Structure detection in a turbulent Rayleigh–Taylor instability

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Outline

I. Motivations: understanding for modeling

II. Generic two-field (two-structure) approach

III. A structure detection approach: contrast–filter–bin

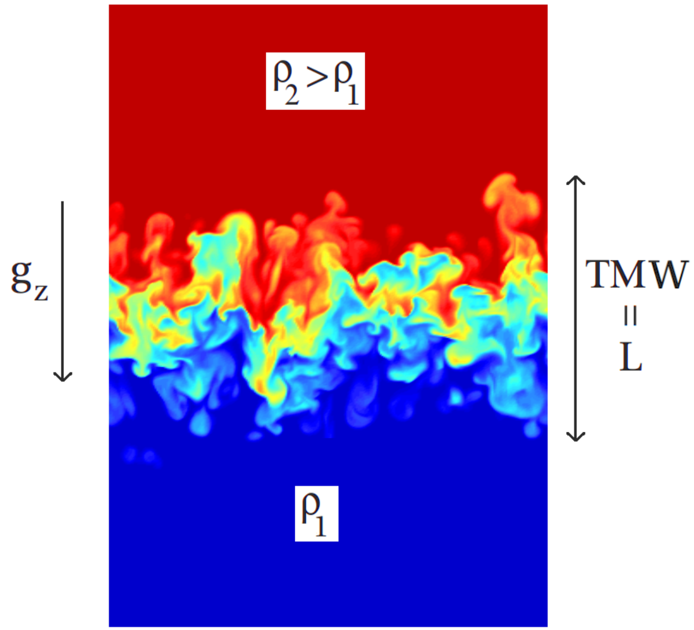
IV. “Preliminary” results, conclusions

Rayleigh–Taylor instability

Unstable stratification of two fluids:

- Densities ρ_2 and ρ_1,
- Atwood number $\mathrm{At} = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$
- Acceleration $g_z(t)$ (possibly time dependent),
- Mixing width $L(t)$,
- Academic limit:

  incompressible, plane infinite, infinite Reynolds. . .



Our basic need: understand and model $L(t) = \mathfrak{F}\left[\, g(t) \,\right]$

and other correlations, turbulent energies, etc.

with "RANS" emphasis for applications in ICF, astro- and geophysics, engineering. . .

Here constant g and vanishing Atwood. "Looks simple. . ."

Hints on large scale structures: empirical knowledge

- Buoyancy–drag equation for $g(t) > 0$ (work horse model in the field):

$$L'' = C_B \, \mathrm{At} \, g - C_D \frac{L'^2}{L}$$

works surprisingly well even for highly variable $g(t)$
(see talk by B.-J. Gréa on Wednesday morning),
means that there is a form of "internal mixing momentum" $\sim L'$ in the system.

- Bulk "0D" two-fluid energy budget analysis of experiments and DNS/LES.

- Bubble competition models from various groups up to 1990's.

- Two-fluid and two-structure models developed by D.L. Youngs at AWE (UK) up to 1990's.

- Tentative Heterogeneous-k–ε model of A.V. Polyonov (1989).

- Severe difficulties encountered in “simple models” if $g(t)$ reverses.

- Modeling difficulties similar to those of counter-gradient fluxes in combustion.

- Motivated development of two-fluid turbulent combustion model by D.B. Spalding (1986).

- Concept also applied to turbulent intermittency by Libby (1975)
  both transition and boundaries between laminar and turbulent states.

- 2SFK : 2-Fluid 2-Structure turbulent model developed by A. Llor et al. at CEA (2001),
  however, validated but not calibrated (M. Al Dahhan's terminology)...

