GDR Turbulence

Poitiers, 15-17 octobre 2012

« Turbulence compressible en astrophysique »

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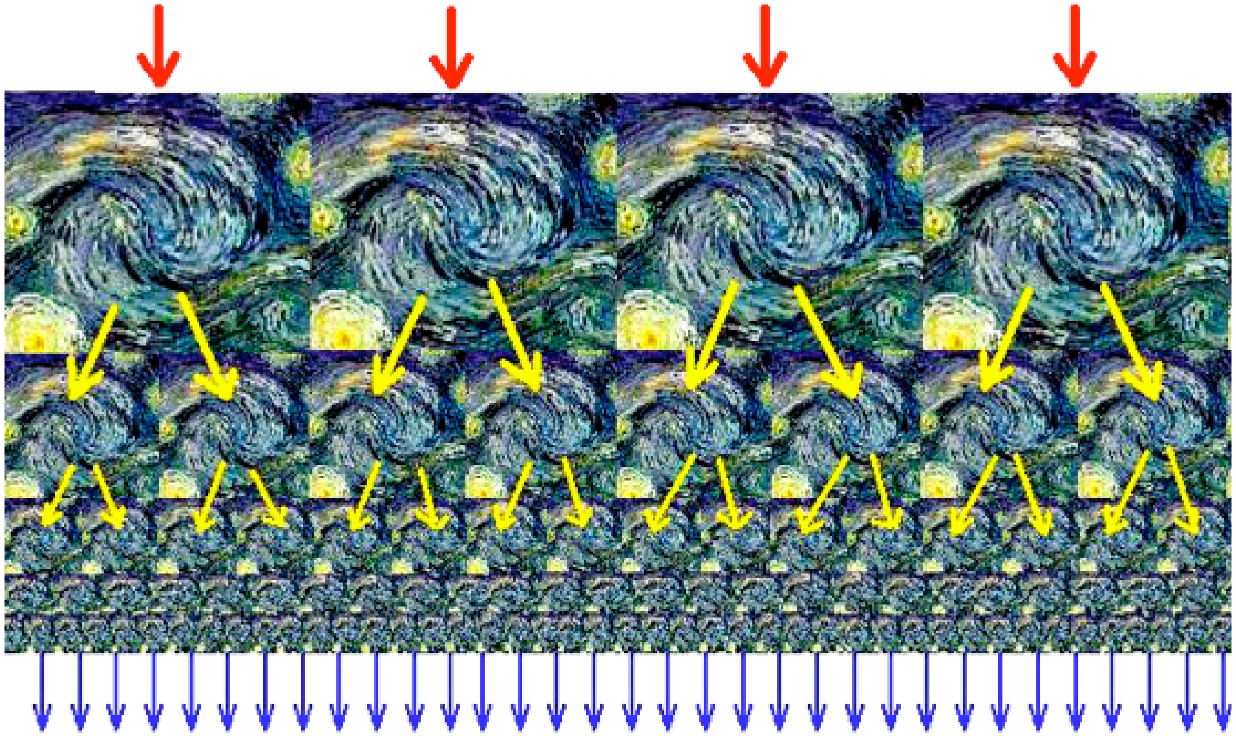
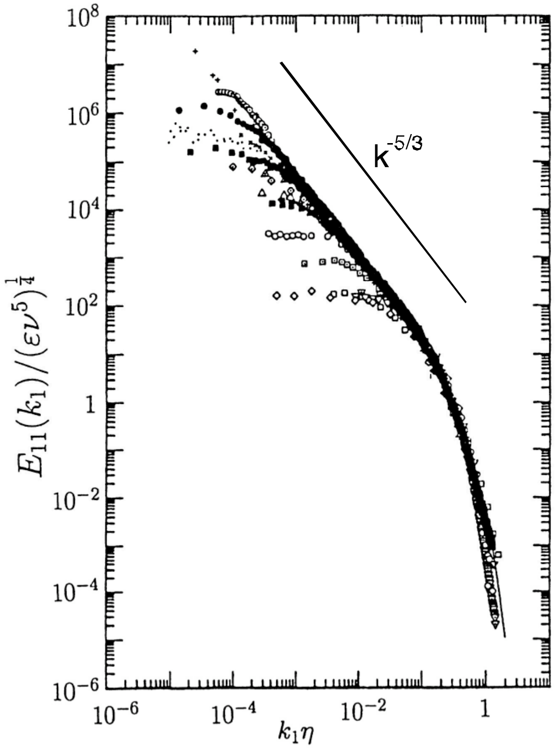
Picture for incompressible turbulence

