

# Landau's big scale invariants in free turbulent decay: old ideas revisited, new ideas for $\varepsilon$ modeling

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## Outline

I. Motivations; Landau's angular momentum invariance

II. Langevin stochastic equation of Landau's invariant

III. Relationship between correlations at big scales and relaxation

IV. Decay of slab, tube and spot of turbulence; impact on  $\varepsilon$ 

II and III in:

• A. Llor, "Langevin equation of big structure dynamics in turbulence: Landau's invariant in the decay of homogeneous isotropic turbulence," *European Journal of Mechanics B/Fluids* **30**,480 (2011).

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after preliminary versions:

Proceedings of the 8th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements, Marseille, France, June 9th–11th, 2010

arXiv:physics/0612220v1 [physics.flu-dyn] (2006).

## Motivation

Numerous reasons to investigate turbulence decay.

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Here focus on understanding meaning of \varepsilon equation (dissipation rate),
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for instance and foremost in RANS models.

According to Pope (2000):

"The dissipation equation is frequently blamed for poor performance of a model. For many flows, much improved performance can be obtained by altering the model constants ( $C_{\varepsilon 1}$  or  $C_{\varepsilon 2}$ ) or by adding correction terms. No correction to the dissipation equation that is effective in all flows has been found."

S.B. Pope (2000) Turbulent flows, §11.10, pp.461-462

First step: understand turbulence relaxation in simplest case:

Homogeneous Isotropic Turbulence (HIT).

 $k \propto t^{-n}$ ,  $n \approx \begin{bmatrix} 1.2 \ (\pm .1) & \text{experiments,} \\ \text{less than 1, up to 1.45} & \text{simulations.} \end{bmatrix}$ 

Introduced and studied since 1930's,

brilliantly interpreted by Landau (1944), but result incompatible with exp. data, revisited recently (Davidson 2000–2009, Llor 2006, 2011).

#### Self-similar decay of HIT

Since Kolmogorov (1940), if HIT decays in a self-similar way,

then it has to preserve some invariant quantity, I.

Self-similarity means self-similarity of all relevant average quantities,

first of all, the spectrum over energy containing range:

 $E(t,\kappa) \propto \mathbf{k}(t) \ \ell(t) \ \mathcal{E}[\ell(t)\kappa]$ 

where  $\kappa$  is wave number.

If  $\ell$  is integral length scale and k turbulent energy, general invariant is  $I = k\ell^m$ 

Note: common but misleading wisdom is that "HIT eventually forgets initial conditions and then must behave in a self-similar way." NOT TRUE IN GENERAL!

As  $\frac{d}{dt}I = 0$ , and  $\frac{d}{dt}\ell \propto \sqrt{k}$  or  $\frac{d}{dt}k \propto -k^{3/2}/\ell$  by dimensional analysis, solving ODEs yields:

$$k \propto t^{-n},$$
  $n = \frac{2m}{2+m},$   $\ell \propto t^{\theta},$   $\theta = \frac{2}{2+m},$ 

Kolmogorov assumed m = 5 from Loitsyanskii (1939) and thus  $n = \frac{10}{7} \approx 1.429$ , significantly off experimental results.

Now, what are I and m, and how are they produced?

Kármán–Howarth equation and Loitsyanskii's invariant

[Navier–Stokes equation] $(x) \otimes u(x + r)$ ,  $\downarrow \downarrow$ homogeneity, isotropy, incompressibility, calculation, no new physics!  $\downarrow \downarrow$ Kármán–Howarth equation

$$\partial_t \big[ k(t) r^4 f(t,r) \big] = k^{3/2}(t) \partial_r \big[ r^4 K(t,r) \big] - 2\nu k(t) \partial_r \big[ r^4 f'(t,r) \big],$$

where f and K are the normalized two-point double and triple longitudinal velocity correlation functions:

$$f(r) = \frac{u_r(x)u_r(x+r)}{(2/3k)},$$
  

$$K(r) = \frac{u_r^2(x)u_r(x+r)}{(2/3k)^{3/2}}.$$

Now, integrate over r = 0 to  $\infty$  and neglect  $\nu$  (high Reynolds number)

$$\partial_t I = \partial_t k \int_0^\infty f(r) r^4 dr = k^{3/2} \Big[ r^4 K(r) \Big]_0^\infty = 0, \quad \text{if} \quad K(r) \sim o(r^{-4}),$$

 $\Rightarrow$  invariance of Loitsyanskii's integral.