More hints on large scale structures: visual evidence

512^3 DNS from "Alpha-group" (2004) : ten different codes, same results

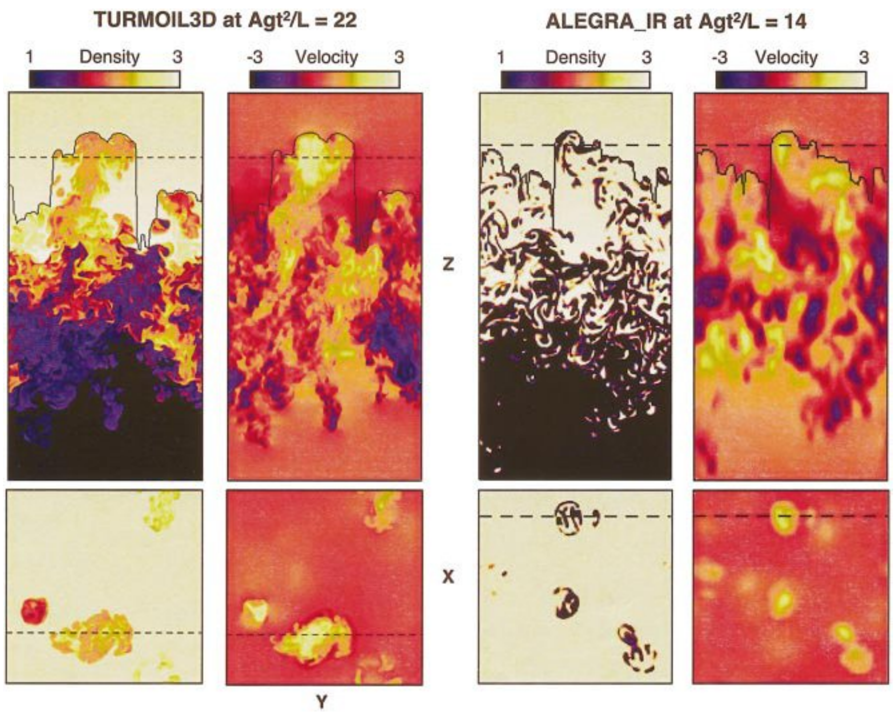
With material diffusion (miscible fluids)

$\Downarrow$

reduced At number,

&

buoyancy with local relative density contrast.



Without material diffusion (immiscible fluids)

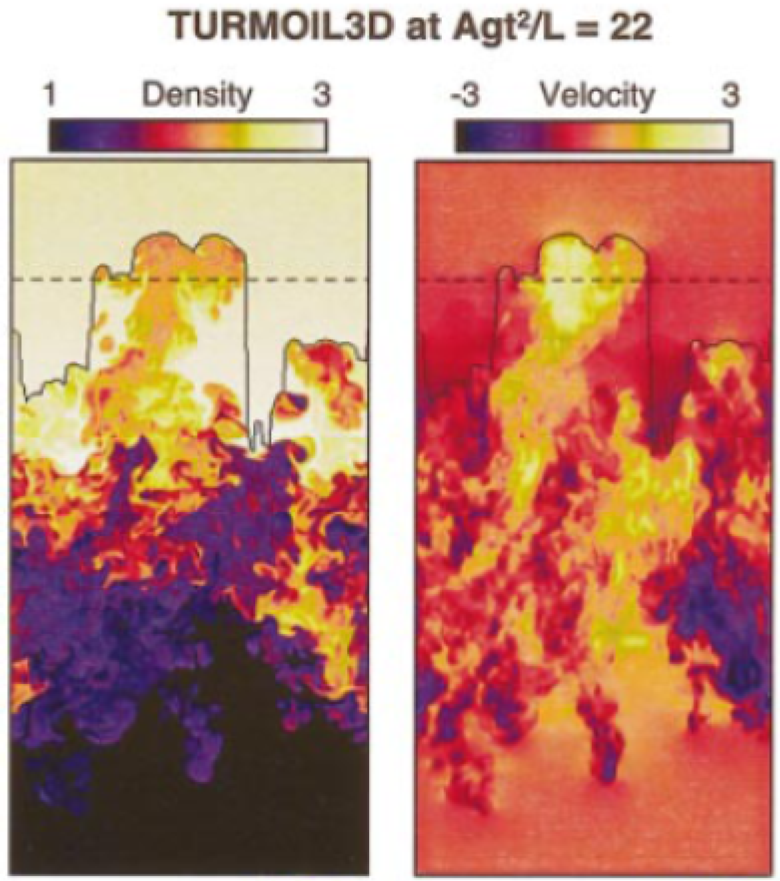
$\Downarrow$

growth identical to miscible case,

&

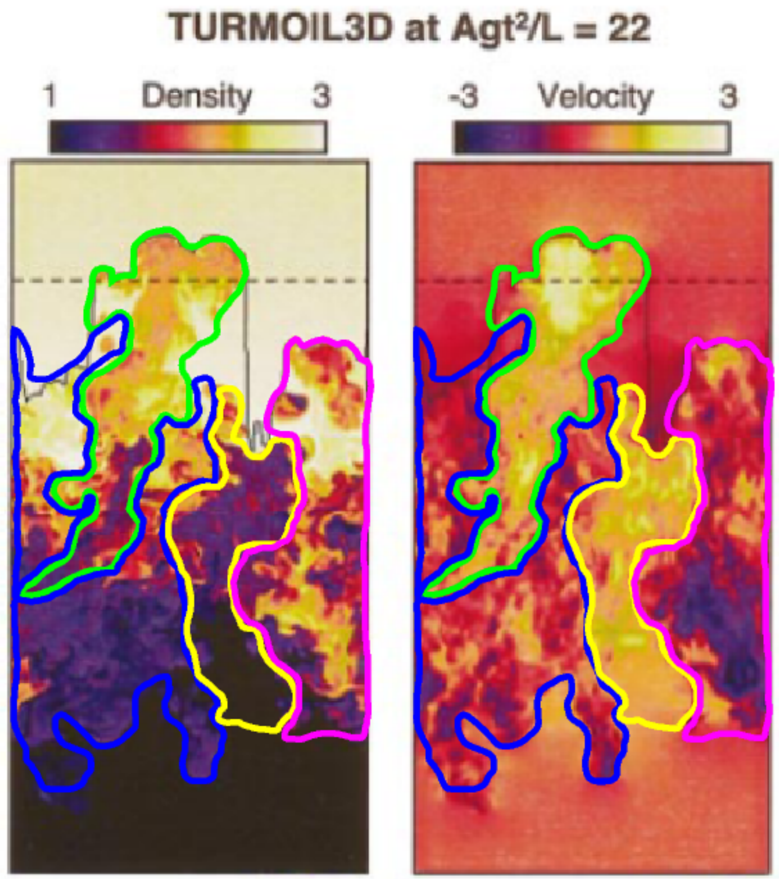
same buoyancy with effective relative density contrast.

