Hysteresis of a pendulum subjected to aerodynamic forces - The role of turbulence

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The pendulum: still many open questions

- Coupling with fluid mechanics
  - Wake interactions \((\text{Bolster et al., 2010})\)
  - Added mass effects \((\text{Neil et al., 2007})\)
  - Influence of surrounding turbulence on equilibrium position
  - Influence of surrounding turbulence on oscillations
  - ...

\[ \text{Diagram of pendulum} \]
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- ...

Forces on the pendulum

- Weight
- Aerodynamic force
  - Drag
  - Lift

\[ \vec{F}_W = -mg\vec{e}_y \]
\[ \vec{F}_{aero} = \vec{F}_D + \vec{F}_L \]
\[ \vec{F}_D = \frac{1}{2}C_D\rho S_D U^2 \vec{e}_z \]
\[ \vec{F}_L = \frac{1}{2}C_L\rho S_D U^2 \vec{e}_y \]
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Forces on the pendulum

At equilibrium

From the equilibrium position we can measure the aerodynamic force (lift + drag) normal to the plate

\[
F_{\text{aero}} = \vec{F}_D + \vec{F}_L
\]

Drag and lift coefficients depend on angle of attack and of turbulence rate. How? How does this affect the equilibrium of the pendulum?

Drag

Lift

\[
F_D = \frac{1}{2}C_D \rho S_D U^2 \bar{e}_z
\]

\[
F_L = \frac{1}{2}C_L \rho S_D U^2 \bar{e}_y
\]

\[
N_{\text{aero}} = mg \sin \alpha_{eq} \frac{L_{OG}}{r}
\]

From the equilibrium position we can measure the aerodynamic force (lift + drag) normal to the plate
Experimental setup - 1 (Laminar)

\[ \alpha_{eq} = f(U) \]

\[ u'/U = 5\% \]

High precision angle sensor

Mean flow direction

L = 75 cm
Experimental setup -2: (Moderate turbulence rate)

High precision angle sensor

$u' / U = 3\%$
Experimental setup - 3 (high turbulence rate)

High precision angle sensor

$u'/U = 20\%$
Experimental setups

High precision angle sensor

$L = 75 \text{ cm}$

mean flow direction
Laminar case at increasing speed

Equilibrium angle of the pendulum

Forbidden angles

Sudden jump of equilibrium position

Range of «forbidden angles»
Laminar case at increasing speed

Equilibrium angle of the pendulum

Forbidden angles

Lift dominates
The plate «flies» in the wind

Drag dominates
The wind pushes the plate

«Stall» of the plate

$u'/U =$
Laminar case at decreasing speed
Equilibrium angle of the pendulum

Hysteretic behavior (bi-stability of the pendulum)
Extended «Lift dominated» branch
almost not «forbidden angles» in the overall
Laminar case at **decreasing** speed

Equilibrium angle of the pendulum

\[
\text{weight}(\alpha) = mgL_{OG} \sin \alpha \vec{e}_x
\]

\[
\text{aero}(\alpha) = -\frac{1}{2} L_{OD} \rho S_d C_N(\alpha) U^2 \vec{e}_x
\]
Terminal case at decreasing speed

Equilibrium angle of the pendulum

\[ \text{weight}(\alpha) = mgL_{OG} \sin \alpha \hat{e}_x \]

\[ \text{aero}(\alpha) = -\frac{1}{2} L_{OD} \rho S_d C_N(\alpha) U^2 \hat{e}_x \]
Laminar case at decreasing speed

Equilibrium angle of the pendulum

\[ h_t = mgLO_G(1 - \cos(\alpha)) \]

\[ E_{tot} = E_{weight} + E_{kin} \]
Hysteretic aerodynamic force

\[ R_{e_p} = \frac{Ud}{\nu}, \quad x \times 10^4 \]

\[ C_{D}^{0} = 1.17 \]
Influence of turbulence rate

At moderate turbulence level

- Drag branch unchanged
- Lift is reduced (plate flies less)
- Hysteretis is reduced

\[ R_{ep} = \frac{U_d}{\nu} \]
Influence of turbulence rate

Moderate turbulence level

At high turbulence rate

- Increase of drag
- Reduction of lift (disk does not fly)
- Hysteretic disappears
Take home message

We still have to learn from the pendulum (good system to probe fluid mechanics issues)

A plate pendulum is bi-stable in a laminar flow with a hysteretic transition from drag to lift dominated equilibrium.

Turbulence:
- bi-stability is reduced
- lift is reduced
- drag is increased

\[ \text{Oscillations around the hysteretic angles (laminar)} \]
\[ \text{disappear at 20\% of turbulence (no turbulent flight)} \]

Oscillations in drag and lift dominated regions
\[ u' / U = 3\% \]

\[ u' / U = 20\% \]