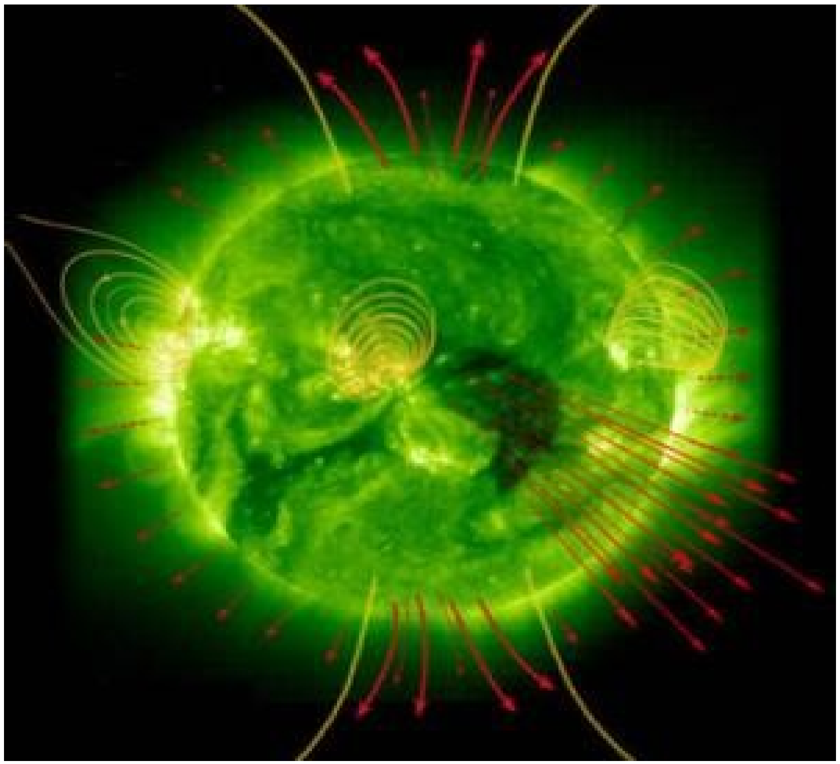
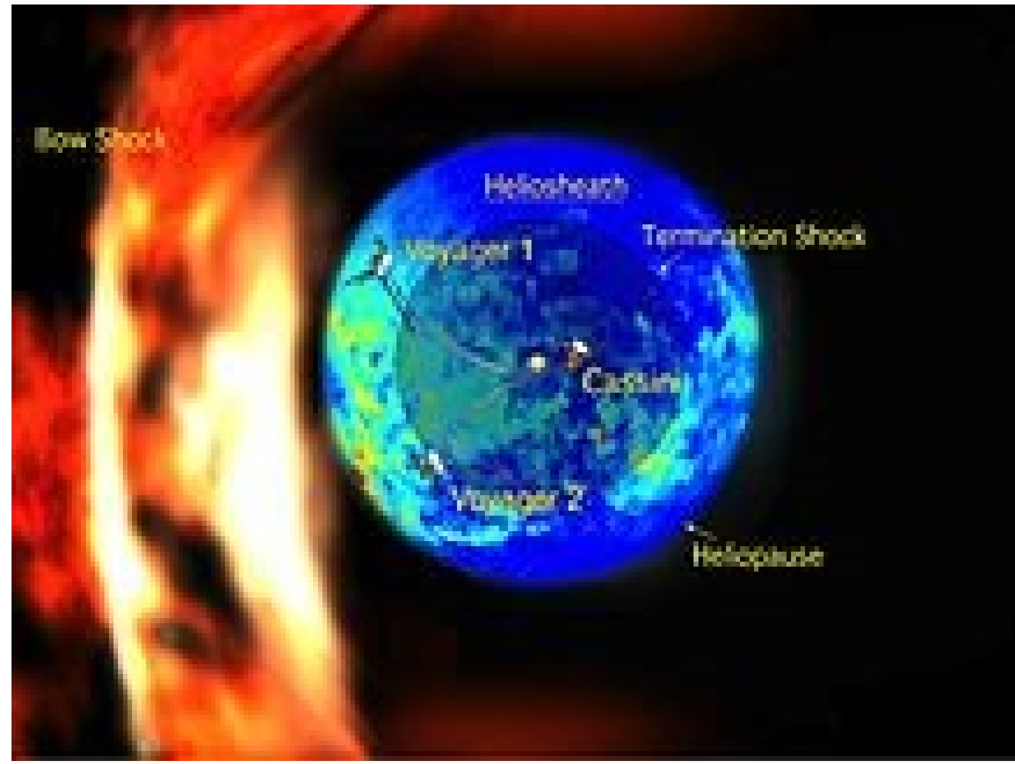
Compressible MHD Turbulence