“Hand made” structure detection



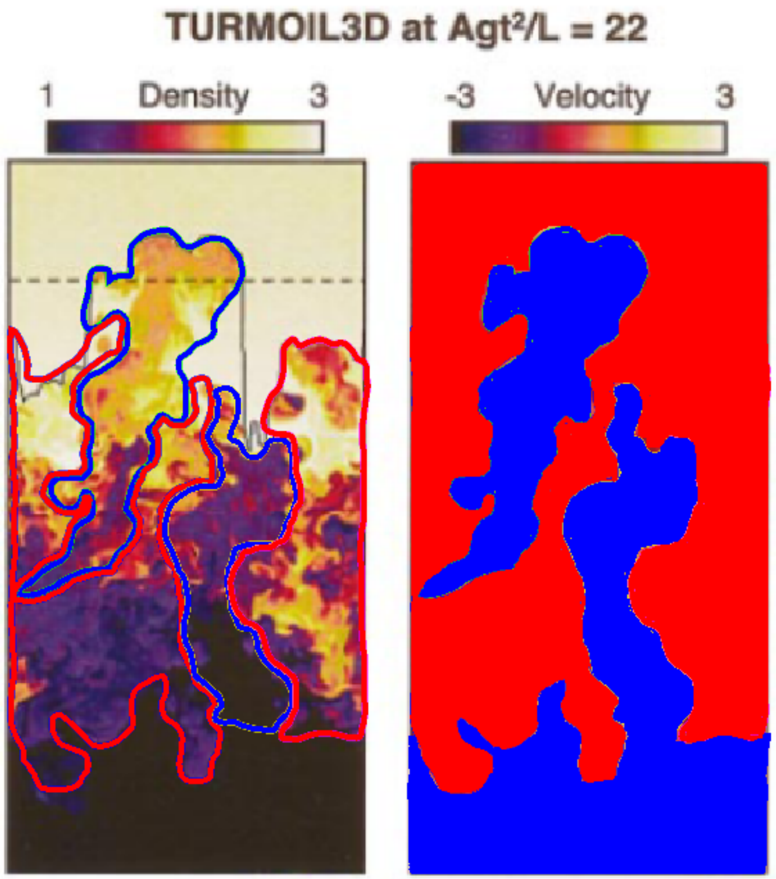
From density and velocity maps. . .

"Hand made" structure detection



. . . use contrasts to visually draw "boundaries". . .

“Hand made” structure detection



...and regroup into presence fields $b^{\pm} = 0, 1$ for upward and downward moving.

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Structure conditioned RANS equations

- RANS is simple ensemble averaging $\widetilde{a} = \overline{\rho a}/\overline{\rho}$:

$$\partial_t(\rho a) + (\rho a u_i)_{,i} = -\phi^a_{i,i} + s^a, \qquad \Rightarrow \qquad \partial_t(\overline{\rho}\,\widetilde{a}) + (\overline{\rho}\,\widetilde{a}\,\widetilde{u}_i)_{,i} = -\overline{\rho a'' u''_i}_{\,i,i} - \overline{\phi^a}_{i,i} + \overline{s^a}.$$

- Two-structure approach is now structure-field-conditioned ensemble averaging:

$$b^{\pm}(t,x) \text{ presence fields of structures } + \text{ and } -, \qquad \Rightarrow \qquad \boxed{A^{\pm} = \overline{\rho b^{\pm} a}/\overline{\rho b^{\pm}}}.$$

