On the measurement of small-scale turbulence

R A Antonia

The University of Newcastle

Australia

<ε> is important since it “plays” several complex roles

- It represents the rate at which the mean turbulent energy is dissipated by the small scales of motion

- It measures the rate at which energy is fed into the large scale motion

- It also indicates the rate at which the energy is transferred down the cascade

$$(1/ < u^2 >) d < u^2 > / dt \sim U_0 / L_0$$

$$< \varepsilon > \sim U_0^3 / L_0$$

…allows the experimenter to ‘guesstimate’

$$\eta \equiv (\nu^3 / < \varepsilon >)^{1/4}$$

Full expressions

$$\langle \varepsilon \rangle = \nu[2\langle u_{1,1}^2 \rangle + 2\langle u_{2,2}^2 \rangle + 2\langle u_{3,3}^2 \rangle + \langle u_{1,2}^2 \rangle + \langle u_{2,1}^2 \rangle + \langle u_{1,3}^2 \rangle$$
$$+ \langle u_{3,1}^2 \rangle + \langle u_{2,3}^2 \rangle + \langle u_{3,2}^2 \rangle + 2\langle u_{1,2}u_{2,1} \rangle + 2\langle u_{1,3}u_{3,1} \rangle + 2\langle u_{2,3}u_{3,2} \rangle]$$

$$\langle \omega^2 \rangle = \langle u_{1,2}^2 \rangle + \langle u_{2,1}^2 \rangle + \langle u_{1,3}^2 \rangle + \langle u_{3,1}^2 \rangle + \langle u_{2,3}^2 \rangle + \langle u_{3,2}^2 \rangle$$
$$-2\langle u_{1,2}u_{2,1} \rangle - 2\langle u_{1,3}u_{3,1} \rangle - 2\langle u_{2,3}u_{3,2} \rangle$$

Considerable simplification if local isotropy is assumed

$$\langle \varepsilon \rangle_{iso} = 15\nu \langle u_{1,1}^2 \rangle \Big|$$

$$\langle \omega^2 \rangle_{iso} = 15 \langle u_{1,1}^2 \rangle \Big|$$

$$< \varepsilon_\theta >_{iso} = 3\nu_\theta < \theta_{,1}{}^2 >$$

Benchmarks for testing small-scale measurements

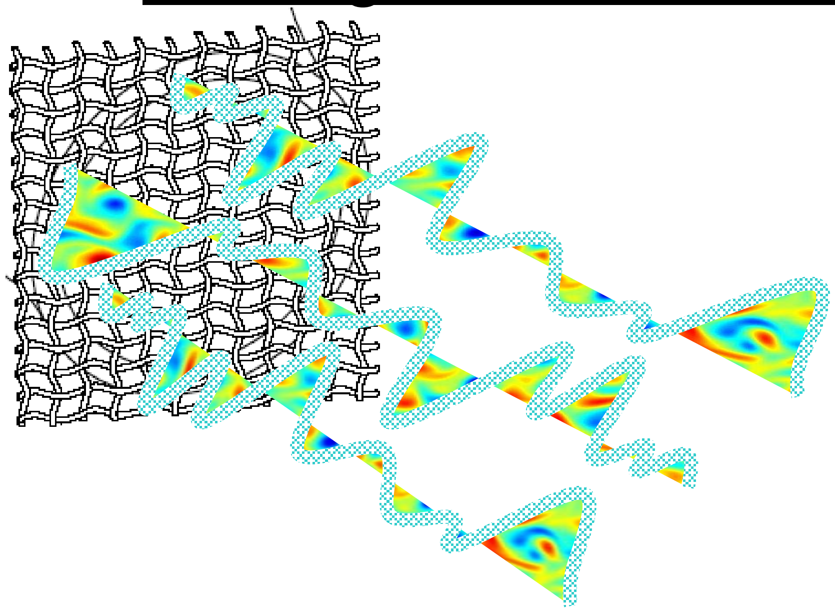
- Homogeneous Isotropic Turbulence (HIT) …important theoretical results

- Use of Direct Numerical Simulations (DNS)

- Intercomparison between different measurement techniques

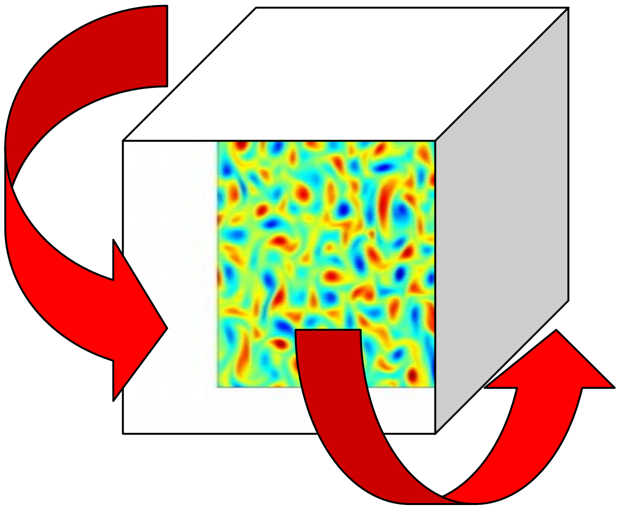
- Other strategies ? e.g. use of synthetic signals

Realizations of homogeneous isotropic turbulence



Grid turbulence – **SPATIALLY DECAYING**

- approximately homogeneous and isotropic
- low turbulence intensity (for passive grids)



Box turbulence – **FORCED or TEMPORALLY DECAYING**

- homogeneous and isotropic

Comparison with grid turbulence data is useful since $< \varepsilon >$ and $< \varepsilon_\theta >$ can be measured accurately in this flow.

$$< \varepsilon > \; = - U \frac{d < q^2 >}{dx}$$

$$< \varepsilon_\theta > \; = -U \frac{d < \theta^2 >}{dx}$$

…with the ‘caveat’ that obtaining a ‘correct’ answer may not be enough

Many attempts at comparing with grid turbulence data

- HWA e.g. Schedvin Stegen Gibson (1974) Wallace (1986)
  Tsinober Kit Dracos (1992) Antonia Zhou Zhu(1998)

- LDV e.g. Michelet Antoine Lemoine Mahouast (1998)

- PIV e.g. Westerweel Dabiri Gharib (1997) Poelma Westerweel Ooms (2006)

but one can't help wonder why this is not done
…as a matter of course

Smolyakov and Tkachenko (1983)

A major problem in the experimental investigation of small-scale turbulence is the interpretation of experimental data which are always distorted by the averaging effect of the transducers.

“The problem of reconstruction of the true fluctuations from the recorded transducer signal belongs to the class of the so-called ill-posed problems and can only be solved approximately.”

“In fact, the a priori knowledge about the turbulence field that is necessary for the solution of the problem must be greater than the amount of information obtained as a result of our measurements.Unfortunately,the experimenter cannot easily break out of this vicious circle.”

“dog chasing tail” syndrome ?

One of the major difficulties…

…is the spatial resolution

$$\eta / L = C_1 \operatorname{Re}^{-3/4}$$

$$\operatorname{Re} = UL / \nu$$

$$\eta_B / L = C_1 \operatorname{Re}^{-3/4} Sc^{-1/2}$$

strain-limited diffusion layer $\lambda_D \sim (D / \gamma)^{1/2}$ $\gamma = (<\varepsilon> / \nu)^{1/2}$

$$\lambda_D / L = C_2 \operatorname{Re}^{-3/4} Sc^{-1/2}$$

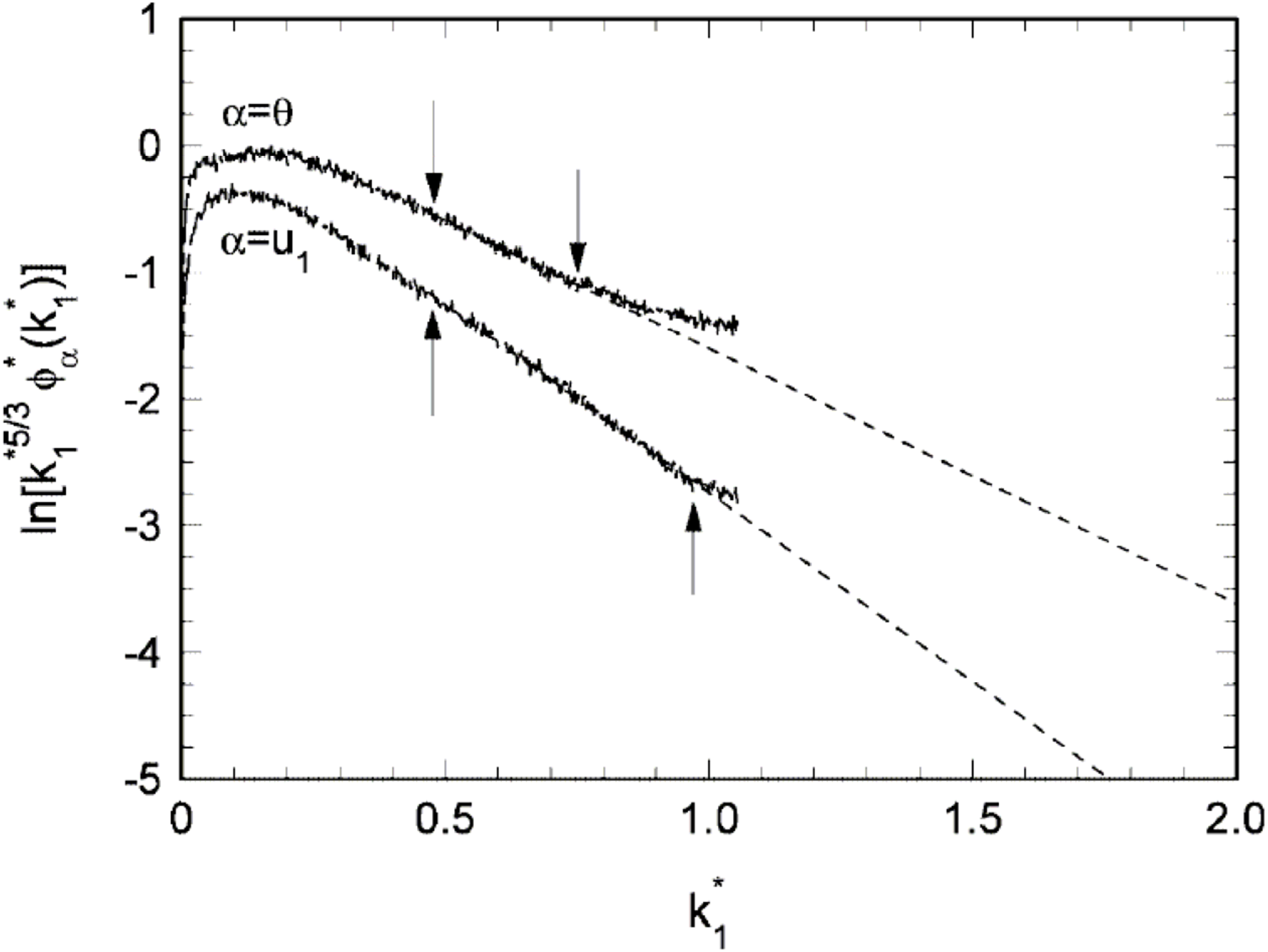
e.g. along axis of circular jet $U \equiv U_0$ $L \equiv \delta$

$C_1 = 1.7$ (Antonia Satyaprakash Hussain 1980)

$C_2 = 11.2$ (Buch Dahm 1998)

$C_2 = 25$ (Miller Dimotakis 1991)

but there are many other sources of errors !!



α=θ
α=u1
ln[k1*5/3 φα*(k1*)]
k1*
1
0
-1
-2
-3
-4
-5
0
0.5
1.0
1.5
2.0

…but “ life was not meant to be easy”

ex prime minister of Australia

Main topics to be covered

1. HIT developments

   a. scale-by-scale energy and scalar variance budgets
   b. implications for large and small separations

2. Spectral corrections - validation ?

3. Correction for HWA data

4. Correction for PIV data

Kolmogorov (1941) or K41

- cascade of energy from large to small scales

- small scales

  become statistically independent of the large scales

  are homogeneous and isotropic

  have UNIVERSAL characteristics

…independence on initial conditions

Karman-Howarth (1938) equation

$$-\frac{1}{3}\left(\frac{\partial}{\partial r}+\frac{4}{r}\right)<(\delta u)^3> \;=\; \frac{4}{3}<\varepsilon>-2\nu\left(\frac{\partial^2}{\partial r^2}+\frac{4}{r}\frac{\partial}{\partial r}\right)<(\delta u)^2>+\frac{\partial}{\partial t}<(\delta u)^2>$$

Integrate with respect to r

$$-<(\delta u)^3>=\frac{4}{5}<\varepsilon>r \;-\; 6\nu\frac{\partial}{\partial r}<(\delta u)^2>$$

$$+\;\frac{3}{r^4}\int_0^r s^4\frac{\partial}{\partial t}<(\delta u)^2>ds$$

Saffman (1968) Danaila Anselmet Zhou Antonia (1999)

If $\partial / \partial t$ is of order $< \varepsilon > / < u^2 >$

then $\partial < (\delta u)^2 > / \partial t$ is negligible provided

$$\frac{< (\delta u^*)^2 >}{R_\lambda} \ll 1 \quad \text{or} \quad \frac{< (\delta u)^2 >}{< u^2 >} \ll 1$$

These requirements are met if R_λ is very large or $r \ll L$

Kolmogorov’s equation (1941)

$$-\left\langle (\delta u)^3 \right\rangle + 6\nu \frac{d}{dr}\left\langle (\delta u)^2 \right\rangle = \frac{4}{5}\langle \varepsilon \rangle r$$

or in a more general form (Antonia Ould-Rouis Anselmet Zhu 1997)

$$-\left\langle (\delta u)(\delta q)^2 \right\rangle + 2\nu \frac{d}{dr}\left\langle (\delta q)^2 \right\rangle = \frac{4}{3}\langle \varepsilon \rangle r$$

where $\left\langle (\delta q)^2 \right\rangle \equiv \left\langle (\delta u)^2 \right\rangle + \left\langle (\delta v)^2 \right\rangle + \left\langle (\delta w)^2 \right\rangle$

Kolmogorov’s 4/5 “law”

In the inertial range, ν can be neglected...

$$-\left\langle (\delta u)^3 \right\rangle = \frac{4}{5} \langle \varepsilon \rangle r$$

and

$$-\left\langle (\delta u)(\delta q)^2 \right\rangle = \frac{4}{3} \langle \varepsilon \rangle r$$

Batchelor (1947) assumed that

$$<(\delta u)^2> = u_K^2 \ f\left(\frac{r}{\eta}\right)$$

$$<(\delta u)^3> = u_K^3 \ g\left(\frac{r}{\eta}\right)$$

After substituting in Kolmogorov's equation

$$-g\left(\frac{r}{\eta}\right) = \frac{4}{5}\frac{r}{\eta} - 6f'\left(\frac{r}{\eta}\right)$$

which is consistent with the first similarity hypothesis

Moffatt (2002)

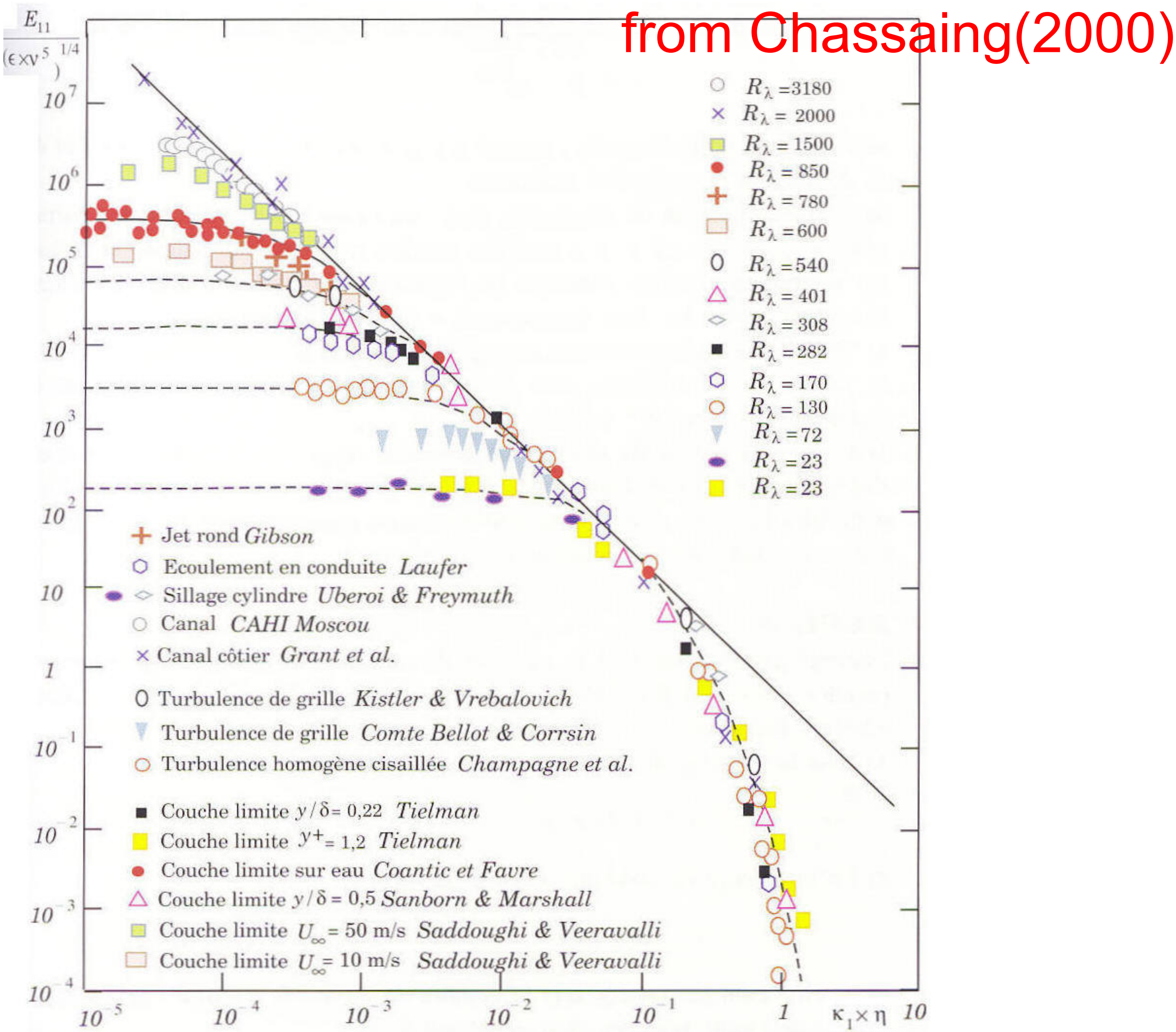
re : 'momentous' 1961 meeting in Marseille

"…Kolmogorov gave his lecture, which I recall was in the sort of French that was as incomprehensible to the French themselves as to the other participants."

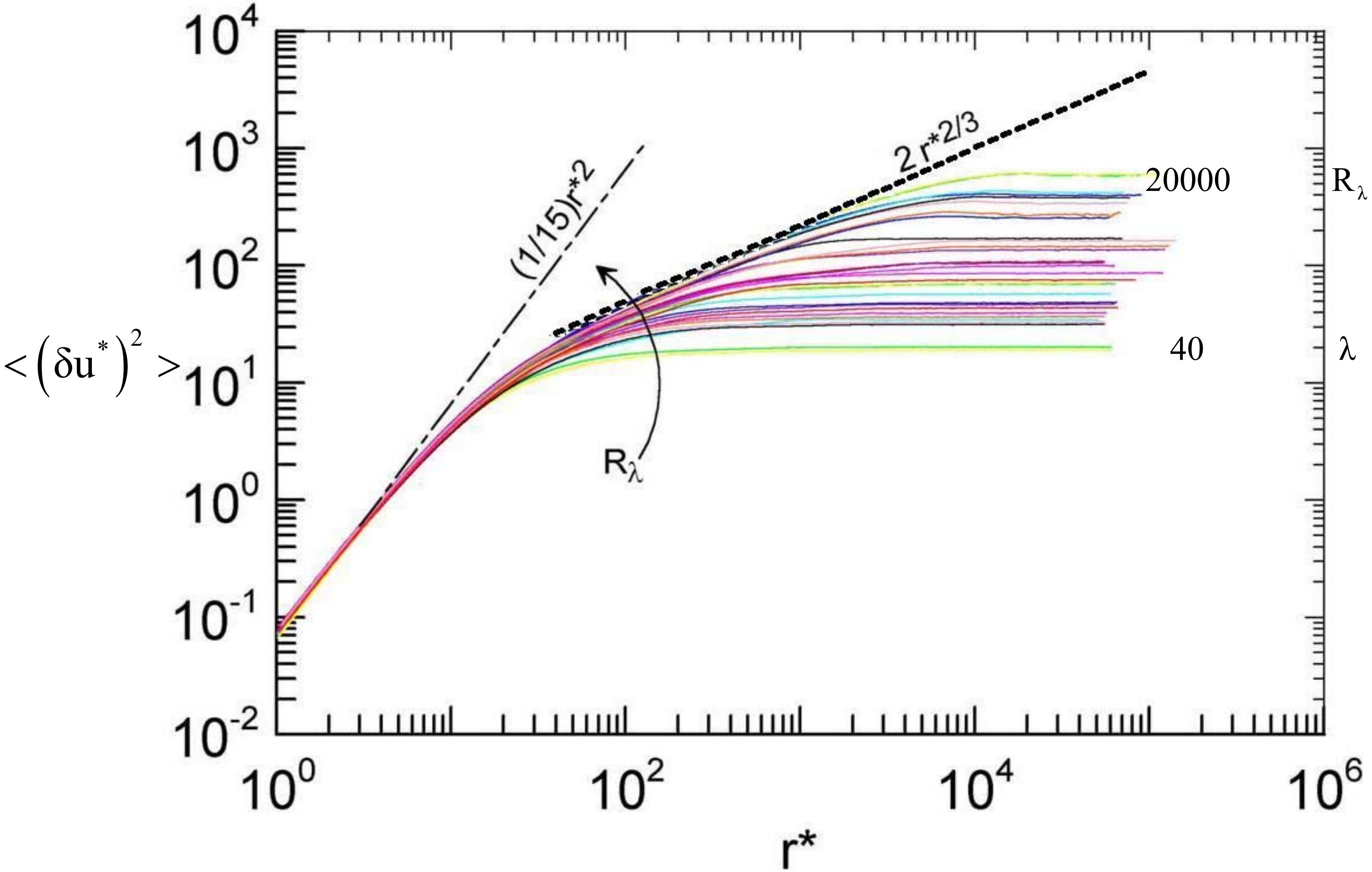
"…it was perhaps the explicit revelation that all was not well with Kolmogorov's theory that finally led him [Batchelor] to abandon turbulence in favour of other fields."