#### Landau's interpretation of Loitsyanskii's invariant

Landau (1940) gave intuitive explanation of Loitsyanskii's invariant as variance of per volume angular momentum at big scales:

$$m{H}(m{D}) = \int_{V(D)} m{r} imes m{u} \, \mathrm{d}^3 m{r} \sim \ell \sqrt{k} imes \ell^3 imes \sqrt{rac{D^3}{\ell^3}},$$

thus:

$$I = k \int_0^\infty f(r) r^4 \mathrm{d}r \approx \lim_{D \to \infty} \frac{\langle \boldsymbol{H}(\boldsymbol{D})^2 \rangle}{V(D)} \propto k\ell^5$$



For large enough D, torque becomes negligible (surface to volume ratio),

and thus *I* should be invariant.

Brilliant insight (only seven sentences in textbook without equations), ... but "wrong result" because effective calculation not done!

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Landau's interpretation in mathematical form: Langevin equation of angular momentum at big scales

From Euler equation (viscosity irrelevant)  $\rho \partial_t u_i + \rho(u_i u_j)_{,j} + p_{,i} = 0$ , volume integration yields Langevin like equation:

$$\frac{\mathsf{d}}{\mathsf{d}t} H_i^V = T_i^V,$$

where fluctuating angular momentum and torque are (pressure eliminates):

$$H_i^V(t) = \int_V \epsilon_{ijk} r_j u_k \,\mathrm{d}^3 r, \qquad \qquad T_i^V(t) = -\oint_{\partial V} \epsilon_{ijk} r_j u_k u_l \,\sigma_l \,\mathrm{d}^2 r.$$

Therefore :

$$\frac{\mathsf{d}}{\mathsf{d}t} \, \overline{H_i^V H_i^V} = 2 \, \overline{T_i^V H_i^V}.$$

T and H must normally be expressed from u(t, r), but yields usual Kármán–Howarth equation, whereas spirit of Landau's approach is in direct correlation between T and H.

$$T_i^V \stackrel{?}{=} T_i^V (H_i^V)$$
 so  $\overline{T_i^V (\boldsymbol{u}) H_i^V (\boldsymbol{u})} \rightsquigarrow \overline{T_i^V (H_i^V) H_i^V} \rightsquigarrow \overline{H_i^V H_i^V} (k, \ell, D)$ .

# First scalings: $\overline{H_i^V H_i^V}$ and $\overline{T_i^V H_i^V}$

Simple but tedious calculations yield (V dropped from now on)

$$\overline{H_i H_i} = \iint_V \epsilon_{ijk} \epsilon_{ij'k'} r_j r_{j'} \overline{u_k(r) u_{k'}(r')} \, \mathrm{d}^3 r \, \mathrm{d}^3 r'$$

$$= \dots$$

$$= D^{4} \frac{\pi^2}{6} k \int_0^D \left(1 - \frac{s^2}{D^2}\right) \left(1 - \frac{2s^2}{D^2}\right) f(s) \, s^3 \, \mathrm{d}s,$$

and

$$\overline{T_i H_i} = -\oint \int_V \epsilon_{ijk} \epsilon_{ij'k'} r_j r_{j'} \overline{u_k(r) u_l(r) u_{k'}(r')} \sigma_l d^2 r d^3 r'$$
  
= ...  
=  $D \frac{4}{6} \frac{\pi^2}{6} k^{3/2} \int_0^D \left(1 + \frac{3s^2}{D^2} - \frac{6s^4}{D^4}\right) K(s) s^2 ds.$ 

Depending on convergence of integrals of f and K, scaling as at least  $D^4$  of both terms, instead of Landau's original estimates of  $D^3$  and  $D^2$ , but still no new physics (~KH equation)!