- Choice of $b^{\pm}$ is here $\boxed{\text{free}}$. First (naive) poor man's guess is structures = fluids, $b^{\pm} = c^{2,1}$.
- In general, $b^{\pm}$is continuous: $\partial_t b^{\pm} + b^{\pm}_{,i}\,\boxed{w_i} = 0$,

  where w_i describes displacement of structure boundary. Hence:

$$\partial_t(\alpha^{\pm}\rho^{\pm}A^{\pm}) + (\alpha^{\pm}\rho^{\pm}A^{\pm}U^{\pm}_i)_{,i} = \boxed{-\overline{b^{\pm}_{,i}\rho a(w_i - u_i)}} - (\overline{b^{\pm}\rho a u^{\pm}_i})_{,i} - (\overline{b^{\pm}\phi^a_i})_{,i} + \boxed{\alpha^{\pm}_{,i}\Phi^a_i} + \boxed{\overline{b^{\pm}_{,i}\phi'^a_i}} + \alpha^{\pm}S^{a\pm},$$

- Per structure equations equivalent to single fluid (with turbulent fluxes),

  on per structure presence probabilities $\alpha^{\pm}$, densities $\rho^{\pm}$, velocities $\vec{U}^{\pm}$,

  velocity fluctuations $\vec{u}^{\pm} = \vec{u} - \vec{U}^{\pm}\ldots$

  but with supplementary "volume" $\boxed{\alpha^{\pm}_{,i}}$ and "interfacial" $\boxed{b^{\pm}_{,i}}$ exchange terms.

Directed energy: quantitative justifications for two-structure modeling

- Two-structure approach
  splits the total turbulent kinetic energy k
  into directed k_d and per-structure turbulent $k^{\pm}$:

$$\overline{\rho} k = \underbrace{\alpha^+ \rho^+ k^+ + \alpha^- \rho^- k^-}_{\text{Per } \pm \text{ structure}} + \underbrace{\frac{\alpha^+ \rho^+ \alpha^- \rho^-}{\overline{\rho}} (\boldsymbol{U}^+ - \boldsymbol{U}^-)^2 / 2}_{\text{Directed } k_d}$$

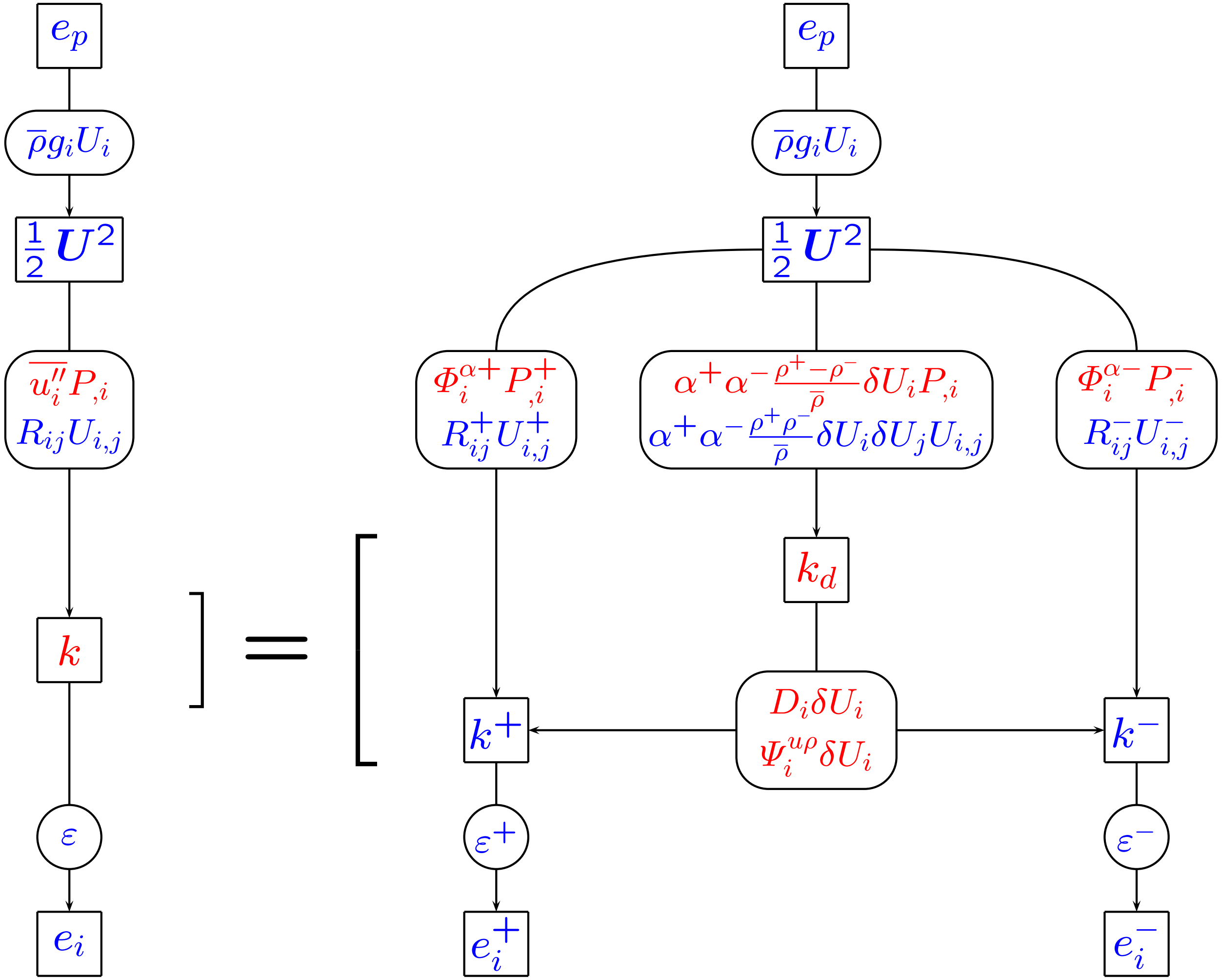
- k_d directly related to growth of mixing layer as $\delta \boldsymbol{U} \approx L'/2$
  (directed energy $\neq$ anisotropy of Reynolds stress tensor).

