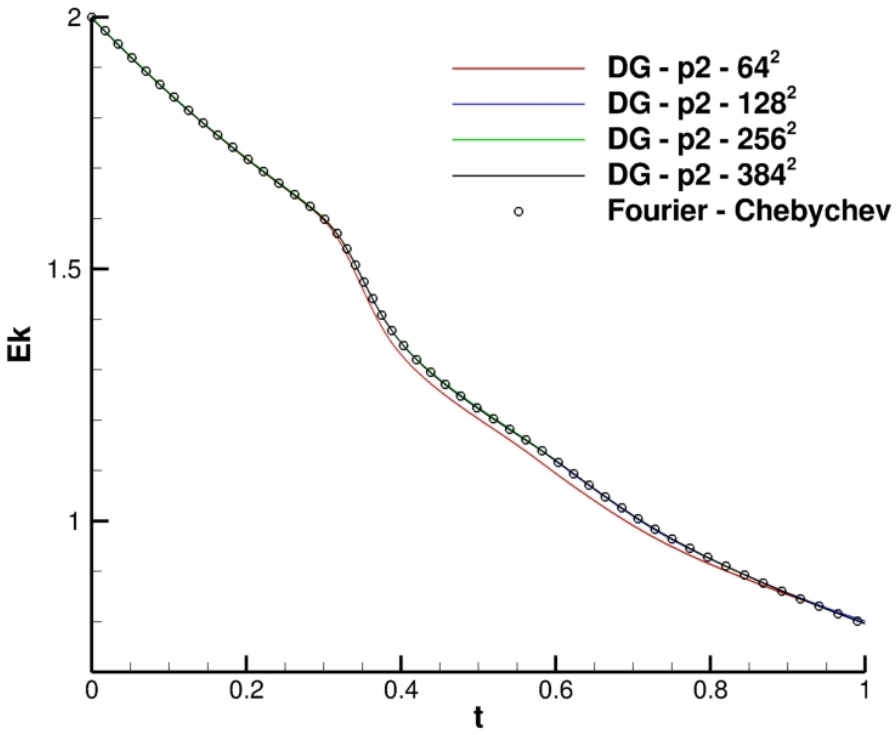
[3] Isosurfaces of ω_z, Spectral DNS (Keetels et al., JCP,2007)

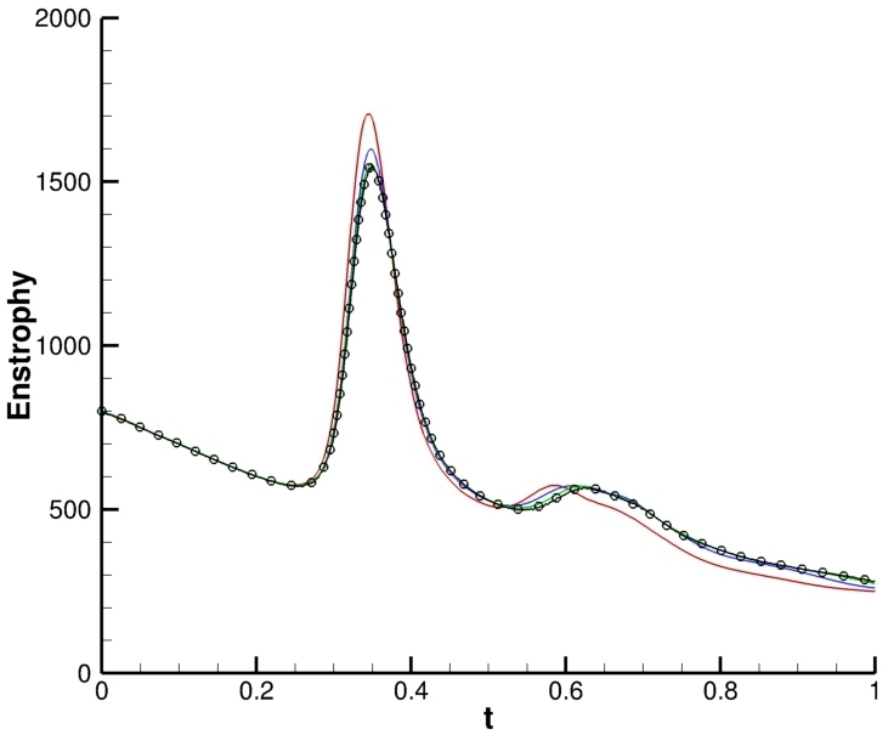
Mesh convergence : detail of computations