#### Two different approaches to close $T_i$ and $\overline{T_iH_i}$

• Mean correlation of  $T_i$  with  $H_i$ : introduce  $\langle T_i \rangle_H$ , the *H*-conditional or sub-ensemble mean of  $T_i$ . Isotropy and weak  $T_i - H_i$  coupling yield

$$\langle T_i \rangle_H = -\omega H_i.$$

• Slow + fast decomposition of  $T_i$ : introduce  $T_i^*$ , the "fast" component of  $T_i$ . Isotropy and weak  $T_i$ - $H_i$  coupling yield

$$T_i = \begin{vmatrix} -\Omega H_i + T_i^*, \\ -\omega H_i + T_i'. \end{vmatrix}$$

with  $\overline{T_i^*(t)T_i^*(t_0)}pprox 0$  for  $|t-t_0|\gtrsim \ell/\sqrt{k}$ 

$$\overline{T_{i}(t)T_{i}(t_{0})} = \left( \begin{array}{c} \underbrace{\Omega(t)\Omega(t_{0}) \overline{H_{i}(t)H_{i}(t_{0})}}_{\text{slow}} - \Omega(t) \overline{H_{i}(t)T_{i}^{*}(t_{0})} - \Omega(t_{0}) \overline{T_{i}^{*}(t)H_{i}(t_{0})} + \overline{T_{i}^{*}(t)T_{i}^{*}(t_{0})}, \\ \\ \underbrace{Slow} \\ \underbrace{\omega(t)\omega(t_{0}) \overline{H_{i}(t)H_{i}(t_{0})}}_{\text{slow}} - \omega(t) \overline{H_{i}(t)T_{i}^{'}(t_{0})} - \omega(t_{0}) \overline{T_{i}^{'}(t)H_{i}(t_{0})} + \overline{T_{i}^{'}(t)T_{i}^{'}(t_{0})}, \\ \\ \underbrace{Slow} \\ \underbrace{Slow} \\ \\ \underbrace{Slow} \\ \underbrace{$$

#### Viscous closure

From:

$$\langle T_i \rangle_H = -\oint_{\partial V} \epsilon_{ijk} r_j \langle u_k u_l \rangle_H \sigma_l d^2 \boldsymbol{r} = -\oint_{\partial V} \epsilon_{ijk} r_j \langle (u_k - \langle u_k \rangle_H) (u_l - \langle u_l \rangle_H) \rangle_H \sigma_l d^2 \boldsymbol{r},$$

a "turbulent viscosity" assumption ( $C_{\mu} \stackrel{?}{\approx} .09$  at the moment)

$$\langle (u_i - \langle u_i \rangle_H) (u_j - \langle u_j \rangle_H) \rangle_H = \frac{2}{3} k \, \delta_{ij} - C_\mu \ell \sqrt{k} \, (\langle u_i \rangle_{H,j} + \langle u_j \rangle_{H,i}),$$

... and a weak coupling assumption

$$\langle u_i(\boldsymbol{r}) \rangle_H = M_{ij}(\boldsymbol{r}) H_j,$$
 with  $\overline{u_i(\boldsymbol{r}) H_j} = M_{ij}(\boldsymbol{r}) \overline{H_k H_k}/3,$ 

where

$$\overline{u_i(\boldsymbol{r})H_j} = \int_V \epsilon_{jkl} r_k \,\overline{u_i(\boldsymbol{r})u_l(\boldsymbol{r}')} \,\mathrm{d}^3\boldsymbol{r} = \dots = \frac{\pi}{12} \,k \,\epsilon_{ijk} r_k \int_{|D/2-r|}^{D/2+r} \mathfrak{P}_2\Big(\frac{D^2}{r^2}, \frac{s^2}{r^2}\Big) f(s) \,rs \,\mathrm{d}s.$$