- With poor man's structure fields ($=$ fluids, no modeling!):
  $k_d \approx k/100$ for Kelvin–Helmholtz shear layer,
  but $k_d \approx k/4$ for Rayleigh–Taylor mixing layer!!!

- Part of turbulence production (modeler's nightmare) is now closed exactly: buoyancy

$$\overline{u_i''} P_{,i} = \Phi_i^{\alpha +} P_{,i} + \Phi_i^{\alpha -} P_{,i} + \alpha^+ \alpha^- \frac{\rho^+ - \rho^-}{\overline{\rho}} (\vec{U}^+ - \vec{U}^-) P_{,i}$$

Directed energy: production and dissipation path (high Re)



Existing two-structure models for RT

So far only two models have been developed:

- D.L. Youngs' model at AWE, UK (1984, 1989, 1991, 1995),
  2 fluid masses, 2 structure masses, 2 momentum, 2 internal energies,
  but 1 turbulent energy, and 1 (integral) length scale.

- CEA's 2SFK model, France (2001, 2003, 2010),
  2 fluid masses, 2 structure masses, 2 momentum, 2 internal energies,
  and 2 turbulent energies, and 2 turbulent dissipations.

Despite equation "thicket," introduce surprisingly few new constants: only three ($\sim C_B, C_D$).

Both "validated" and "calibrated" indirectly on global experimental and simulation data.
Makes them not up to "usual" model standards, though both give good results.

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DNS for exploring structure detection

Solution of the incompressible Navier-Stokes equation (same as SSVARTs):

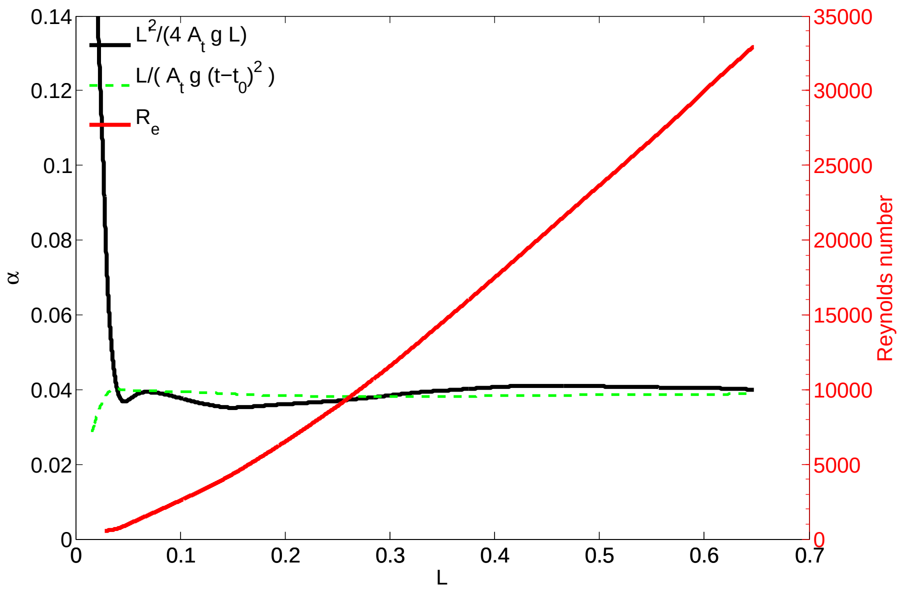
- TurbMix3D, modified version of SURFER2,
- Navier-Stokes with concentration equation,
- Finite-volume method, 2nd order in space and time, V-Cycle Poisson solver,
- Parallelized using MPI-2,
- Runs on Titane computer, up to 256 processors (CCRT at CEA),
- All tests on standard RT at $\mathrm{At} = .1$.
- With variable viscosity "trick" of SSVARTs for maximal Reynolds.