Batchelor (1961)

“The theory is an asymptotic one, and its predictions hold with increasing accuracy (if the theory is correct) as $R_\lambda \to \infty$, but no theoretical estimate has been made of the actual value of R_λ needed for a given degree of accuracy.”



from Chassaing(2000)
$E_{11}/(\epsilon \times \nu^5)^{1/4}$
$R_\lambda = 3180$
$R_\lambda = 2000$
$R_\lambda = 1500$
$R_\lambda = 850$
$R_\lambda = 780$
$R_\lambda = 600$
$R_\lambda = 540$
$R_\lambda = 401$
$R_\lambda = 308$
$R_\lambda = 282$
$R_\lambda = 170$
$R_\lambda = 130$
$R_\lambda = 72$
$R_\lambda = 23$
$R_\lambda = 23$
Jet rond *Gibson*
Ecoulement en conduite *Laufer*
Sillage cylindre *Uberoi & Freymuth*
Canal *CAHI Moscou*
Canal côtier *Grant et al.*
Turbulence de grille *Kistler & Vrebalovich*
Turbulence de grille *Comte Bellot & Corrsin*
Turbulence homogène cisaillée *Champagne et al.*
Couche limite $y/\delta = 0{,}22$ *Tielman*
Couche limite $y+ = 1{,}2$ *Tielman*
Couche limite sur eau *Coantic et Favre*
Couche limite $y/\delta = 0{,}5$ *Sanborn & Marshall*
Couche limite $U_\infty = 50$ m/s *Saddoughi & Veeravalli*
Couche limite $U_\infty = 10$ m/s *Saddoughi & Veeravalli*
$\kappa_1 \times \eta$



$$R_\lambda = \frac{<u^2>^{\frac{1}{2}}\lambda}{\nu}$$

$$\lambda = \frac{<u^2>^{\frac{1}{2}}}{<(\partial u/\partial x)^2>^{\frac{1}{2}}}$$

Pearson and Antonia (2001)

Grid turbulence

(Danaila Anselmet Zhou Antonia 1999)

$$-<(\delta u)^3> \;=\; \frac{4}{5}<\varepsilon>r \;-\; 6\nu\frac{\partial}{\partial r}<(\delta u)^2> \;+ I_u$$

where $$I_u \;\equiv\; \frac{3U}{r^4}\int_0^r s^4\,\frac{\partial}{\partial x}\,<(\delta u)^2>ds$$

At large r

$$<(\delta u)^3> \to 0 \qquad <(\delta u)^2> \to 2<u^2>$$

$$\boxed{<\varepsilon> = -\frac{U}{2}\frac{d<q^2>}{dx}}$$

if $<q^2> = 3<u^2>$

Grid turbulence

Danaila, Anselmet and Antonia (2002)

Transport equation for $\left\langle (\delta q)^2 \right\rangle$

$$-\left\langle (\delta u)(\delta q)^2 \right\rangle + 2\nu \frac{\mathrm{d}}{\mathrm{d}r}\left\langle (\delta q)^2 \right\rangle - I_q = \frac{4}{3}\langle \varepsilon \rangle r$$

where $$I_q \equiv \frac{U}{r^2}\int_0^r s^2 \frac{\partial}{\partial x}\left\langle (\delta q)^2 \right\rangle \mathrm{d}s$$

Equation can be interpreted as a scale-by-scale energy budget

At large r, $$< \varepsilon > \; = \; -\frac{\mathrm{U}}{2}\frac{\mathrm{d} < \mathrm{q}^2 >}{\mathrm{dx}}$$

- Transport equation for $<(\delta\theta)^2>$

Danaila Anselmet Zhou Antonia (1999)

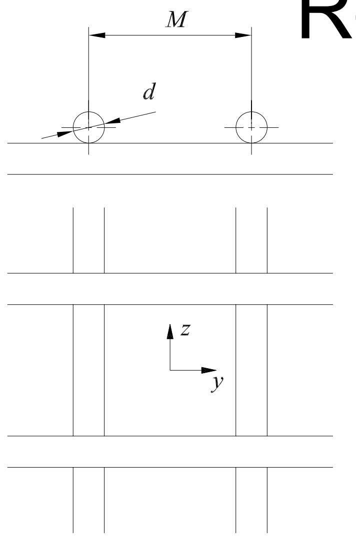
$$-<(\delta u)(\delta\theta)^2> = \frac{4}{3}<\varepsilon_\theta>r - 2\nu_\theta\frac{\partial}{\partial r}<(\delta\theta)^2> + I_\theta$$

$$I_\theta \equiv \frac{U}{r^2}\int_0^r s^2\frac{\partial}{\partial x}<(\delta\theta)^2>ds$$

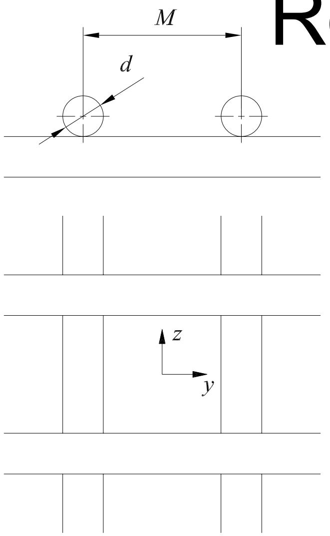
At large r

$$<\varepsilon_\theta> = -\frac{U}{2}\frac{d<\theta^2>}{dx}$$

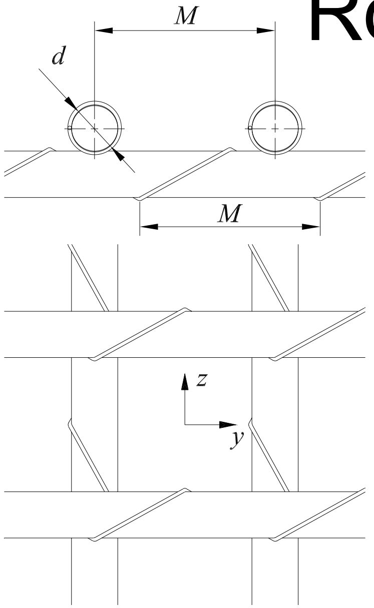
Rd35



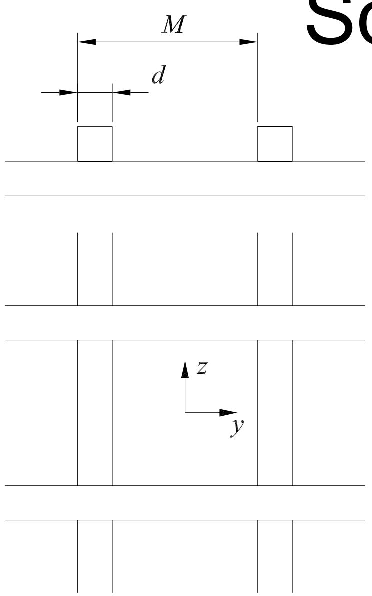
Rd44



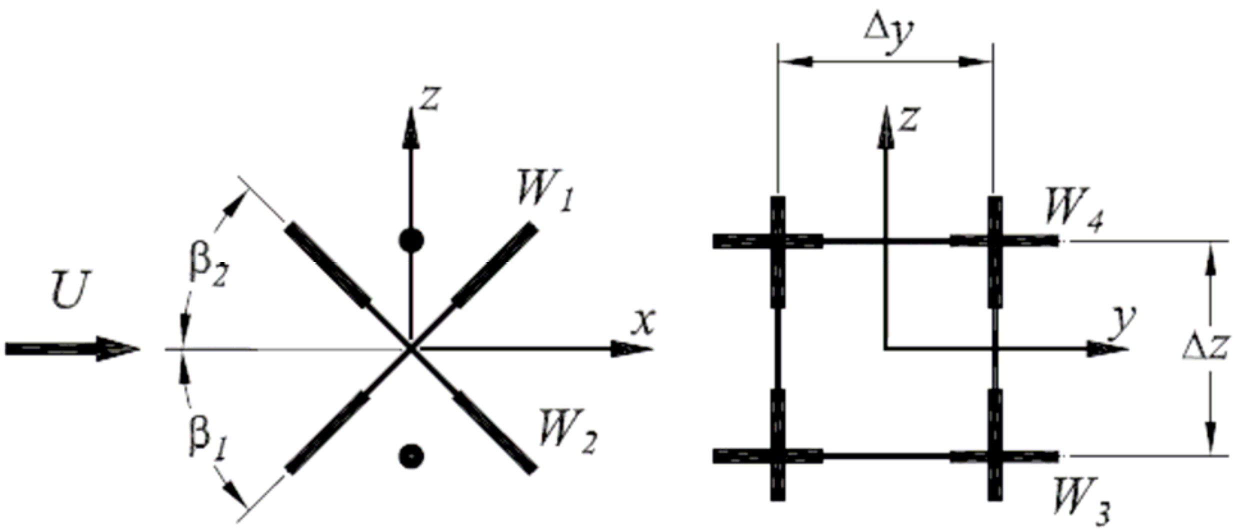
Rd44w



Sq35



Lavoie (2006)



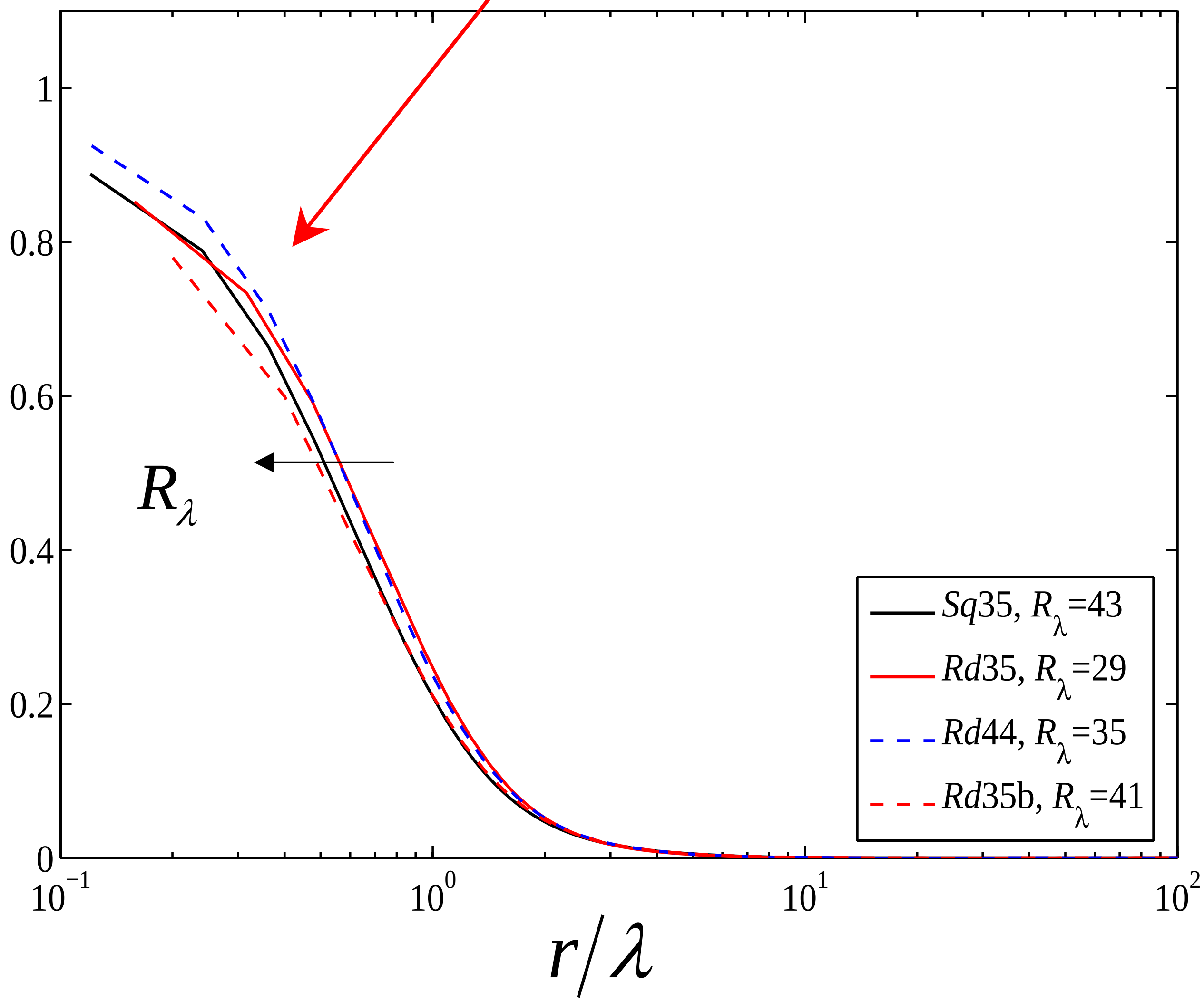
Top view *(y direction into page)*

Front view *(x direction out of page)*

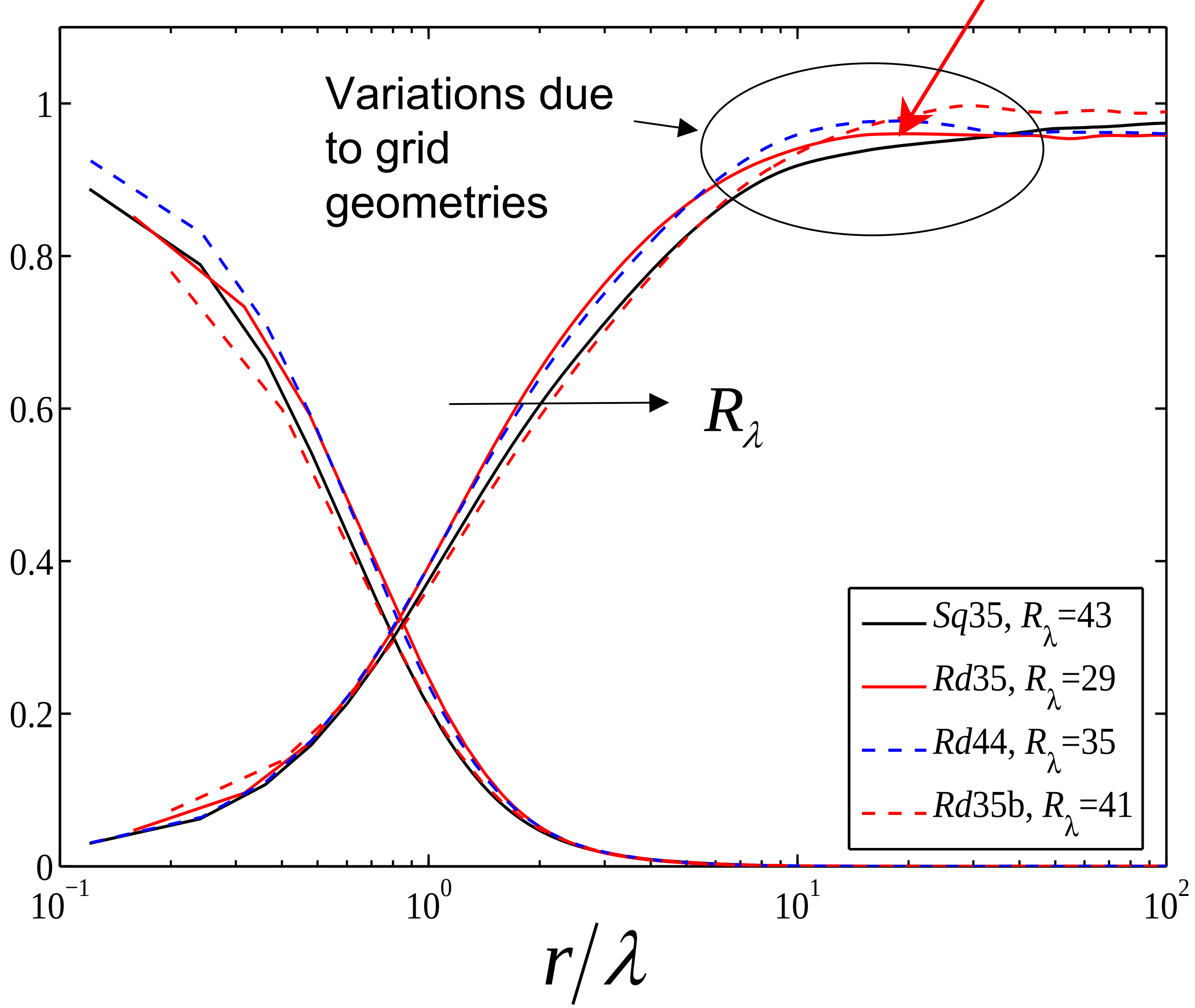
Experimental conditions

	Grid	Bar shape	M (mm)	d (mm)	σ	R_M	η (mm)	Δx^*	Δy^*	Δz^*
□	*Sq35*	Square	24.76	4.76	0.35	10,400	0.21–0.49	1.3–1.9	2.0–4.8	3.2–7.6
○	*Rd35*	Round	24.76	4.76	0.35	10,400	0.24–0.58	1.4–1.9	1.7–4.2	2.8–6.7
⊗	*Rd35b*	Round	24.76	4.76	0.35	19,700	0.15–0.36	2.1–3.2	2.8–6.7	4.4–10
◇	*Rd44*	Round	24.76	6.35	0.44	10,400	0.23–0.54	1.2–2.1	1.8–4.3	3.0–7.0
△	*Rd44w*	Round	24.76	6.35	0.44	10,400	0.23–0.56	1.2–2.0	1.8–4.3	2.8–7.0

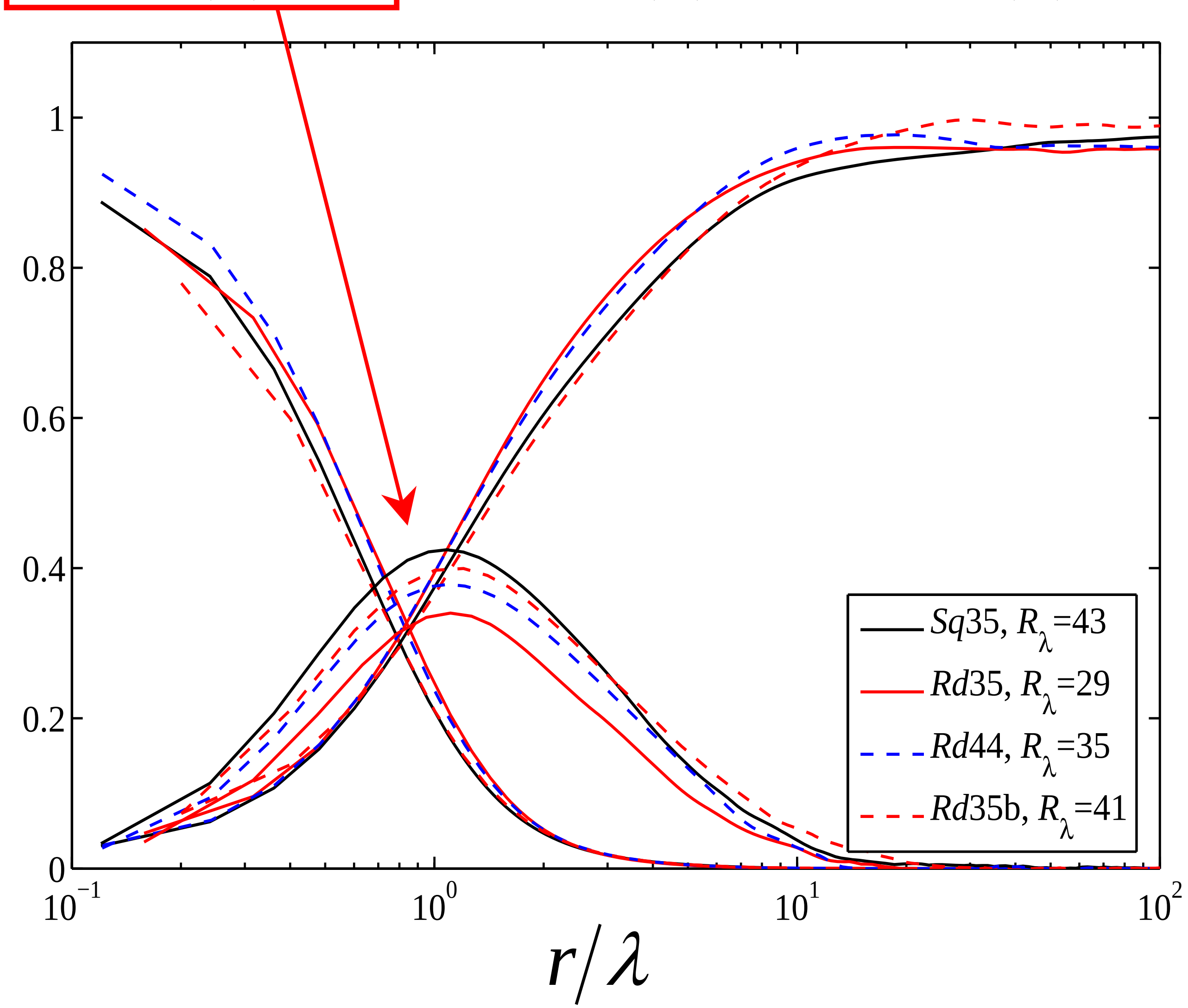
$$-\frac{3}{4}\frac{\left\langle(\delta u)(\delta q)^2\right\rangle}{\langle\varepsilon\rangle r}+\frac{3}{4}\frac{2\nu\, d\left\langle(\delta q)^2\right\rangle/dr}{\langle\varepsilon\rangle r}-\frac{3}{4}\frac{I_q}{\langle\varepsilon\rangle r}=1$$



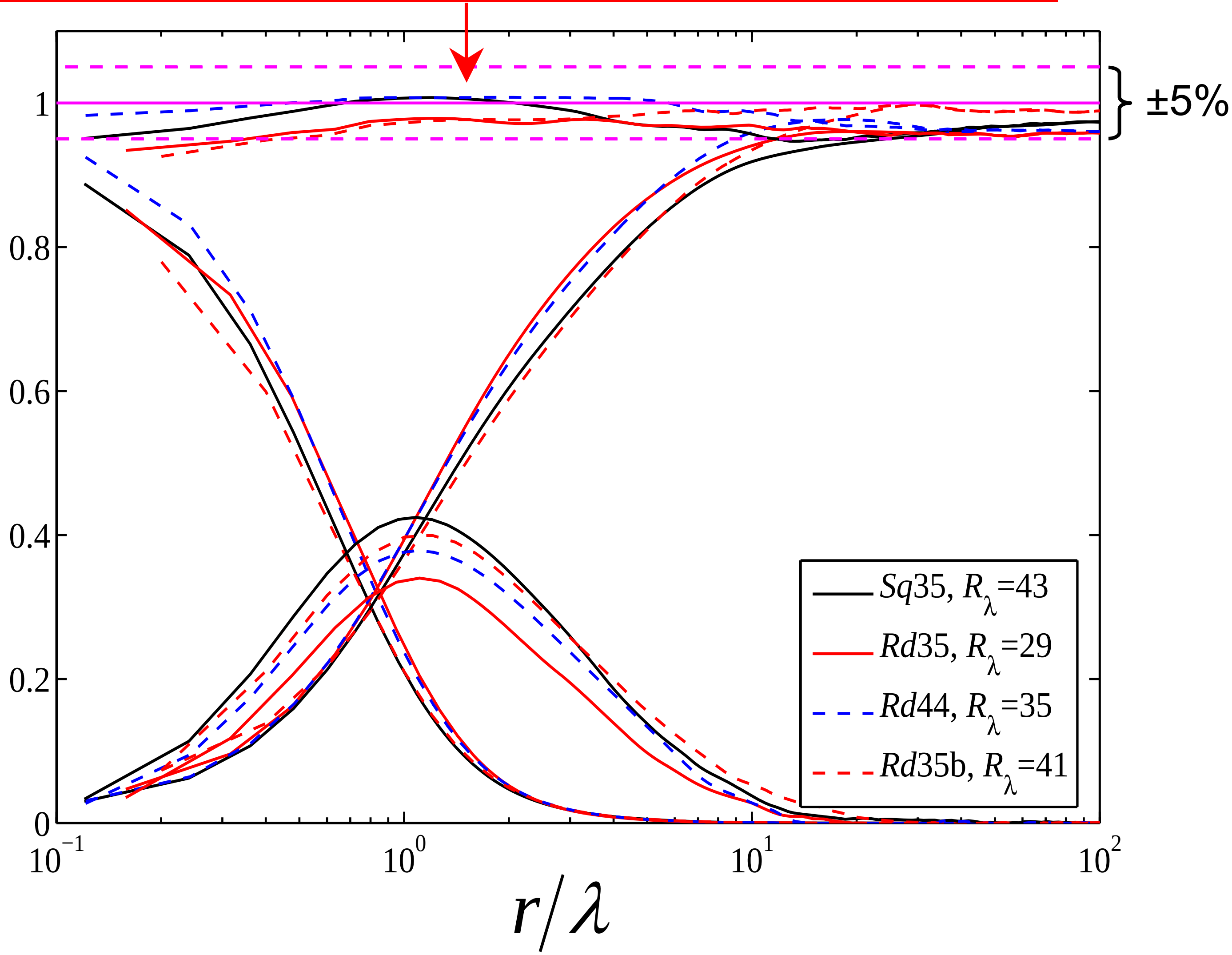
$$-\frac{3}{4}\frac{\left\langle (\delta u)(\delta q)^2 \right\rangle}{\langle \varepsilon \rangle r}+\frac{3}{4}\frac{2\nu\, d\left\langle (\delta q)^2 \right\rangle / dr}{\langle \varepsilon \rangle r}-\frac{3}{4}\frac{I_q}{\langle \varepsilon \rangle r}=1$$