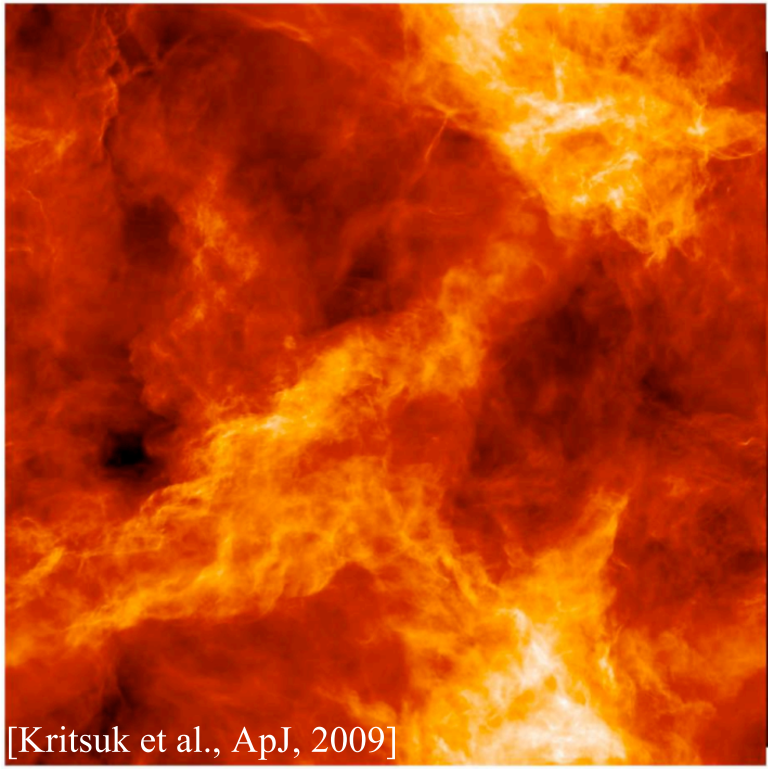




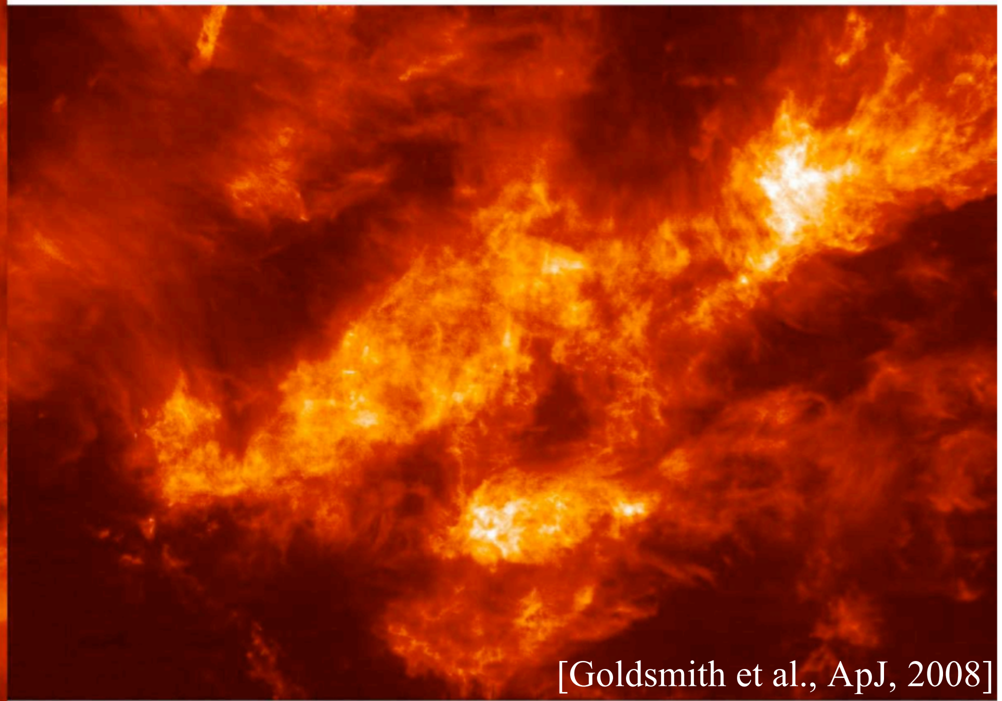
Is it the good picture for compressible turbulence ?