Leads eventually to

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\overline{H_i H_i}}{(\pi^2/6) D^4} = -\frac{2\omega \overline{H_i H_i}}{(\pi^2/6) D^4} = \frac{C_{\mu}}{4} \ell k^{3/2} \left[ D^2 f(D) + \int_0^D \left( 1 + \frac{6s^2}{D^2} - \frac{18s^4}{D^4} \right) f(s) s \, \mathrm{d}s \right]$$

Incidentally: profiles of  $\langle u(r) \rangle_H$ 

Interesting profiles of reduced conditional velocity

$$\mathfrak{U}(r) = rac{\langle u_{\perp}(r_{\perp}) 
angle_{H}}{\sqrt{2k}} = rac{\mathsf{3}\,\overline{u_{\perp}(r_{\perp})H_{\parallel}}}{\sqrt{2k\,\overline{H_{i}H_{i}}}},$$

for  $f(s) \sim s^{-m}$ , and small  $\ell/D = .1$  to .01.



Weak coupling approximation is well supported.

Solid like rotation of sphere for m = 3 (Saffman correlation).

Shear layer at sphere's surface for m = 5 (Batchelor correlation).

#### Noise closure

Difficulties:  $T_i^{V*}(t)$  not well-defined (slow-fast separation), non white, non stationary...

Requires various "reasonable" assumptions, mainly  $\Omega/\omega$  constant—but no "turbulent viscosity."

Therefore (less trivial,  $C_{ au} \stackrel{?}{pprox} {}^1\!/{8}$  estimated) :

$$\frac{\mathrm{d}}{\mathrm{d}t} \overline{H_i(t)H_i(t)} = -2 \,\Omega(t) \,\overline{H_i(t)H_i(t)} + 2 \,\overline{T_i^*(t)H_i(t)}$$

$$= -2 \,\Omega(t) \,\overline{H_i(t)H_i(t)} + 2 \int_{-\infty}^t \overline{T_i^*(t)T_i^*(t_1)} \, e^{-\int_{t_1}^t \Omega(t_2)\mathrm{d}t_2} \, \mathrm{d}t_1$$

$$= -2 \,\Omega(t) \,\overline{H_i(t)H_i(t)} + 2 \,\tau^*(t) \,\overline{T_i^*(t)T_i^*(t)},$$

$$= \dots$$

$$= \dots$$

$$= \dots$$

$$= -2 \,\tau \,\overline{T_i(t)T_i(t)}$$

$$= -2 \,C_\tau \,\frac{\ell}{\sqrt{k}} \,\overline{T_i(t)T_i(t)}.$$

Now

$$\begin{aligned} \tau \overline{T_i T_i} &= \tau \oint_V \epsilon_{ijk} \epsilon_{ij'k'} r_j r_{j'} \overline{u_k(r) u_l(r) u_{k'}(r') u_{l'}(r')} \, \sigma_l \sigma_{l'} \, \mathrm{d}^2 r \, \mathrm{d}^2 r' \\ &= \dots \\ &= D^4 \frac{\pi^2}{6} \frac{4C_\tau}{3} \, \ell k^{3/2} \int_0^D \left[ \left( 1 - \frac{2s^2}{D^2} \right) \left( 1 - \frac{3s^2}{D^2} \right) g_2(s) + \left( 1 - \frac{4s^2}{D^2} \right) \left( 1 - \frac{s^2}{D^2} \right) h_2(s) \\ &- \frac{s^2}{D^2} \left( 1 - \frac{s^2}{D^2} \right) \left[ f_1(s) - 2g_1(s) + h_1(s) \right] \right] s \, \mathrm{d}s, \end{aligned}$$

with five two-point fourth-order normalized velocity correlation functions (r - r') along z

 $g_2 = (zx|zx), \quad h_2 = (xy|xy), \quad f_1 = (zz|zz), \quad g_1 = (zz|xx), \quad h_1 = (xx|yy).$ 

Old "quasi-normal" trick, here very good (with second-order transverse g(s) = f(s) + sf'(s)/2):  $g_2(s) = f(s) g(s), \quad h_2(s) = g(s) g(s), \quad f_1(s) - 2g_1(s) + h_1(s) = 2f(s) f(s).$ 