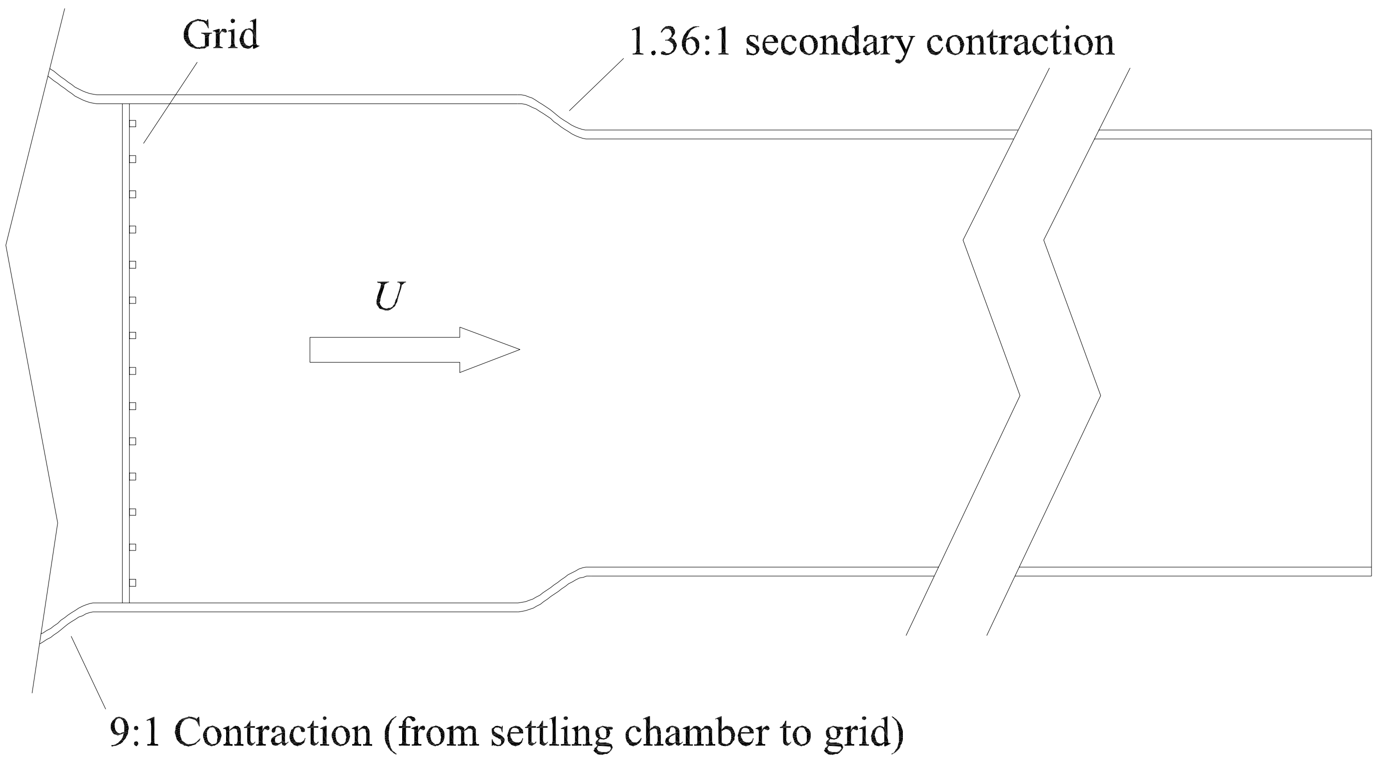
$$-\frac{3}{4}\frac{\left\langle(\delta u)(\delta q)^2\right\rangle}{\langle\varepsilon\rangle r}+\frac{3}{4}\frac{2\nu\, d\left\langle(\delta q)^2\right\rangle/dr}{\langle\varepsilon\rangle r}-\frac{3}{4}\frac{I_q}{\langle\varepsilon\rangle r}=1$$



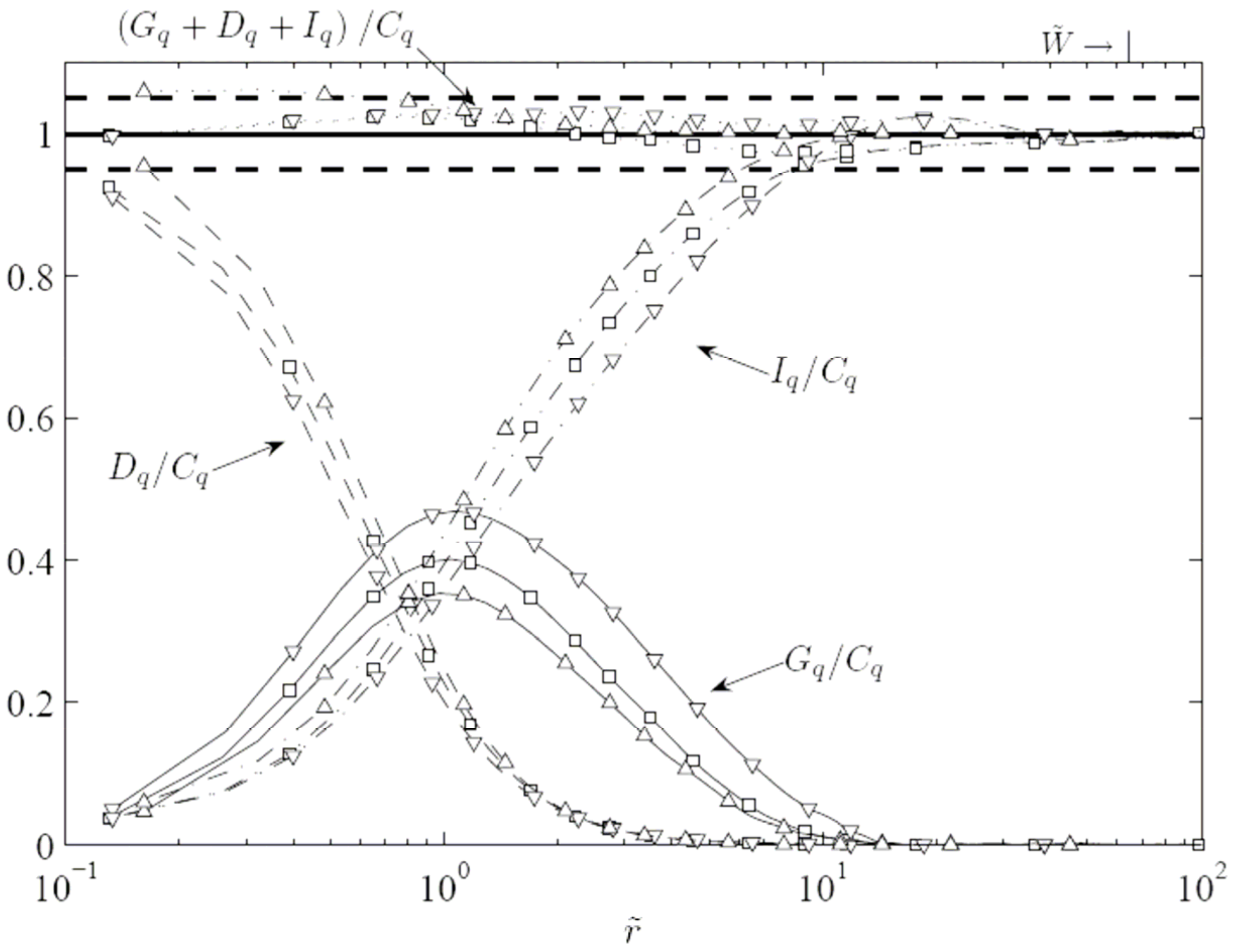
$$-\frac{3}{4}\frac{\left\langle (\delta u)(\delta q)^2 \right\rangle}{\langle \varepsilon \rangle r} + \frac{3}{4}\frac{2\nu\, d\left\langle (\delta q)^2 \right\rangle / dr}{\langle \varepsilon \rangle r} - \frac{3}{4}\frac{I_q}{\langle \varepsilon \rangle r} = 1$$



... with a secondary contraction...



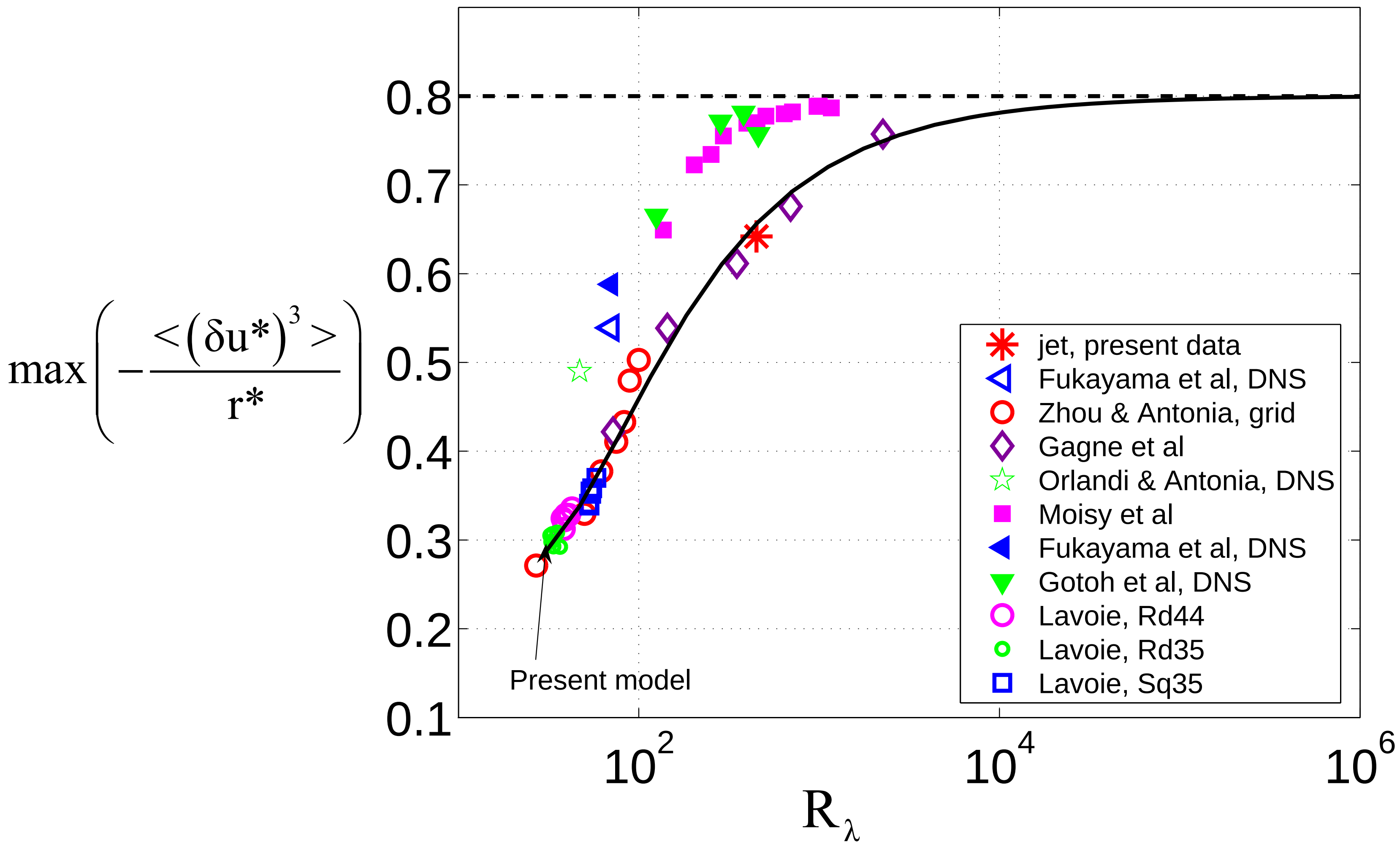
Effect of contraction on scale-by-scale budget



$Sq35$ (□), $Rd44w$ (△) and $Bk38$

At what R_λ does the 4/5 "law" hold?

Antonia & Burattini (2005)



Model for $\langle(\delta u)^2\rangle$

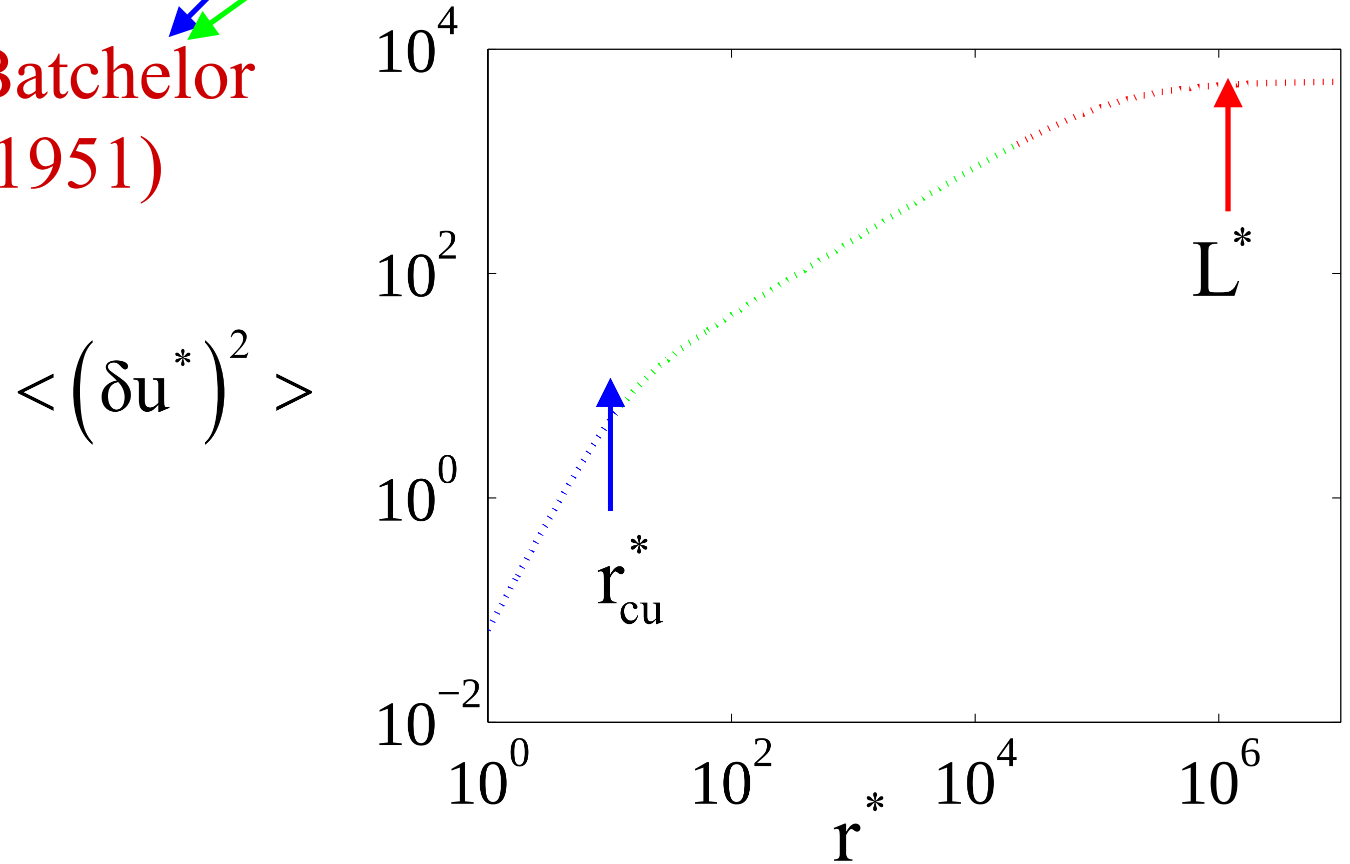
$$<(\delta u^*)^2> = \frac{r^{*2}}{15} \frac{\left(1+\frac{r^*}{L^*}\right)^{2c-2}}{\left(1+\left(\frac{r^*}{r_{cu}^*}\right)^2\right)^c}$$

$$c = 1-\zeta_{u2}/2$$

$$r_{cu}^* = (15C_{u2})^{\frac{3}{2}}$$

Dhruva, Kurien, Sreenivasan (2000)

Batchelor (1951)



- For “decaying-type” turbulence, the “4/5” (or “4/3”) law may be satisfied for $R_{\lambda} \,\square\, 10^{6}$.

- With “forcing”, $R_{\lambda} \,\square\, 10^{3}$ may be sufficient.

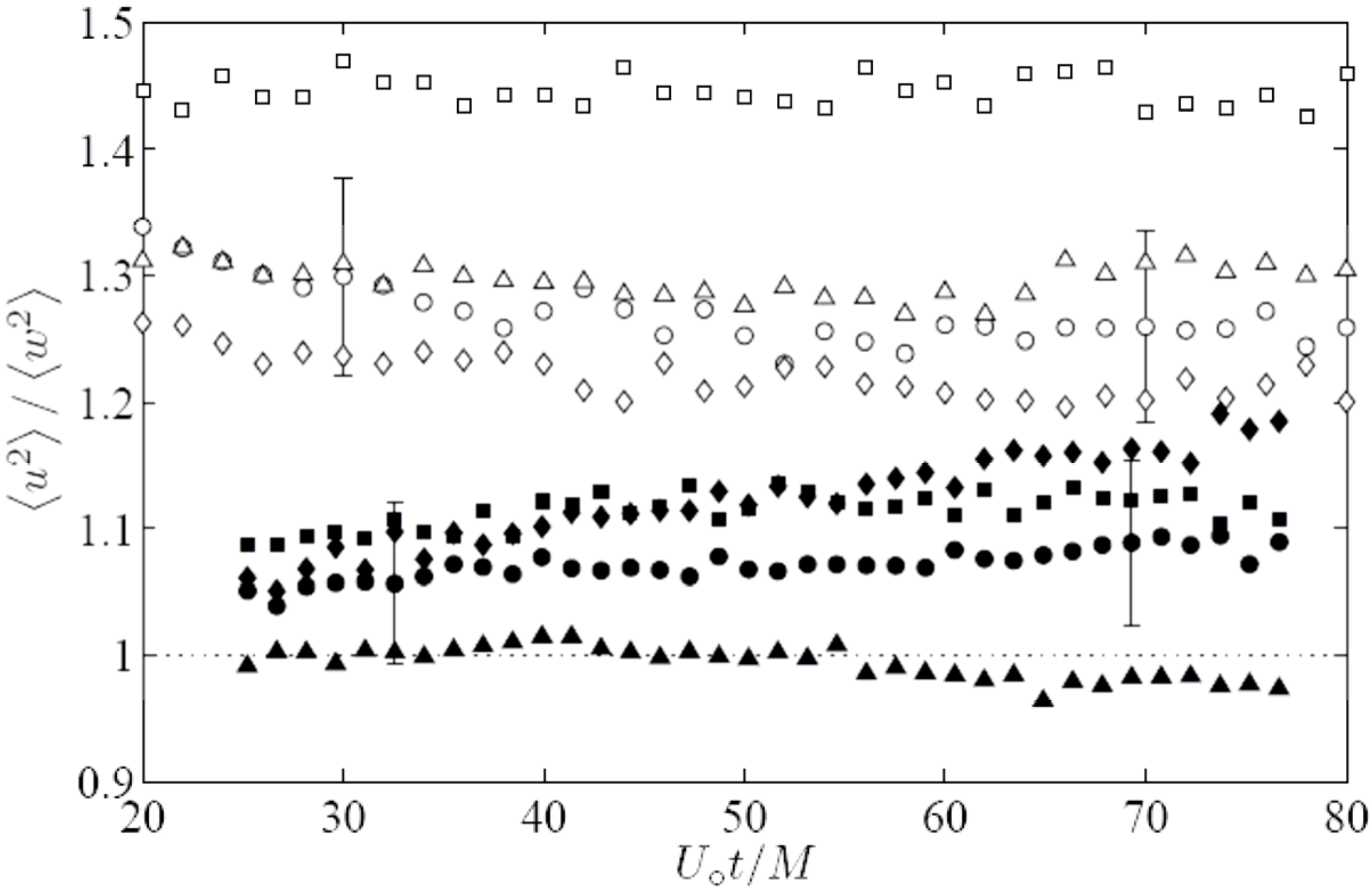
A brief digression…

effect of the contraction on large and small scales

Lavoie (2006)

Lavoie Djenidi Antonia (2007)