Computation	Elements	Nb. DOFs
DG $64p2$	64^2	192^2
DG $128p2$	128^2	384^2
DG $256p2$	256^2	768^2
DG $384p2$	384^2	1152^2
Fourier-Chebychev	-	2048×1024

- Key points
  - Evolution of the mean energy and enstrophy, vorticity contours
  - Representation of the collision process

Mesh convergence : Evolution of energy and enstrophy



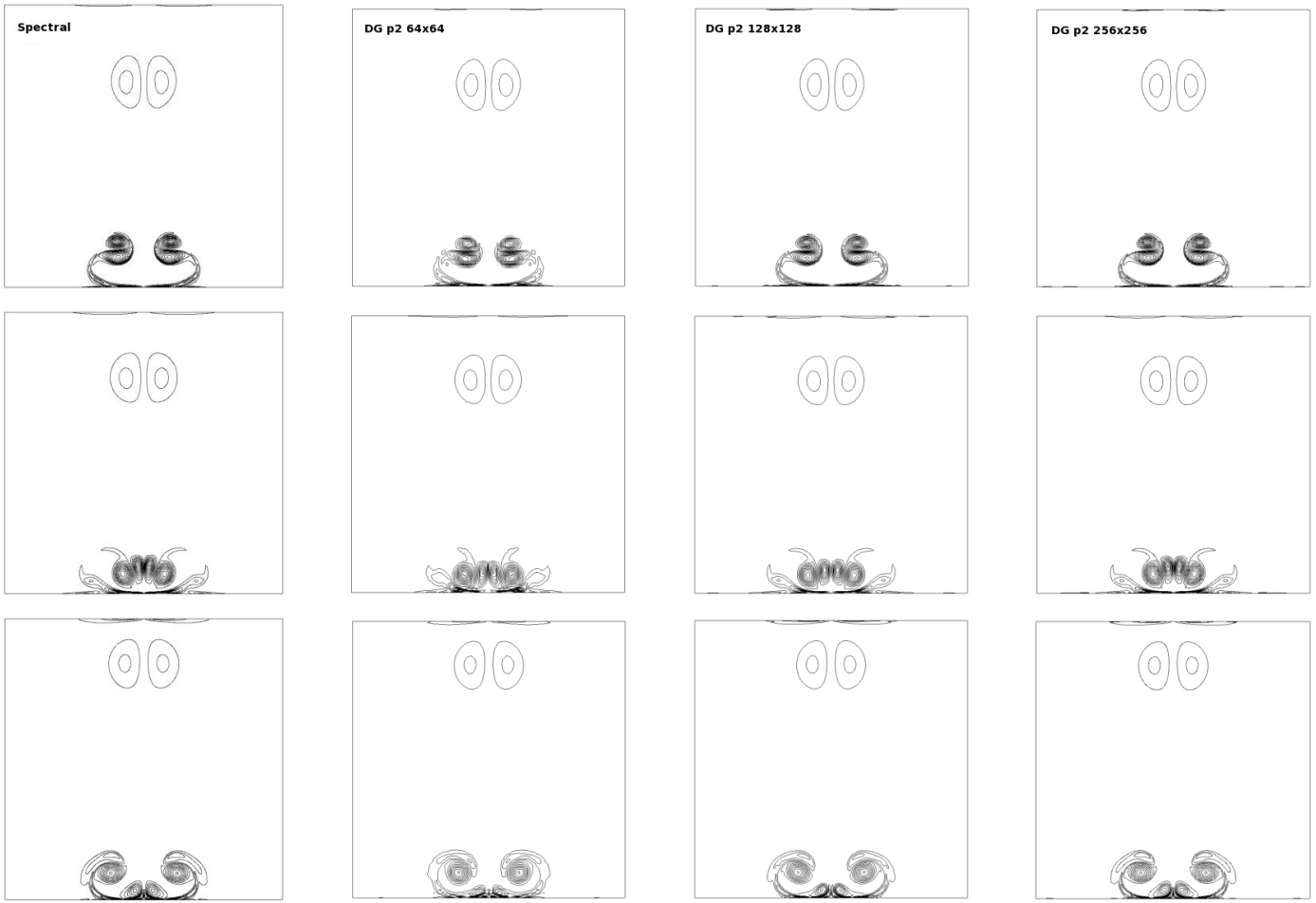


Mesh convergence : Vorticity plots

$t = 0.5$

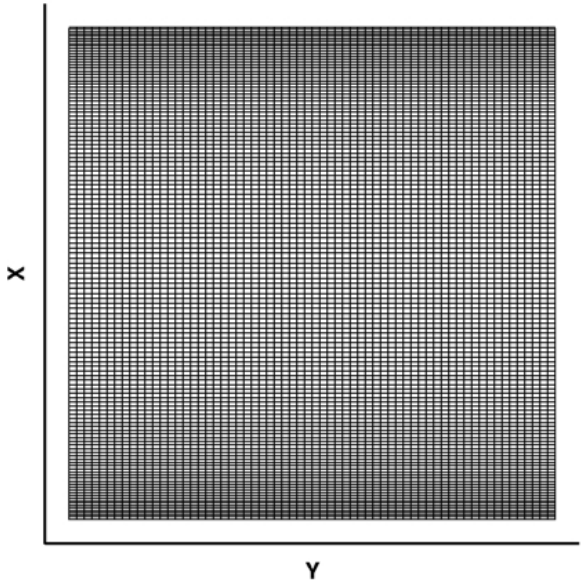
$t = 0.6$