Hence

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\overline{H_i H_i}}{(\pi^2/6) D^4} = -\frac{2\tau \overline{T_i T_i}}{(\pi^2/6) D^4} = \frac{8C_\tau}{3} \ell k^{3/2} \int_0^D \left[ \mathcal{O}(1)f(s) g(s) + \mathcal{O}(1)g^2(s) - \mathcal{O}(\frac{s^2}{D^2})f^2(s) \right] s \,\mathrm{d}s$$

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Expansion at  $D \rightarrow \infty$  of  $\overline{H_i H_i}$  evolution equation

Expand f(s) at big scales (2 < m < 4, can be extended)

$$f(t,s) = f_m(t) (s/\ell)^{-m} + o(s^{-4})$$
 for  $s \gtrsim \ell(t),$   
$$\Rightarrow g(t,s) = (1 - m/2) f_m(t) (s/\ell)^{-m} + o(s^{-4})$$

Then [two apparently different closures of same process  $\rightarrow$  fluctuation-dissipation theorem]

$$\mathsf{d}_{t} \frac{\overline{H_{i}H_{i}}}{(\pi^{2}/6) D^{4}} = \begin{vmatrix} -\frac{2\omega\overline{H_{i}H_{i}}}{(\pi^{2}/6) D^{4}}, & \text{viscous closure,} \\ -\frac{2\tau\overline{T_{i}T_{i}}}{(\pi^{2}/6) D^{4}}, & \text{noise closure,} \end{vmatrix}$$

expands in D as

$$\begin{aligned} \frac{\overline{H_iH_i}}{(\pi^2/6)D^4} &= \frac{2mf_mk\ell^m D^{4-m}}{(4-m)(6-m)(8-m)} + k D^0 \int_0^\infty \left[ f(s) - f_m \left(\frac{\ell}{s}\right)^m \right] s^3 ds + o(D^0), \\ \frac{2\omega\overline{H_iH_i}}{(\pi^2/6)D^4} &= \frac{C_\mu\sqrt{k}}{2\ell} k\ell^2 D^0 \int_0^\infty f(s) s \, ds + o(D^0), \\ \frac{2\tau\overline{T_iT_i}}{(\pi^2/6)D^4} &= \frac{8C_\tau\sqrt{k}}{3\ell} k\ell^2 D^0 \int_0^\infty \left[ g^2(s) + f(s)g(s) \right] s \, ds + o(D^0), \end{aligned}$$

#### First consequence: threshold of self-similar relaxation

Behavior of f at big scales determines invariance conditions:

$$m < 4$$
  
 $d_t(f_m k \ell^m) = 0,$   
 $4 < m$   
 $d_t \int_0^\infty k f(s) s^3 ds = ....$   
Proves "permanence of big structures" for  $m < 4.$ 

Adding a self-similarity assumption to m < 4 case

 $d_t f_m = 0$  then  $d_t (k \ell^m) = 0$ 

Invariance is only marginal at m = 4, and m = 5 (Batchelor) is NOT invariant.

Now, coefficients in ... very small, make m = 5 "almost invariant", with apparent n above 4/3 (m = 4), but below 10/7 Kolmogorov (m = 5). A next order effect: viscosity coefficient  $C_{\mu}$ 

Consider self-similar case at m < 4; identifying  $D^0$  terms yields

$$\frac{m-4}{m}\int_0^\infty \left[f(s)-f_m(\frac{\ell}{s})^m\right]\frac{s^3\mathsf{d}s}{\ell^4} = \frac{C_\mu}{2}\int_0^\infty f(s)\frac{s\,\mathsf{d}s}{\ell^2},$$

Expression depends critically on behavior of f(s) at "intermediate" and big scales,  $s \ge \ell$ , where theoretical, experimental, and numerical data are practically nonexistent ( $f_m = ?$ ).