Interstellar turbulence

2048^3, DNS, Mach 6, no gravity
isothermal hydro turbulence



Interstellar cloud Taurus
(^{12}CO J=1-0 map)

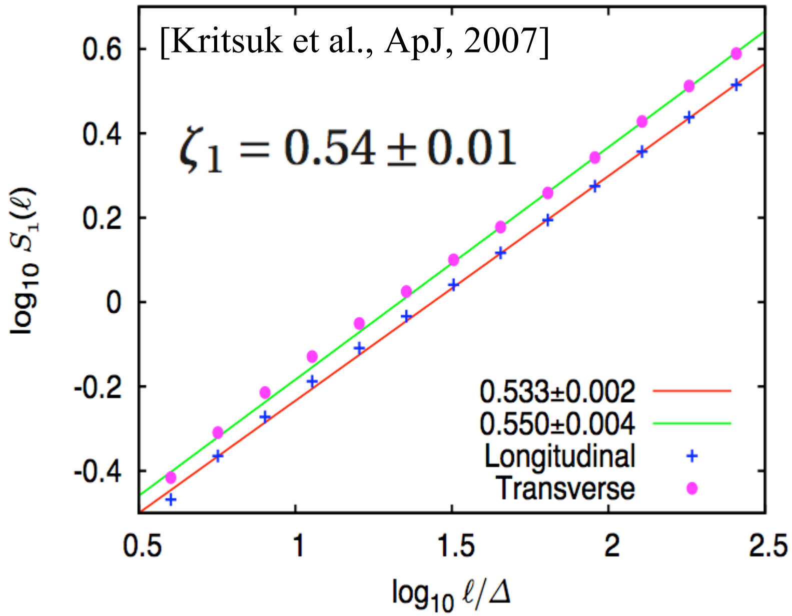


Density structures are morphologically similar

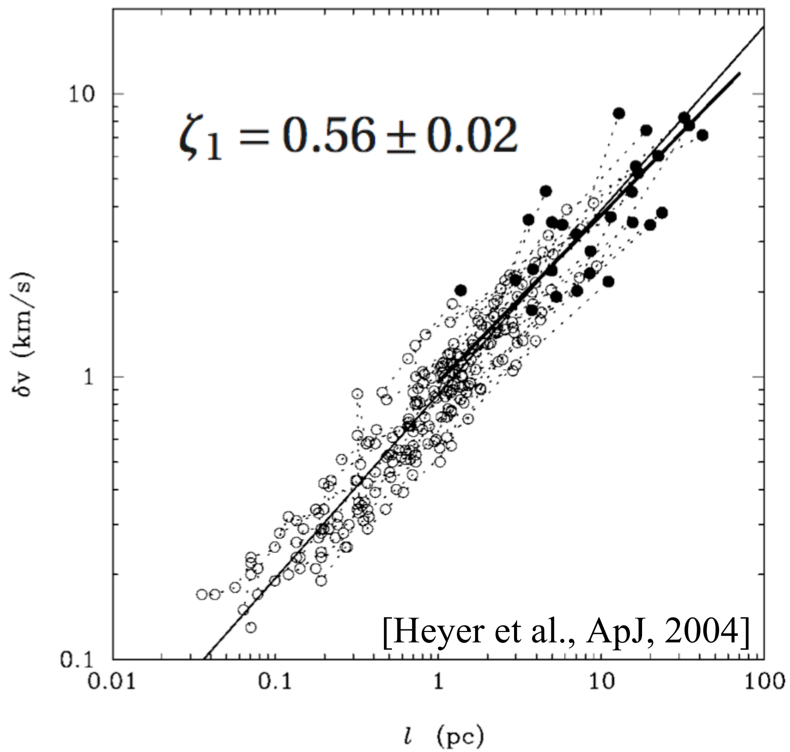
First-order structure functions of velocity:

$$S_1(u,\ell) \equiv \langle|\delta u|\rangle = u_0 \ell^{\zeta_1}$$

1024^3, DNS, Mach 6, no gravity isothermal HD turbulence

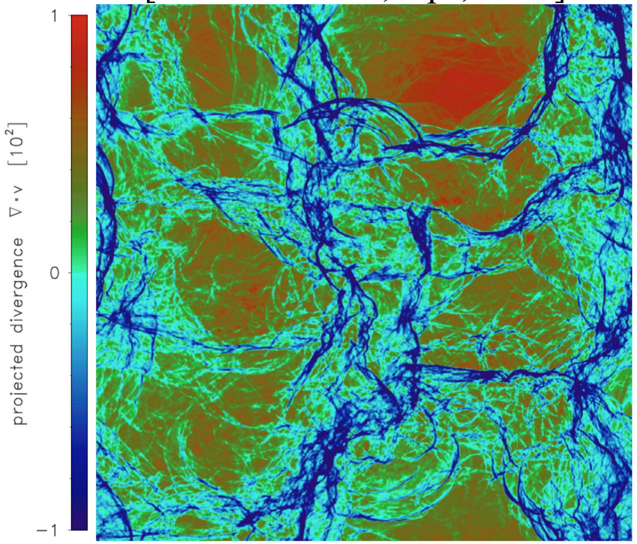


Sample of 27 interstellar clouds



First-order structure function with similar slopes

$\nabla \cdot \mathbf{u}$ [Federrath et al., ApJ, 2010]



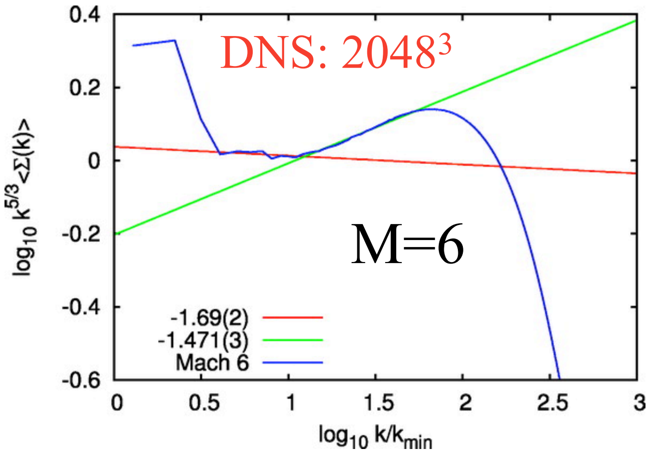


FIG. 19.—Time-averaged power spectrum of the density-weighted velocity $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$ compensated by $k^{5/3}$. The straight lines represent the least-squares fits to the data for $\log k/k_{\rm min} \in [0.5, 1.1]$ and $\log k/k_{\rm min} \in [1.2, 1.8]$. The inertial subrange slope is in excellent agreement with the model prediction. [*See the electronic edition of the Journal for a color version of this figure.*]

[Kritsuk et al., ApJ, 2007]