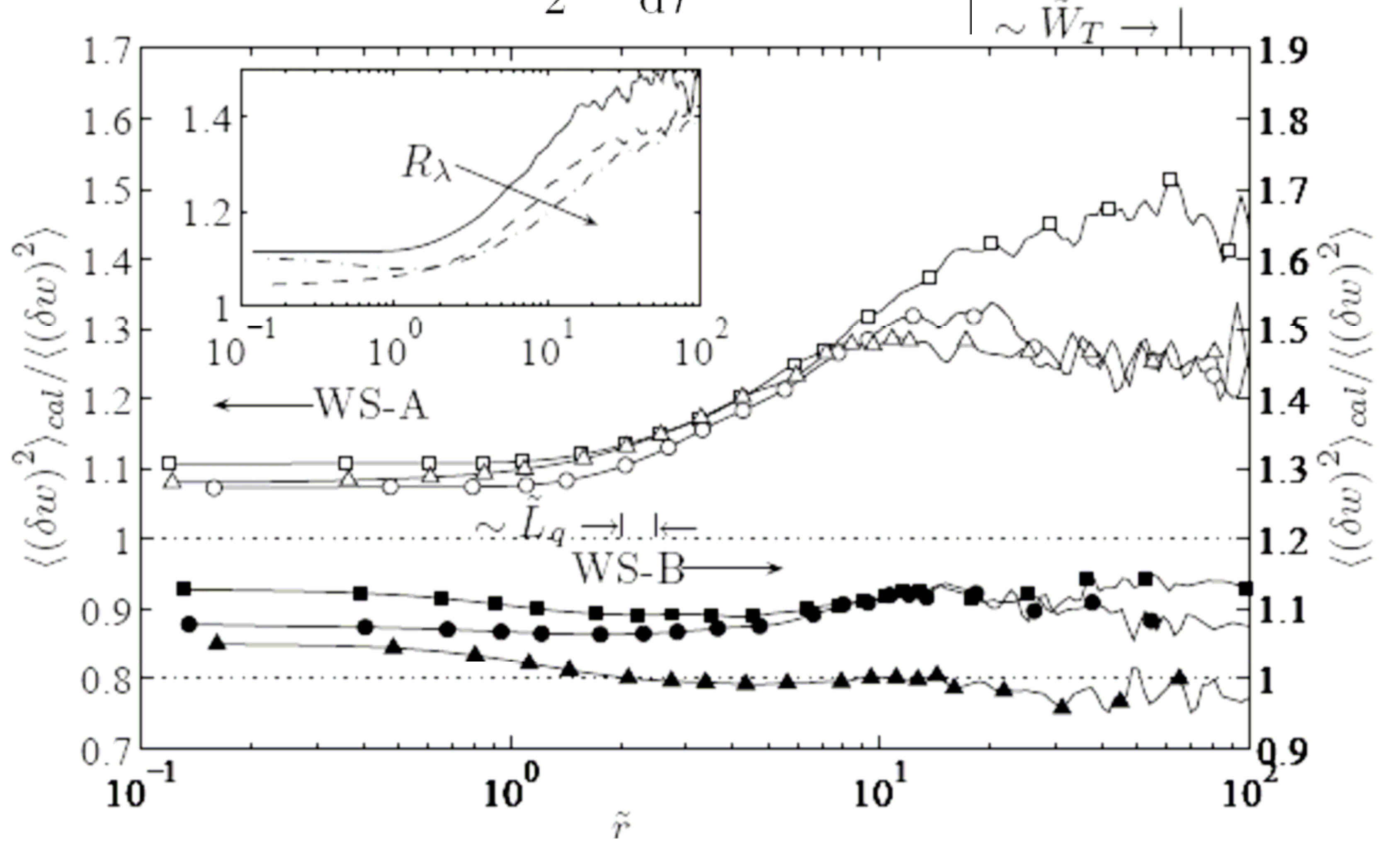
Effect of contraction on ratio $\langle u^2 \rangle / \langle w^2 \rangle \ (\equiv K)$



$Sq35$ (□), $Rd35$ (○), $Rd44$ (◇) and $Rd44w$ (△)

Effect of contraction on isotropy

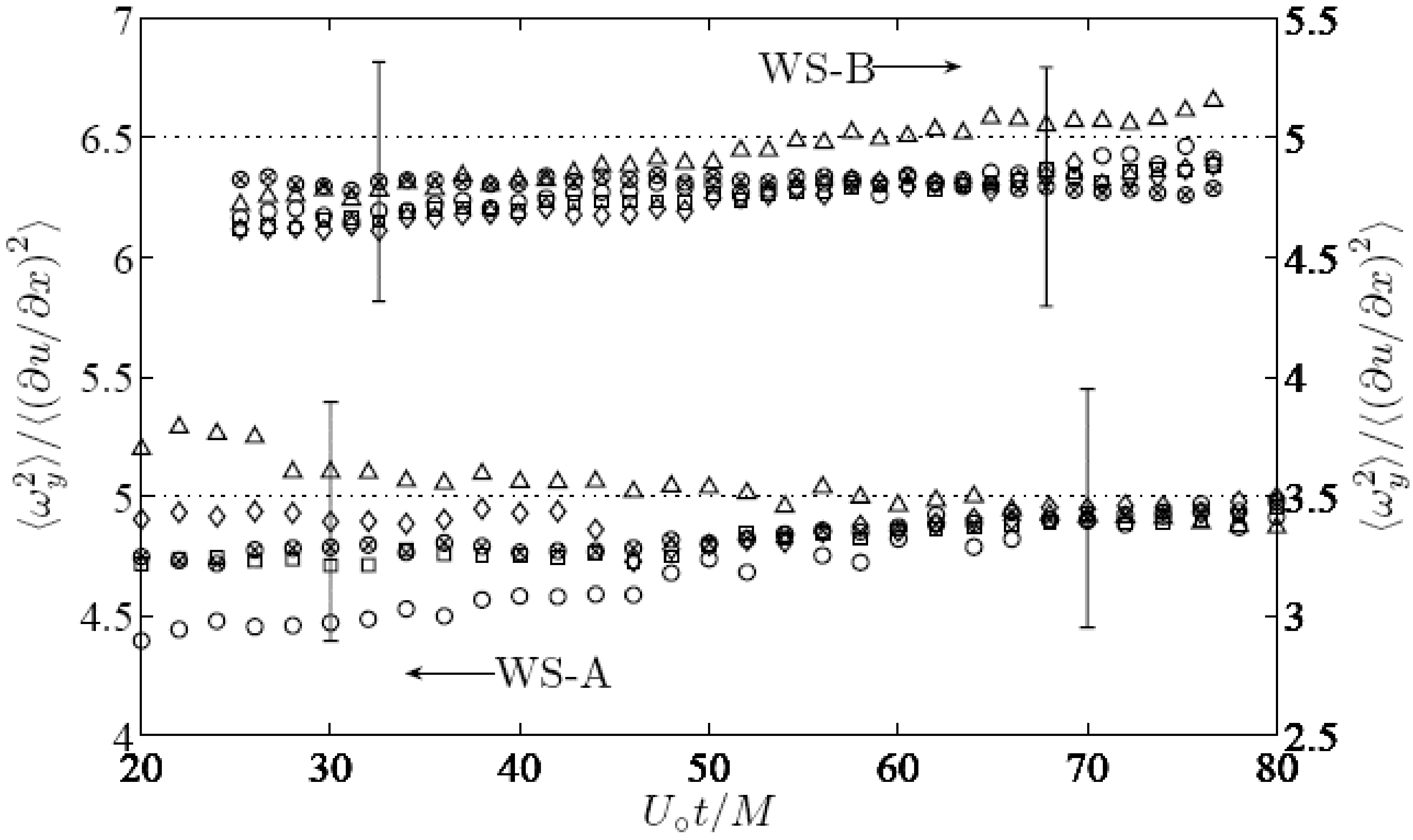
$$\langle(\delta w)^2\rangle_{cal} = \frac{r}{2}\frac{\mathrm{d}\langle(\delta u)^2\rangle}{\mathrm{d}r} + \langle(\delta u)^2\rangle$$



$Sq35$ (□), $Rd35$ (○), $Rd44$ (◊) and $Rd44w$ (△)

Effect of contraction on lateral mean square vorticity

$$<\omega_y{}^2> = 5 < (\partial u / \partial x)^2 >$$



When $r \to 0$ Antonia Zhou Danaila Anselmet (2000)

$$-\frac{1}{35\nu} U \frac{d<\varepsilon>}{dx} = <\left(\frac{\partial u}{\partial x}\right)^3> + 2\nu <\left(\frac{\partial^2 u}{\partial x^2}\right)^2>$$

$$U \frac{d<\varepsilon>}{dx} = \frac{7}{3\times 15^{1/2}} \frac{<\varepsilon>^{3/2}}{\nu^{1/2}} \left(S - 2\frac{G}{R_\lambda}\right)$$

(Batchelor & Townsend 1947)

$$S \equiv <(\partial u/\partial x)^3> / <(\partial u/\partial x)^2>^{3/2}$$

$$G = <(\partial^2 u/\partial x^2)^2><u^2> / <(\partial u/\partial x)^2>^2$$

$$R_\lambda \equiv \lambda \langle u^2\rangle^{1/2}/\nu \qquad \lambda \equiv \langle u^2\rangle^{1/2}/\langle(\partial u/\partial x)^2\rangle^{1/2}$$

Antonia Zhou Danaila Anselmet (2000)

When $r \to 0$

$$-\frac{U}{15\alpha}\frac{d\langle \epsilon_\theta \rangle}{dx} = \left\langle \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \theta}{\partial x}\right)^2 \right\rangle + 2\frac{\alpha}{3}\left\langle \left(\frac{\partial^2 \theta}{\partial x^2}\right)^2 \right\rangle$$

$$U\frac{d\langle \epsilon_\theta \rangle}{dx} = \frac{\langle \epsilon_\theta \rangle \langle \epsilon \rangle^{1/2}}{\nu^{1/2}}\left[\left(\frac{5}{3}\right)^{1/2} S_T - \frac{10}{9}\frac{G_\theta}{\mathrm{Pe}}\mathrm{Pr}^{1/2} R^{-1/2}\right]$$

$$S_T \equiv -\langle (\partial u/\partial x)(\partial \theta/\partial x)^2 \rangle / \langle (\partial u/\partial x)^2 \rangle^{1/2} \langle (\partial \theta/\partial x)^2 \rangle$$

$$G_\theta \equiv \langle (\partial^2 \theta/\partial x^2)^2 \rangle \langle \theta^2 \rangle / \langle (\partial \theta/\partial x)^2 \rangle^2$$

$$\mathrm{Pe} \equiv \langle u^2 \rangle^{1/2} \lambda_\theta / \alpha$$

$$R \equiv \frac{\langle \theta^2 \rangle}{\langle \varepsilon_\theta \rangle} \cdot \frac{\langle \varepsilon \rangle}{\langle q^2 \rangle}$$

Coefficients of destruction of energy and scalar dissipation rates

$$G = \frac{15}{7}\left(1 + \frac{1}{n}\right) - \frac{SR_\lambda}{2}$$

$$\langle q^2 \rangle = A x_1^{-n}$$

$$\frac{G_\theta}{R} = \frac{9}{10}\left(\frac{m+1}{n}\right) - \frac{9}{10} S_T R_\lambda$$

$$\langle \theta^2 \rangle = B x_1^{-m}$$

$$\langle u_{1,11}^2 \rangle = \int_0^\infty k_1^4 \phi_{u_1}(k_1)\,\mathrm{d}k_1$$

$$\langle \theta_{,11}^2 \rangle = \int_0^\infty k_1^4 \phi_\theta(k_1)\,\mathrm{d}k_1$$

When R_λ is sufficiently large

$$\langle u_{1,1}^3 \rangle = -2\nu \langle u_{1,11}^2 \rangle$$

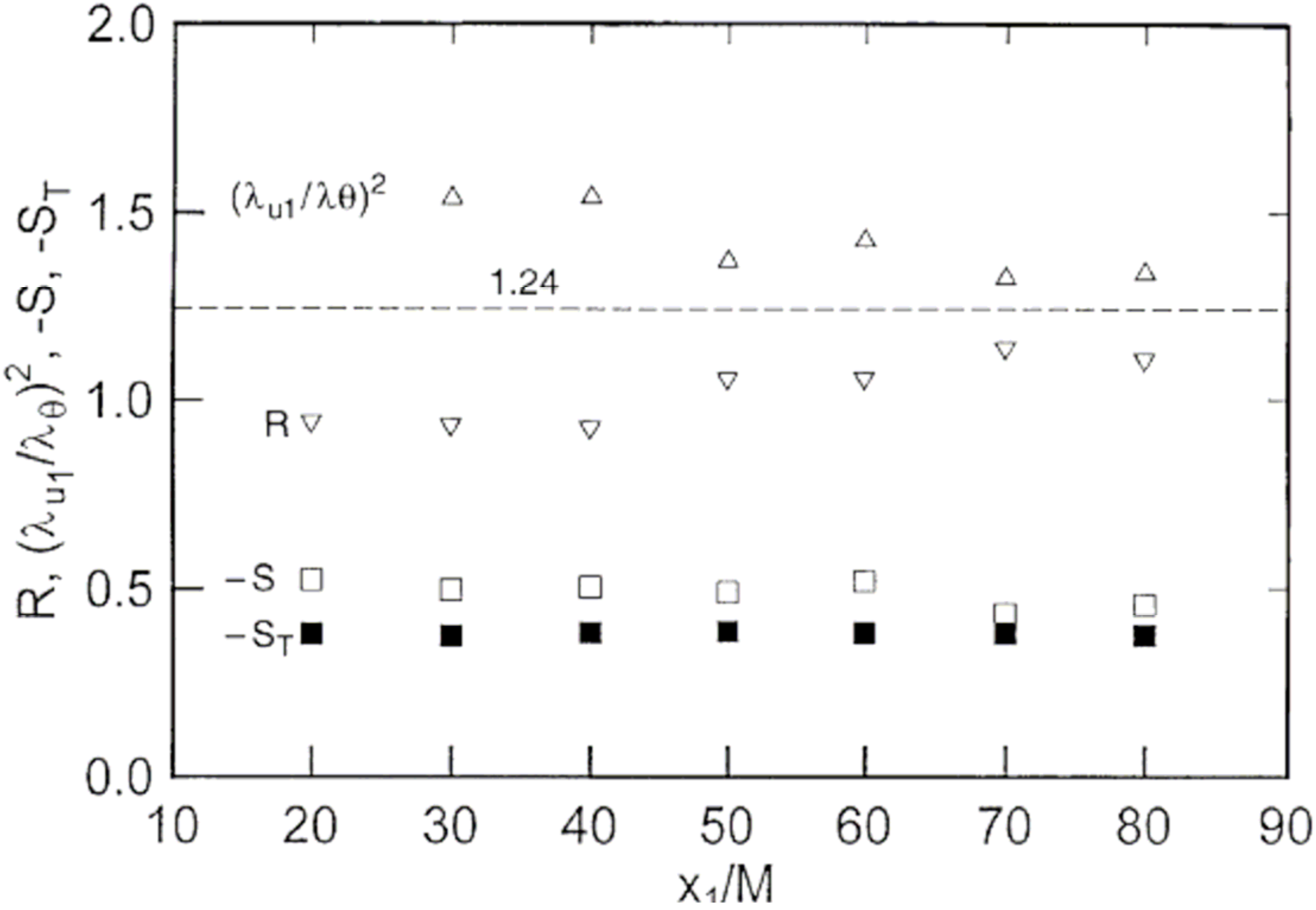
$$\langle u_{1,1}\, \theta_{,1}^2 \rangle = -\frac{2c}{9}\, \alpha \,\langle \theta_{,11}^2 \rangle$$

$$c \approx 2.5$$

Zhou Antonia Danaila Anselmet (2000)

Measured skewnesses in grid turbulence

Zhou Antonia Anselmet Danaila (2000)



Spectral Domain

- Consider enstrophy equation

$$\frac{\partial}{\partial t}\left(k^2 E\right) = k^2 T - 2\nu k^4 E$$

- After integrating wrt k, this can be rewritten as

$$S = S_1 + S_2$$

- Where S is the velocity derivative skewness

$$\text{i.e. } S \equiv \frac{<(\partial u/\partial x)^3>}{<(\partial u/\partial x)^2>^{3/2}} = -\frac{3(30)^{1/2}}{14} \frac{\int_0^\infty k^2 T dk}{\left[\int_0^\infty k^2 E dk\right]^{3/2}}$$

$$S_1 = -\frac{(30)^{1/2}}{14}\frac{\frac{\partial}{\partial t}\int_0^\infty k^2 E dk}{\left[\int_0^\infty k^2 E dk\right]^{3/2}} = \frac{30}{7R_\lambda}\frac{m-1}{m}$$

$$S_2 = -\frac{3(30)^{1/2}}{14}\frac{\nu\int_0^\infty k^4 E dk}{\left[\int_0^\infty k^2 E dk\right]^{3/2}}$$

$$= -\frac{3(30)^{1/2}}{14R_\lambda}\frac{\int_0^\infty \tilde{k}^4 \tilde{E} d\tilde{k}}{\left[\int_0^\infty \tilde{k}^2 \tilde{E} d\tilde{k}\right]^{3/2}}$$

George (1992)
or G92

$$\therefore SR_\lambda = \text{constant}$$

- With Kolmogorov-normalisation

$$S_2 = - - \frac{3(30)^{1/2}}{14} \frac{\int_0^\infty k^{*4} E^* dk^*}{\left[\int_0^\infty k^{*2} E^* dk^*\right]^{3/2}}$$

- i.e. G92 is not consistent with **K41**

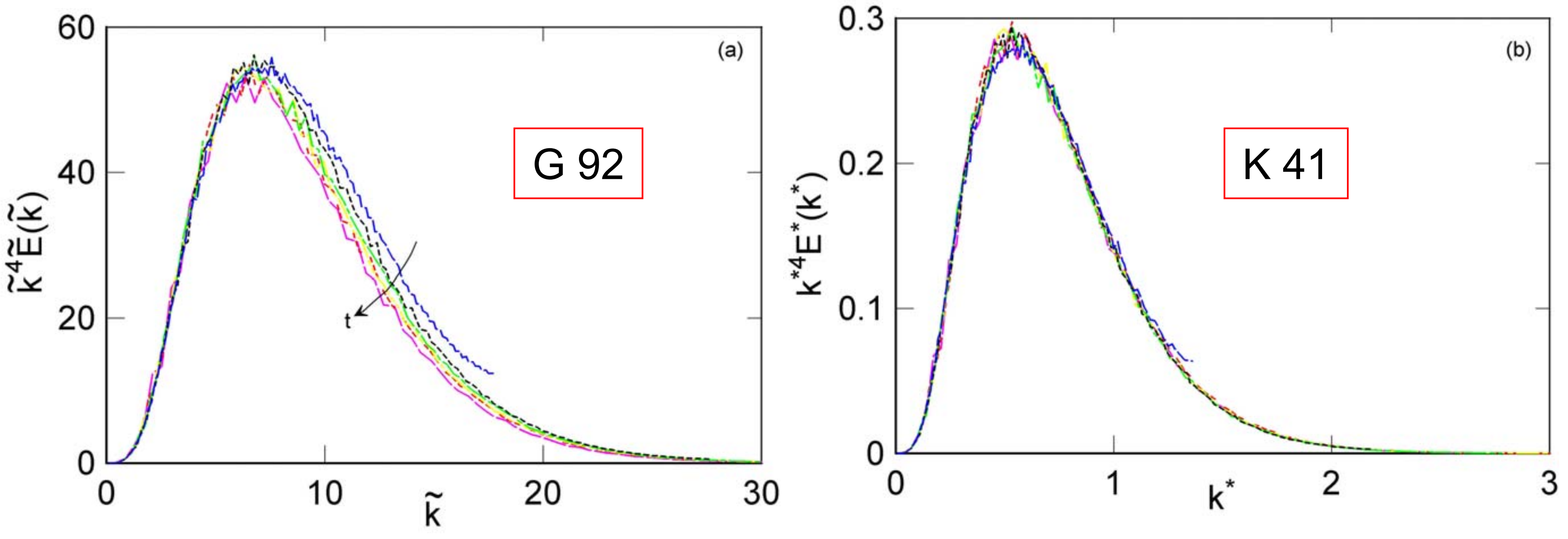
$$S_\theta(t) = -\frac{2}{15^{1/2}} \frac{\int_0^\infty k^2 T_\theta(k,t)\,\mathrm{d}k}{\left[\int_0^\infty k^2 E(k,t)\,\mathrm{d}k\right]^{1/2} \left[\int_0^\infty k^2 E_\theta(k,t)\,\mathrm{d}k\right]} = S_{\theta_1}(t) + S_{\theta_2}(t)$$

$$S_{\theta_1}(t) = -\frac{2}{15} \frac{\partial/\partial t \int_0^\infty k^2 E_\theta(k,t)\,\mathrm{d}k}{\left[\int_0^\infty \mathrm{k}^2 E(k,t)\,\mathrm{d}k\right]^{1/2} \left[\int_0^\infty k^2 E_\theta(k,t)\,\mathrm{d}k\right]}$$

$$S_{\theta_2}(t) = -\frac{4}{15} \nu_\theta \frac{\int_0^\infty k^4 E_\theta(k,t)\,\mathrm{d}k}{\left[\int_0^\infty \mathrm{k}^2 E(k,t)\,\mathrm{d}k\right]^{1/2} \left[\int_0^\infty k^2 E_\theta(k,t)\,\mathrm{d}k\right]}$$

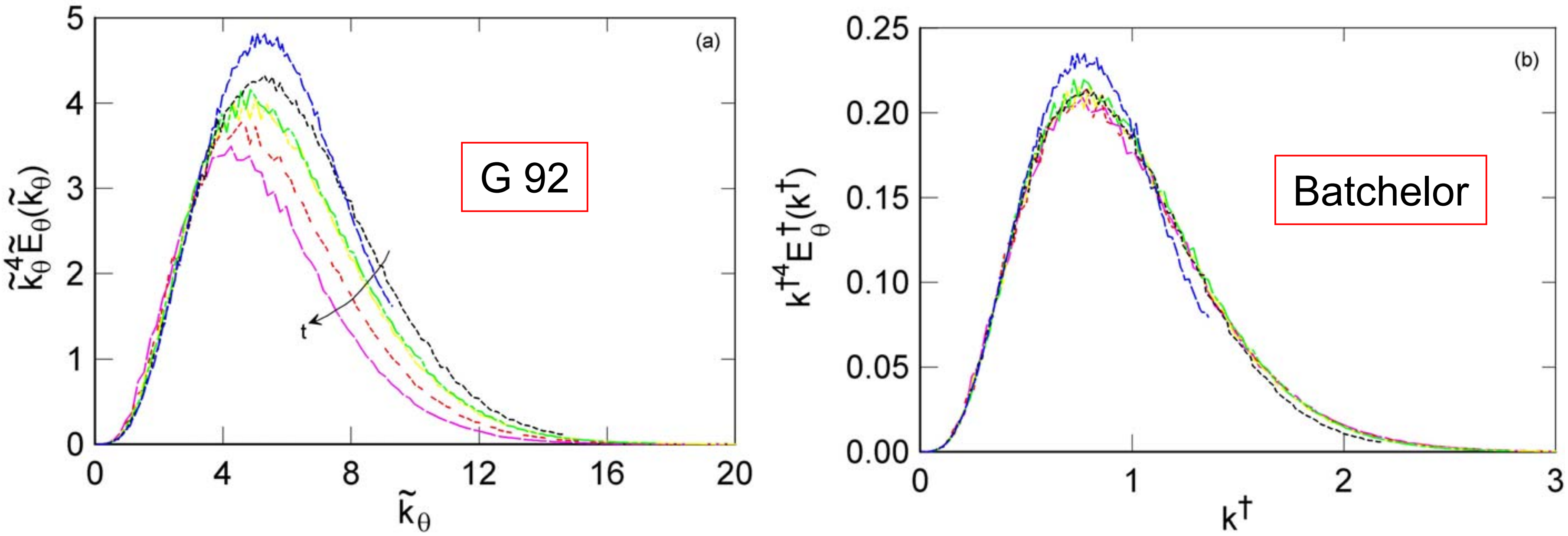
K41 superior to G92

Antonia & Orlandi (2004)



