



Equivalence of two hybrid temporal-LES methods based on elliptic blending

Investigation in separated flows

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INTRODUCTION

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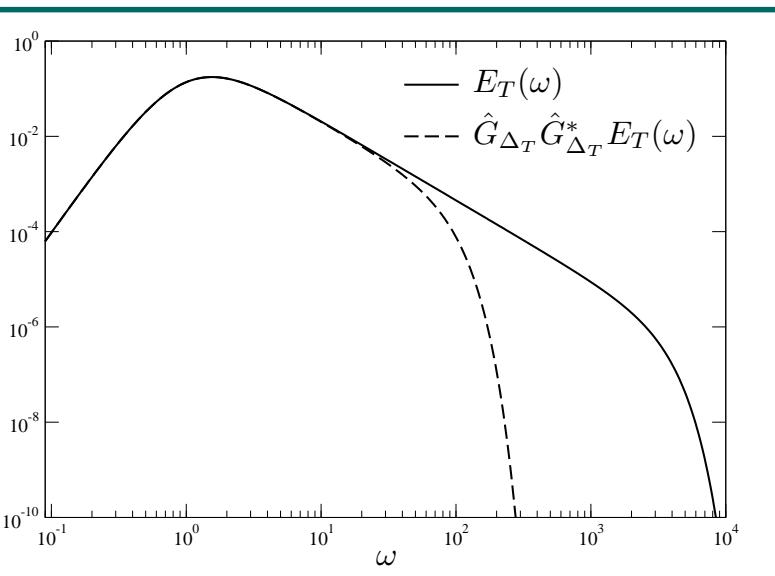
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- ✓ For inhomogeneous, stationary flows (statistical average independent of t): consistency issue addressed in the frame of temporal filtering \Rightarrow Hybrid Temporal LES/RANS (Fadai-Ghotbi, Friess, Manceau, Gatski, and Borée, 2010)

FORMALISM

- ✓ Temporal filtering approach within the LES formalism: TLES
(Pruett, 2000; Pruett, Gatski, Grosch, and Thacker, 2003)

$$\tilde{\mathbf{u}}(\mathbf{x}, t) = \int_{-\infty}^0 G_{\Delta_T}(\tau) \mathbf{u}(\mathbf{x}, t + \tau) d\tau \quad \Rightarrow \quad \frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}^{SFS}}{\partial x_j}$$



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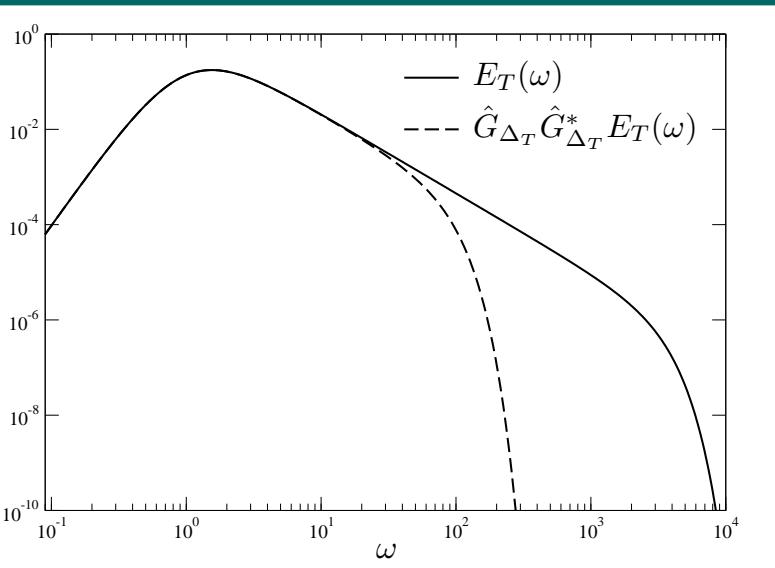
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RANS and TLES equations are form-invariant \Rightarrow provides a consistent basis for RANS-TLES hybridization



