RANS MODEL DEVELOPMENT IN SUPERSONIC BOUNDARY LAYER FLOW WITH AND WITHOUT SHOCKS

DÉVELOPPEMENT D'UN MODÉLE RANS EN COUCHE LIMITE D'ECOULEMENT SUPERSONIQUE AVEC ET SANS CHOCS

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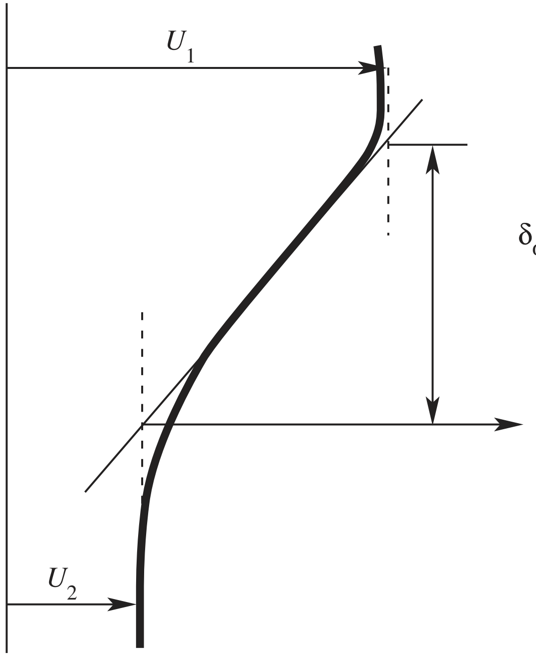
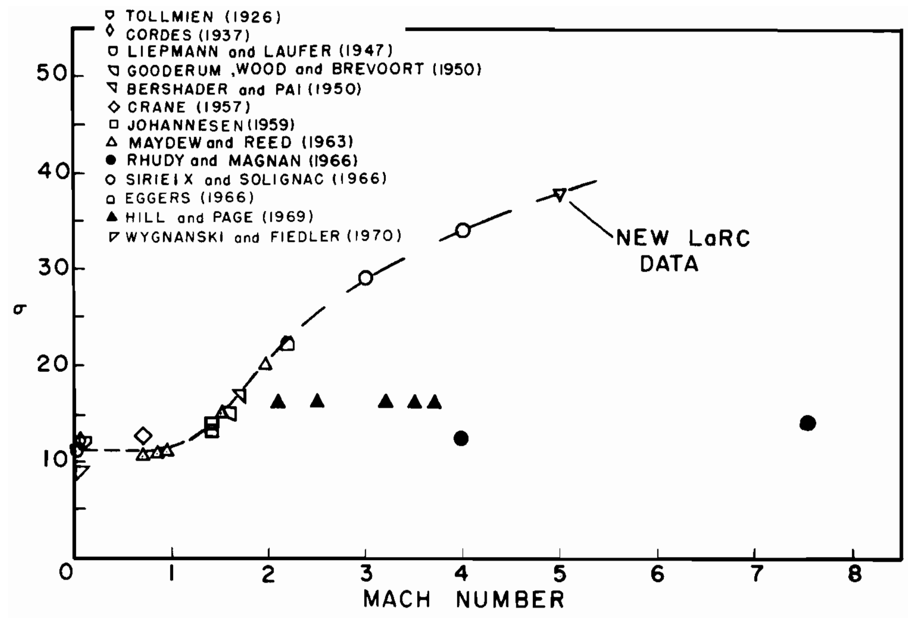
SOME BACKGROUND

- RANS: Most commonly used prediction method
  - Variable density extensions incompressible forms
- Compressibility models
  - Dilatation models
  - Influence and role of pressure fluctuations
- Reduced mixing-layer spreading rate with convective Mach number motivated early research
  - Early studies assumed changes in density distribution important

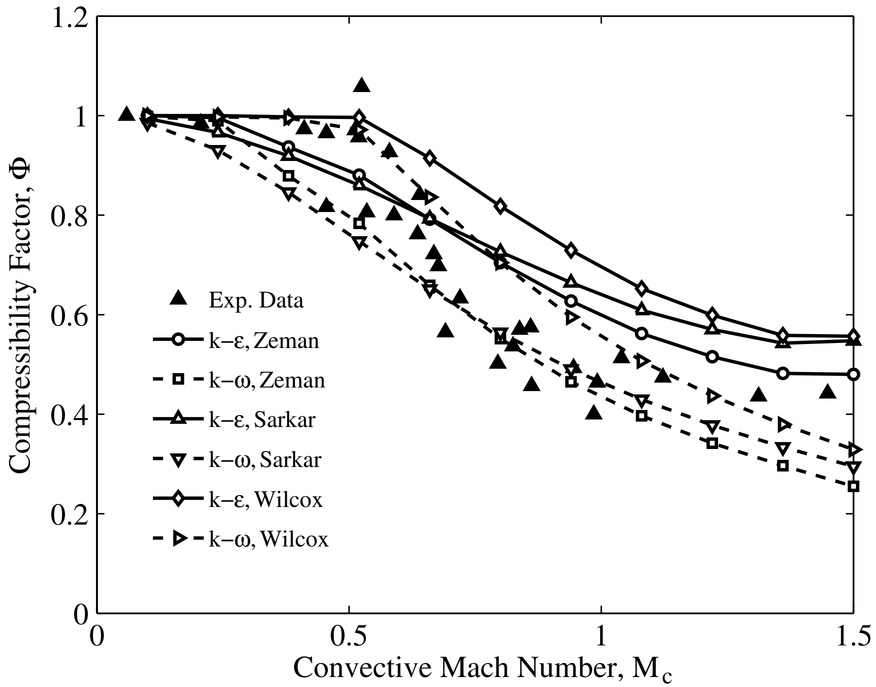
SOME BACKGROUND





Schematic of self-similar mixing layer longitudinal mean velocity profile, and variation of spreading rate with Mach number $U_2 = 0$: $\bullet$, values for incompressible flow, and other symbols for compressible mixing layers. (Birch and Eggers, 1972)

SOME BACKGROUND



$$\left(\frac{d\delta}{dx}\right)_c = \Phi\left(\frac{d\delta}{dx}\right)_i$$

Growth rate from different experimental mixing layer studies selected by Barone et al. (2006).

OUTLINE

Data from DNS of ZPG Supersonic BL Flow, $M_\infty = 2.25$

- Without Shock
  - Equilibrium modeling considerations
  - Mean velocity, thermodynamic variable distributions across BL
  - Turbulent velocity, thermodynamic and scalar flux correlation distributions across BL
  - Kinetic energy, energy dissipation rate, temperature variance and scalar flux budgets across BL
- With shock
  - Non-equilibrium modeling considerations
  - Strength of impinging shock/flow separation
  - Streamwise relaxation effects

PHYSICAL PARAMETERS

Adiabatic

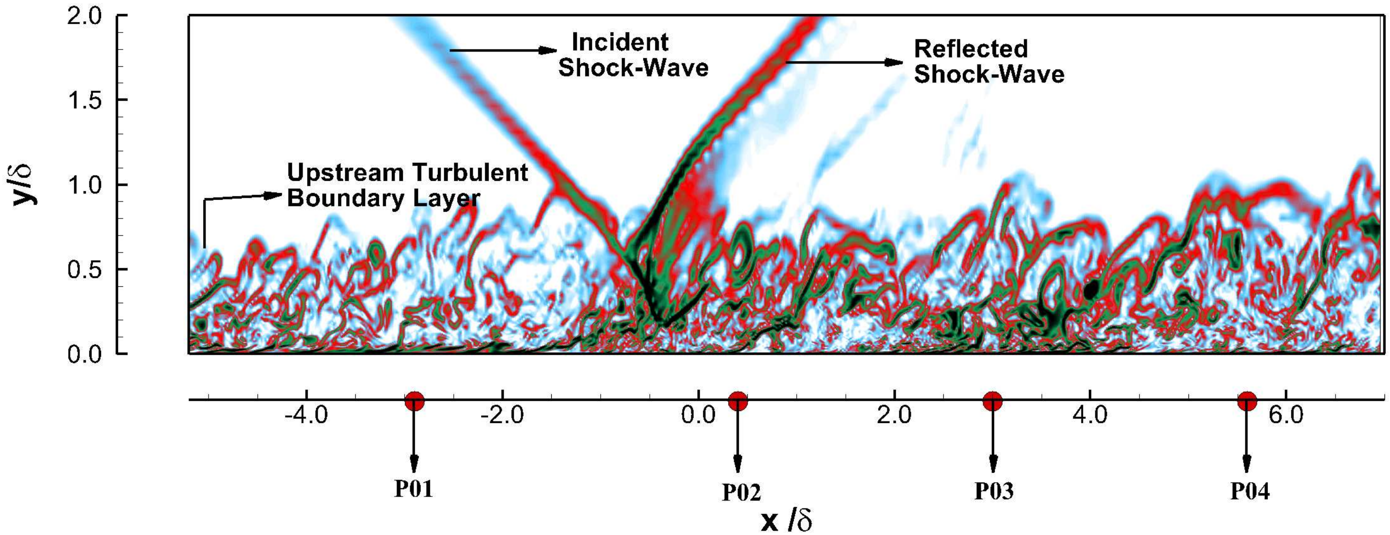
- Freestream Mach Number M_∞ =2.25
- Freestream unit Reynolds number Re/m. = 25×10^6
- Momentum Thickness Reynolds number Re_θ = 3706
- Shape factor $H = 3.40$
- $q_w = 0$, $T_{aw} = 342$K, $T_r = 323$K, $T_\infty = 170$K, $r = 0.89$
- $(-B_q, M_\tau) = (0.0, 0.077)$