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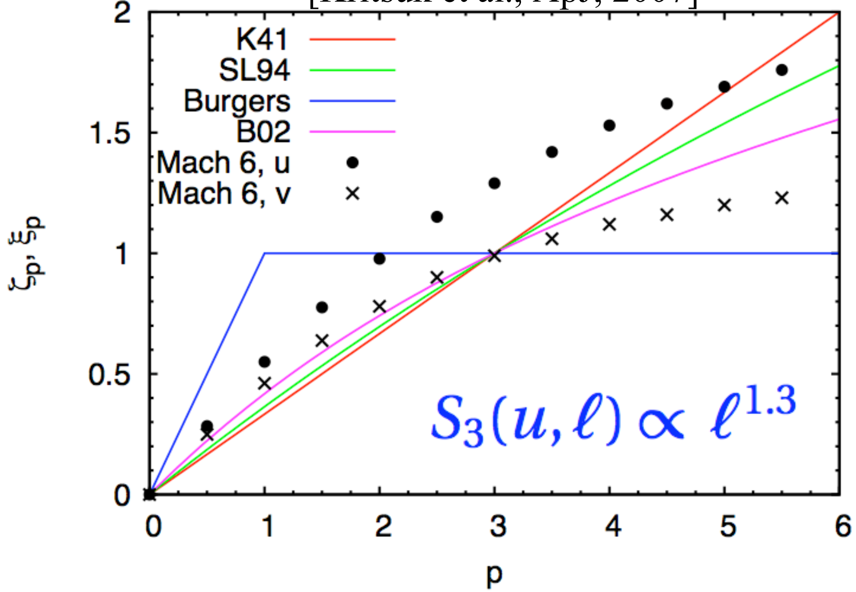

Density-weighted velocity, $\mathbf{v} \equiv \mathbf{u}\,\rho^{1/3}$, seems to have a universal behavior

Why ?

Is the Scaling of Supersonic Turbulence Universal?

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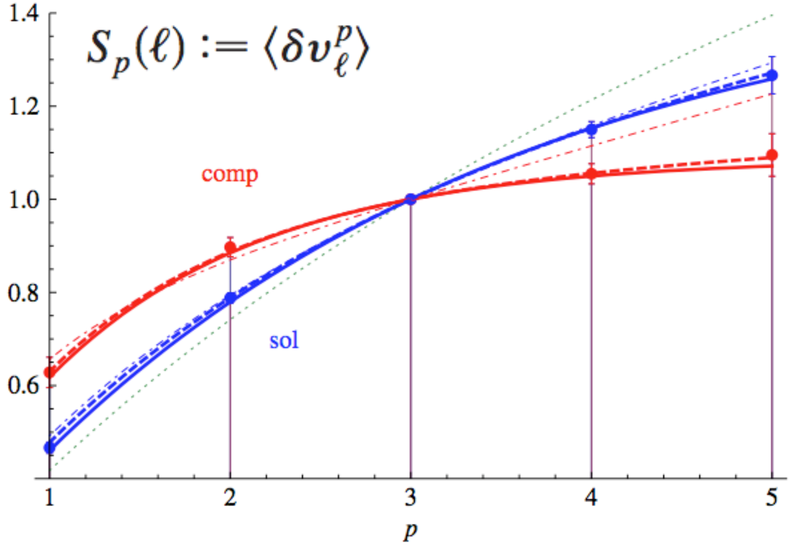
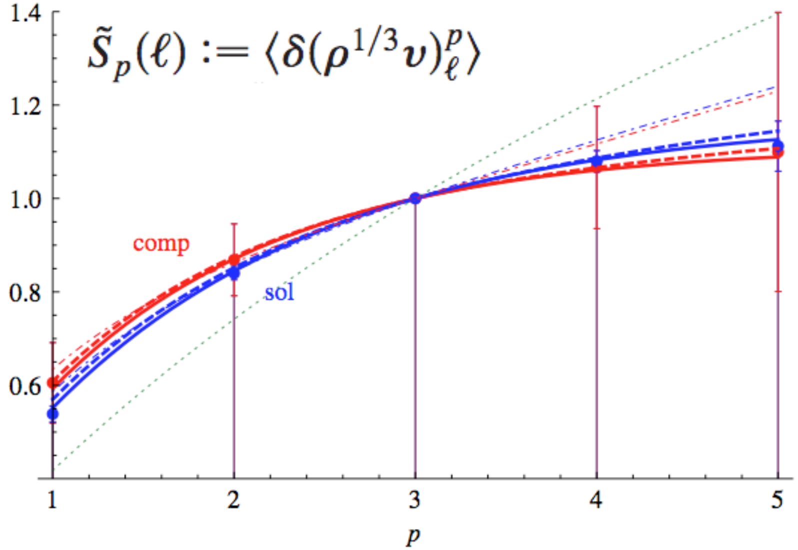
$$Z_p := \frac{\zeta_p}{\zeta_3}$$





Is there a Kolmogorov law in compressible turbulence ?