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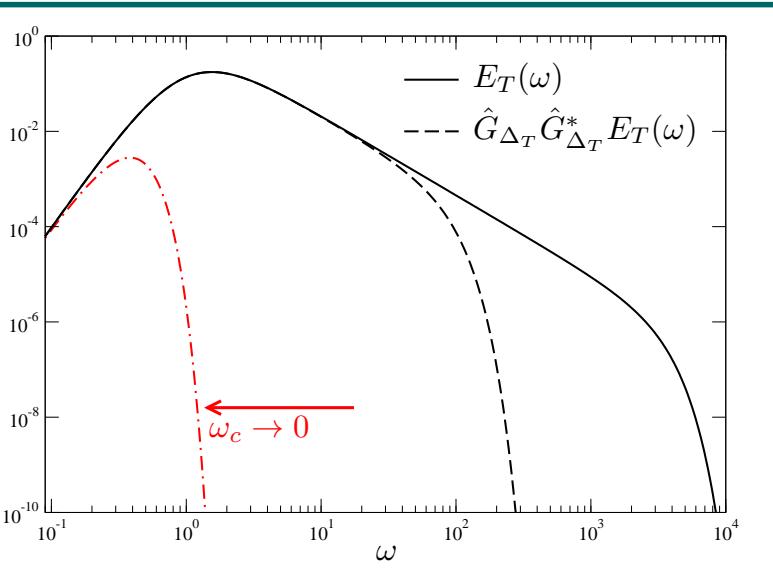
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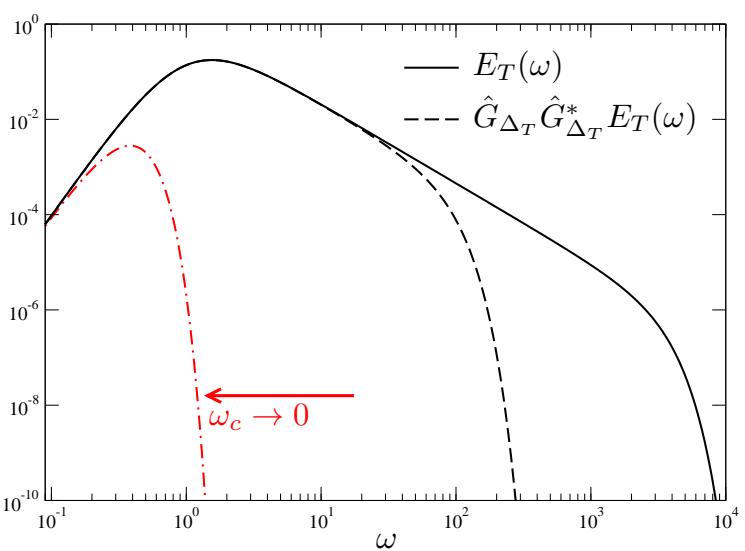
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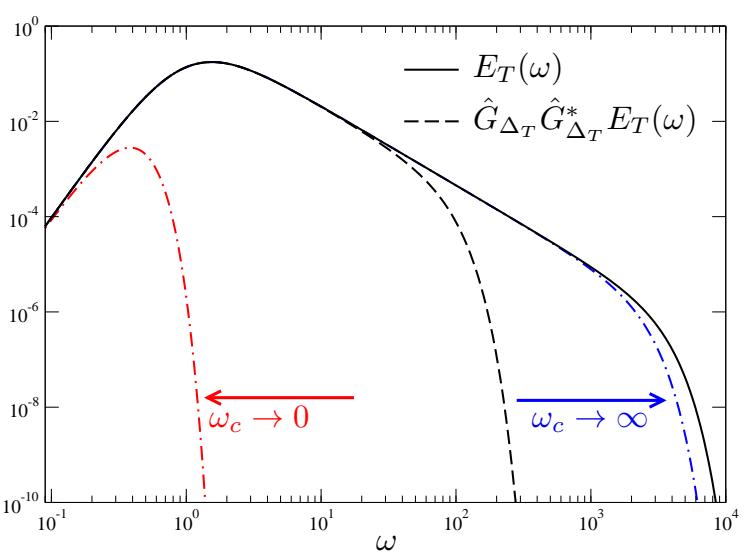
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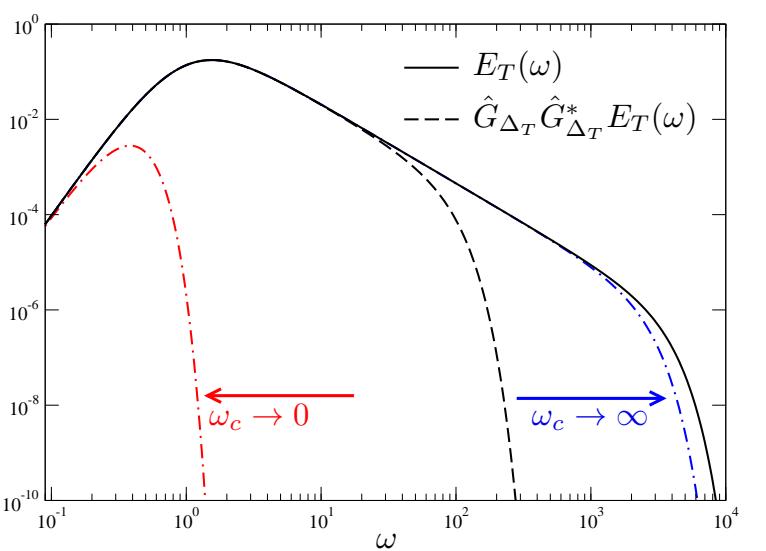
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✓ $\omega_c \rightarrow \infty$: SFS stresses vanish

\Rightarrow Navier-Stokes equations recovered

CONTROL OF THE RANS \longleftrightarrow LES TRANSITION

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- ✓ Partial integration over each zone of the transport equation for $E_T(\mathbf{x}, \omega)$

$$\begin{aligned}\frac{Dk_m}{Dt} &= P_m - \varepsilon_m + D_m \\ \frac{D\varepsilon}{Dt} &= C_{\varepsilon_1} \frac{\varepsilon}{k_m} P_m - \underbrace{\left[C_{\varepsilon_1} + r(C_{\varepsilon_2} - C_{\varepsilon_1}) \right]}_{C_{\varepsilon_2}^*(\omega_c)} \frac{\varepsilon^2}{k_m} + D_\varepsilon\end{aligned}$$