Isothermal

- Freestream Mach Number M_∞ =2.25
- Freestream unit Reynolds number Re/m. = 25×10^6
- Momentum Thickness Reynolds number Re_θ = 3798
- Shape factor $H = 2.65$
- $T_w = 230$K, $T_{aw} = 342$K, $T_w/T_{aw} = 0.67$, $T_\infty = 170$K
- $(-B_q, M_\tau) = (0.017, 0.079)$

$$B_q = \frac{q_w}{\overline{\rho}_w c_p u_\tau \widetilde{T}_w}, \qquad M_\tau = \frac{u_\tau}{c_w}$$

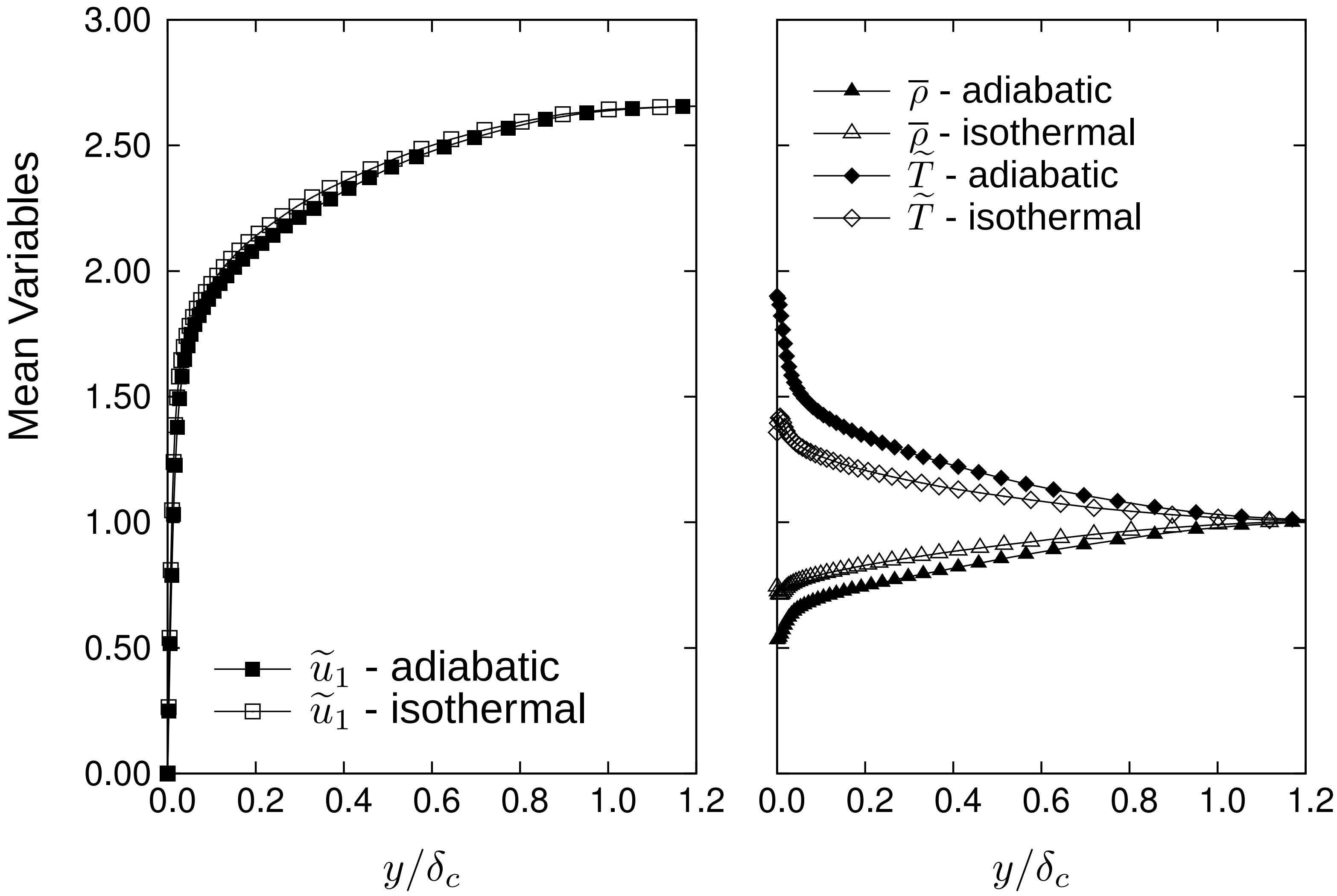
SHOCK CASE: DIAGNOSTIC POSITIONS



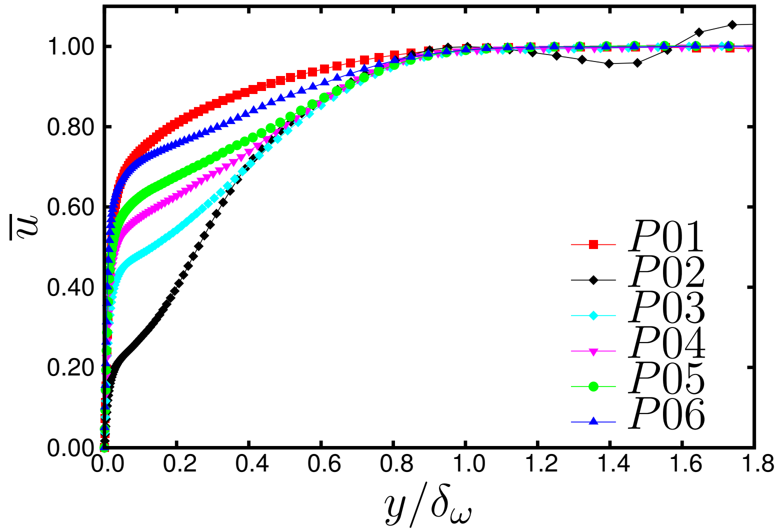
Diagnostic plane positions (x_i, impingement position of incident shock-wave; δ_c^o, boundary layer thickness at station $P01$).

$Plane$	$xpos = (x - x_i)/\delta_c^o$	
	Adiabatic	Isothermal
$P01$	-2.9	-2.9
$P02$	0.4	0.4
$P03$	3.0	3.0
$P04$	5.6	5.6
$P05$	8.2	8.2
$P06$	16.7	16.7

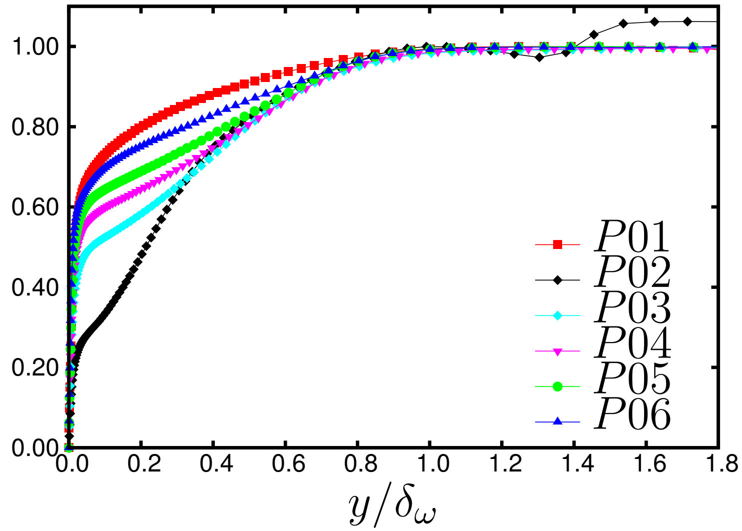
MEAN FIELD


Mean Variables
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.0 0.2 0.4 0.6 0.8 1.0 1.2
$\widetilde{u}_1$ - adiabatic
$\widetilde{u}_1$ - isothermal
y/δ_c
$\overline{\rho}$ - adiabatic
$\overline{\rho}$ - isothermal
$\widetilde{T}$ - adiabatic
$\widetilde{T}$ - isothermal
0.0 0.2 0.4 0.6 0.8 1.0 1.2
y/δ_c

MEAN VELOCITY RELAXATION



Adiabatic



Isothermal

WALL AND SEMI-LOCAL SCALING

- Proper choice of scaling dictated by collapse of correlation data
  - Usual viscous normalization