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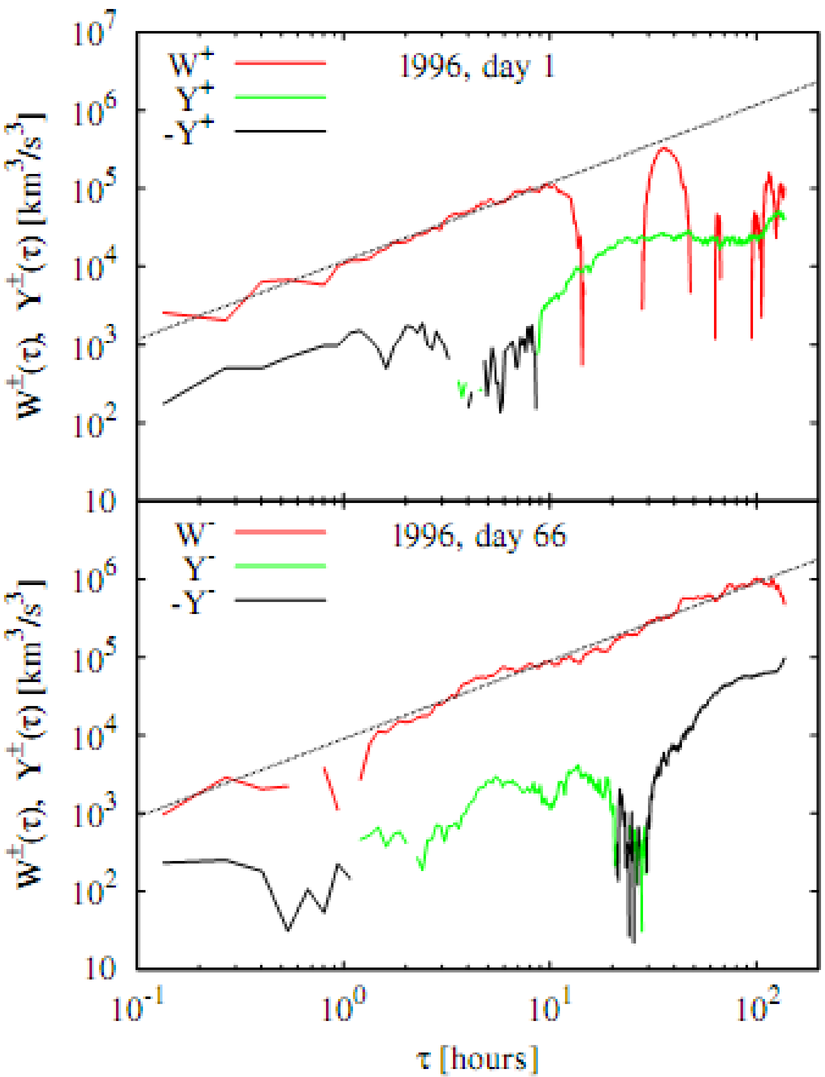
Outline of the presentation

- Indications from observational studies : Solar wind turbulence
- Basic equations of MHD using compressible Elsässer variables
- Invariants in compressible MHD
- Hypotheses for exact relation
- Exact relation : Simplification under strong $\mathbf{B}_0$ approximation
- Conclusion