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= measure of the energy partition among resolved and subfilter scales

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$\frac{dk_m}{dt} = P_m - \varepsilon + D_m$ $\frac{d\varepsilon}{dt} = C_{\varepsilon 1} \frac{\varepsilon}{k_m} P_m - C_{\varepsilon 2}^*(\omega_c) \frac{\varepsilon^2}{k_m} - D_\varepsilon$	$\frac{dk_m}{dt} = P_m - \psi(\omega_c) \varepsilon + D_m$ $\frac{d\varepsilon}{dt} = C_{\varepsilon 1} \frac{\varepsilon}{k_m} P_m - C_{\varepsilon 2} \frac{\varepsilon^2}{k_m} - D_\varepsilon$
$C_{\varepsilon 2}^*(\omega_c) = C_{\varepsilon 1} + r(\omega_c) (C_{\varepsilon 2} - C_{\varepsilon 1})$	$\psi(\omega_c) = \frac{k_m^{3/2}/\varepsilon}{L(\omega_c)}$

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- ✓ Equivalence (Manceau et al., 2010):
 ψ function that gives the same energy partition as the $C_{\varepsilon 2}^*$ function?

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- ✓ By integration: Equivalence $\Rightarrow \psi = 1 + \left(\frac{C_{\varepsilon 2}}{C_{\varepsilon 1}} - 1 \right) \left(1 - r^{\frac{C_{\varepsilon 1}}{C_{\varepsilon 2}}} \right)$

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$$\checkmark \quad \psi = \frac{k_m^{3/2}/\varepsilon}{L} \quad \Rightarrow \quad L = \frac{r^{3/2}}{1 + \left(\frac{C_{\varepsilon 2}}{C_{\varepsilon 1}} - 1 \right) \left(1 - r^{\frac{C_{\varepsilon 1}}{C_{\varepsilon 2}}} \right)} L_{int} \quad \text{where} \quad L_{int} = \frac{k^{3/2}}{\varepsilon}$$

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✓ Evaluation of r as a function of ω_c

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$U_s = \sqrt{U^2 + \gamma k}$ is the sweeping velocity (Tennekes, 1975)

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- ✓ Cutoff frequency = maximum observable frequency $\omega_c = \min \left(\frac{\pi}{dt}; \frac{U_s \pi}{\Delta} \right)$

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- ✓ EB-RSM (Manceau and Hanjalić, 2002) : a single scalar α accounting for wall effects

$$\triangleright \alpha + L^2 \nabla^2 \alpha = 1 \quad \phi_{ij}^* - \varepsilon_{ij} = \alpha^3 (\phi_{ij}^* - \varepsilon_{ij})_w + (1 - \alpha^3) (\phi_{ij}^* - \varepsilon_{ij})_h$$

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▷ H-TLES :

$$\begin{cases} \frac{d\tau_{ijSFS}}{dt} &= P_{ijSFS} + \phi_{ijSFS}^* + D_{ijSFS}^T + D_{ijSFS}^\nu - \frac{k_{SFS}^{3/2}}{L\varepsilon_{SFS}} \varepsilon_{ijSFS} \\ \frac{d\varepsilon_{SFS}}{dt} &= \text{standard} \end{cases}$$

DERIVATION OF AN EQUIVALENT DES SUBFILTER STRESS MODEL

- ✓ EB–RSM (Manceau and Hanjalić, 2002) : a single scalar α accounting for wall effects

$$\triangleright \alpha + L^2 \nabla^2 \alpha = 1 \quad \phi_{ij}^* - \varepsilon_{ij} = \alpha^3 (\phi_{ij}^* - \varepsilon_{ij})_w + (1 - \alpha^3) (\phi_{ij}^* - \varepsilon_{ij})_h$$

- ✓ Modified EB–RSM model

▷ H-TLES :

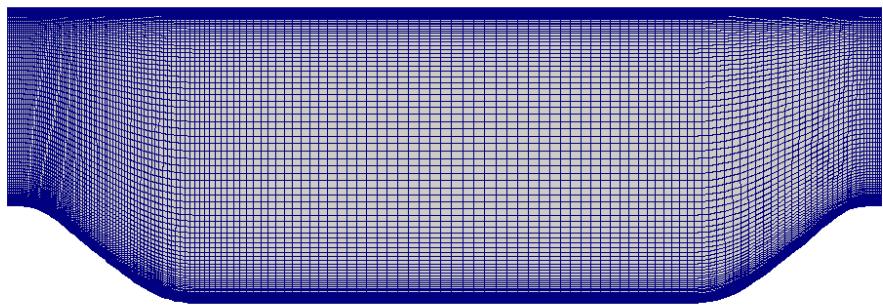
$$\begin{cases} \frac{d\tau_{ijSFS}}{dt} &= P_{ijSFS} + \phi_{ijSFS}^* + D_{ijSFS}^T + D_{ijSFS}^\nu - \frac{k_{SFS}^{3/2}}{L\varepsilon_{SFS}} \varepsilon_{ijSFS} \\ \frac{d\varepsilon_{SFS}}{dt} &= \text{standard} \end{cases}$$