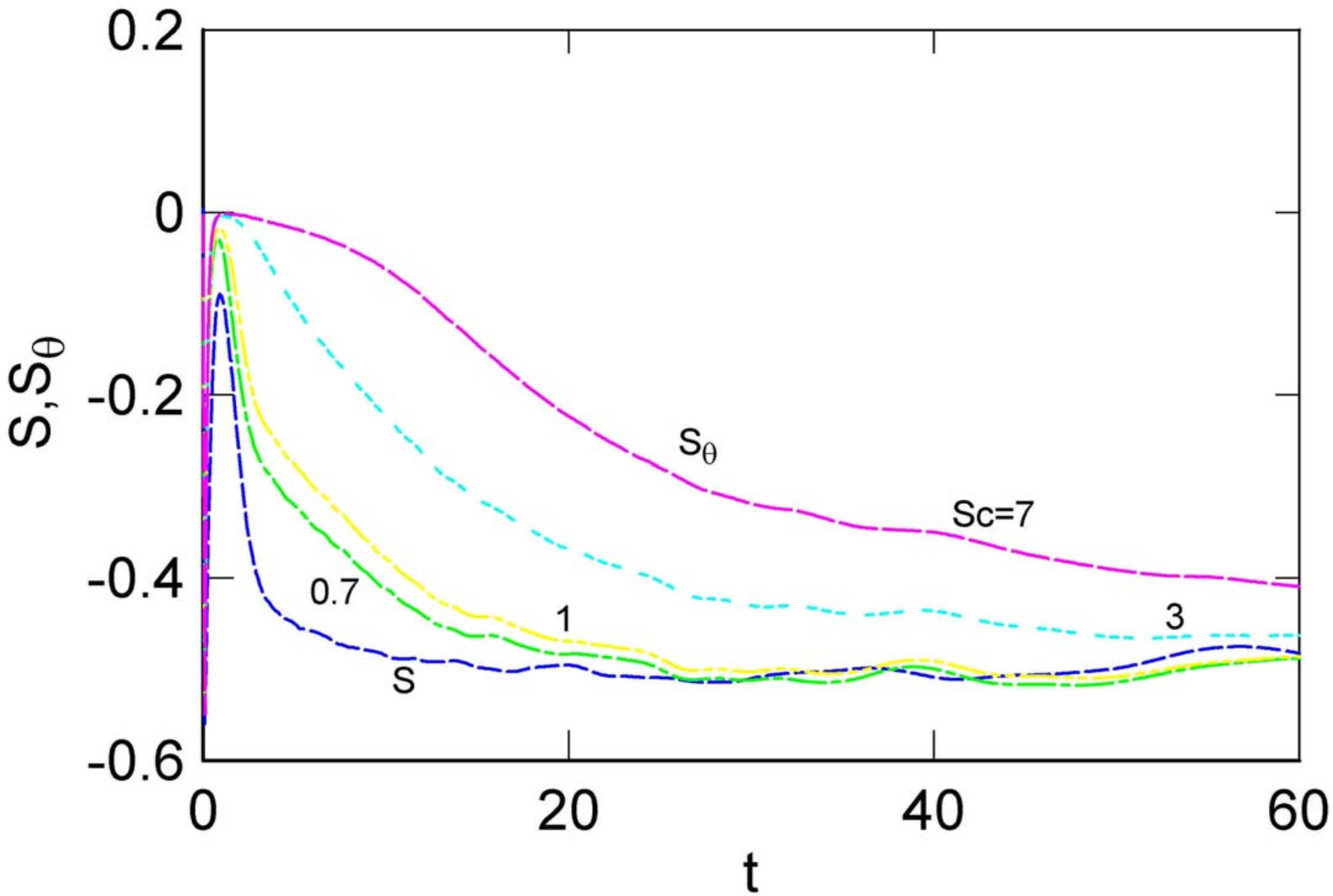
Batchelor normalization superior to G92

Antonia & Orlandi (2004)

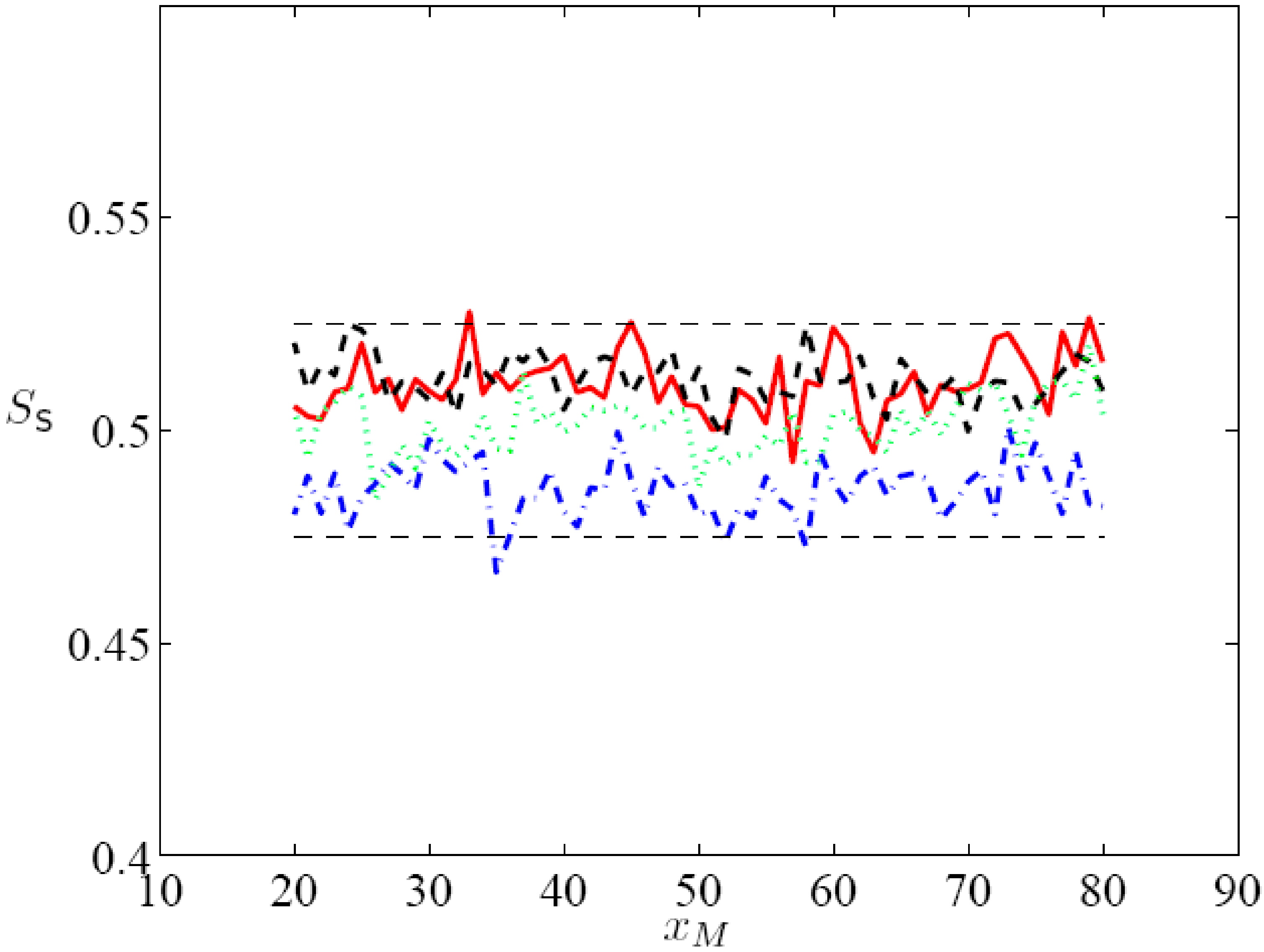


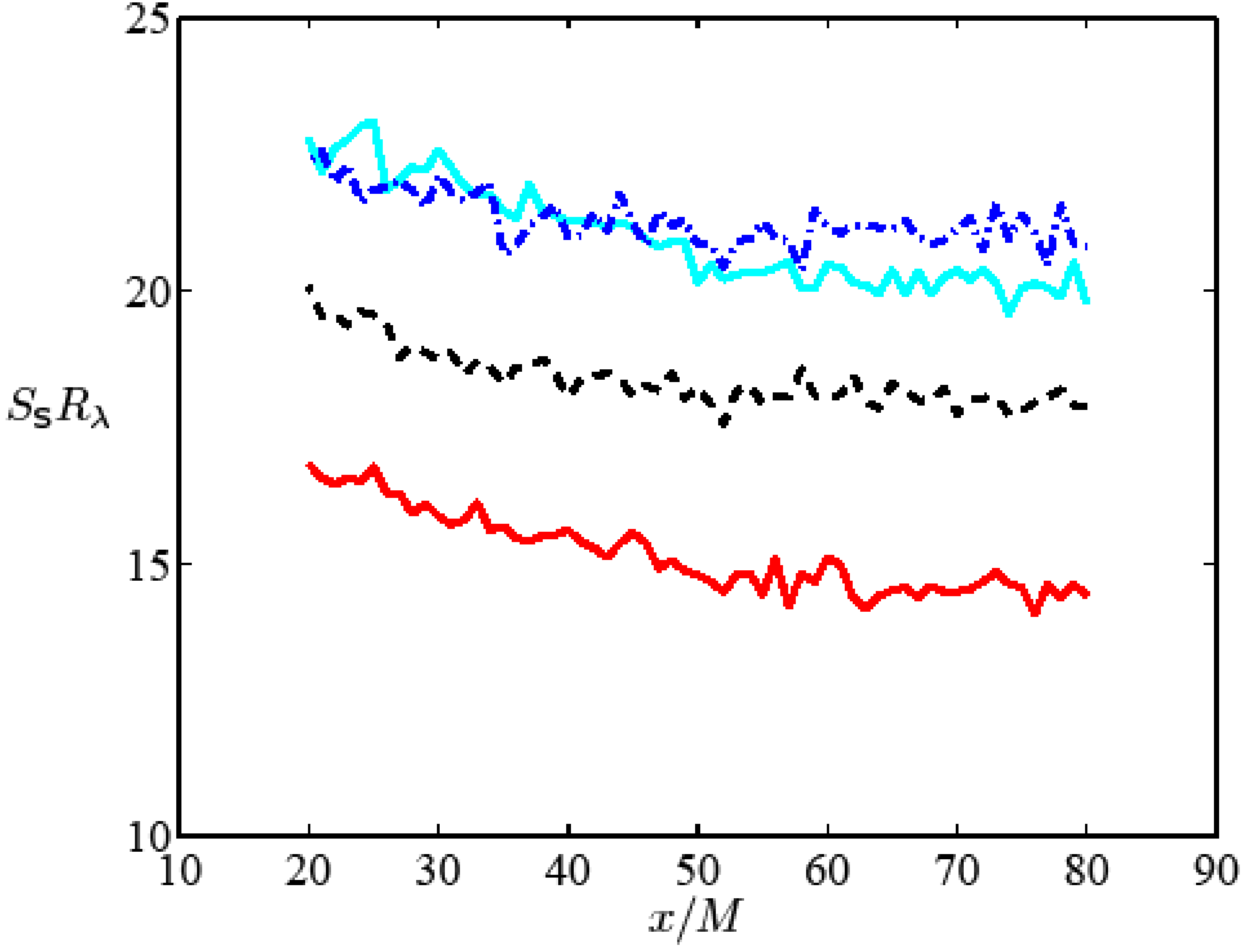
Temporal evolution of skewnesses (DNS)

Antonia & Orlandi (2004)



Measured skewness downstream of the grids





25
20
15
10
$S_S R_\lambda$
10
20
30
40
50
60
70
80
90
x/M

Methodology for correcting spectra

for the imperfect spatial resolution of hot (and cold) wires

Uberoi Kovasznay (1953)

Wyngaard (1968,1969,1971)

Fourier-Stieltjes representations of velocity and scalar fields

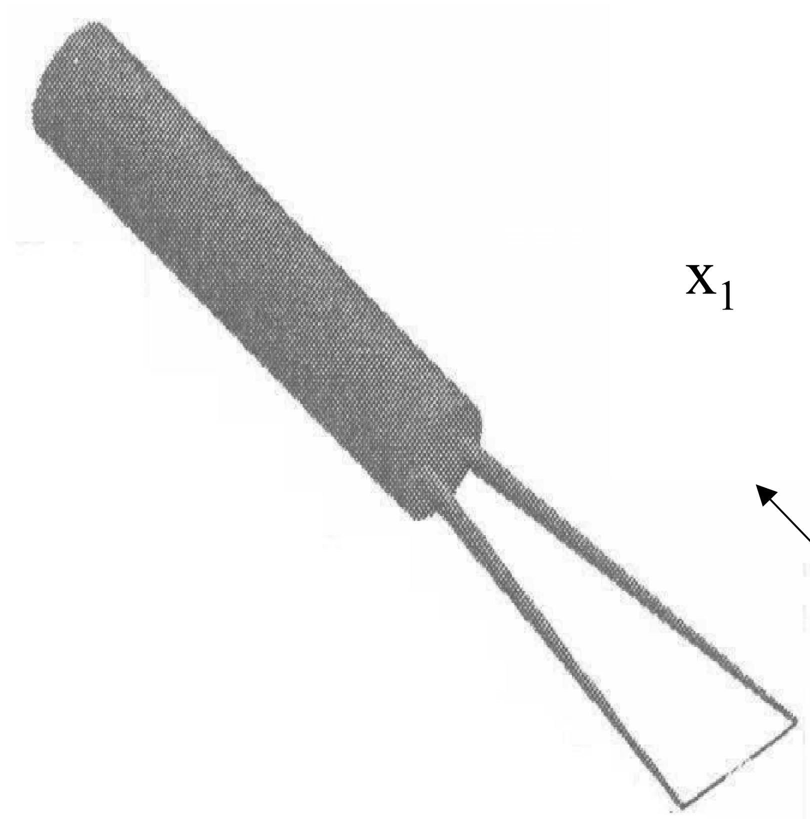
$$u_i(\boldsymbol{x}) = \int_{-\infty}^{+\infty} e^{j\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}Z_i(\boldsymbol{k})$$

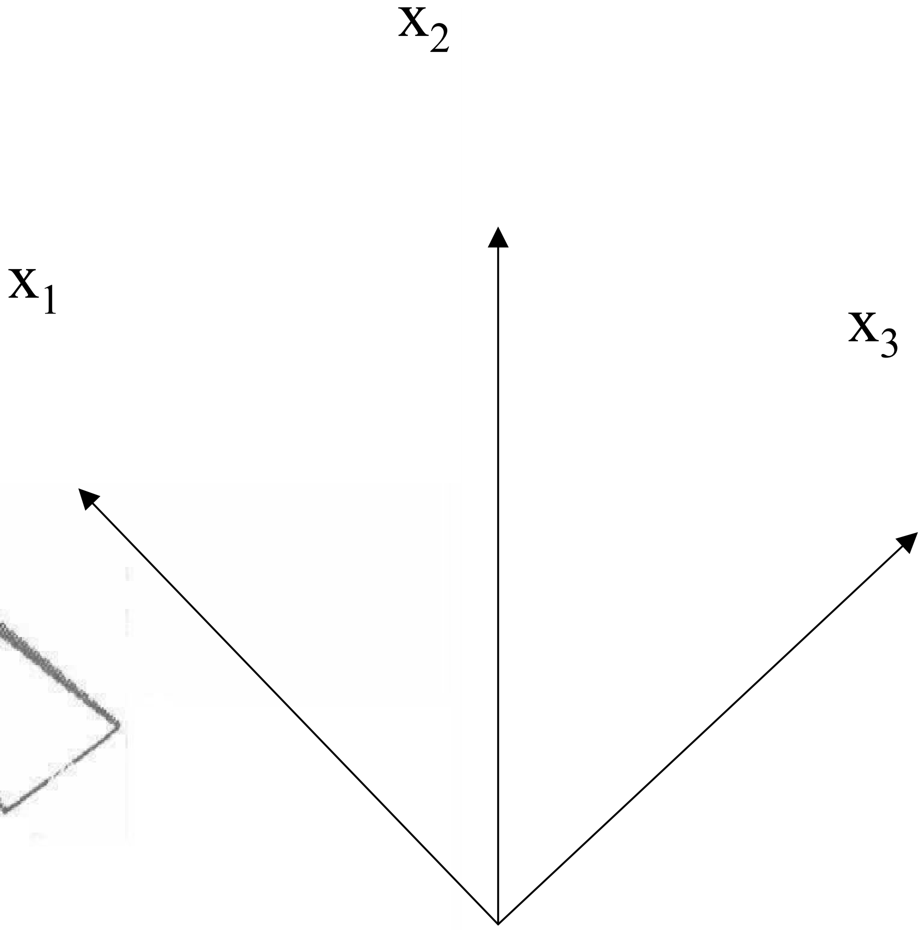
$$\theta(\boldsymbol{x}) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}Z_\theta(\boldsymbol{k})$$

$$\mathrm{d}Z_i^M(\boldsymbol{k}) = A\,\mathrm{d}Z_i(\boldsymbol{k})$$

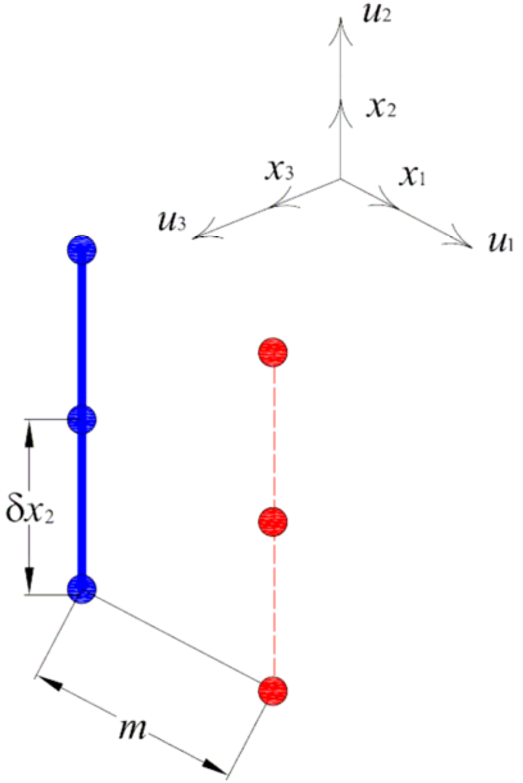
↑ Filter function

Single hot wire (SW)





SW



$$f = U_1 / m$$

Filter transfer function for a single hot wire

$$A = LM$$

$$L = \frac{\sin(\mathbf{k} \cdot \mathbf{l}/2)}{(\mathbf{k} \cdot \mathbf{l}/2)} = \frac{\sin(k_2 l/2)}{(k_2 l/2)}$$

$$M = \frac{\sin(\mathbf{k} \cdot \mathbf{m}/2)}{(\mathbf{k} \cdot \mathbf{m}/2)} = \frac{\sin(k_1 m/2)}{(k_1 m/2)}$$

Measured Spectra

$$\Phi_{ii}{}^{M}(k) = A^2 \Phi_{ii}(k)$$

$$R_{u_i}(k_1) = \frac{\phi_{u_i}^{M}(k_1)}{\phi_{u_i}(k_1)} = \frac{\iint \Phi_{ii}^{M}(\boldsymbol{k})\,\mathrm{d}k_2\,\mathrm{d}k_3}{\iint \Phi_{ii}(\boldsymbol{k})\,\mathrm{d}k_2\,\mathrm{d}k_3}$$

$$r_{u_{im}} = \frac{\left\langle u_{i,m}^{2} \right\rangle^{M}}{\left\langle u_{i,m}^{2} \right\rangle} = \frac{\int_{-\infty}^{+\infty} \phi_{u_{i,m}}^{M}\left(k_1\right)\mathrm{d}k_1}{\int_{-\infty}^{+\infty} \phi_{u_{i,m}}\left(k_1\right)\mathrm{d}k_1}$$

$$r_{\theta,m} = \frac{\int_{-\infty}^{+\infty} \phi_{\theta,m}^{M}(k_1)\,\mathrm{d}k_1}{\int_{-\infty}^{+\infty} \phi_{\theta,m}(k_1)\,\mathrm{d}k_1}$$

Assuming isotropy

$$\Phi_{ij}(\boldsymbol{k}) = \Phi_{ij}(k) = \frac{E(k)}{4\pi k^4}\left(k^2\delta_{ij} - k_i k_j\right)$$

$$\phi(\boldsymbol{k}) = \phi(k) = \frac{\Gamma(k)}{4\pi k^2}$$

$$E(k) = k^2 \left(\partial^2 \phi_{11} / \partial k_1^2\right)_{k_1 = k} - k\left(\partial \phi_{11} / \partial k_1\right)_{k_1 = k}$$

$$\Gamma(k) = -k\left(\frac{\partial \phi_\theta}{\partial k_1}\right)_{k_1 = k}$$

$$R_{u_{i,m}}(k_1,\Delta x_m)=\frac{4\iint_{-\infty}^{\infty}\sin^2\left(\frac{k_m\Delta x_m}{2}\right)\frac{E(k)}{4\pi k^4}(k^2-k_i^2)\,dk_2\,dk_3}{(\Delta x_m)^2\iint_{-\infty}^{\infty}k_m^2\frac{E(k)}{4\pi k^4}(k^2-k_i^2)\,dk_2\,dk_3}$$

$$R_{\theta,m}(k_1,\Delta x_m)=\frac{4}{(\Delta x_m)^2}\frac{\iint_{-\infty}^{\infty}\sin^2\left(\frac{k_m\Delta x_m}{2}\right)\frac{\Gamma(k)}{4\pi k^2}\,dk_2\,dk_3}{\iint_{-\infty}^{\infty}k_m^2\frac{\Gamma(k)}{4\pi k^2}\,dk_2\,dk_3}$$