Here assume piecewise  $C^1$  connection between inertial range and pure  $\xi^{-m}$  at big scales ( $\xi = C_k s/\ell$ )

$$f^{(2)}(\xi) = \begin{vmatrix} 1 - \xi^{2/3} + \frac{m}{2(m+2)} \xi^2 & \text{for } \xi \le \xi_c = \left(\frac{3m/2}{m+2/3}\right)^{3/2} \\ \frac{4/3}{(m+2)(m+2/3)} \left(\frac{\xi_c}{\xi}\right)^m & \text{for } \xi > \xi_c. \end{vmatrix}$$

(assumptions supported by the scarce known results).

Then

$$C_{\mu}^{(2)}(m) = C_{k}^{-2} \frac{9m^{2}(m-2)}{7(m+2/3)^{3}}.$$

Results are relatively independent of precise  $f(\xi)$  profile (if not mixed at big scales)



Amplitude  $f_m$  is relatively constant,

and  $C_{\mu}(3)$  is close to the "accepted" empirical value of .09.



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Saffman's projection procedure

Saffman (1967) proposed procedure to generate a (random) velocity field:

- give impulse field, constrained at will, *i*
- take divergence free component of  $i: u = i \nabla q$ , where  $\Delta q = \nabla \cdot i$



Examples:

Simple impulse fields can be produced for any type of infrared spectrum power  $\mu$ .

Now, Saffman's procedure preserves H, and also the D-scaling of T

#### Saffman's procedure and angular momentum

One does not need complicated maths to obtain D-scalings (back of the envelope):

- contributions to variances from different structures are fully uncorrelated.
- watch for torque scaling (no in-structure cancellations on surface).

Then variances of angular momentum and torque are:



Thus, when  $D \to \infty$ ,  $\overline{H_i H_i}$  becomes invariant if it grows faster than  $D^4$ .

Saffman's correlation in  $E \propto \kappa^2$  is invariant with  $I = k\ell^3$ .

Batchelor's correlation in  $E\propto\kappa^4$  is not invariant,

Loitsyanskii's integral is not constant and Kolmogorov's  $n = \frac{10}{7}$  cannot be reached.

Impulsive approach on layer, tube, and spot

Question: is it possible to analyze inhomogeneous situations:



Detailed calculations (Kármán-Howarth) are untractable.

But big scale behavior with impulsive approach could.

What do we need? See pictures:

- symmetrical control volume (to cancel pressure),
- negligible surface effects at big scales,
- thus check (or adapt) to cancel influence of laminar regions (technicality).

Decay exponents for layer, tube, and spot

"Back of the envelope" calculations again.

Batchelor type correlations appear as never invariant.

Results for Saffman correlations and marginally invariant correlations:

Geometry	HIT		Layer (IRM)		Tube	
d (dilution)	0		1		2	
Invariant type	Saffman	Maximum	Saffman	Maximum	S. & B.	Spin
m	3	4	4	5	7	6
n	6/5	4/3	4/3	10/7	14/9	3/2
$1-n/2~(\ell$ growth)	2/5	1/3	1/3	2/7	2/9	1/4
$C_{\varepsilon 2}$	11/6	7/4	2	15/8	19/10	2

Some of these values have been found independently by Chasnov and Inogamov,

but with numerous assumptions and calculations.

"0D" reduction of  $k-\varepsilon$  model for HIT, layer (RM), tube, and spot

Most models reduce to usual  $k-\varepsilon$  for free turbulent decay.

When reduced by "0D" averaging over mixing zone,  $k-\varepsilon$  yields coupled ODEs:

$$\frac{\mathrm{d}}{\mathrm{d}t}k = -d\frac{\frac{\mathrm{d}}{\mathrm{d}t}\ell}{\ell}k - \varepsilon,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = -d\frac{\frac{\mathrm{d}}{\mathrm{d}t}\ell}{\ell}\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^{2}}{k},$$

where, in dilution terms, d = 0, 1, 2, 3 for HIT, layer, tube and spot.