▷ T-PITM :

$$\begin{cases} \frac{d\tau_{ijSFS}}{dt} &= \text{standard} \\ \frac{d\varepsilon_{SFS}}{dt} &= C_{\varepsilon 1} \frac{P_{SFS} \varepsilon_{SFS}}{k_{SFS}} - C_{\varepsilon 2}^* \frac{\varepsilon_{SFS}^2}{k_{SFS}} + \text{Diff.} \end{cases}$$

FLOW OVER A PERIODIC HILL

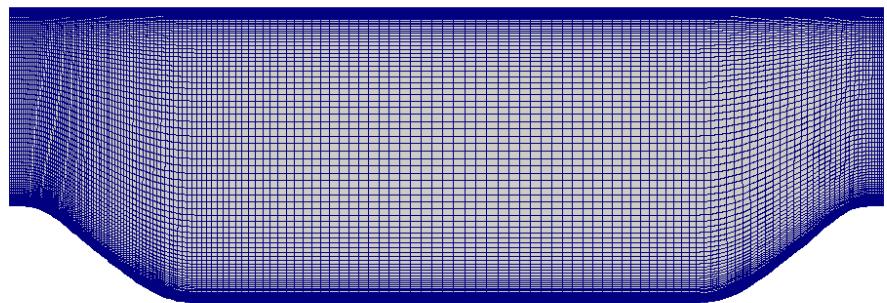
CONFIGURATION AND NUMERICS



- ✓ Periodically constricted channel

FLOW OVER A PERIODIC HILL

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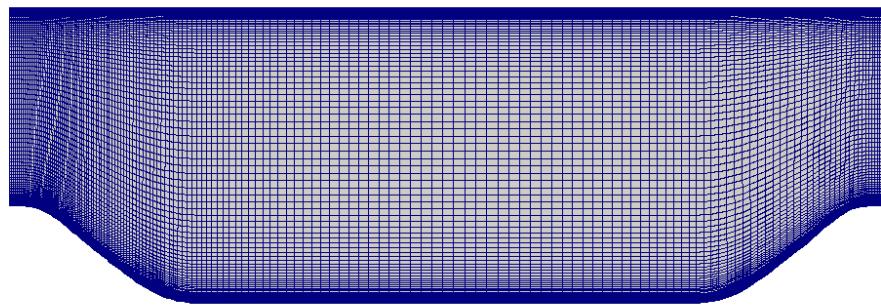


✓ 2 meshes

- ✓ Periodically constricted channel
- ✓ $Re_b = U_b h / \nu = 10595$

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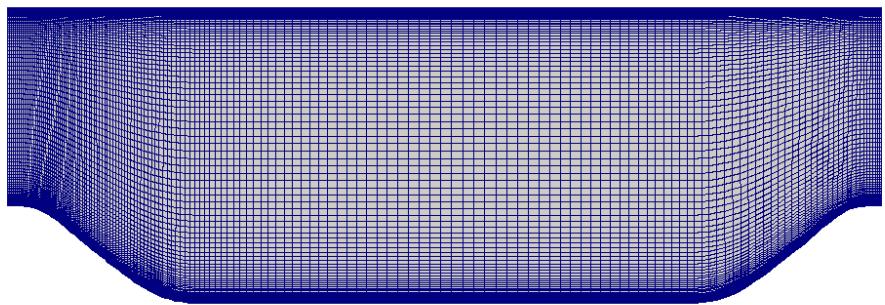