For m = 1 (independently of i)

$$R_{u_{i,1}}(k_1,\Delta x_m)=R_{\theta,1}(k_1,\Delta x_m)=\frac{\sin^2\left(k_1\Delta x_1/2\right)}{\left(k_1\Delta x_1/2\right)^2}$$

independently of isotropy or of E(k) or $\Gamma(k)$

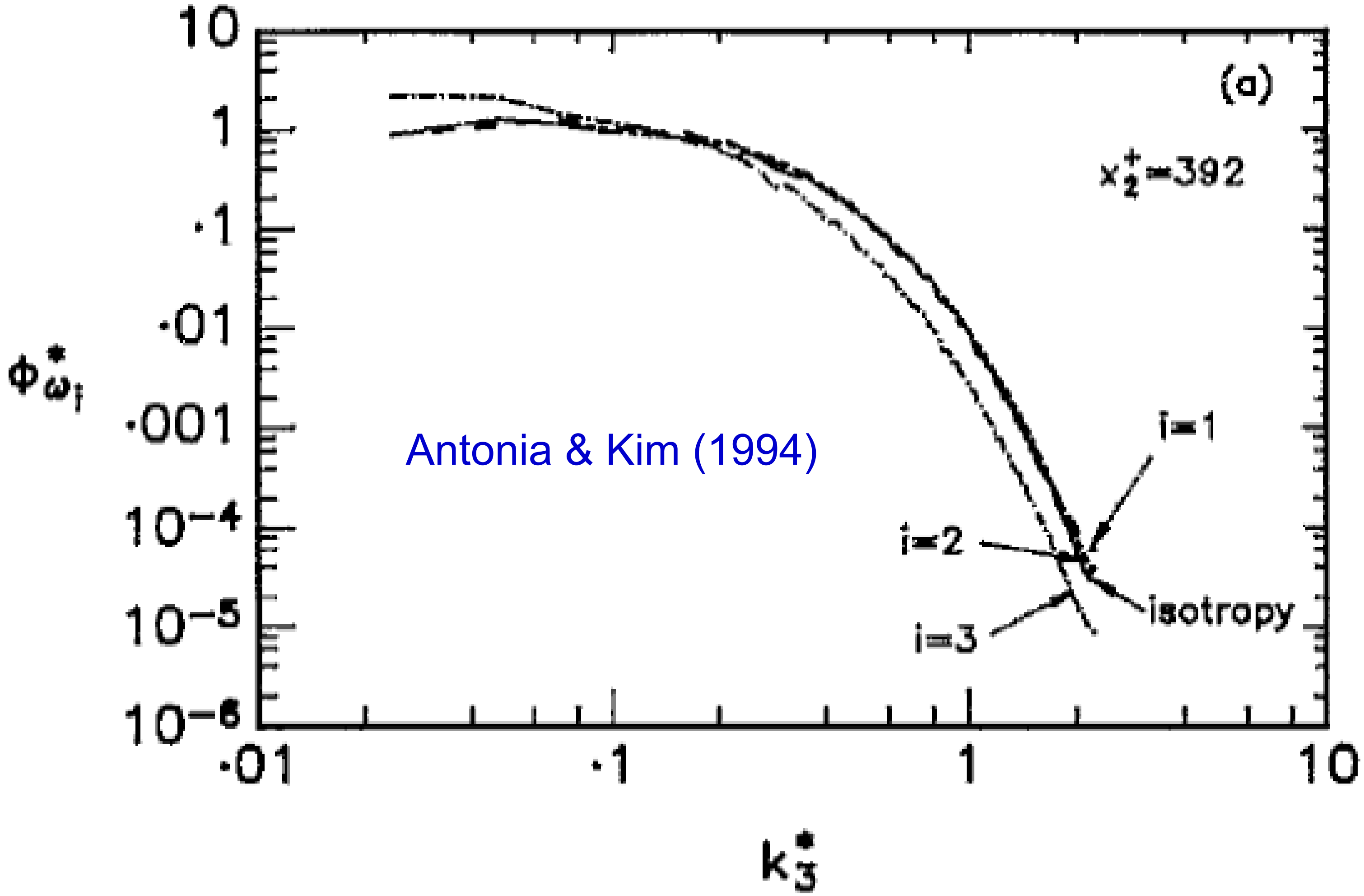
…with only a weak dependence on R_λ

Key assumptions

- Local isotropy

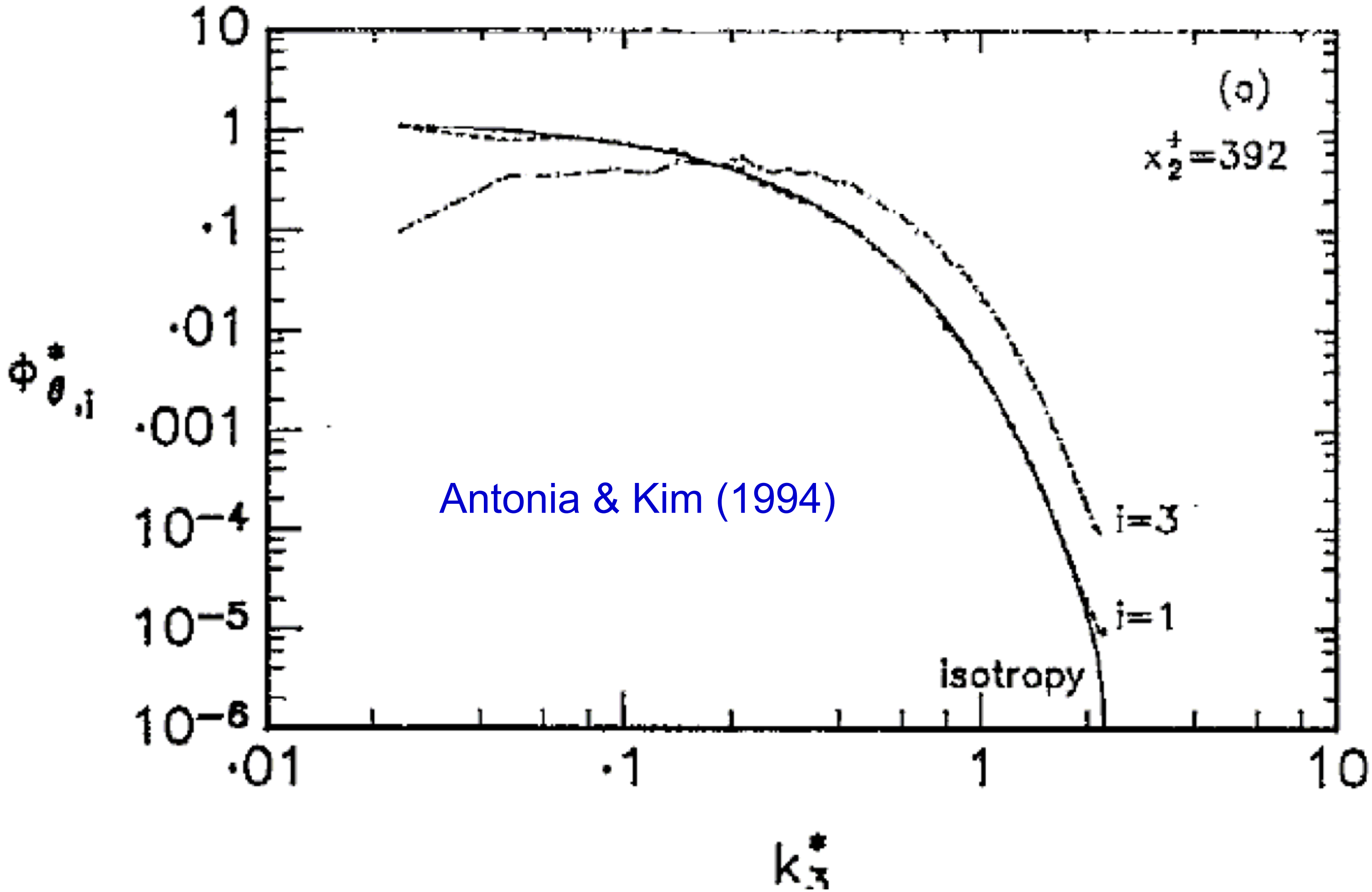
- Specific knowledge of $E(k)$

$$\phi_{\omega_1}(k_3) = \phi_{\omega_2}(k_3) = \frac{1}{2}\left(\phi_{\omega_3} - k_3 \frac{d\phi_{\omega_3}}{dk_3}\right)$$

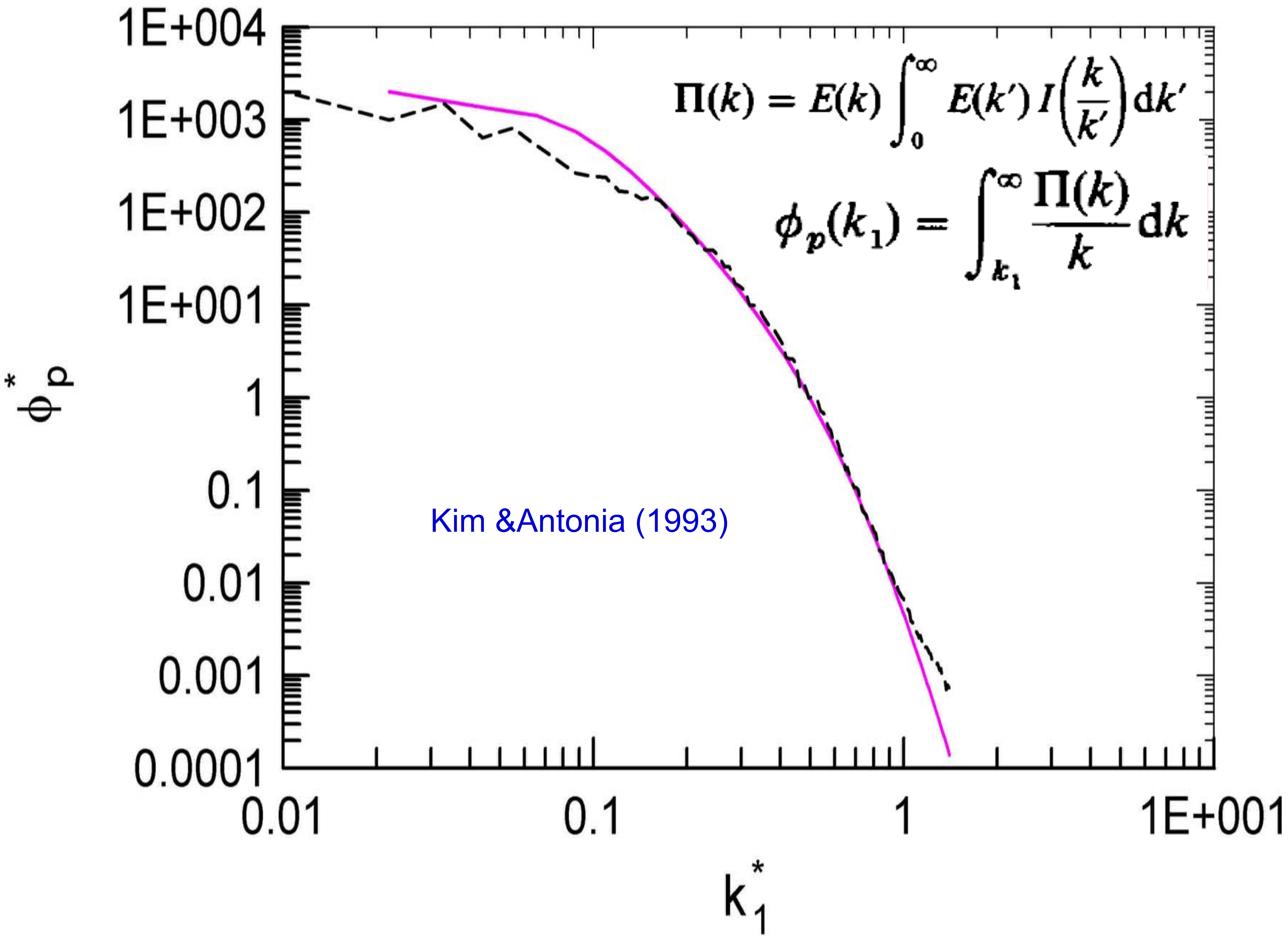


Antonia & Kim (1994)

$$\phi_{\theta,1} = \phi_{\theta,2} = \int_{k_3}^{\infty} k^{-1} \phi_{\theta,3}(k)\, dk$$

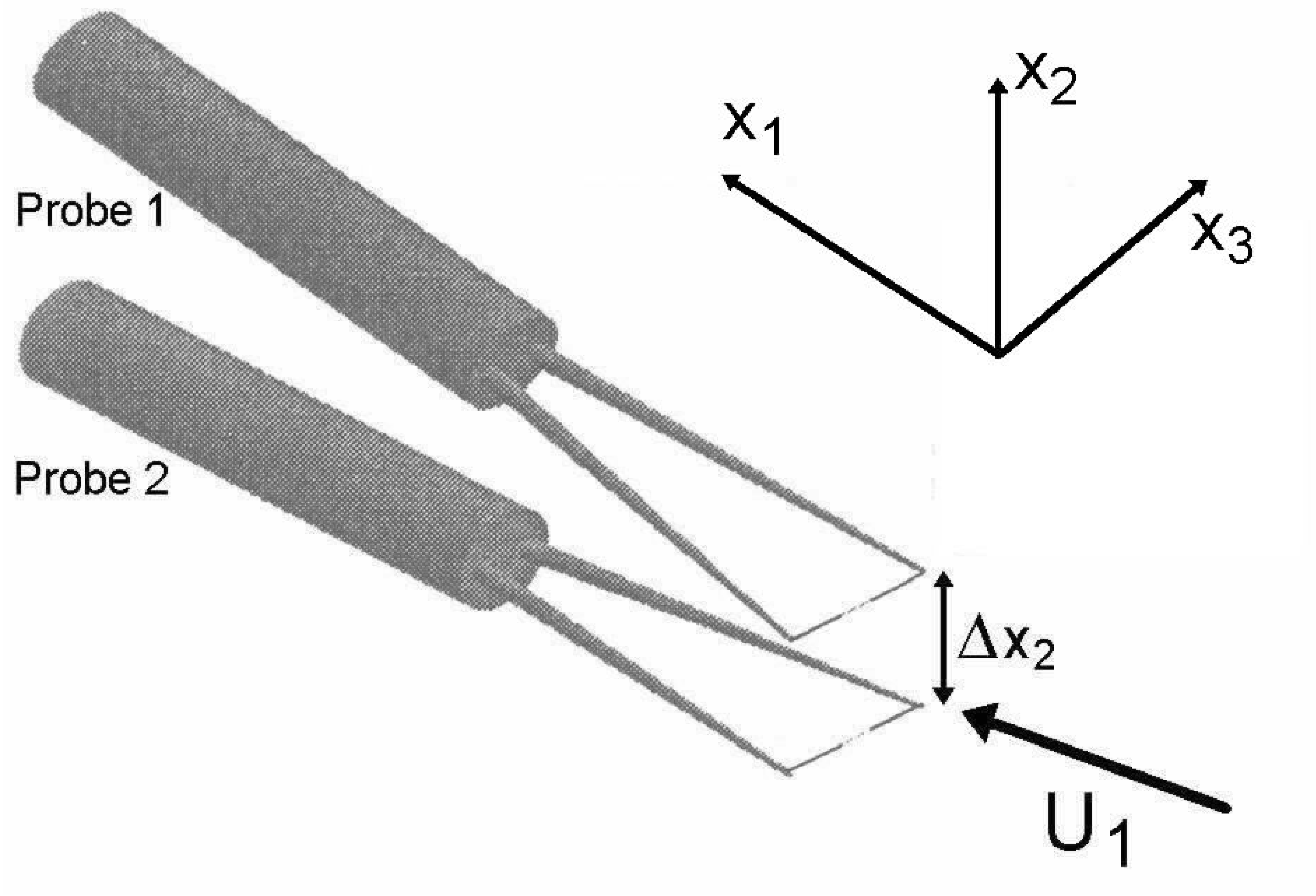


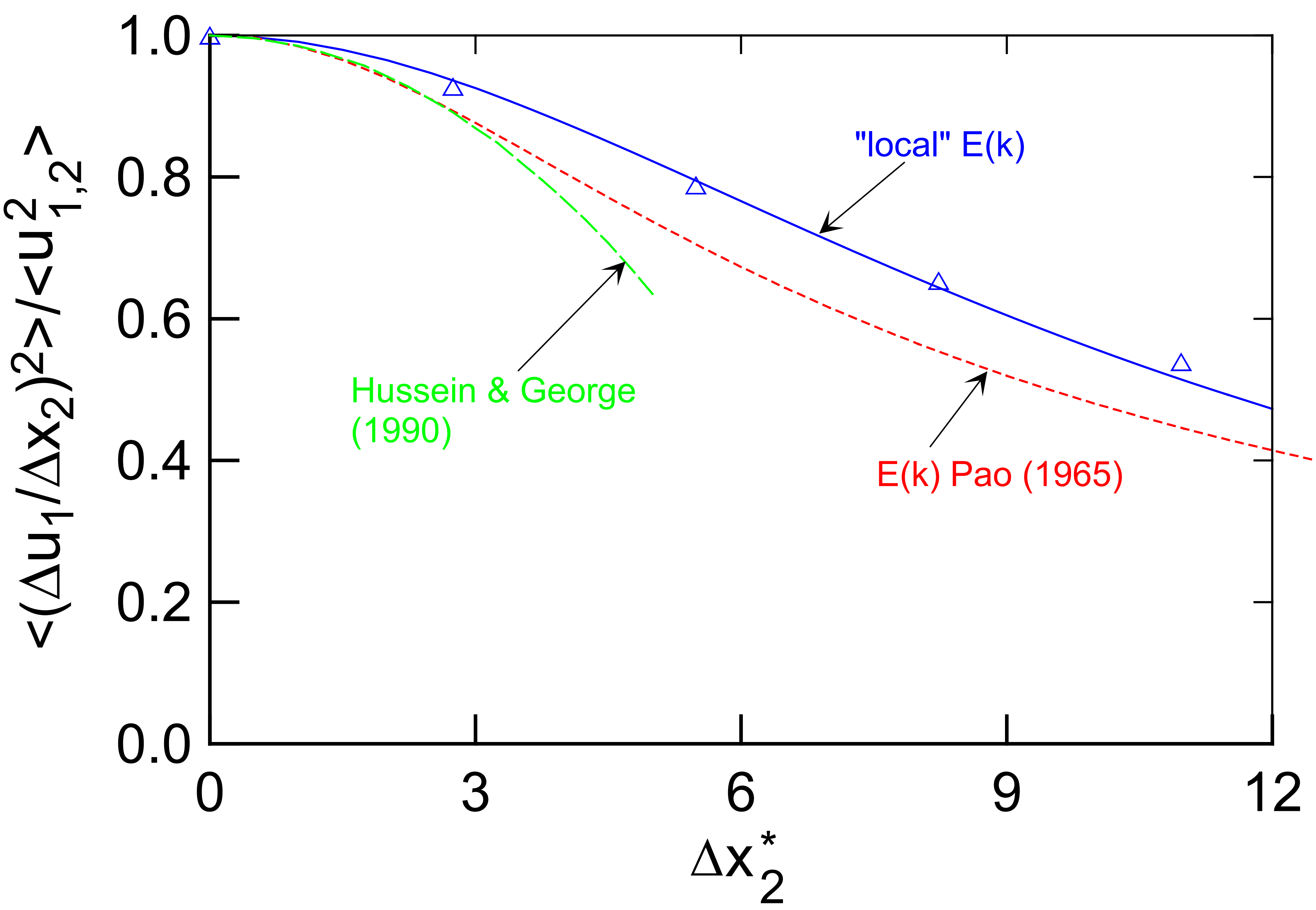
Antonia & Kim (1994)



$$\Pi(k) = E(k)\int_0^\infty E(k')\, I\left(\frac{k}{k'}\right) \mathrm{d}k'$$
$$\phi_p(k_1) = \int_{k_1}^\infty \frac{\Pi(k)}{k}\mathrm{d}k$$
Kim &Antonia (1993)
ϕ_p^*
k_1^*
1E+004
1E+003
1E+002
1E+001
1
0.1
0.01
0.001
0.0001
0.01
0.1
1
1E+001

Parallel hot (or cold) wires





1.0
0.8
0.6
0.4
0.2
0.0
0
3
6
9
12
$<(\Delta u_1/\Delta x_2)^2>/<u_{1,2}^2>$
Δx_2^*
"local" E(k)
Hussein & George
(1990)
E(k) Pao (1965)

$$E(k) = k^2 \left(\frac{\partial^2 \phi_{u_1}}{\partial k_1^2} \right)_{k_1 = k} - k \left(\frac{\partial \phi_{u_1}}{\partial k_1} \right)_{k_1 = k}$$

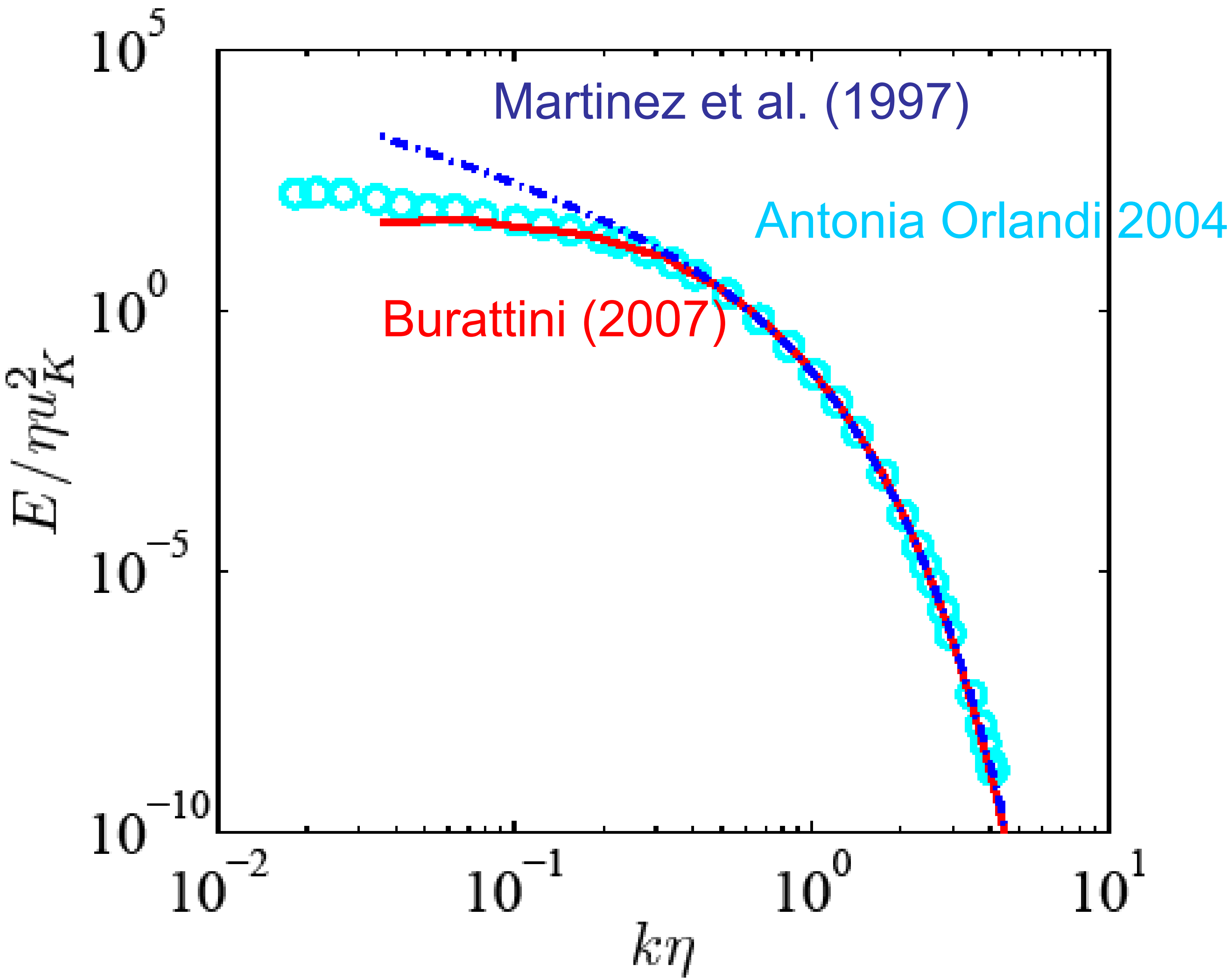
$$E(k) = -k \left(\frac{\partial \phi_q}{\partial k_1} \right)_{k_1 = k}$$

$$\phi_q(k_1) = \phi_{u_1}(k_1) + \phi_{u_2}(k_1) + \phi_{u_3}(k_1)$$

Martinez Chen Doolen Kraichnan Wang & Zhou (1997)

$$\frac{E(k\eta)}{u_K^2\eta} = (k\eta)^a \exp(-b(k\eta))$$

3D energy spectrum (DNS vs model)