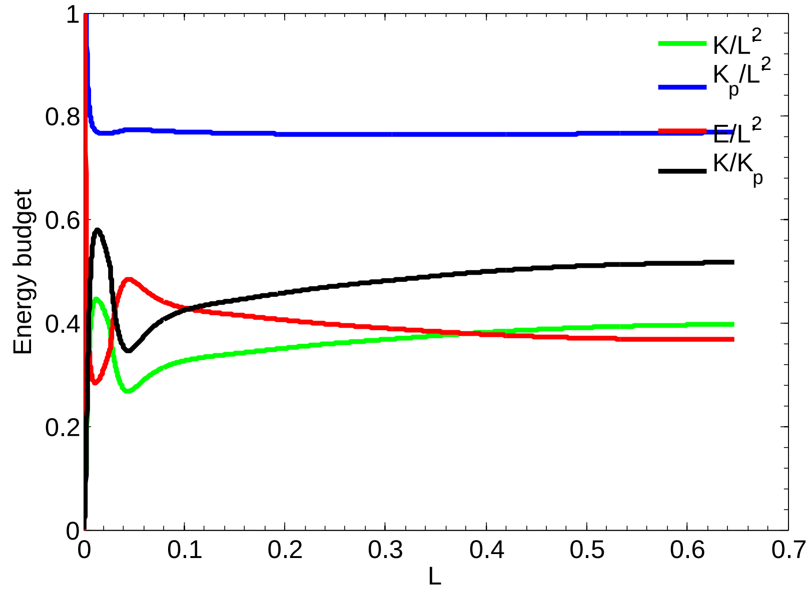
Complemented with simply modifiable filtering equations of passive quantities.

Resolution:

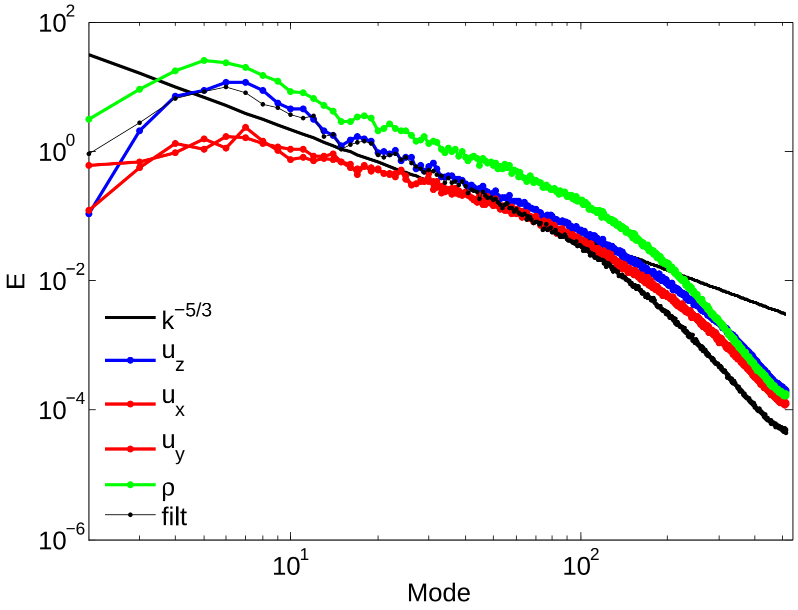
- 128^3 for quick testing,
- 256^3 to 512^3 for adjusting,
- 1024^3 for modeling reference.

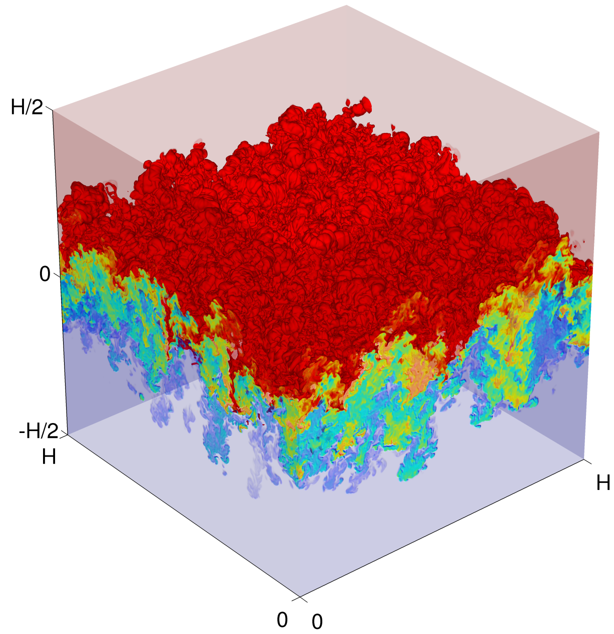
Base 1024^3 simulation consistent with previous results





Usual growth coefficient, Reynolds number, and energy ratios. . .