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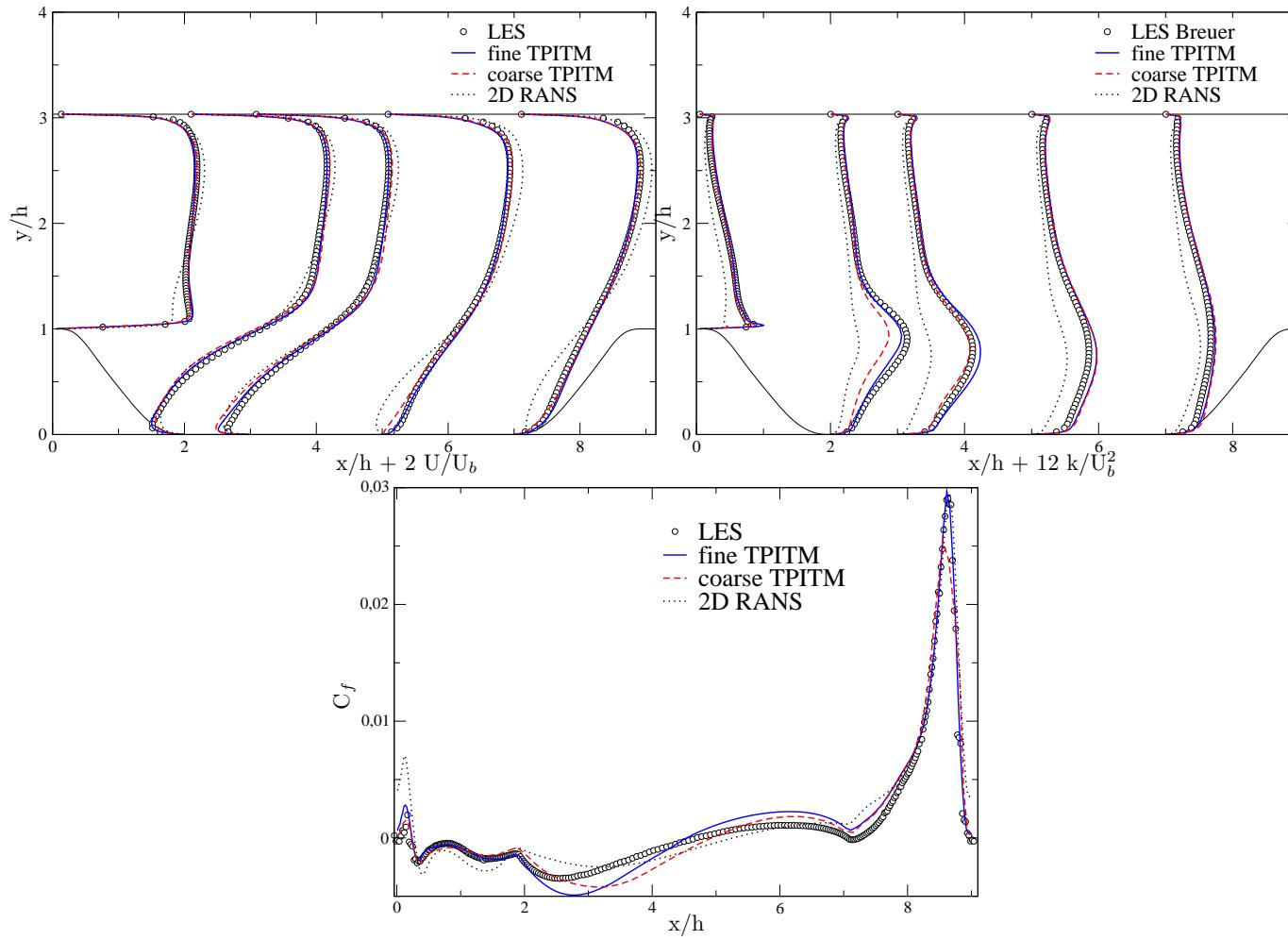
NUMERICS

- ✓ Code Saturne: *open source* finite volume solver developed by EDF
- ✓ CDS convection scheme for resolved momentum + UDS for _{SFS} quantities

FLOW OVER A PERIODIC HILL

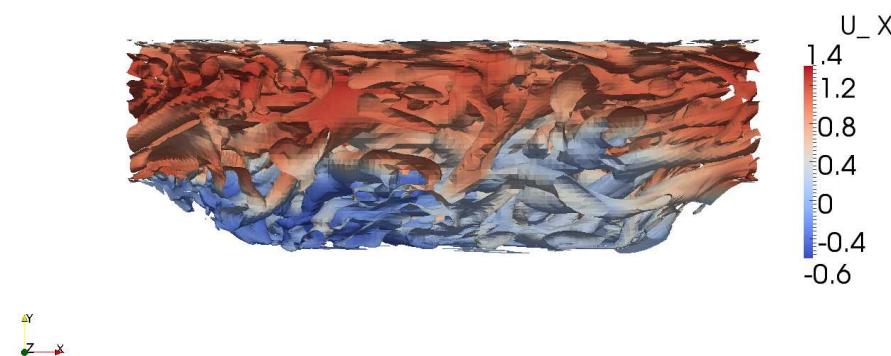
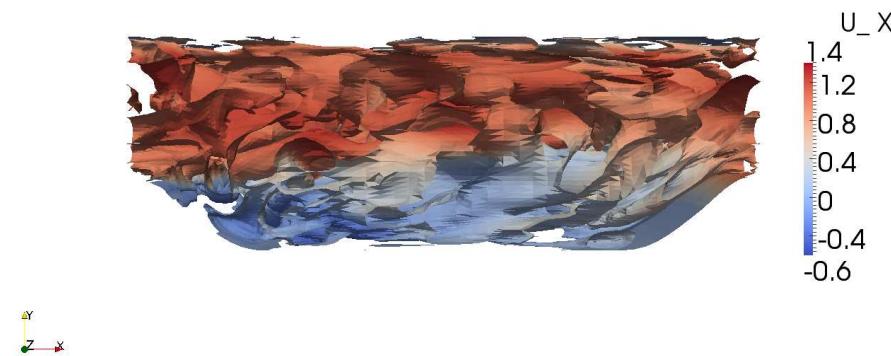
EFFECTS OF GRID REFINING