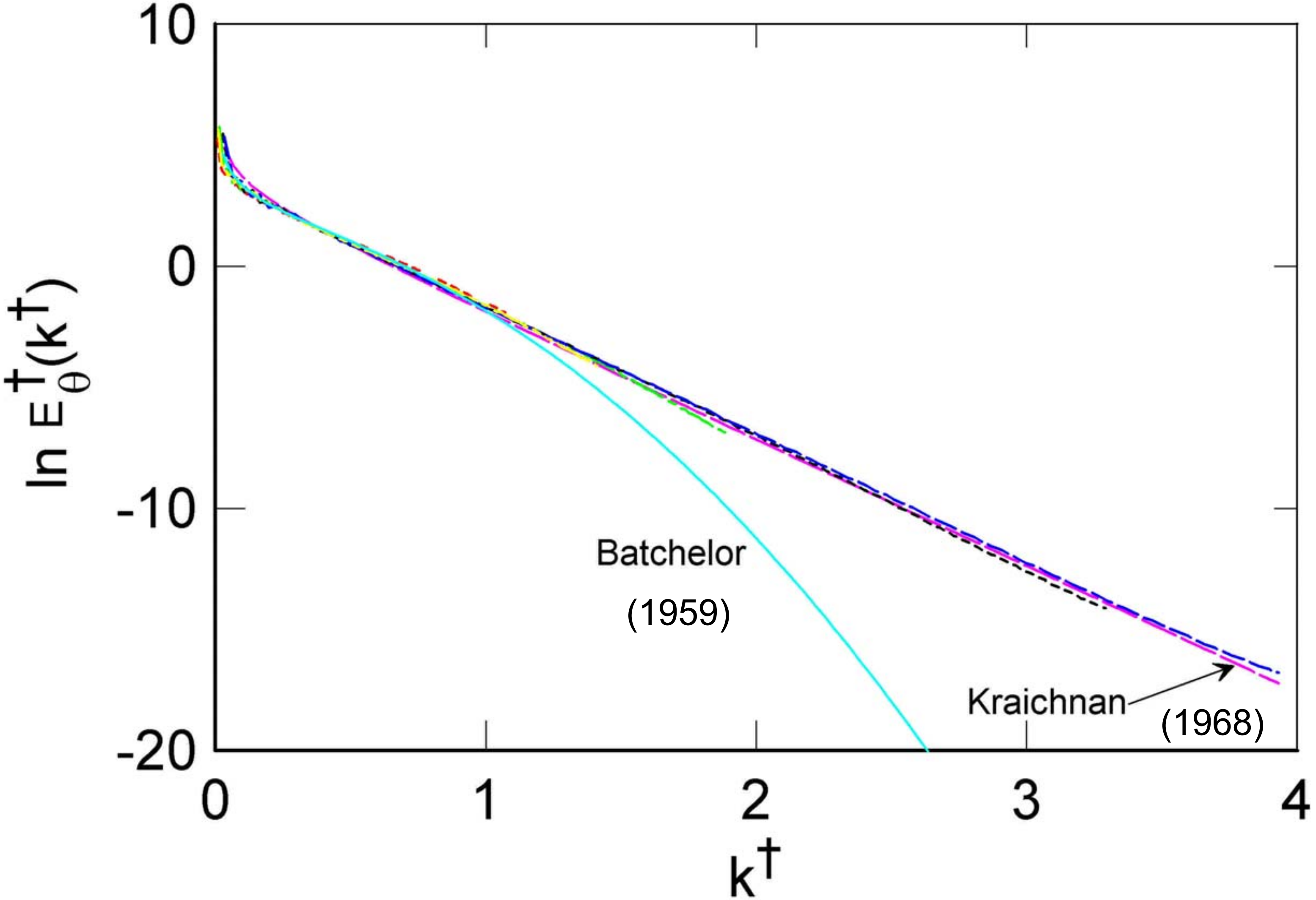
Kraichnan (1968)

$$\frac{E_\theta(k\eta_B)}{\theta_B^2\eta_B} = q(k\eta_B)^{-1}\left(1 + (6q)^{1/2}k\eta_B\right)\exp\left(-(6q)^{1/2}k\eta_B\right)$$

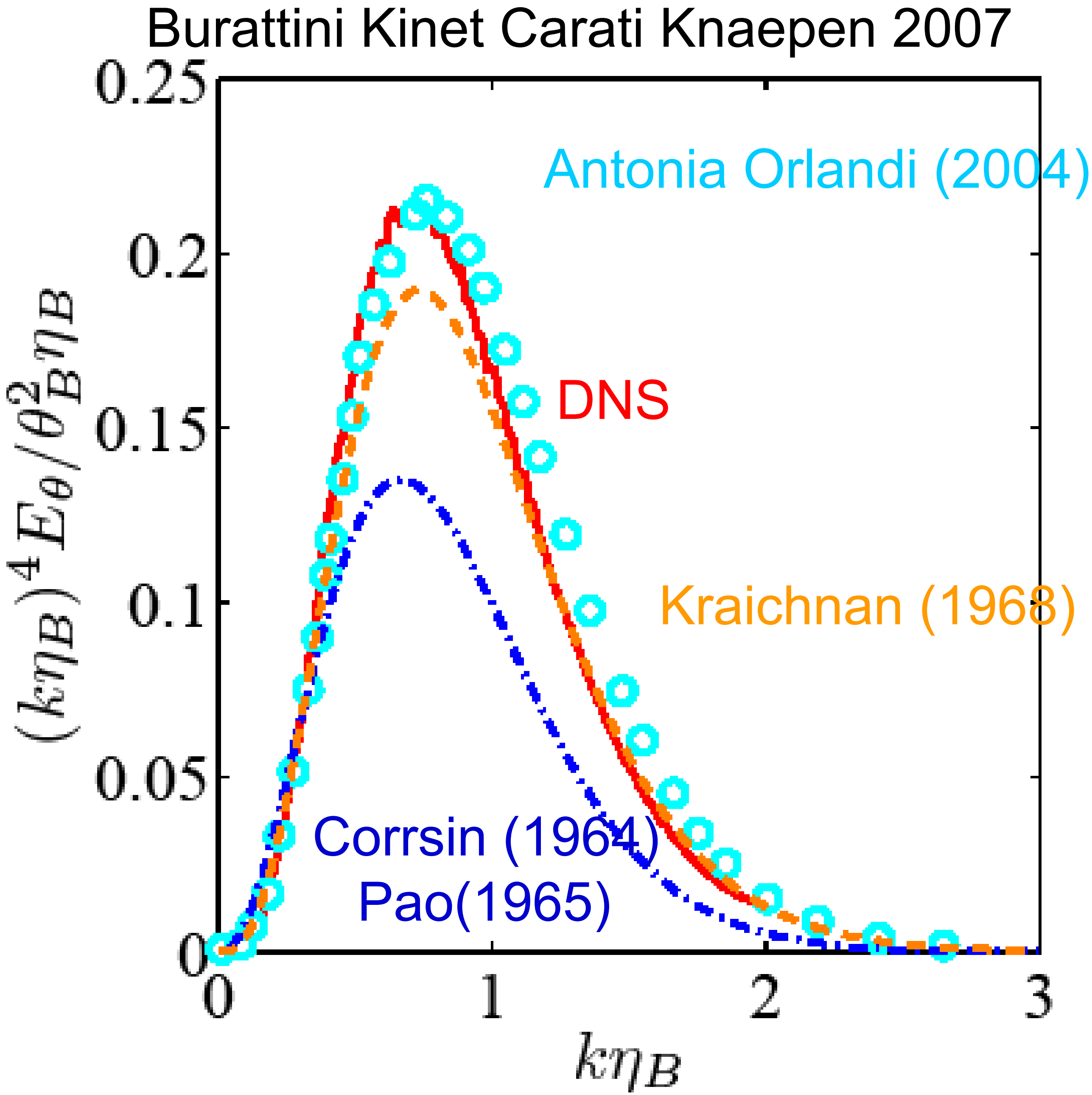
$$q = 2\sqrt{5}$$

3D scalar spectrum

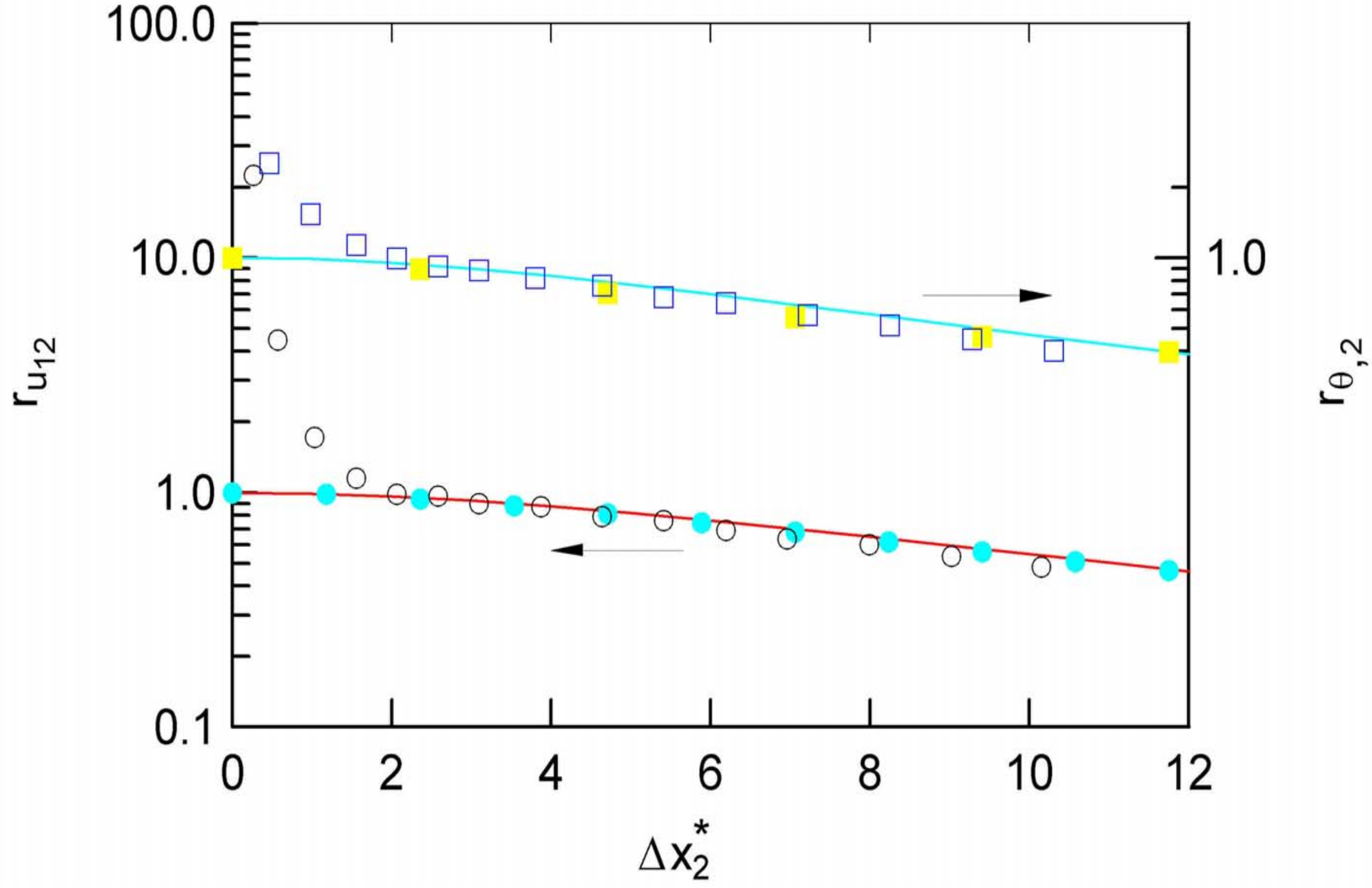
Antonia Orlandi (2004)



3D scalar spectrum (DNS vs model)

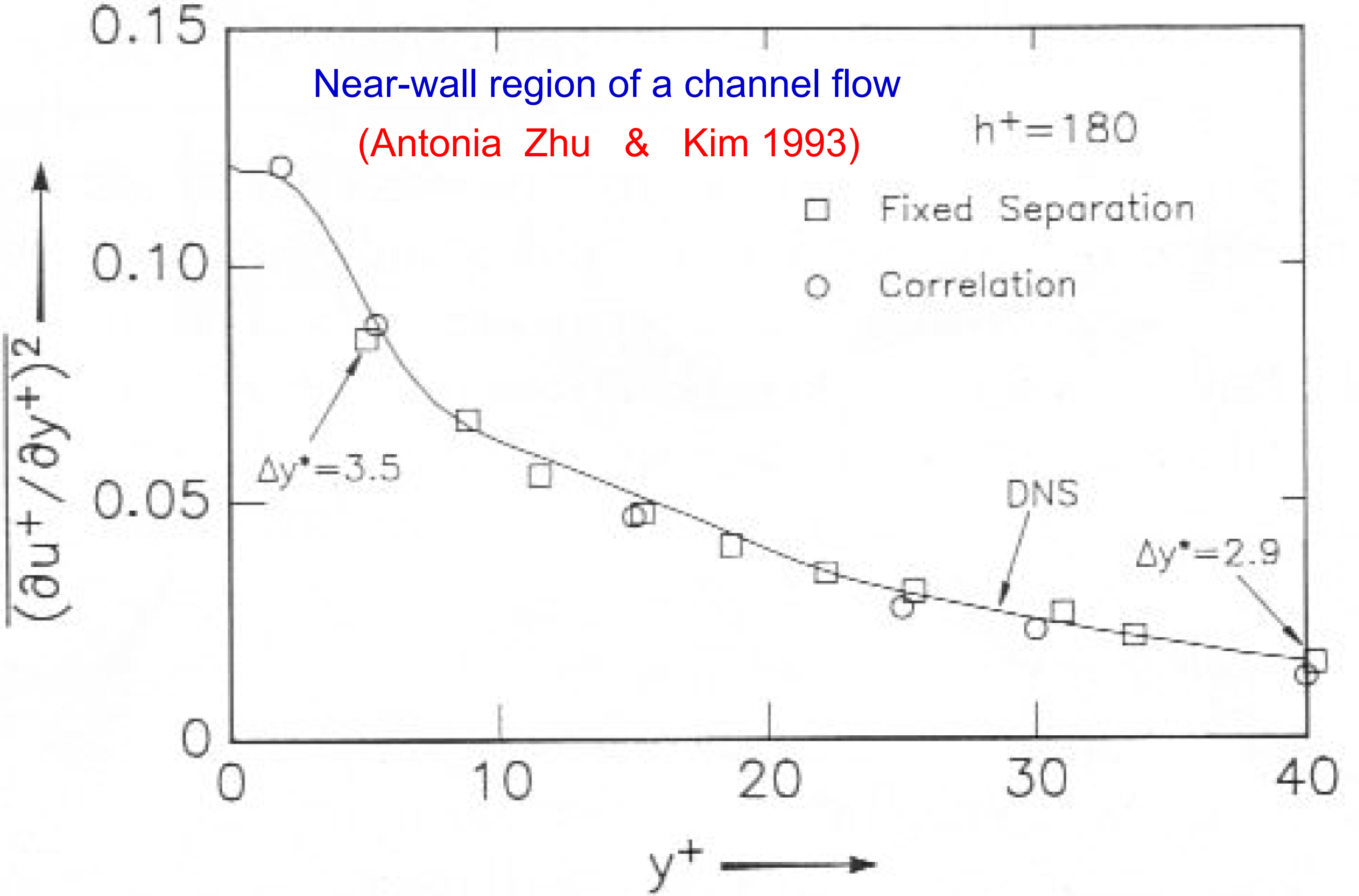


Effect of separation on the derivative variances

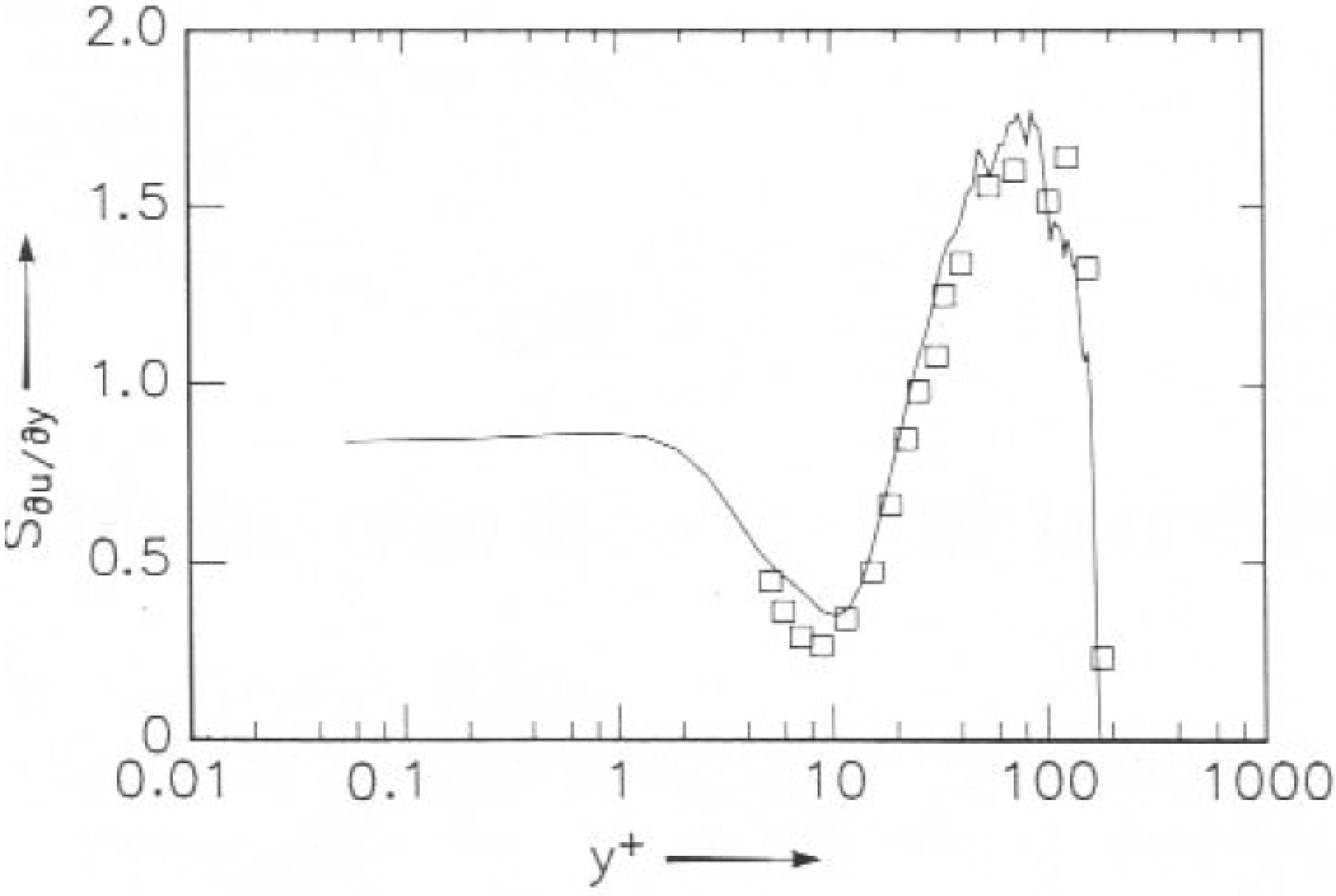


(——), calculations; (■,●); DNS; (□,○) measurement.

What happens when homogeneity and isotropy are violated ?