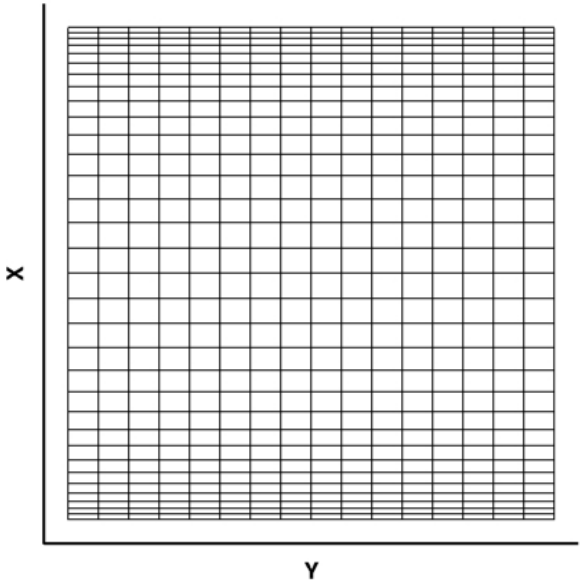
$t = 0.8$



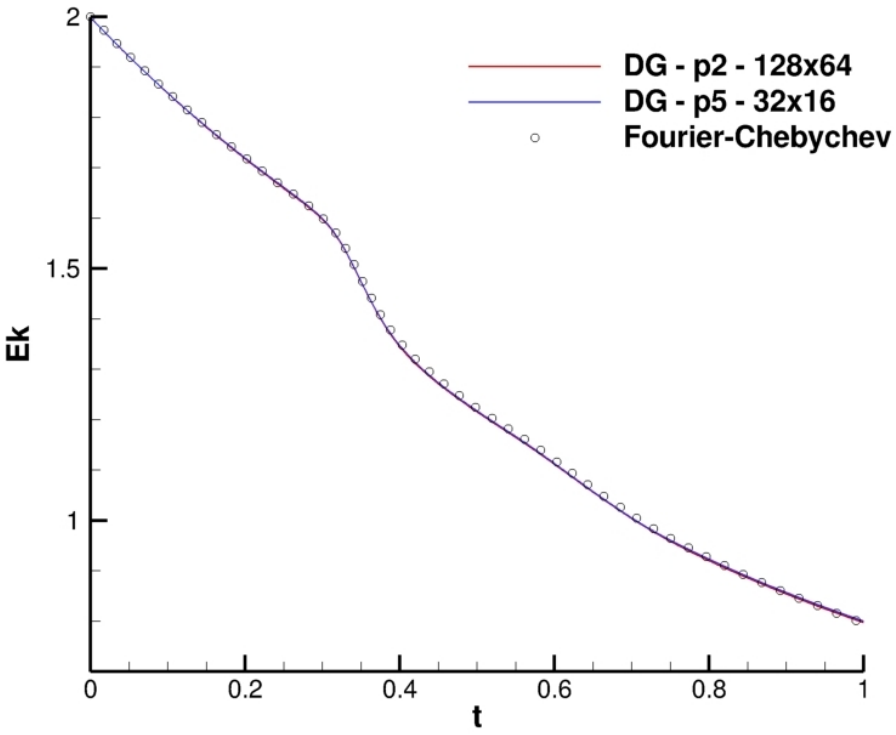
Stretched meshes

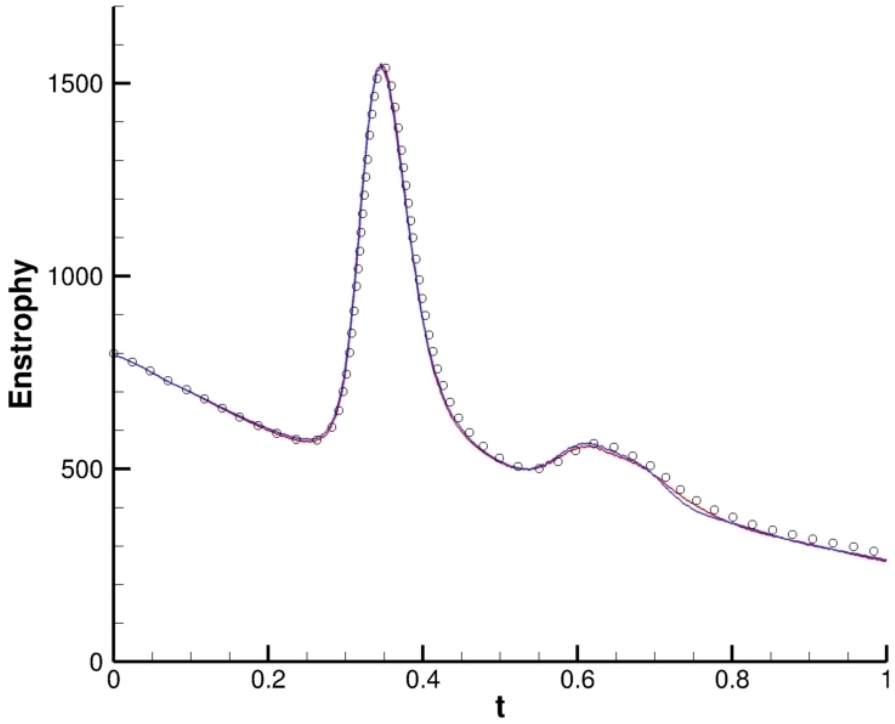
Computation	Order	Mesh	Nb. Dofs
DG $p2$	3	128×64	73728
DG $p5$	6	32×16	18432