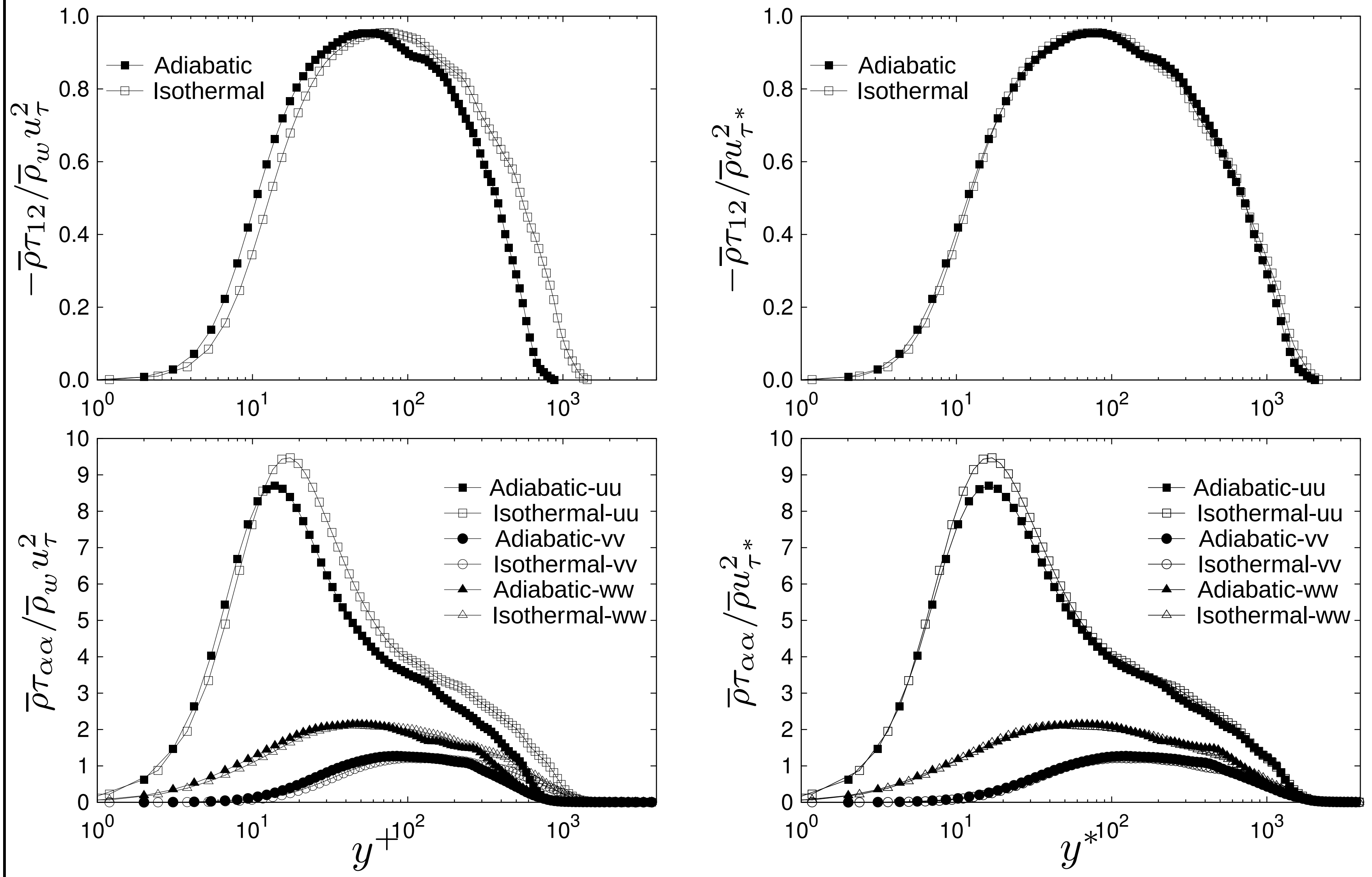
$$u_\tau = \sqrt{\tau_w/\overline{\rho}_w}, \qquad \overline{\nu}_w = \overline{\mu}_w/\overline{\rho}_w$$

  - Semi-local viscous normalization

$$u_{\tau^\star} = \sqrt{\tau_w/\overline{\rho}}, \qquad \overline{\nu} = \overline{\mu}/\overline{\rho}$$

- Need to account for density variations across boundary layer

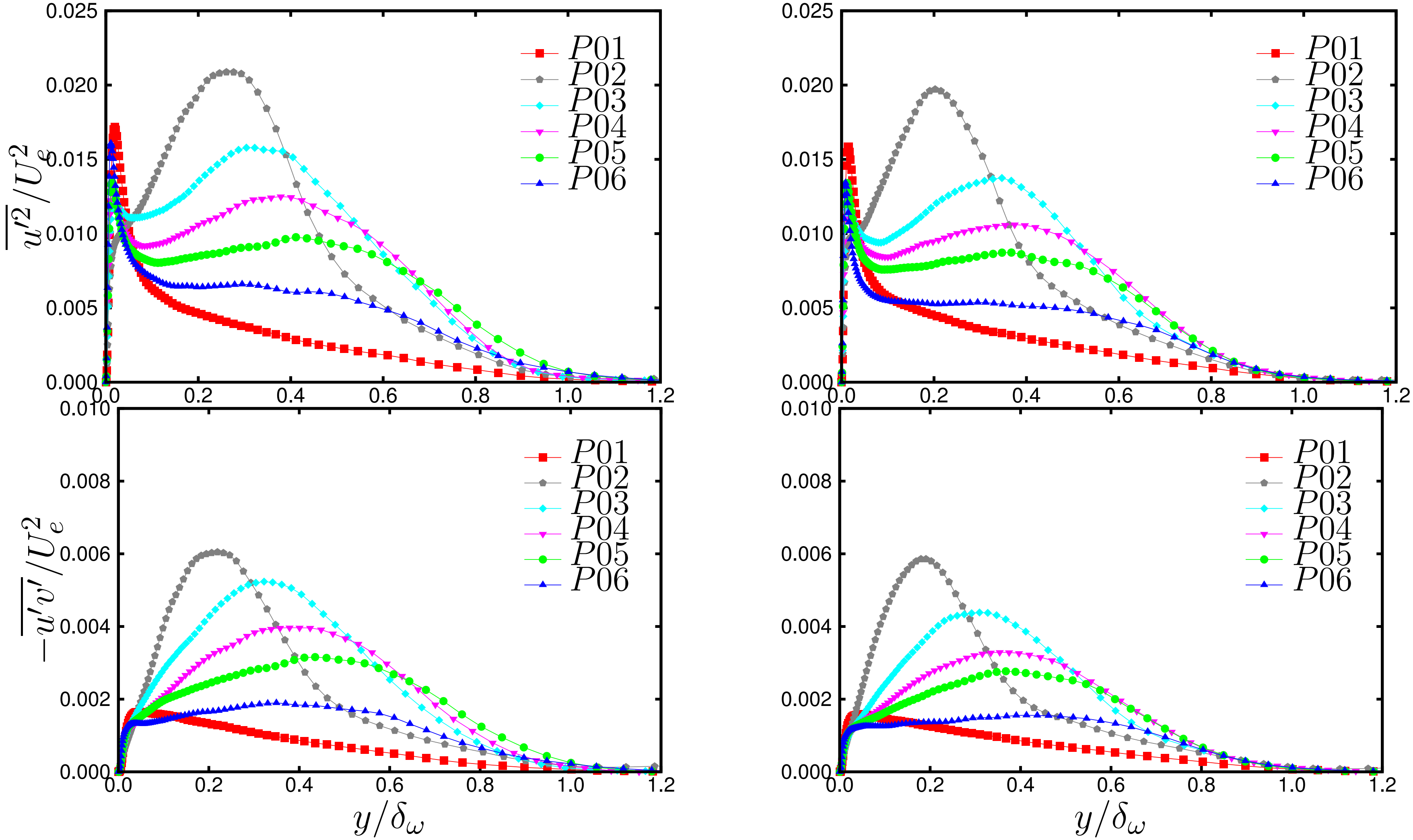
WALL AND SEMI-LOCAL SCALING



REYNOLDS STRESS DOWNSTREAM RELAXATION

Adiabatic

Isothermal



Turbulent Reynolds stress distributions across boundary layer

TURBULENT KINETIC ENERGY BUDGET

$$-A_K + P_K + \Pi_K + T_K + D_K - \epsilon + M_K = 0 \; ,$$

$A_K = \overline{\rho}\widetilde{u}_j \left(\dfrac{\partial K}{\partial x_j} \right)$ Advection (negligibly small)

$P_K = -\overline{\rho}\tau_{ik}\widetilde{S}_{ki}$ Production

$\Pi_K = \overline{p' s'_{kk}}$ Pressure-dilatation

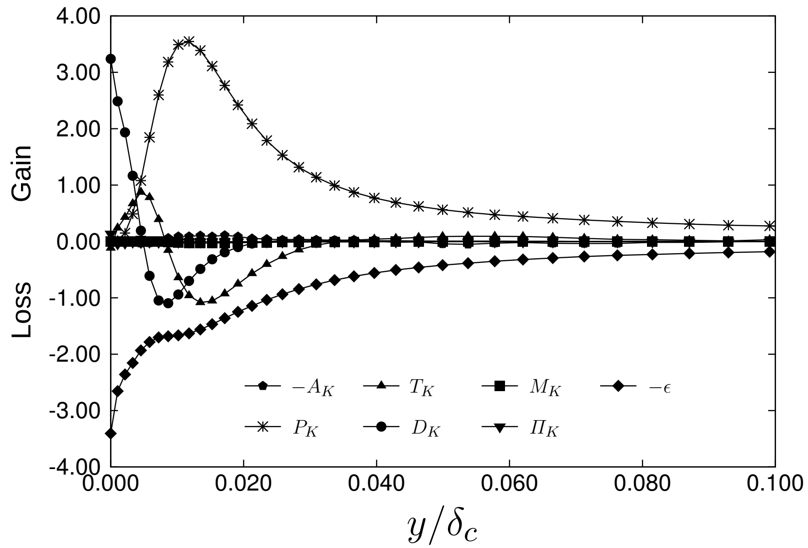
$T_K = -\dfrac{\partial}{\partial x_k} \left[\dfrac{\overline{\rho}\widetilde{u''_i u''_i u''_k}}{2} + \overline{p' u'_i}\delta_{ik} \right]$ Turbulent transport

$D_K = -\dfrac{\partial}{\partial x_k} \left[\overline{\sigma'_{ik} u'_i} \right]$ Viscous diffusion

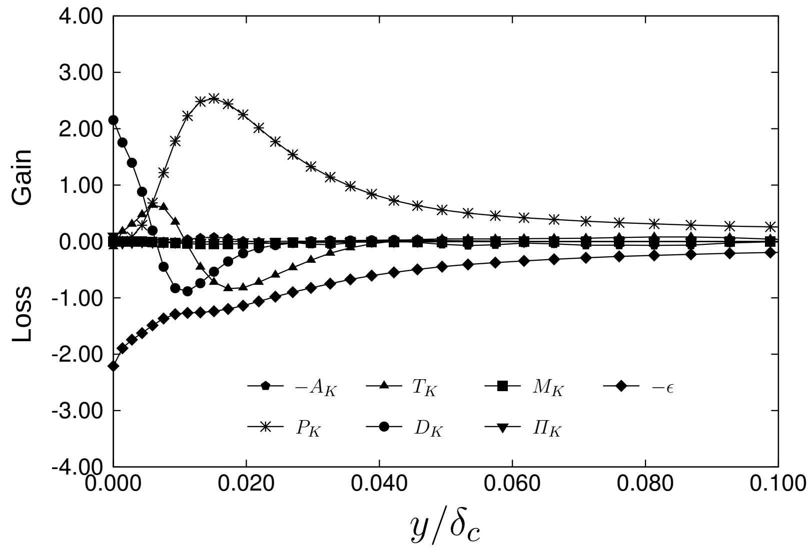
$\epsilon = \overline{\sigma'_{ik} s'_{ki}}$ Dissipation