- ✓ T-PITM results for a coarse and a fine grid, vs 2D RANS and LES



FLOW OVER A PERIODIC HILL

- ✓ Isocontours of zero-Q criterion : coarse & fine grid

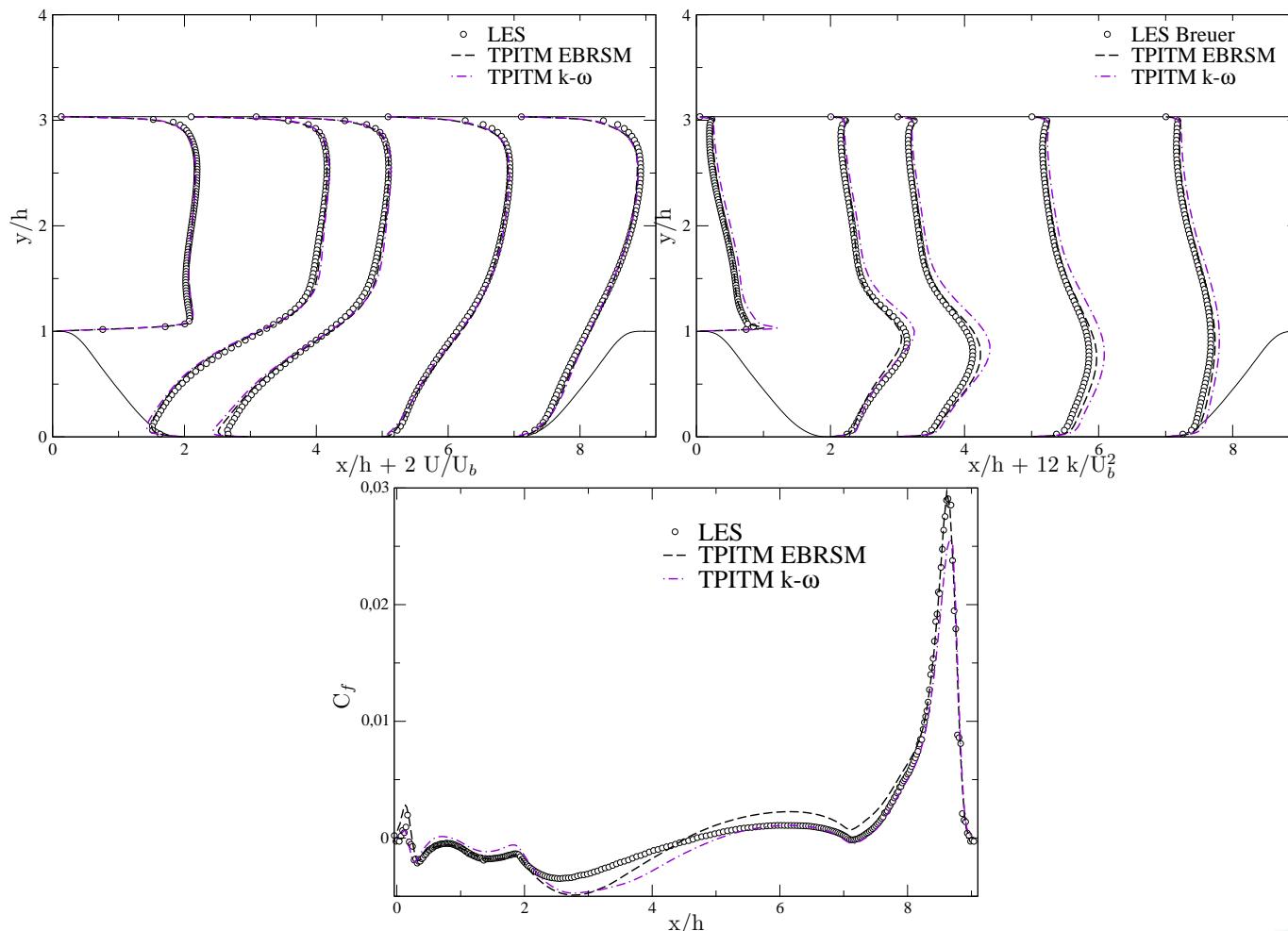


⇒ refining the grid allows resolving more structures

FLOW OVER A PERIODIC HILL

EFFECTS OF THE SUBFILTER CLOSURE MODEL

- ✓ T-PITM results for coarse grid : EB-RSM vs $k-\omega$ of (Bentaleb and Manceau, 2011)