Central Problems Of Fast Solar Wind



- Presence of scaling law despite strong **v-B** correlation(Carbone et al. 2009, Ulysses data)
- Incompressible scaling for only 1/3 of a time period of study

$$Y^{\pm}(l) \equiv <|\delta \boldsymbol{z}^{\pm}|^2 \delta z_{\parallel}^{\mp}> = -\frac{4}{3}\epsilon^{\pm} l$$

$$(where\; \boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \boldsymbol{B} \;\text{ and }\; \epsilon = the\; rate\; of\; energy\; injection)$$

- Almost ¾ part of the period, an approximative scaling observed

$$W^{\pm}(l) \equiv <|\delta \boldsymbol{w}^{\pm}|^2 \delta w_{\parallel}^{\mp}>/<\rho> = -\frac{4}{3}\epsilon^{\pm} l$$

$$(where\; \boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \frac{\boldsymbol{B}}{\sqrt{\mu_0 \rho}} \;\text{ and }\; \boldsymbol{w}^{\pm} = \rho^{1/3} \boldsymbol{z}^{\pm})$$

- Necessity of an exact relation to identify the suitable variable

Basic Equations

- Compressible Ideal MHD with isothermal plasma:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{v}) = 0 \qquad \text{continuity equation}$$

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}.\nabla \boldsymbol{v}\right) = -\nabla P + (\boldsymbol{j} \times \boldsymbol{B}) + \boldsymbol{F}\,(\textit{forcing term})$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \qquad \text{Faraday's law}$$

$$P = C_s^2 \rho \qquad \text{isothermal assumption}$$

$$\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{0} \qquad \text{idealised Ohm's law}$$

Compressible Elsässer Variables

- Incompressible MHD → Solenoidal Elsässer fields

$$z^{\pm} = v \pm \frac{B}{\sqrt{\mu_0 \rho_0}}$$

- Compressible MHD → **Compressible Elsässer variables** (Marsch & Mangeney, 1987)

$$z^{\pm} = v \pm \frac{B}{\sqrt{\mu_0 \rho}}$$

- Basic equations :

$$\left(\text{with } c_s^2 = \textit{sound velocity} \text{ and } \frac{D^{\pm}}{Dt} \equiv \frac{\partial}{\partial t} + (z^{\pm}.\nabla)\right)$$

$$\frac{D^{\pm}}{Dt}\ln\rho = -\frac{1}{2}\nabla.(3z^{\pm} - z^{\mp})$$

$$\frac{D^{\mp}}{Dt}z^{\pm} = \pm\frac{1}{4}(z^{+} - z^{-})\frac{D^{\pm}}{Dt}\ln\rho - \frac{1}{8}\nabla(z^{+} - z^{-})^{2} - [c_s^2 + \frac{1}{8}(z^{+} - z-)^{2}]\nabla\ln\rho$$

General Study Of Compressible MHD Flow - Invariants

- Boundary conditions: **v = j = 0** at every point on the surface of the chosen fixed volume
- Invariants : (under above conditions)

i) Mass $(\int \rho \, d\tau)$

ii) Total energy $(\int [\rho(\frac{v^2}{2} + C_s^2 \ln \rho) + \frac{B^2}{2\mu_0}] \, d\tau)$

t

iii) Cross-Helicity $(\int \boldsymbol{v}.\boldsymbol{B} \, d\tau)$

iv) Magnetic Helicity $(\int \boldsymbol{A}.\boldsymbol{B} \, d\tau)$

- Kinetic helicity $(\int (\boldsymbol{v}.\omega) \, d\tau)$ **&**

  Compressible pseudo energies $(\int \boldsymbol{w}^{\pm} \, d\tau = \int \frac{\rho}{2} \boldsymbol{z}^{\pm}.\boldsymbol{z}^{\pm} \, d\tau)$

  are **NOT invariants** of compressible MHD turbulence

Basic Assumptions

- Fully developed turbulence
- Assumption of very large Reynold's numbers (Re and Rm)
- Statistical homogeneity
- Stationarity
- Forcing only in Navier-Stokes equation and at large scales
- The existence of an "Inertial zone" which is hardly affected by the forcing nature