$M_K = \overline{\rho' u'_i} \left(\dfrac{\partial \overline{p}}{\partial x_i} - \dfrac{\partial \overline{\sigma}_{ik}}{\partial x_k} \right)$ Mass flux

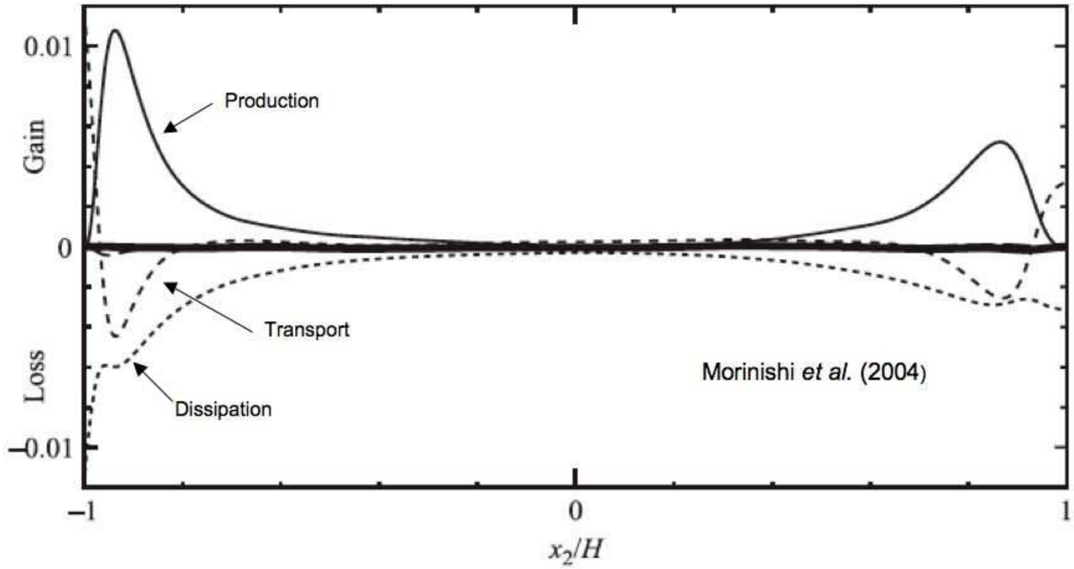
TURBULENT KINETIC ENERGY BUDGET



Isothermal



Adiabatic



TURBULENT STRESS ANISOTROPY EQUATION

$$
\begin{aligned}
\frac{Db_{ij}}{Dt} - \frac{D_K}{K}\left[d_{ij}^{(t-\mu)} - b_{ij}\right] \quad = \quad & -\left[\frac{P_K}{\epsilon} + \frac{\Pi_K}{\epsilon} + \frac{M_K}{\epsilon} - 1\right]\frac{b_{ij}}{\tau} + \frac{d_{ij}^{(\epsilon)}}{\tau} \\
& + \frac{1}{2K}\left[\left(\Pi_{ij} - \frac{2\delta_{ij}}{3}\Pi_K\right) + \left(M_{ij} - \frac{2\delta_{ij}}{3}M_K\right)\right] \\
& - \frac{2}{3}\left(\widetilde{S}_{ij} - \frac{\delta_{ij}}{3}\widetilde{S}_{ii}\right) + \left(b_{ik}\widetilde{W}_{kj} - \widetilde{W}_{ik}b_{kj}\right) \\
& - \left(b_{ik}\widetilde{S}_{kj} + \widetilde{S}_{ik}b_{kj} - \frac{2}{3}\left[\mathbf{b}{:}\widetilde{\mathbf{S}}\right]\delta_{ij}\right)
\end{aligned}
$$

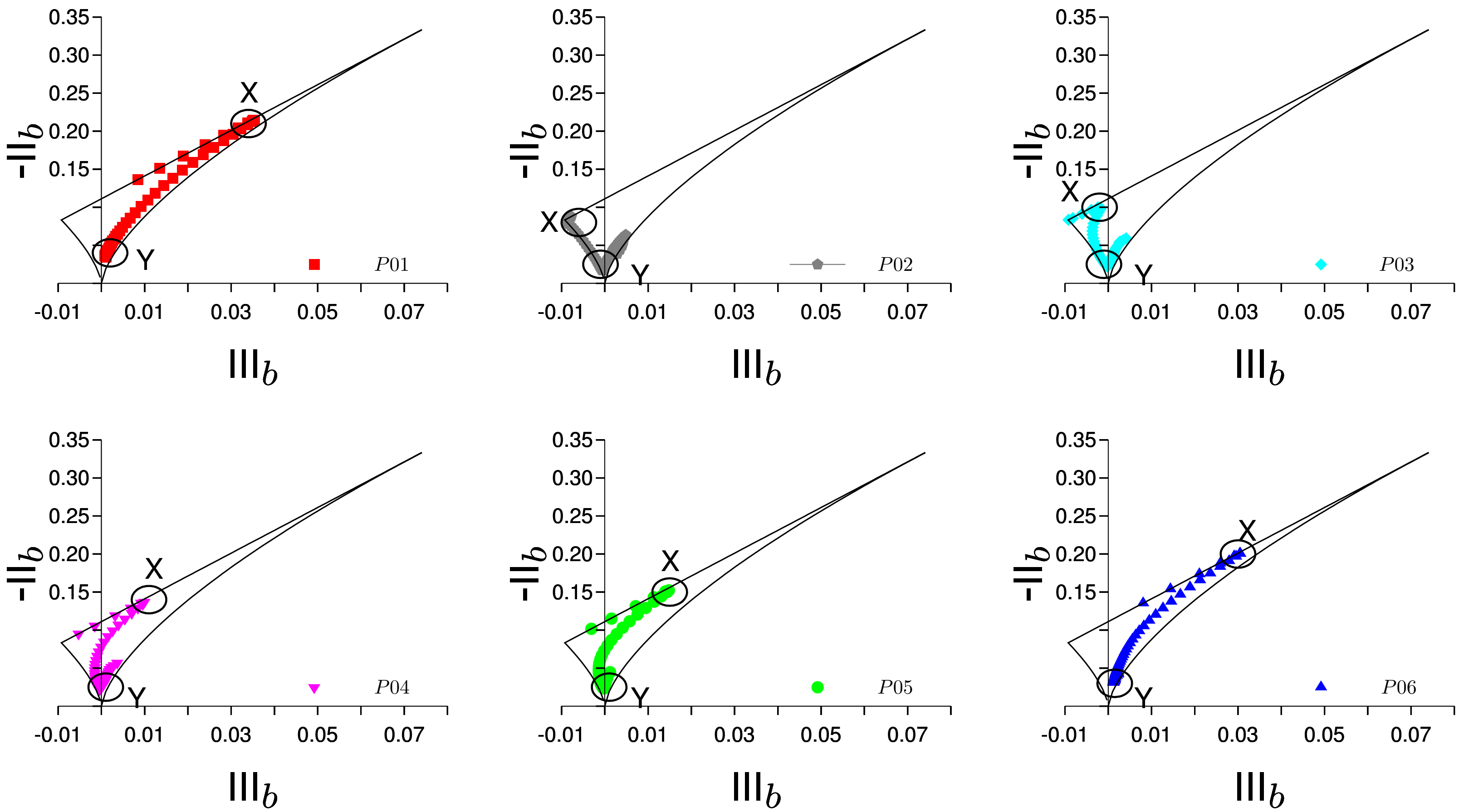
where

$$
d_{ij}^{(t-\mu)} = \frac{D_{ij}}{2D_K} - \frac{\delta_{ij}}{3}, \qquad \tau = K/\epsilon, \qquad d_{ij}^{(\epsilon)} = \frac{\epsilon_{ij}}{2\epsilon} - \frac{\delta_{ij}}{3}
$$

TURBULENT STRESS ANISOTROPY EQUATION

$$
\begin{aligned}
\frac{Db_{ij}}{Dt} - \frac{D_K}{K}\left[d_{ij}^{(t-\mu)} - b_{ij}\right] \;=\; & -\left[\frac{P_K}{K} + \frac{\mathit{\Pi}_K}{K} + \frac{M_K}{K}\right] b_{ij} + \frac{\epsilon}{K}\left[d_{ij}^{(\epsilon)} + b_{ij}\right] \\
& + \frac{\mathit{\Pi}_K}{K} d_{ij}^{(\mathit{\Pi})} + \frac{M_K}{K} d_{ij}^{(M)} \\
& - \frac{2}{3}\left(\widetilde{S}_{ij} - \frac{\delta_{ij}}{3}\widetilde{S}_{ii}\right) + \left(b_{ik}\widetilde{W}_{kj} - \widetilde{W}_{ik} b_{kj}\right) \\
& - \left(b_{ik}\widetilde{S}_{kj} + \widetilde{S}_{ik} b_{kj} - \frac{2}{3}\left[\mathbf{b}{:}\widetilde{\mathbf{S}}\right]\delta_{ij}\right)
\end{aligned}
$$