... density and velocity spectra, $c = .9$ iso-surface.

[density17Bone.mp4]

Mimicking visual eduction: contrast, filter, bin

Numerous approaches have been tried, not mentioned here.

So far, best is to mimic visual eduction:

- chose a "good" contrasting field $\phi(t,x,y,z)$, $\phi = u_z$ (ρ possible, k hopefully...),
- perform a Lagrangian time filtering, $\boxed{\widetilde{\phi}(t,x,y,z)}$,
  to introduce a memory effect and produce bi-modality,
- separate structures according to optimized threshold $\widetilde{\phi}_c(t,z)$ on $\widetilde{\phi}(t,z)$ (Otsu 1979).

Thus "binning"

$$\widetilde{\phi}(t,x,y,z) > \widetilde{\phi}_c(t,z) \quad \Rightarrow \quad b^+(t,x,y,z) = 1, \quad b^-(t,x,y,z) = 0 \tag{1}$$

$$\widetilde{\phi}(t,x,y,z) < \widetilde{\phi}_c(t,z) \quad \Rightarrow \quad b^+(t,x,y,z) = 0, \quad b^-(t,x,y,z) = 1 \tag{2}$$

Numerous filtering approaches have been tried, not mentioned here.

So far, best filtering is to follow typical scales of energy containing eddies

$$\underbrace{\partial_t \widetilde{\Phi} + u_i\, \partial_i \widetilde{\Phi}}_{\text{Lagrangian derivative}} \;=\; \underbrace{C_\Phi\, \widetilde{\omega}\, (\Phi - \widetilde{\Phi})}_{\text{Filtering term}}$$

where :

- $\widetilde{\Phi}$: filtered $\Phi = k, \varepsilon, u_z$,
- u_i : local velocity,
- $\widetilde{\omega} = \widetilde{\varepsilon}/\widetilde{k}$: filtered turbulence turnover frequency,
- $\widetilde{k}$: filtered local turbulent kinetic energy,
- $\widetilde{\varepsilon}$: filtered dissipation of k,
- C_Φ : adjustable coefficient for best bi-modality.

Besides the filtering of u_z, filtering of k and ε is required to produce $\widetilde{\omega}$.

Thus, three constants need to be adjusted C_u, C_k, C_ε,

in order to maximize the bi-modality coefficient of $\widetilde{u_z}$.

[dens_struc_wif.mp4]

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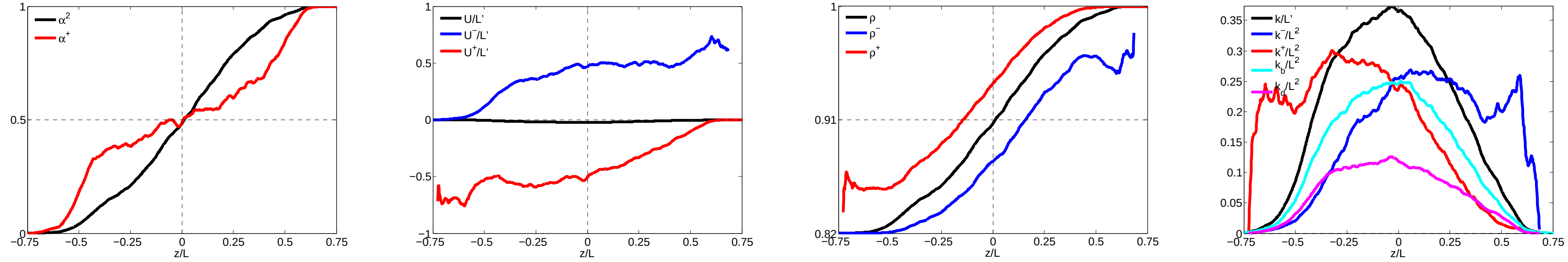
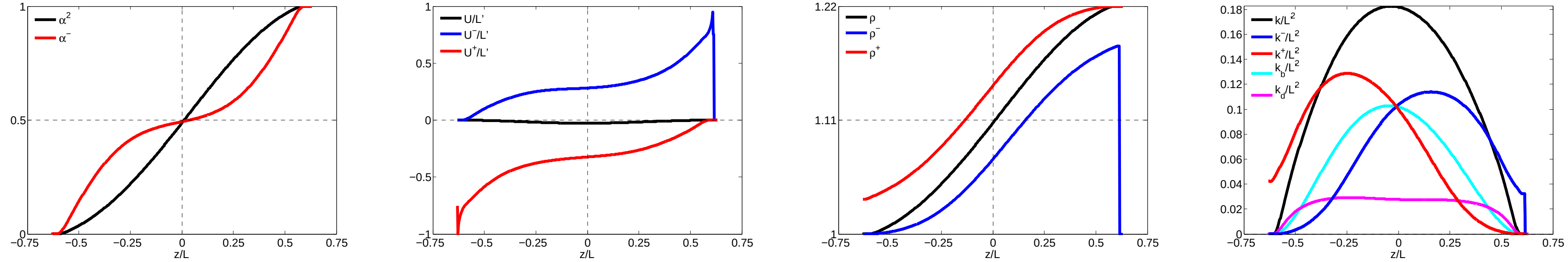
Profiles of simple two-structure statistics