- Incompressible HD: $-\frac{4}{3}\varepsilon r = \langle (\delta u_i)^2 \delta u_r \rangle$ [Kolmogorov, 1941]

- Incompressible MHD: $-\frac{4}{3}\varepsilon^{\pm} r = \langle \delta z_L^{\mp} (\delta z^{\pm})^2 \rangle$ [Politano & Pouquet, PRE, 1998]

- Incompressible Hall MHD: [Galtier, PRE, 2008]

$$-\frac{4}{3}\varepsilon^T r = \langle [(\delta \mathbf{v})^2 + (\delta \mathbf{b})^2] \delta v_r \rangle - 2\langle [\delta \mathbf{v} \cdot \delta \mathbf{b}] \delta b_r \rangle \; \overbrace{+4 d_I \langle [(\mathbf{J} \times \mathbf{b}) \times \delta \mathbf{b}]_r \rangle}^{\text{Hall effect}}$$

- Compressible fluids ? …

Compressible isothermal turbulence

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) = 0,$$

$$\partial_t (\rho \mathbf{u}) + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}\mathbf{u}) = -\boldsymbol{\nabla} P + \mu \Delta \mathbf{u} + \frac{\mu}{3} \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{u}) + \mathbf{f},$$

$$P = C_s^2 \rho$$

$$\begin{cases} E = \rho u^2/2 + \rho e \\ e = C_s^2 \ln(\rho/\rho_0) \end{cases}$$

- Homogeneous medium
- ε becomes constant when Re → +∞ (ε is the mean energy dissipation rate)
- Stationary turbulence

→ $$\frac{\mathcal{R}(\mathbf{r}) + \mathcal{R}(-\mathbf{r})}{2} = \langle E \rangle - \frac{1}{4} \langle \delta(\rho \mathbf{u}) \cdot \delta \mathbf{u} \rangle - \frac{1}{2} \langle \delta \rho \delta e \rangle,$$

$\mathcal{R}(\mathbf{r}) \equiv \langle \rho \mathbf{u} \cdot \mathbf{u}'/2 + \rho e' \rangle \equiv \langle R \rangle$: 2 points correlation function

Exact relation for compressible turbulence

[Galtier & Banerjee, PRL, 2011]

Flux term

$$-2\varepsilon = \nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{\delta(\rho \mathbf{u}) \cdot \delta \mathbf{u}}{2} + \delta\rho\delta e - C_s^2 \bar{\delta}\rho \right] \delta \mathbf{u} + \bar{\delta} e \delta(\rho \mathbf{u}) \right\rangle$$

$$+ \langle (\nabla' \cdot \mathbf{u}')(R - E) \rangle + \langle (\nabla \cdot \mathbf{u})(\tilde{R} - E') \rangle$$

$\bar{\delta}f = (f' + f)/2$

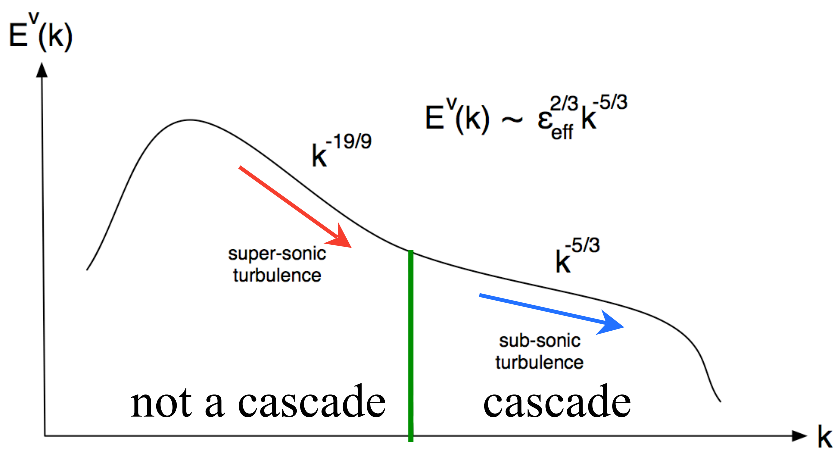
$\delta f = f' - f$

Source term: $\mathcal{S}(r) \simeq -\langle \bar{\delta}(\nabla \cdot \mathbf{u})[\frac{1}{4}\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u} + \frac{1}{2}\delta\rho\delta e] \rangle \sim r^{2/3}$