TURBULENT STRESS ANISOTROPY EQUATION



Adiabatic

PRESSURE CORRELATION TERMS

- Pressure-strain rate correlation

$$
\begin{aligned}
\overline{\rho}\mathit{\Pi}_{ij} &= \frac{2}{3}\overline{p' s'_{kk}}\delta_{ij} + 2\left(\overline{p' s'_{ij}} - \frac{\delta_{ij}}{3}\overline{p' s'_{kk}}\right) \\
&= \frac{2}{3}\overline{\rho}\mathit{\Pi}\delta_{ij} + \overline{\rho}\mathit{\Pi}^{d}_{ij}
\end{aligned}
$$

PRESSURE CORRELATION TERMS

- Pressure-strain rate correlation

$$\begin{aligned} \overline{\rho}\Pi_{ij} &= \frac{2}{3}\overline{p' s'_{kk}}\delta_{ij} + 2\left(\overline{p' s'_{ij}} - \frac{\delta_{ij}}{3}\overline{p' s'_{kk}}\right) \\ &= \frac{2}{3}\overline{\rho}\Pi\delta_{ij} + \overline{\rho}\Pi^d_{ij} \end{aligned}$$

- Pressure-variance equation

$$\overline{\rho}\Pi = -\frac{1}{\overline{c^2}}\frac{D}{Dt}\left(\frac{\overline{p'^2}}{2}\right) - \overline{p' u'_j}\frac{\partial\overline{p}}{\partial x_j} - \frac{\gamma\overline{p'^2}}{\overline{c^2}}\widetilde{S}_{jj} + hot.$$

$$\overline{\rho}\Pi = \begin{cases} \alpha_{Zd}\left(\frac{\overline{p'^2}}{p_e^2} - 1\right)\overline{\rho}\varepsilon M_t + \alpha_{Zc}\frac{\overline{p'^2}}{\gamma\overline{p}}\widetilde{S}_{kk} & \text{Zeman (1991)} \\ \alpha_{Sd}\,\overline{\rho}\varepsilon M_t^2 + \alpha_{Sc}\gamma\,\overline{p}\widetilde{S}_{kk}M_t^4 & \text{Sarkar (1992)} \end{cases}$$

PRESSURE CORRELATION TERMS

- Pressure-strain rate correlation

$$\begin{aligned}\overline{\rho}\Pi_{ij} &= \frac{2}{3}\overline{p's'_{kk}}\delta_{ij} + 2\left(\overline{p's'_{ij}} - \frac{\delta_{ij}}{3}\overline{p's'_{kk}}\right) \\ &= \frac{2}{3}\overline{\rho}\Pi\delta_{ij} + \overline{\rho}\Pi^d_{ij}\end{aligned}$$

- Convective wave equation

$$\left\{M_t^2\left[\frac{\partial}{\partial t} + \widetilde{u}_j\left(\frac{M_g}{M_t}\right)\frac{\partial}{\partial x_j}\right]^2 - \frac{\partial^2}{\partial x_j \partial x_j}\right\}p'$$

$$= 2\left(\frac{M_g}{M_t}\right)\left\{\left[\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial(\rho u''_j)}{\partial x_i}\right] + \left(\frac{M_g}{M_t}\right)\rho'\left[\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial \widetilde{u}_j}{\partial x_i}\right]\right\}$$

$$+\frac{\partial^2}{\partial x_i \partial x_j}\left[\overline{\rho}(u''_i u''_j - \widetilde{u''_i u''_j}) + \rho' u''_i u''_j\right] \ ,$$

PRESSURE CORRELATION TERMS

- Pressure-strain rate correlation

$$\begin{aligned} \overline{\rho}\Pi_{ij} &= \frac{2}{3}\overline{p's'_{kk}}\delta_{ij} + 2\left(\overline{p's'_{ij}} - \frac{\delta_{ij}}{3}\overline{p's'_{kk}}\right) \\ &= \frac{2}{3}\overline{\rho}\Pi\delta_{ij} + \overline{\rho}\Pi^d_{ij} \end{aligned}$$

- Poisson equation – time derivative of density in forcing term

$$\begin{aligned} -\frac{\partial^2 p'}{\partial x_i \partial x_i} = &-\frac{\partial^2}{\partial x_i \partial x_j}\left(\overline{\rho}\widetilde{u''_i u''_j} - \rho u''_i u''_j\right) \\ &+ 2\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial(\rho u''_j)}{\partial x_i} + \rho'\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial \widetilde{u}_j}{\partial x_i} - \frac{D^2\rho'}{Dt^2} \end{aligned}$$

PRESSURE-STRAIN RATE CORRELATION

- Solution for fluctuating pressure field from convective wave equation

$$p'(\mathbf{x},t) = \int_0^t dt' \int d^3\mathbf{x}' G(\mathbf{x}-\mathbf{x}', t-t') f(\mathbf{x}', t')$$

$$\hat{p}(\mathbf{k},t) = \int_0^t dt' \widehat{G}(\mathbf{k}, t-t') \hat{f}(\mathbf{k}, t')$$

where $f(\mathbf{x}', t')$ is right side of wave equation

$$\Pi_{ij}(\mathbf{r},\mathbf{t}) = \left\langle \mathbf{p}'(\mathbf{x}+\mathbf{r},\mathbf{t}) \left(\frac{\partial}{\partial \mathbf{x_j}} \mathbf{u_i''}(\mathbf{x},\mathbf{t}) + \frac{\partial}{\partial \mathbf{x_i}} \mathbf{u_j''}(\mathbf{x},\mathbf{t}) \right) \right\rangle$$

PRESSURE-STRAIN RATE CORRELATION

- Pressure-strain rate correlation from convective wave equation

$$
\begin{aligned}
\Pi^r_{ij} &= 2\overline{\rho}\left(\frac{M_g}{M_t}\right)\int_0^\infty dk \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi\, R(k,\theta,\phi;\tau_I) \\
&\quad \times k_l \widetilde{S}_{ln}\left[E_{ni}(k,\theta,\phi;b_{mn})k_j + E_{nj}(k,\theta,\phi;b_{mn})k_i\right]
\end{aligned}
$$

$$
\begin{aligned}
E_{ij}(\mathbf{k};\mathbf{b}) &= \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) + \frac{E_a(k)}{8\pi k^2}\left(\hat{k}_n b_{nm}\hat{k}_m\right)\left(\delta_{ij} + \hat{k}_i\hat{k}_j\right) \\
&\quad + \frac{E_a(k)}{4\pi k^2}\left[b_{ij} - \left(b_{in}\hat{k}_n\hat{k}_j + \hat{k}_i\hat{k}_n b_{nj}\right)\right]
\end{aligned}
$$

SOLENOIDAL DISSIPATION RATE BUDGET

- $\overline{\rho}\epsilon \approx \overline{\rho}\varepsilon + \overline{\rho}\varepsilon_d + \overline{\rho}\varepsilon_I$

$$\overline{\rho}\varepsilon = \overline{\mu}\overline{w'_{ij}w'_{ij}} = \overline{\mu}\overline{\omega'_i\omega'_i}$$

$$\overline{\rho}\varepsilon_d = \frac{4}{3}\overline{\mu}\overline{s'_{kk}s'_{ll}}$$

$$\overline{\rho}\varepsilon_I = 2\overline{\mu}\left[\frac{\partial^2(\overline{u'_iu'_j})}{\partial x_i\partial x_j} - 2\frac{\partial}{\partial x_k}\left(\overline{u'_k\frac{\partial u'_j}{\partial x_j}}\right)\right]$$

- Contributions involving fluctuating viscosity negligible

SOLENOIDAL DISSIPATION RATE BUDGET

$$-A_\varepsilon + P^1_\varepsilon + P^2_\varepsilon + P^3_\varepsilon + T_\varepsilon + D_\varepsilon - \Upsilon + B_\varepsilon + F_\varepsilon + \frac{\varepsilon}{\overline{\nu}} \frac{D\overline{\nu}}{Dt} = 0$$

$$P^1_\varepsilon \;=\; 2\overline{\nu} \left[\left(\overline{\omega'_i \omega'_k}\, \overline{S}_{ik} - \overline{\omega'_i \omega'_i}\, \overline{S}_{kk} \right) + \left(\overline{\omega'_i s'_{ik}}\, \overline{\Omega}_k - \overline{\omega'_i s'_{kk}}\, \overline{\Omega}_i \right) \right]$$

$$\text{Production mean shear, dilatation, vorticity}$$

$$P^2_\varepsilon \;=\; -2\overline{\nu}\, (\overline{u'_k \omega'_i})\, \frac{\partial \overline{\Omega}_i}{\partial x_k} \qquad \text{Gradient production}$$

$$P^3_\varepsilon \;=\; \overline{\nu} \left(2\overline{\omega'_i \omega'_j s'_{ji}} - \overline{\omega'_i \omega'_i s'_{kk}} \right) \qquad \text{Production due to vortex stretching}$$

$$T_\varepsilon \;=\; -\overline{\nu} \frac{\partial (\overline{u'_k \omega'_i \omega'_i})}{\partial x_k} \qquad \text{Turbulent transport}$$

$$D_\varepsilon \;=\; 2\overline{\nu} \frac{\partial}{\partial x_k} \left[\overline{\frac{\omega'_r}{\rho} \left(e_{rji} \frac{\partial \sigma_{ik}}{\partial x_j} \right)} \right] \qquad \text{Viscous diffusion}$$

SOLENOIDAL DISSIPATION RATE BUDGET

$$-A_\varepsilon + P^1_\varepsilon + P^2_\varepsilon + P^3_\varepsilon + T_\varepsilon + D_\varepsilon - \Upsilon + B_\varepsilon + F_\varepsilon + \frac{\varepsilon}{\overline{\nu}}\frac{D\overline{\nu}}{Dt} = 0$$

$$\Upsilon = 2\overline{\nu}\overline{\left[\frac{\partial}{\partial x_k}\left(\frac{\omega'_r}{\rho}\right)\right]\left(e_{rji}\frac{\partial \sigma_{ik}}{\partial x_j}\right)}, \qquad \text{Viscous destruction}$$