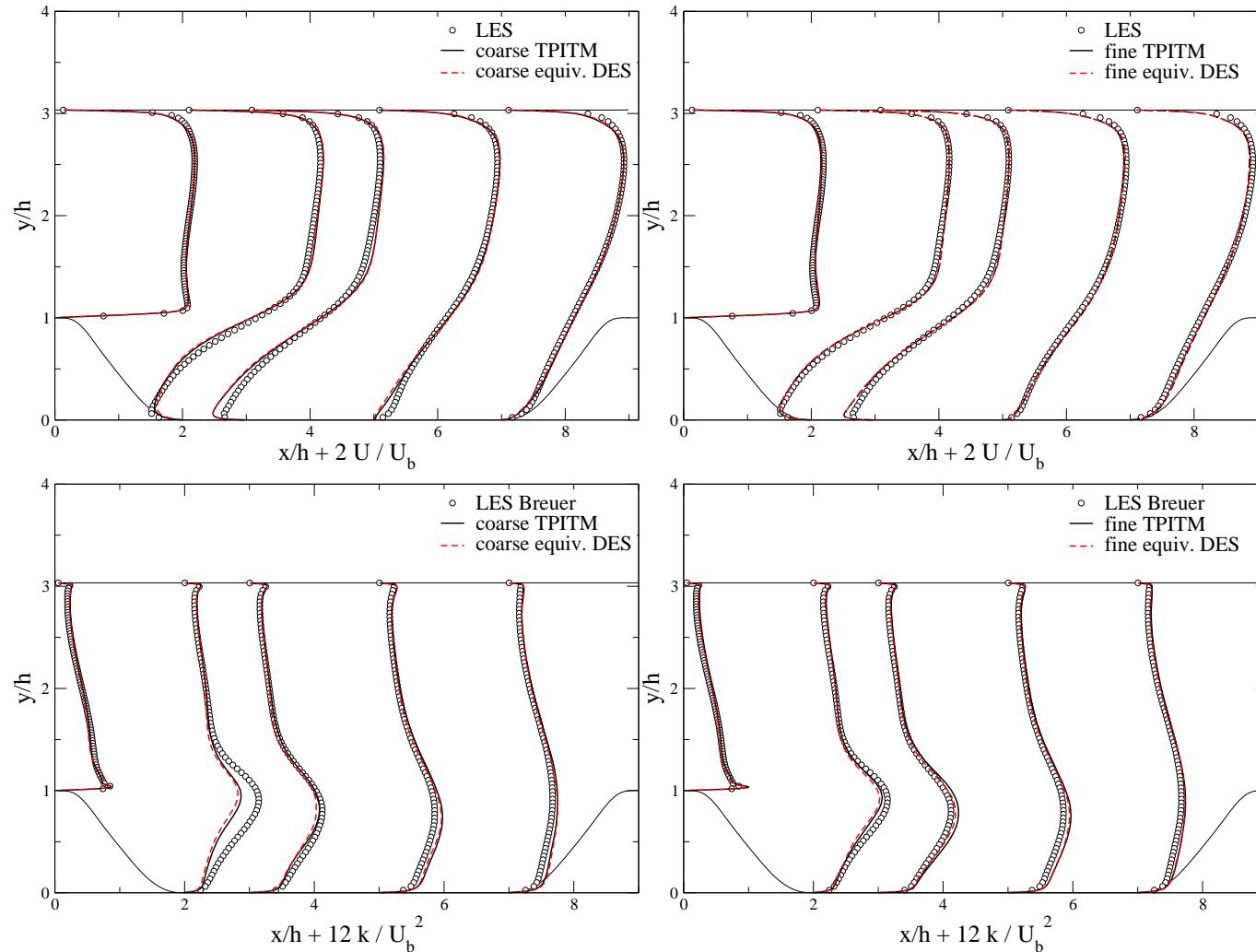


Streamwise velocity, total kinetic energy & friction coefficient

FLOW OVER A PERIODIC HILL

EFFECTS OF THE HYBRID METHOD

- ✓ T-PITM vs. equivalent DES on both grids, same EB-RSM closure



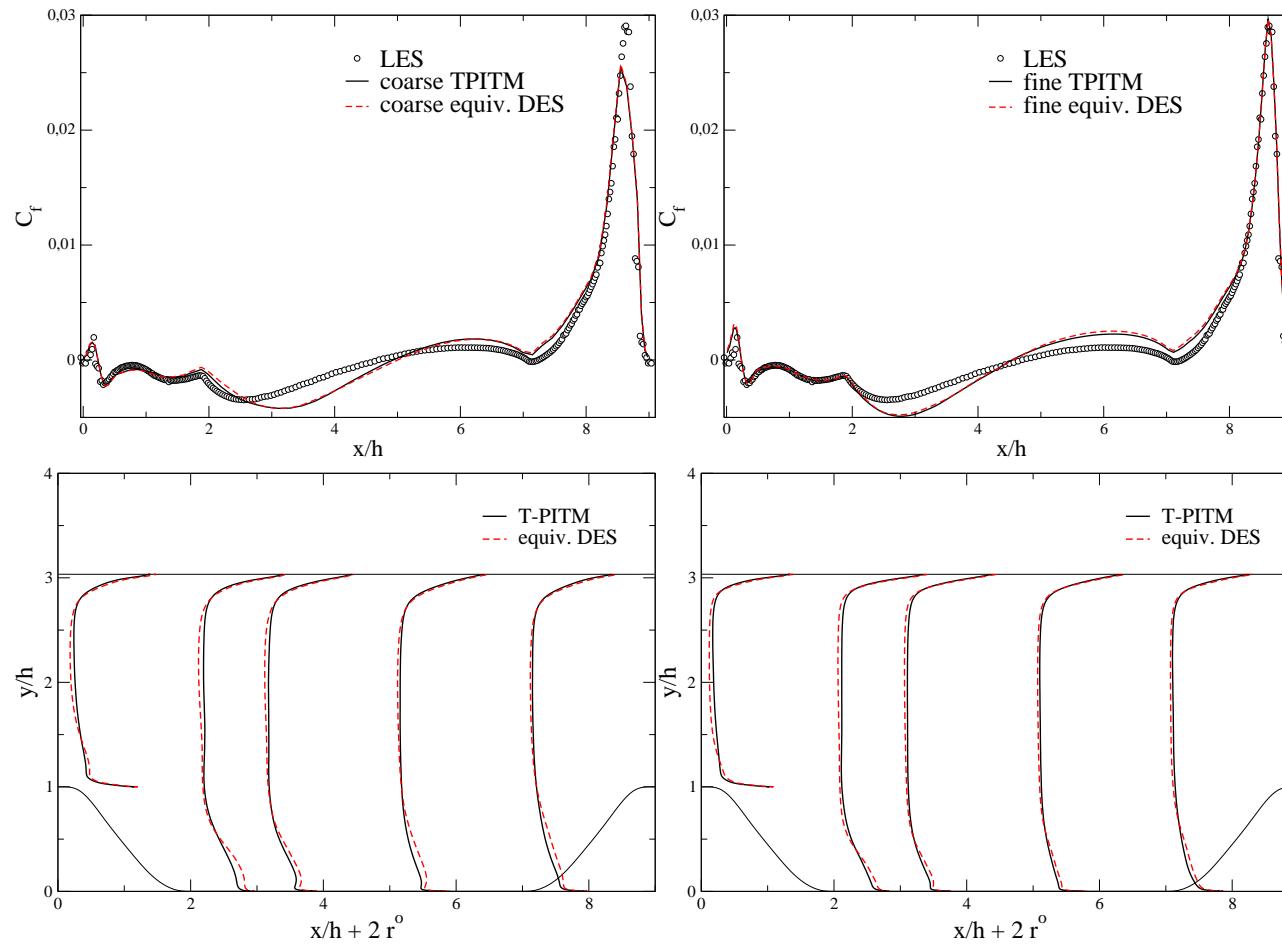
Streamwise velocity & total kinetic energy. Left : coarse grid ; right : fine grid