Evolution of energy and enstrophy



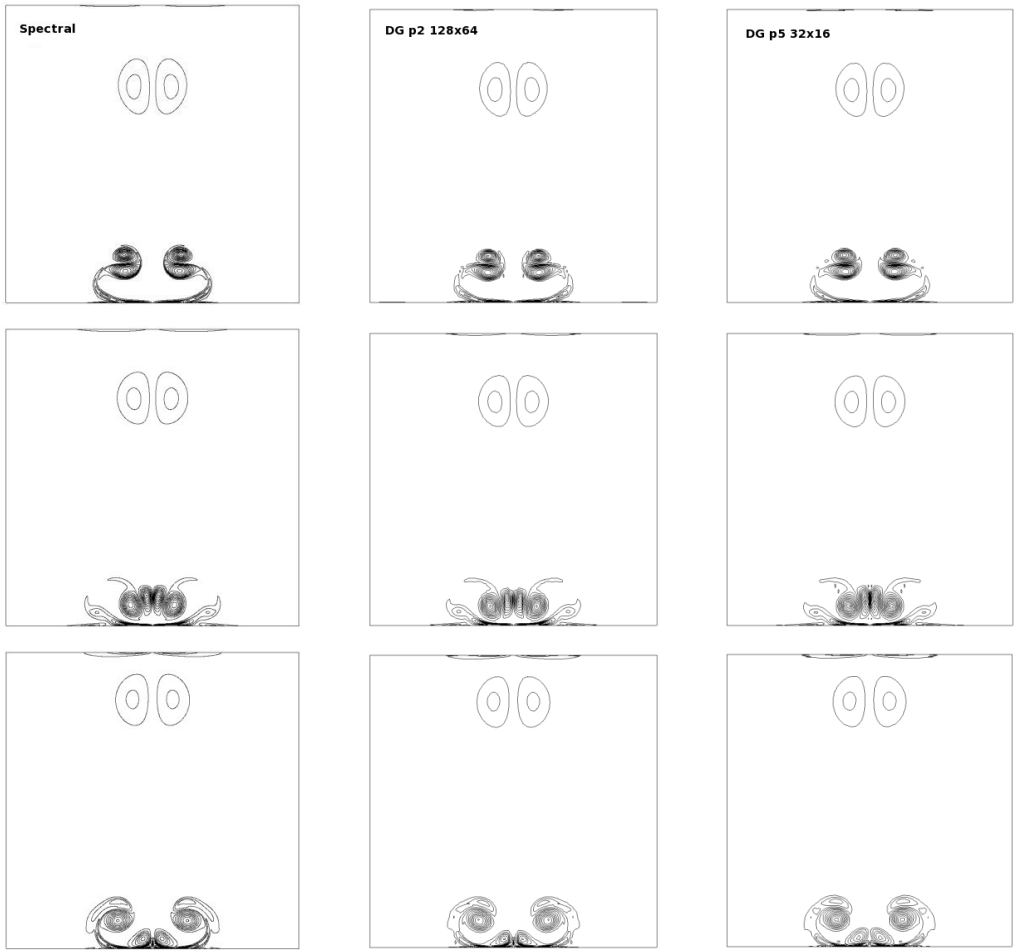


Vorticity plots

$t = 0.5$

$t = 0.6$

$t = 0.8$



Summary

- The DG method is able to represent complex near-wall phenomena
- High-order DG allows for a drastic reduction of the number of DOFs for the representation of the collision process

Conclusion

- DG methods well adapted to DNS of decaying turbulent flows
  - Good representation of structures in physical space
  - Accurate description of the spectral content
  - Good prediction of mean quantities
  - Equivalent to pseudo-spectral methods in terms of accuracy
- DG methods show an interest for wall bounded flows
  - Representation of complex near-wall phenomena