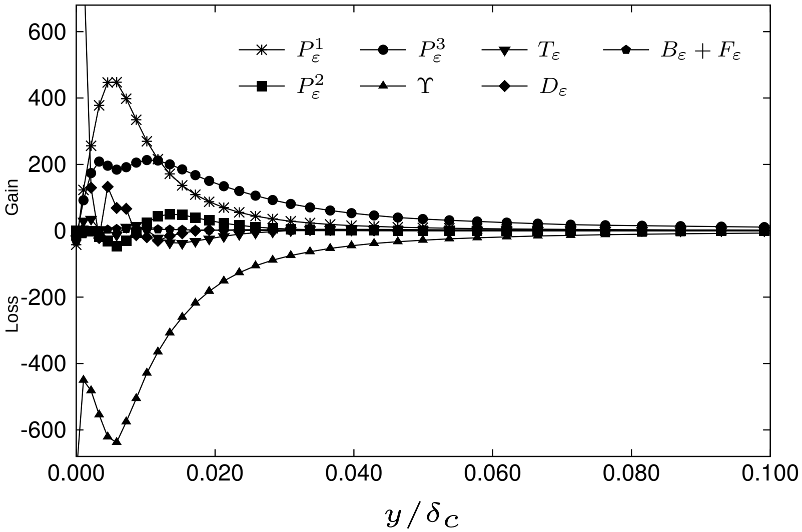
- Compressibility terms

$$B_\varepsilon = -2\overline{\nu}\overline{\left(\frac{\omega'_r}{\rho^2}\right)\left[e_{rij}\frac{\partial \rho}{\partial x_j}\frac{\partial p}{\partial x_i}\right]} \qquad \text{Baroclinic term}$$

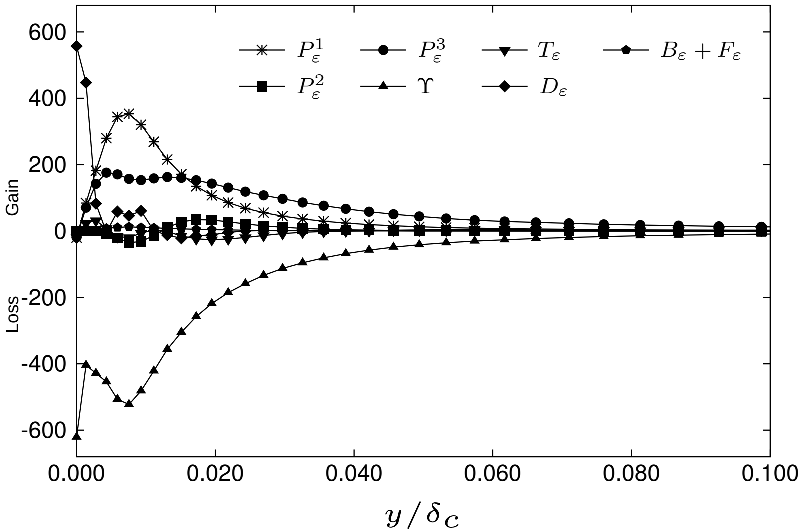
$$F_\varepsilon = 2\overline{\nu}\overline{\left(\frac{\omega'_r}{\rho^2}\right)\left[e_{rji}\frac{\partial \rho}{\partial x_j}\frac{\partial \sigma_{ik}}{\partial x_k}\right]}. \qquad \text{Viscous force term}$$

- Last term on right known

SOLENOIDAL DISSIPATION RATE BUDGET



Isothermal



Adiabatic

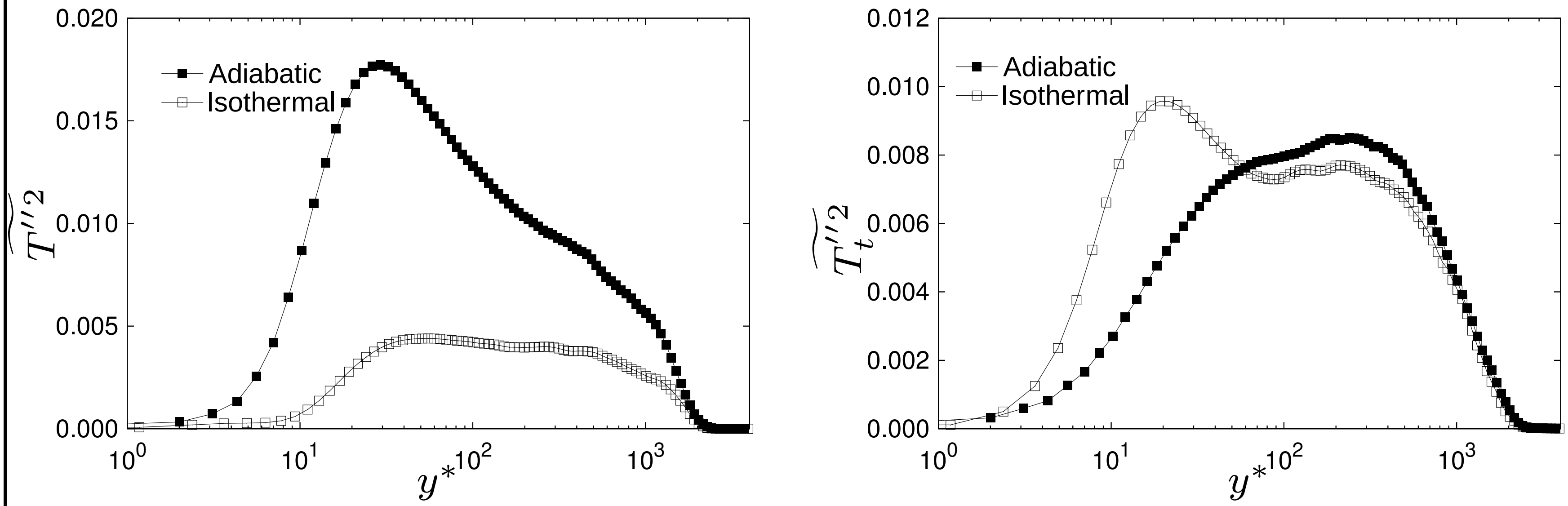
$$P_\varepsilon^1 = 2\overline{\nu}\left[\left(\overline{\omega_i'\omega_k'}\,\overline{S}_{ik} - \overline{\omega_i'\omega_i'}\,\overline{S}_{kk}\right) + \left(\overline{\omega_i' s_{ik}'}\,\overline{\Omega}_k - \overline{\omega_i' s_{kk}'}\,\overline{\Omega}_i\right)\right]$$

$$P_\varepsilon^3 = \overline{\nu}\left(2\overline{\omega_i'\omega_j' s_{ji}'} - \overline{\omega_i'\omega_i' s_{kk}'}\right)$$

$$\Upsilon = 2\overline{\nu}\overline{\left[\frac{\partial}{\partial x_k}\left(\frac{\omega_r'}{\rho}\right)\right]\left(e_{rji}\frac{\partial \sigma_{ik}}{\partial x_j}\right)}$$

- P_ε^3 relatively unaffected by change in wall condition

DISTRIBUTION OF TEMPERATURE (TOTAL) VARIANCES



- Temperature variance
  - Reduction of temperature fluctuations
  - Relatively constant over log-layer region
- Total temperature variance
  - Overshoot in buffer layer
  - Relatively constant and lesser in amplitude log-layer region

TEMPERATURE VARIANCE BUDGET

$$-A_T + P_T + T_T + D_T - \varepsilon_T + C_T^1 + C_T^2 = 0 \ ,$$

$$P_T = -\overline{\rho}\widetilde{u_k'' T''}\frac{\partial \widetilde{T}}{\partial x_k} \qquad \text{Thermal production}$$

$$T_T = \frac{\partial}{\partial x_k}\left(\frac{\overline{\rho}\widetilde{u_k'' T''^2}}{2}\right) \qquad \text{Thermal transport}$$

$$D_T = \left(\frac{\gamma}{c_p}\right)\frac{\partial}{\partial x_k}\left[\overline{k}_T\overline{T''\frac{\partial T'}{\partial x_k}} + \overline{k_T' T''}\frac{\partial \overline{T}}{\partial x_k} + \overline{k_T' T''\frac{\partial T'}{\partial x_k}}\right] \qquad \text{Thermal diffusion}$$

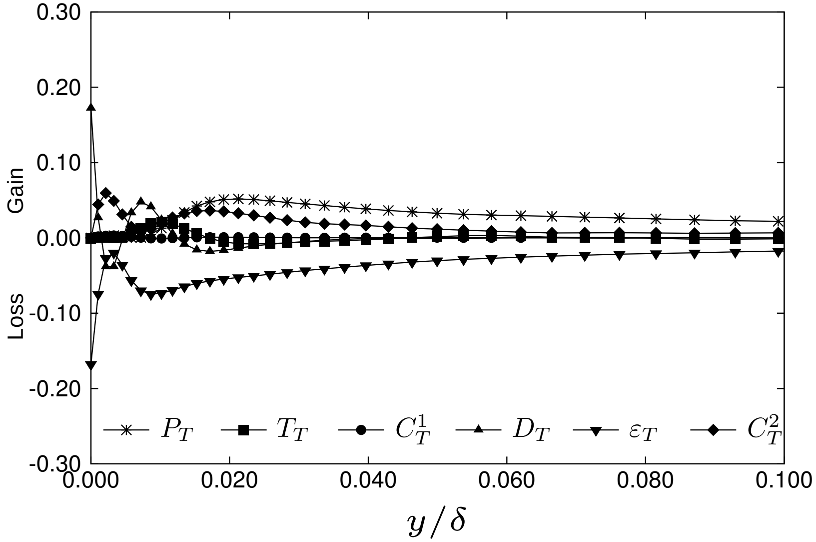
$$\varepsilon_T = \left(\frac{\gamma}{c_p}\right)\left[\overline{k}_T\overline{\frac{\partial T''}{\partial x_k}\frac{\partial T'}{\partial x_k}} + \overline{k_T'\frac{\partial T''}{\partial x_k}}\frac{\partial \overline{T}}{\partial x_k} + \overline{k_T'\frac{\partial T''}{\partial x_k}\frac{\partial T'}{\partial x_k}}\right]$$