 $C_{\varepsilon 2}$  is determined if given flow with invariant m is to be captured:

$$C_{\varepsilon 2} = \frac{3}{2} + \frac{1+d/2}{m-d}$$

Values in previous table for various d and m, show:

- Small variations around  $C_{\varepsilon 2} = 2$ .
- Larger variations are obtained with other "equivalent" variables
- Usual approach

"adjust  $C_{\varepsilon 2}$  to capture HIT and then apply to other flows as mixing layers," is acceptable but not accurate  $(C_{\varepsilon 2} = 11/6 \approx 1.83 \text{ instead of } 2)$ .

• Difference between layer and tube ( $C_{\varepsilon 2} = 2 \text{ or } 19/10$ )

consistent with growth of plane vs round jets.

• Improving  $\varepsilon$  equation thus requires some sensing of TMZ dimensionality

but present findings do not tell us how...

 $(k-\ell, k-\omega, k-\nu...).$ 

Two brain teasers...

#### 1. Influence of dissipation process in decay laws.

Decay laws are independent of spectrum slope in inertial range (i.e.  $E(\kappa) \approx \varepsilon^{2/3} \kappa^{-5/3}$ ): only requirement is preservation of slope, whatever it is, during decay (self-similarity).

Question: why, then, is decay independent of actual nature of dissipation process?

Answer: conservation laws at large scales constrain decay,

similarly to perfectly inelastic collision of two bodies : final state is entirely determined by momentum conservation, regardless of actual dissipative process of excess energy (rotation, vibration, heat, radiation...)

#### 2. Significance of $\varepsilon$ in modeling.

 $\varepsilon$  model equation commonly derived from:

- statistical "one-point" dissipation equation,  $\varepsilon = 2\nu \overline{s_{i,j} s_{i,j}}$  (often),
- spectral flux at integral length scale,  $\varepsilon = C \left( \frac{k^{3/2}}{r} \right) \left( 1 f(r) \right)^{3/2}$  at  $r \ll \ell$  (seldom).

Conclusion: all this is "thin air," since  $\varepsilon$ , as effectively acting in models,

is determined solely by big scale behavior,  $\varepsilon = C \left( k^{3/2}/r \right) f^{-1/m}(r)$  at  $r \gg \ell$ .

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Self-similar decay of turbulent energy spectrum

Self-similarity means self-similarity of all average quantities,

first of all, the spectrum over energy containing range:

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E(t,\kappa) \propto k(t) \ \ell(t) \ \mathcal{E}[\ell(t)\kappa]
```

where  $\kappa$  is wave number.

Self-similarity over many orders of magnitude is required for precise observations. Here, show Lesieur's EDQNM shell-model "simulations."

EDQNM is exact in the infrared limit, around  $\kappa \sim 0$ .

Numerical resolution of EDQNM yields behavior of self-similar infrared spectrum:

 $E(t = 0, \kappa \sim 0) \propto J \kappa^{\mu} \begin{cases} \mu \leq 3 & \text{invariance of } J = I, \text{ "permanence of big structures"} \\ 3 < \mu \leq 4 & J(t) \neq I \text{ time dependent, slow self-similar growth,} \\ 4 < \mu & \text{quick transition to } \mu = 4, \text{ in about } \ell/\sqrt{k} \end{cases}$ 

Dimensional analysis thus yields  $I = k\ell^m$  with  $m = \mu + 1$  for  $\mu \leq 3$ .

The permanence of big structures has been proved recently

(no longer a conjectured principle based on observed behavior, A. Llor, *Eur. J. of Mech. B – Fluids*, in press (2011)).

### Lesieur's et al. (2000) EDQNM model results

Starting from "peaked"  $\kappa^8$  spectrum at t = 0:



 $\kappa^4$  spectrum is produced in a few turn-over times (backscattering).

Notice 12 decades in E,

5 decades in inertial range of  $\kappa$ ,

1 decade in infrared range of  $\kappa$ .

... but  $E(\kappa)/\kappa^4 \sim I(t) = k\ell^5$  is not exactly invariant, and thus n < 10/7:







This is the "permanence of big eddies," each with own invariant.