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Friction coefficient & *observed* energy ratio r^o . Left : coarse grid ; right : fine grid

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 - ▷ as for DES, based on a length-scale modification in the subfilter stress equations

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 - ▷ Satisfactory results : RANS << hybrid \sim LES
 - ▷ Equivalence T-PITM / equiv. DES observed for 2 grids
 - same resolution level + same subfilter closure \Leftrightarrow same statistics

REFERENCES

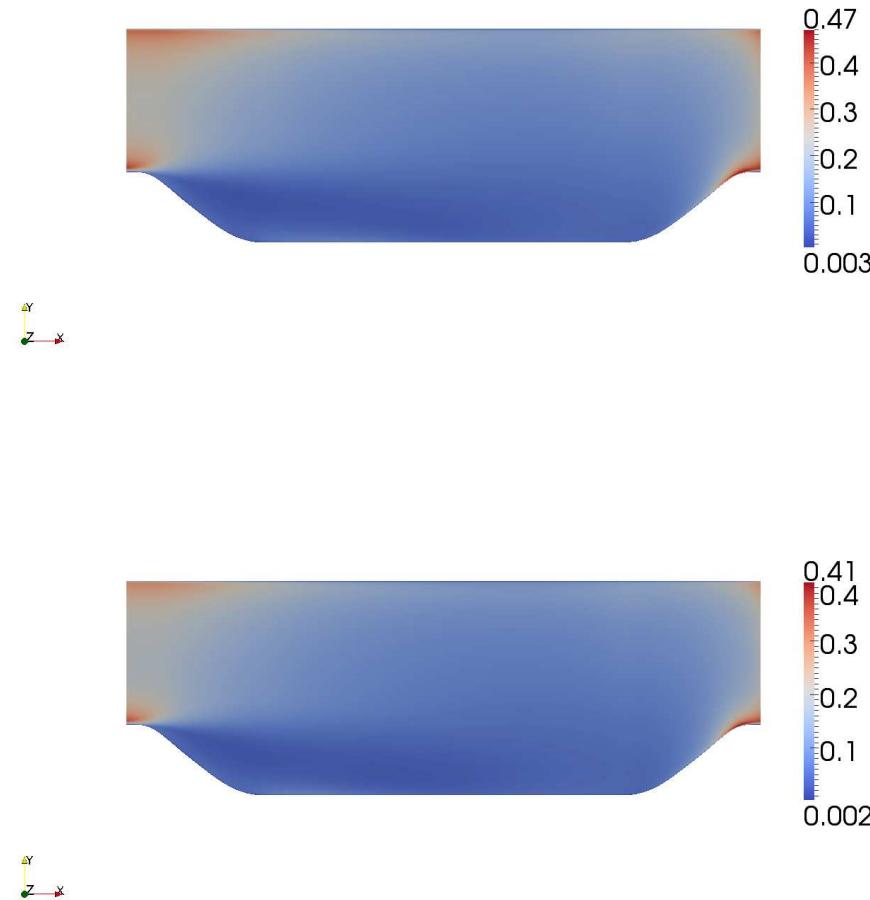
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Temporal filtering and cutoff frequency

- ✓ Contours of $U_S dt / \Delta$: coarse & fine mesh



⇒ convective time scale governs temporal filtering