Thermal dissipation rate

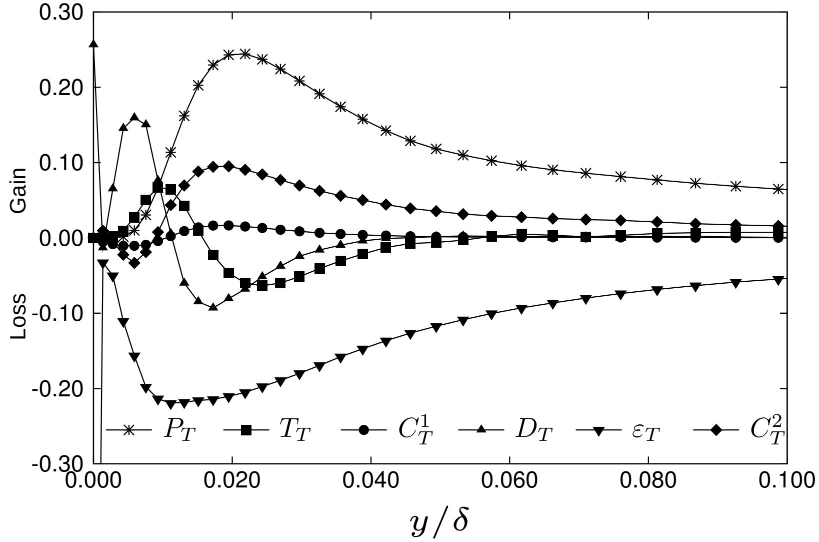
$$C_T^1 = \left(\frac{\gamma}{c_p}\right)\left[\overline{T''}\frac{\partial}{\partial x_k}\left(\overline{k}_T\frac{\partial \overline{T}}{\partial x_k}\right)\right], \quad C_T^2 = -\left(\frac{\gamma}{c_p}\right)\left[\overline{T''\left(p\frac{\partial u_k}{\partial x_k} - \sigma_{ik}\frac{\partial u_i}{\partial x_k}\right)}\right]$$

Thermal conduction — Pressure-dilatation, viscous dissipation

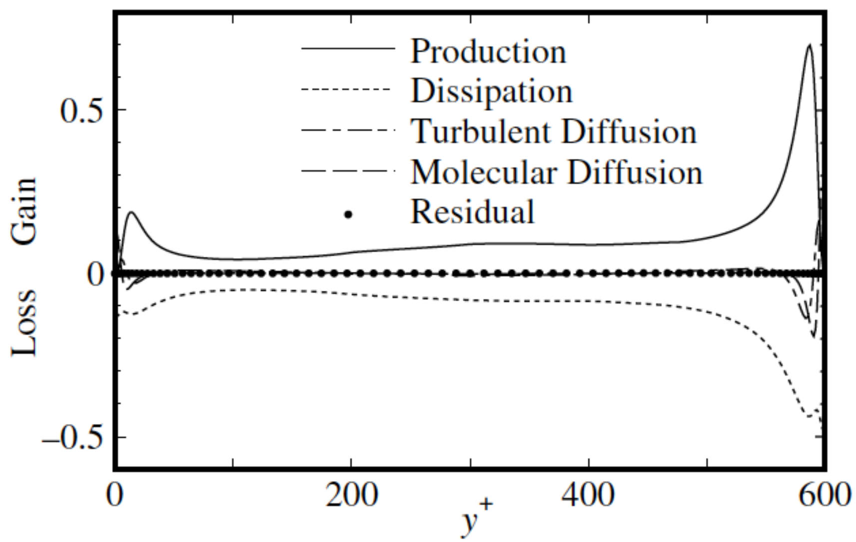
TEMPERATURE VARIANCE BUDGET



Isothermal



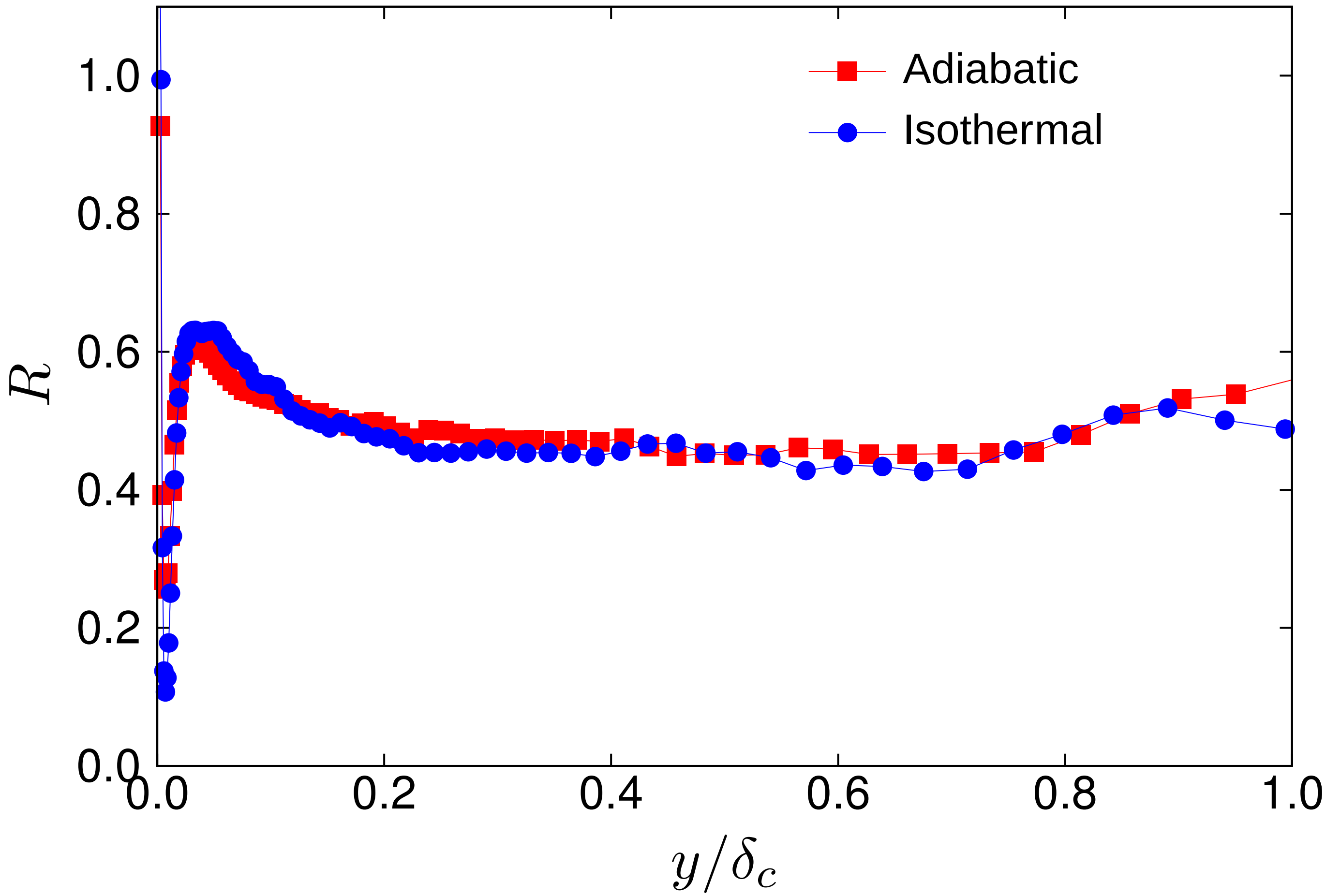
Adiabatic



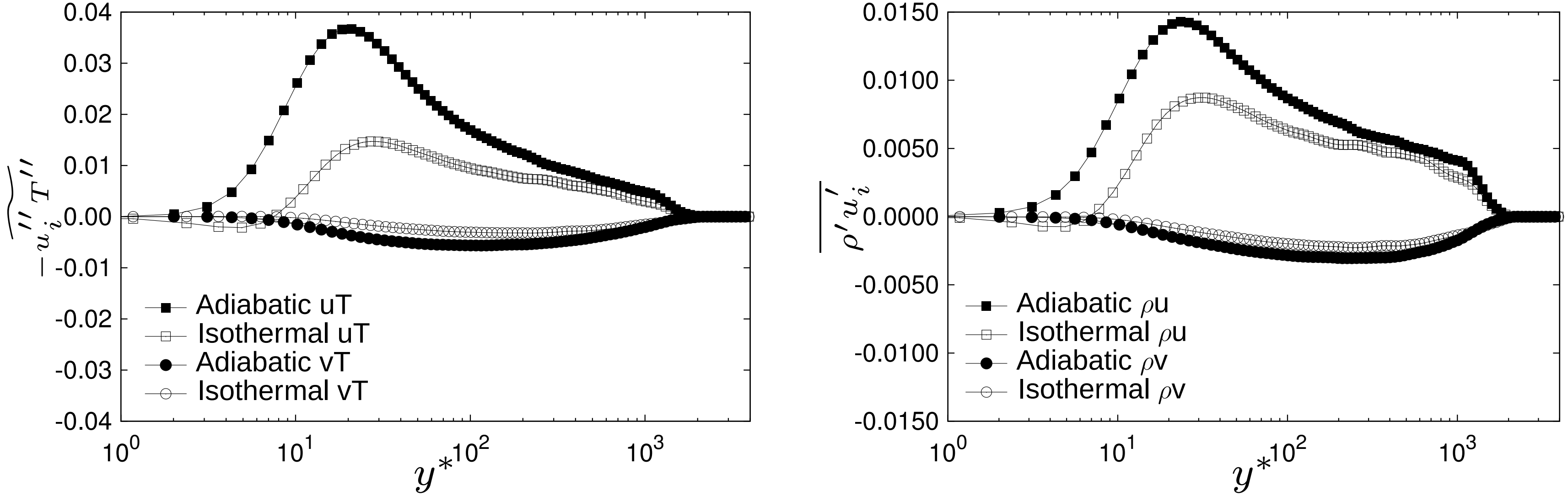
Morinishi et al. IJHMT 2007 ($T_w > T_{ad}$)