DNS: relatively robust with respect to filtering options



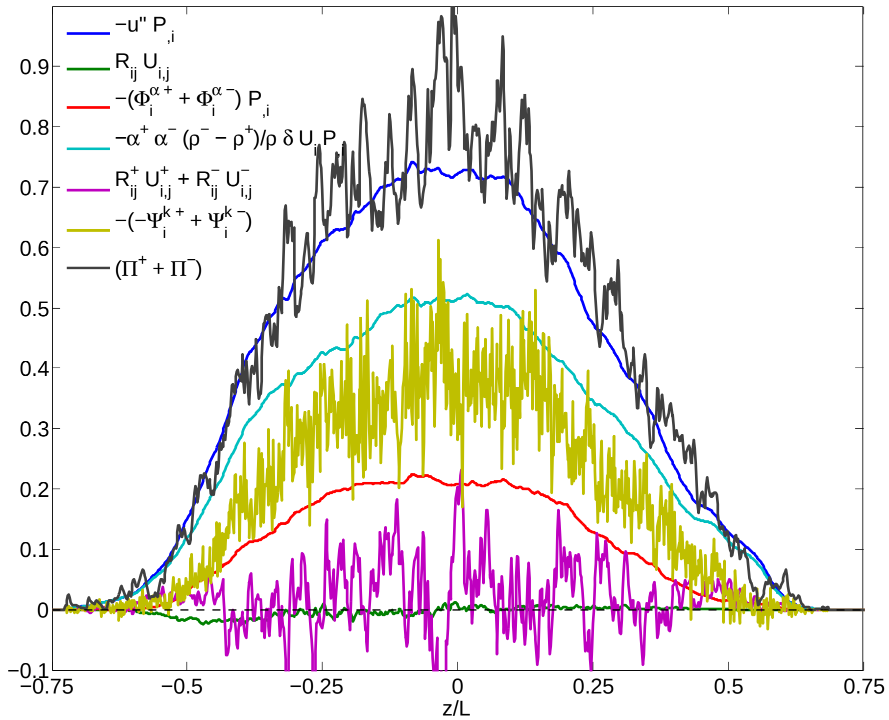
2SFK: one coefficient had to be corrected, for effective Atwood number



α^+, α^2 $\quad$ $U^\pm$ $\quad$ $\overline{\rho}$, $\rho^\pm$ $\quad$ k, $k^\pm$, k_d

$L/H = 0.63$ and $R_e = 32000$.

Profiles of single-fluid and two-structure turbulence productions



$L/H = 0.63$ and $R_e = 32000$.

Confirms dominance of the two-structure production terms in global single-fluid production.

Many analogies with existing structure detection schemes

Existing definition and eduction of structures in turbulence:

- First visual evidence in shear layers by Brown & Roshko (1974);
- Mostly centered on analysis of still pictures (space or space–time lines);
- Usual analysis techniques: vorticity, POD, wavelets, Morse–Smale complex. . .
- Present closest to "Lagrangian Coherent Structures" of Haller (2000);
- Memory effect also found in PDF turbulence modeling Pope (1990's).

Existing conditional Reynolds averaged Navier–Stokes:

- Turbulent transition and edge intermittency by Libby (1975);
- Multi-fluid modeling of turbulent combustion Spalding (1986);
- Almost all the multi-fluid community. . .

Conclusions

- Feasibility of two-structure detection in a Rayleigh–Taylor mixing layer.
- Introduction of an explicit prescription based on contrast–filter–bin approach.
- Put into evidence importance of memory effect.
- Calculation of all the one-point second-order conditional averages.
- Comparison with a 2-structure 2-fluid model, leading to:

  importance of directed effects, as obtained before with poor man's structures,

  confirmation of soundness of model closures,

  and to correction of important model coefficients.
- However, still room for improvement and further understanding:

  not fully universal as does not work on Richtmyer–Meshkov (or KH. . .),

  exchange terms are noisy and not very robust (small difference of large terms).
- Detailed results can be found in R. Watteaux PhD thesis

  (available on line from ENS-Cachan).

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