Exact Relation: (Banerjee & Galtier, article in preparation)

In the inertial zone, with negligible kinetic and magnetic viscosities $(\nu, \eta \to 0)$, we obtain finally

$$-2\epsilon = \frac{1}{2}\nabla_{r}.\langle \delta \boldsymbol{z}^{+}[\frac{1}{2}\delta(\rho \boldsymbol{z}^{-})\delta \boldsymbol{z}^{-} + \delta\rho\delta e] + \delta \boldsymbol{z}^{-}[\frac{1}{2}\delta(\rho \boldsymbol{z}^{+})\delta \boldsymbol{z}^{+} + \delta\rho\delta e] + 2\bar{\delta}(e + \frac{v_A^2}{2})\delta(\rho \boldsymbol{v})\rangle$$

$$-\frac{1}{2}\langle \frac{1}{\beta'}\nabla'.(\rho \boldsymbol{v} e') + \frac{1}{\beta}\nabla.(\rho' \boldsymbol{v}' e)\rangle$$

Flux

Source terms (kinetic + magnetic)

$$+\langle(\nabla.\boldsymbol{v})[R_E' - E' - \frac{\bar{\delta}\rho}{2}(\boldsymbol{v}_A.\boldsymbol{v}_A') - \frac{P'}{2} + \frac{P_M'}{2}]\rangle + \langle(\nabla'.\boldsymbol{v}')[R_E - E - \frac{\bar{\delta}\rho}{2}(\boldsymbol{v}_A.\boldsymbol{v}_A') - \frac{P}{2} + \frac{P_M}{2}]\rangle$$

$$+\langle(\nabla.\boldsymbol{v}_A)[R_H - R_H' + H' - \bar{\delta}\rho(\boldsymbol{v}'.\boldsymbol{v}_A)]\rangle + \langle(\nabla'.\boldsymbol{v}_A')[R_H' - R_H + H - \bar{\delta}\rho(\boldsymbol{v}.\boldsymbol{v}_A')]\rangle$$

where $\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \boldsymbol{v}_A$ are compressible Elsasser variables (Marsch & Mangeney 1987), $\beta = \frac{P}{P_M} = \frac{C_S^2\rho}{(1/2)\rho v_A^2}$,

$$E = \frac{1}{2}\rho(v^2 + v_A^2) + \rho e;\ R_E = \frac{1}{2}\rho(\boldsymbol{v}.\boldsymbol{v}' + \boldsymbol{v}_A.\boldsymbol{v}_A') + \rho e';\ H = \rho \boldsymbol{v}.\boldsymbol{v}_A;\ R_H = \frac{\rho}{2}(\boldsymbol{v}.\boldsymbol{v}_A' + \boldsymbol{v}'.\boldsymbol{v}_A)$$

Strong Field (B0) Approximation

- If we have a constant strong field I**B0**I >> I**δb**I : The total flux term gets reduced to

$$\Phi_{B_0} \simeq \frac{\nabla_r}{2}.\langle\delta\left(\frac{1}{\sqrt{\rho}}\right)\delta\left(\sqrt{\rho}\right)[\boldsymbol{B}_0\times(\delta\boldsymbol{v}\times\boldsymbol{B}_0)]-\delta^2\left(\frac{1}{\sqrt{\rho}}\right)[\delta(\rho\boldsymbol{v}).\boldsymbol{B}_0]\boldsymbol{B}_0\rangle$$

- The source terms do not seem to get more simplified or more indicative.
- No magnetic fluctuation terms in the flux

Discussion

- Self deformation => Alfvén effect is no more valid.
- Compressible pseudo energies are not conserved => exact relation for total energy only.
- The appropriate variable is $\rho^{1/3}\left[z^{+2} z^{-} + z^{-2} z^{+}\right]^{1/3}$
- Two types of flux terms : Characterises plasmas according to its β value.
- Strong B_0 decouples the flux terms into parallel and perpendicular direction w.r.t. B_0 .
- Phenomenological study is essential for constructing the energy spectra.

A looooooong way to go !

Thank you