TIME SCALE RATIO

$$R = \left(\frac{K_T}{\varepsilon_T}\right) / \left(\frac{K}{\varepsilon}\right)$$



DISTRIBUTION OF HEAT AND MASS FLUX



$$\frac{\overline{p'u_i'}}{\overline{p}} = \frac{\widetilde{u_i''T''}}{\widetilde{T}} + \frac{\overline{\rho' u_i'}}{\overline{\rho}} \,.$$

- Heat and mass flux
  - Reduction of streamwise heat flux component
  - Minimal effect on $-\widetilde{v''T''}$ and $\overline{\rho' v'}$ components

HEAT FLUX BUDGETS

$$-A_{Ti} + P^1_{Ti} + P^2_{Ti} + \Phi_{Ti} + D_{Ti} - \varepsilon_{Ti} + C_{Ti} = 0\ ,$$

the production due to the mean temperature and velocity gradients,

$$P^1_{Ti} = -\overline{\rho}\tau_{ik}\frac{\partial\widetilde{T}}{\partial x_k} - \overline{\rho}\widetilde{u''_k T''}\frac{\partial\widetilde{u}_i}{\partial x_k} - \left(\frac{\gamma}{c_p}\right)\left[\overline{p u''_i}\widetilde{S}_{kk} - \overline{u''_i \sigma'_{jk}}\widetilde{S}_{kj}\right]$$

$$\text{Production mean temperature, velocity gradients}$$

$$P^2_{Ti} = \left(\frac{\gamma}{c_p}\right)\left[\overline{u''_i \sigma'_{jk} s''_{kj}} - \overline{u''_i p' s''_{kk}}\right] \qquad \text{Production fluctuating strain rate}$$

$$\Phi_{Ti} = -\left[\overline{T''\frac{\partial p}{\partial x_i}}\right] \qquad \text{Pressure} - \text{scrambling}$$

HEAT FLUX BUDGETS

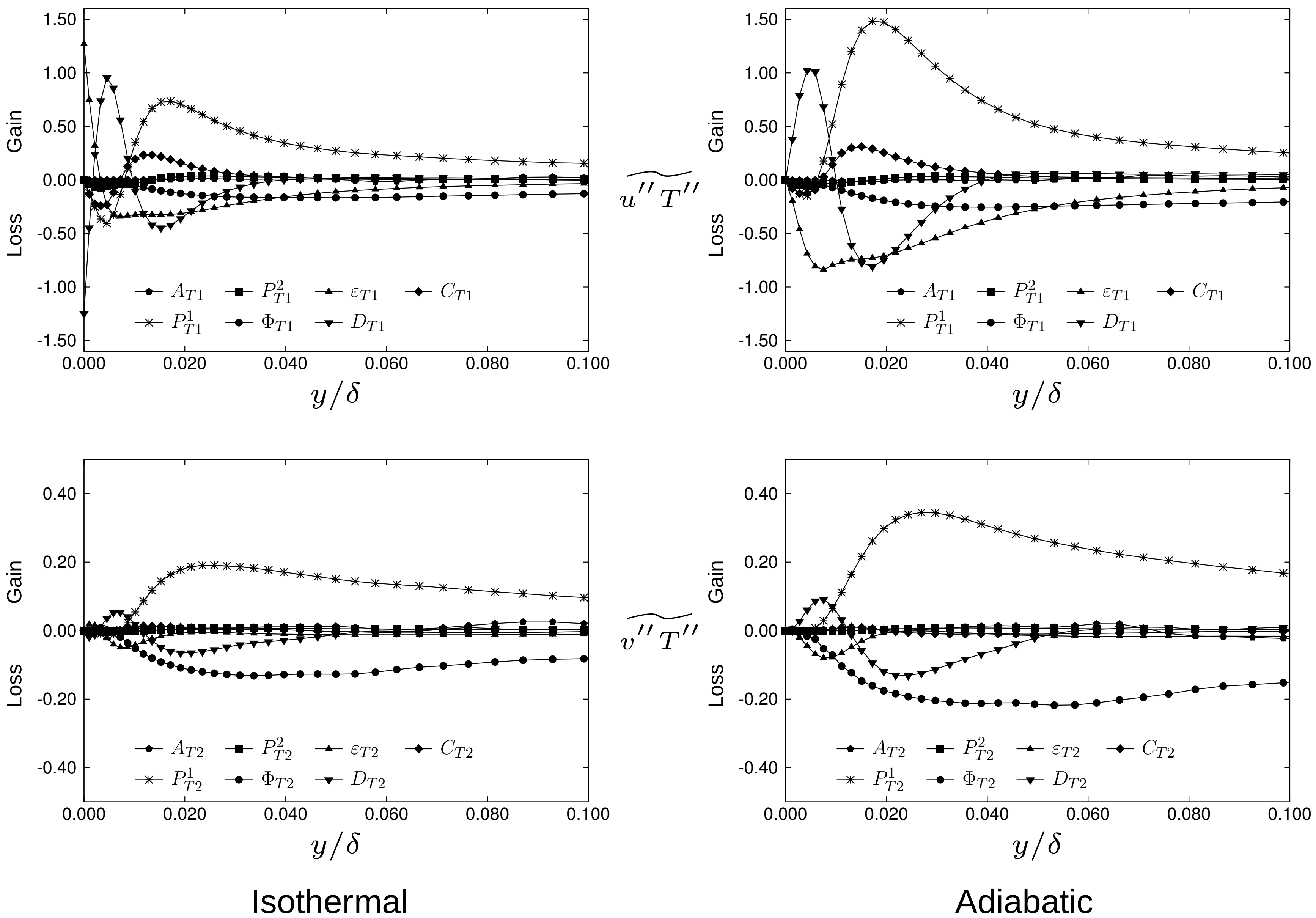
$$-A_{Ti} + P^1_{Ti} + P^2_{Ti} + \Phi_{Ti} + D_{Ti} - \varepsilon_{Ti} + C_{Ti} = 0 \ ,$$

$$D_{Ti} = \left(\frac{\gamma}{c_p}\right) \frac{\partial}{\partial x_k} \left[\overline{k}_T \overline{u''_i \frac{\partial T'}{\partial x_k}} + \overline{k'_T u''_i} \frac{\partial \overline{T}}{\partial x_k} + \overline{k'_T u''_i \frac{\partial T'}{\partial x_k}} \right]$$
$$- \frac{\partial}{\partial x_k} \left[\overline{\rho u''_i T'' u''_k} - \overline{\sigma'_{ik} T''} \right]$$
$$\text{Turbulent, viscous} - \text{thermal transport}$$

$$\varepsilon_{Ti} = \left(\frac{\gamma}{c_p}\right) \left[\overline{k}_T \overline{\frac{\partial u''_i}{\partial x_k} \frac{\partial T'}{\partial x_k}} + \overline{k'_T \frac{\partial u''_i}{\partial x_k}} \frac{\partial \overline{T}}{\partial x_k} + \overline{k'_T \frac{\partial u''_i}{\partial x_k} \frac{\partial T'}{\partial x_k}} \right] + \left[\overline{\sigma'_{ik} \frac{\partial T''}{\partial x_k}} \right]$$
$$\text{Turbulent, viscous} - \text{thermal dissipation}$$

$$C_{Ti} = \overline{T''} \frac{\partial \overline{\sigma}_{ik}}{\partial x_k} + \left(\frac{\gamma}{c_p}\right) \left[\overline{u''_i} \overline{\sigma}_{jk} \widetilde{S}_{kj} + \overline{u''_i s''_{kj}} \left(\overline{\sigma}_{kj} - \overline{p} \delta_{kj} \right) \right] \quad \text{Compressibility}$$

HEAT FLUX BUDGETS



RANS MODEL DEVELOPMENT

- Higher-Order Velocity Correlation Models
  - Pressure-strain rate, pressure-dilation and solenoidal dissipation
  - Variable density extensions for other unknown correlations
  - Wall-proximity modifications
- Temperature Variance Models
  - Variance dissipation rate model
  - Numerous unknown correlations
  - Wall-proximity modifications
- Thermal-flux Models
  - Pressure-scrambling term important
  - Numerous unknown correlations
  - Wall-proximity modifications

RANS MODEL DEVELOPMENT

- Low-Order Velocity Correlation Models
  - Wide variety of two-equation models
  - Heat flux vector
    - Gradient transport models - wall proximity corrections
    - Variable Pr_t effects
    - Variable time scale ratio
- Hybrid Methods
  - Simplest level - provides RANS solution where LES is deficient
  - Complex level - provides linkage between DNS/LES and RANS

References

Barone, M. F., Oberkampf, W. L., Blottner, F. G., 2006. Validation case study: Prediction of compressible turbulent mixing layer growth rate. AIAA J. 44, 1488–1497.

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