ISOTROPIC TURBULENCE

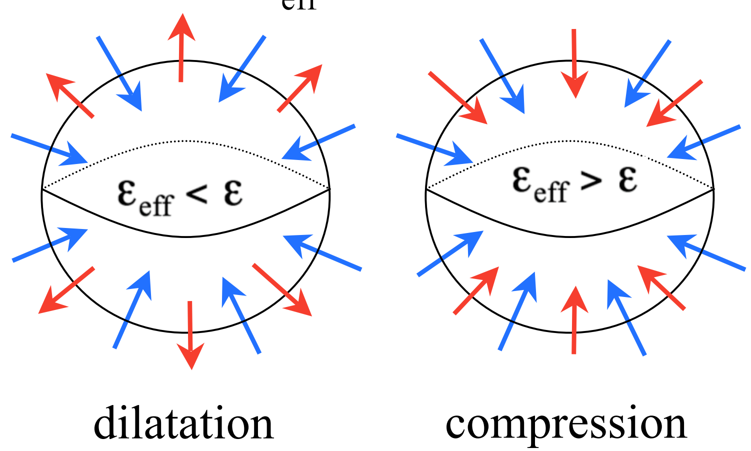
$$-2\varepsilon = \nabla.F + S$$

$v \equiv u\, \rho^{1/3}$



sonic scale : $k_s \sim \langle (\nabla \cdot \mathbf{u}) / \delta \mathbf{u} \rangle$

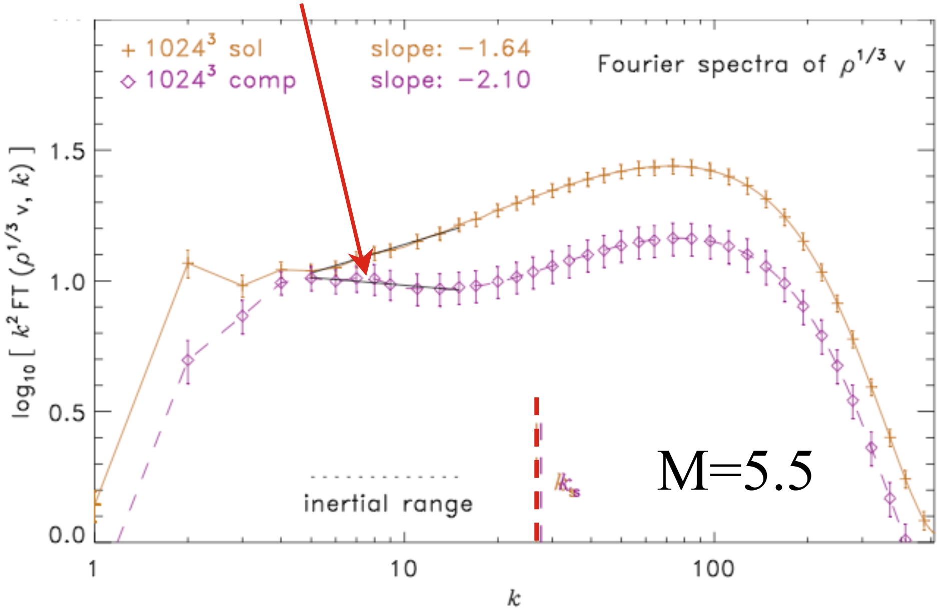
$\varepsilon_{eff} = \varepsilon + S/2$



Comparison with 3D DNS

$E^v \sim k^{-2.1}$

$\mathbf{v} \equiv \mathbf{u}\, \rho^{1/3}$



[Federrath et al., A&A, 2010]

In agreement with the theory (-19/9 = -2.11)

Conclusion

- Obukhov's idea – constancy of the energy flux – is not applicable
- Cascade of eddies is not enough
- Is Burgers prediction ($E^u \sim k^{-2}$) relevant in 3D ?
- One needs further DNS to understand supersonic turbulence
- Generalization to isothermal compressible MHD (next talk)