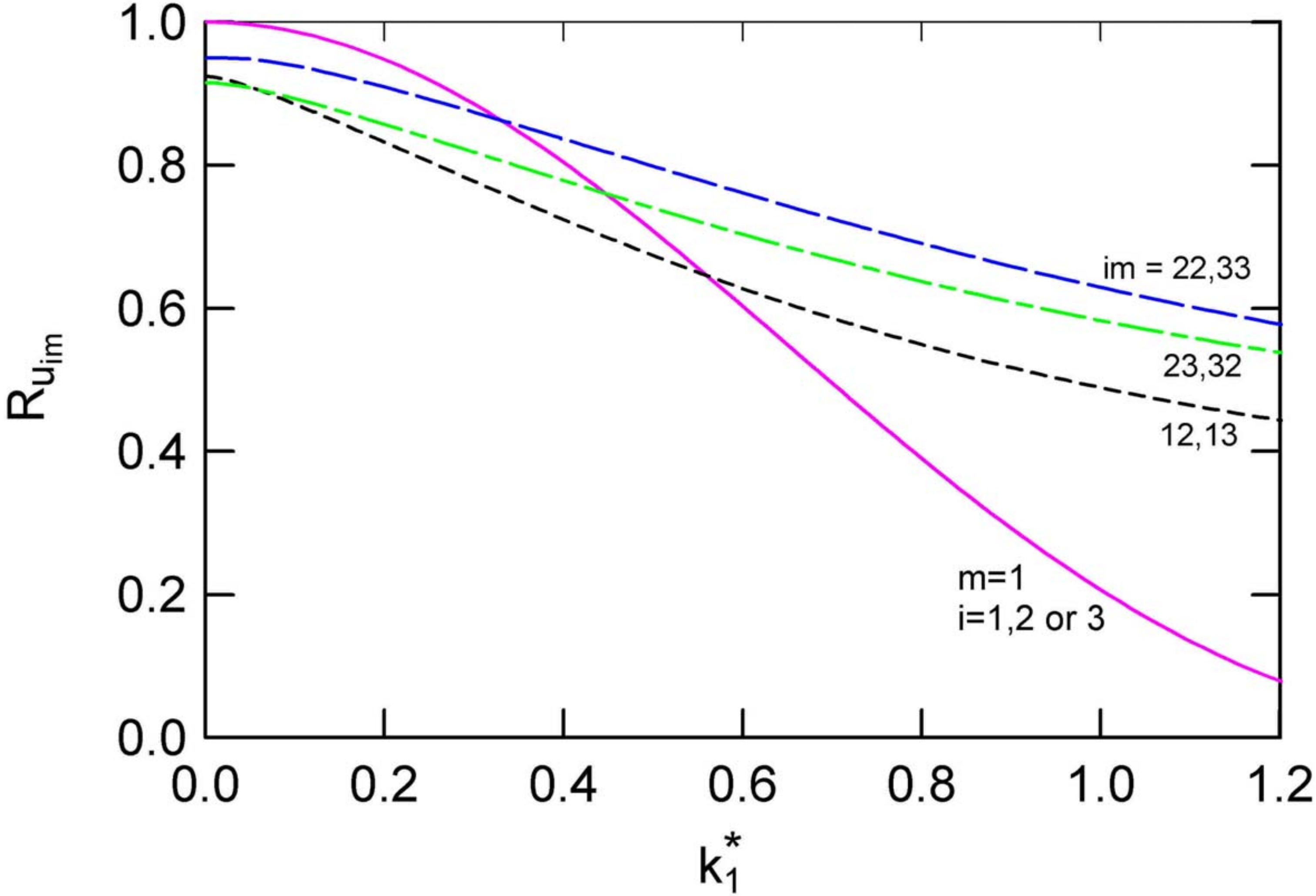


Skewness of lateral velocity derivative



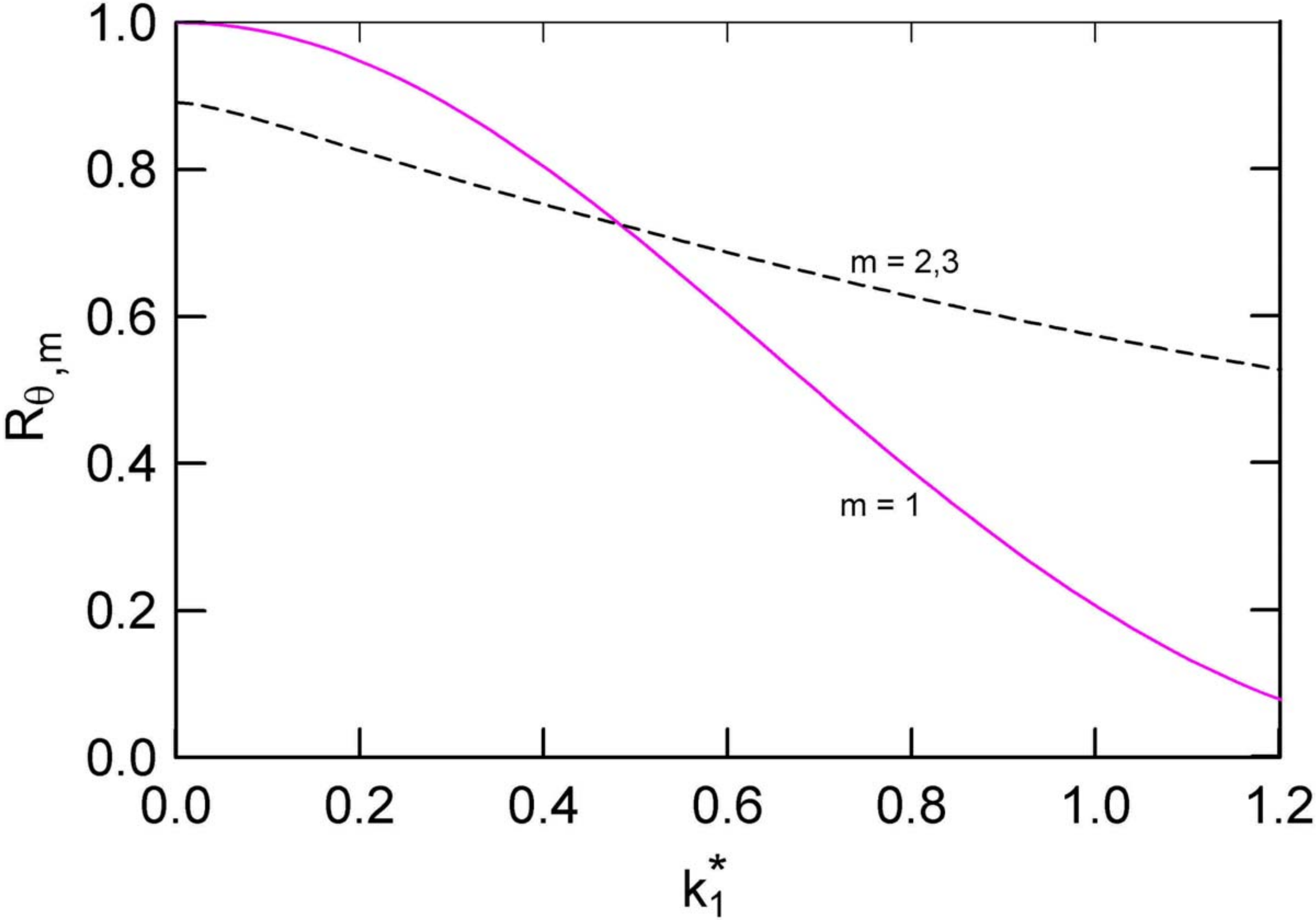
Spectral correction coefficients for velocity derivatives

Zhu & Antonia (1996a)

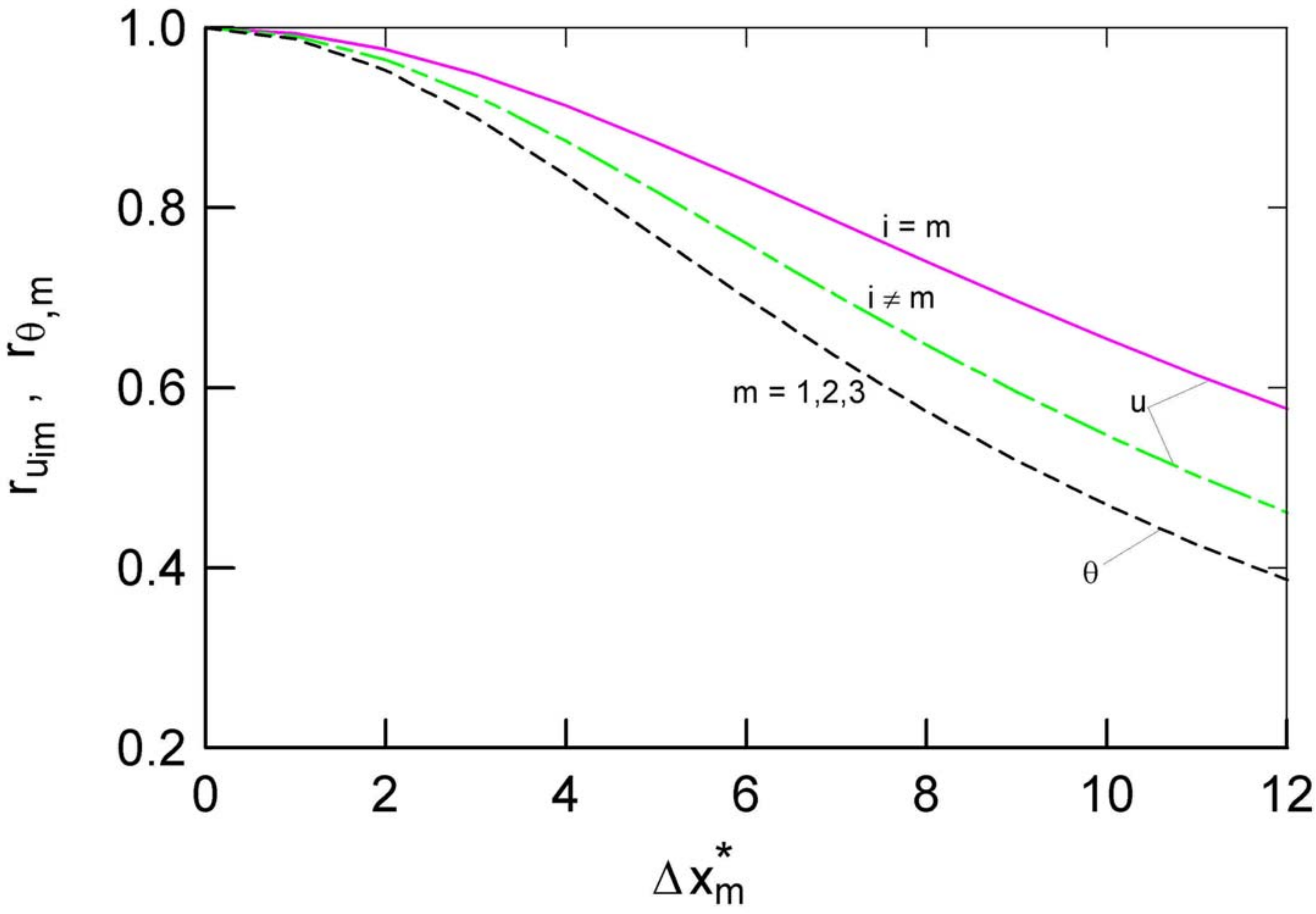


Spectral correction coefficients for scalar derivatives

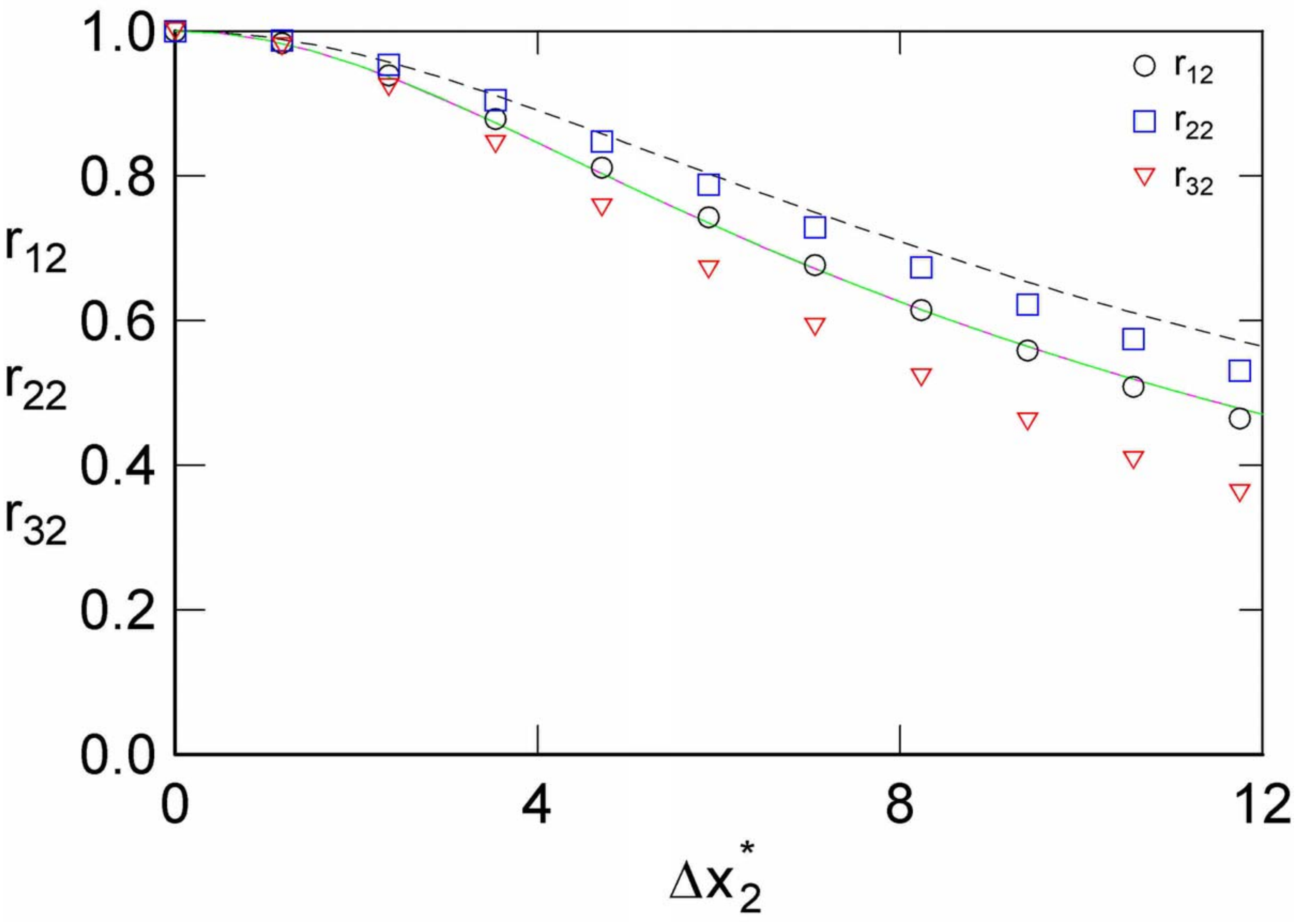
Zhu & Antonia (1996a)



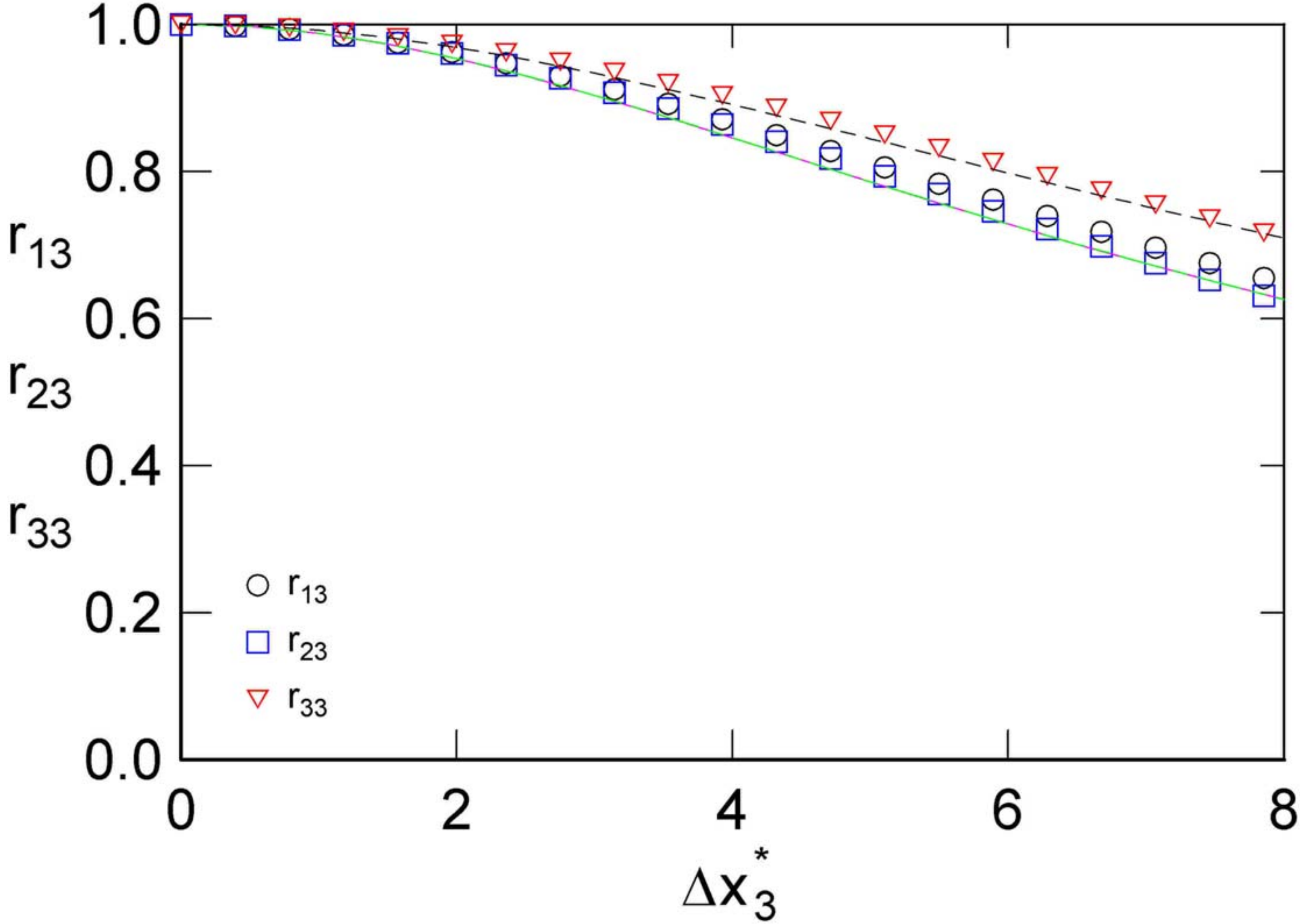
Attenuation of velocity and scalar derivative variances



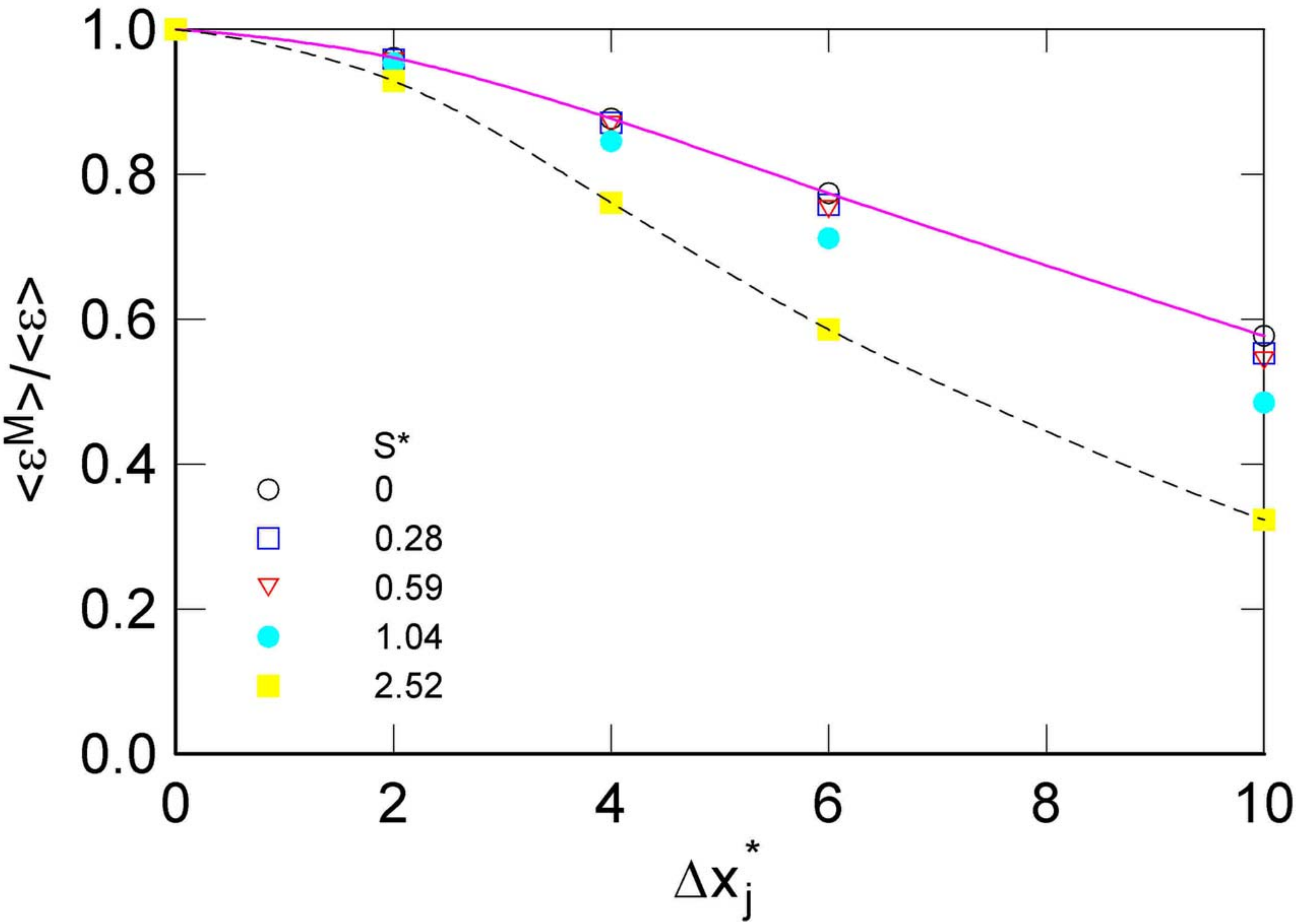
Attenuation of r_{i2} using DNS data (channel flow)



Attenuation of r_{i3} using DNS data (channel flow)

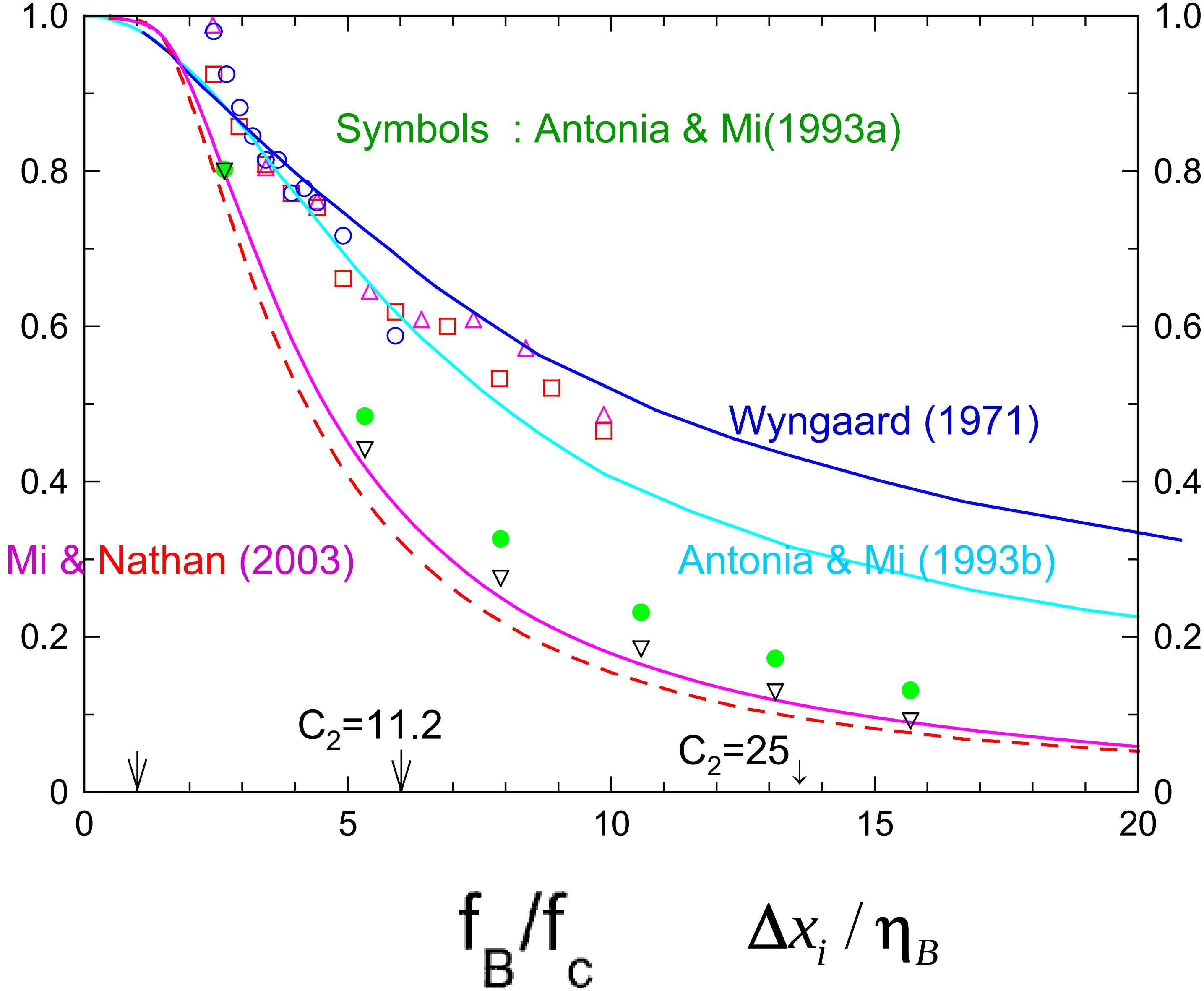


Attenuation of $\langle \epsilon \rangle|$ using DNS data (channel flow)

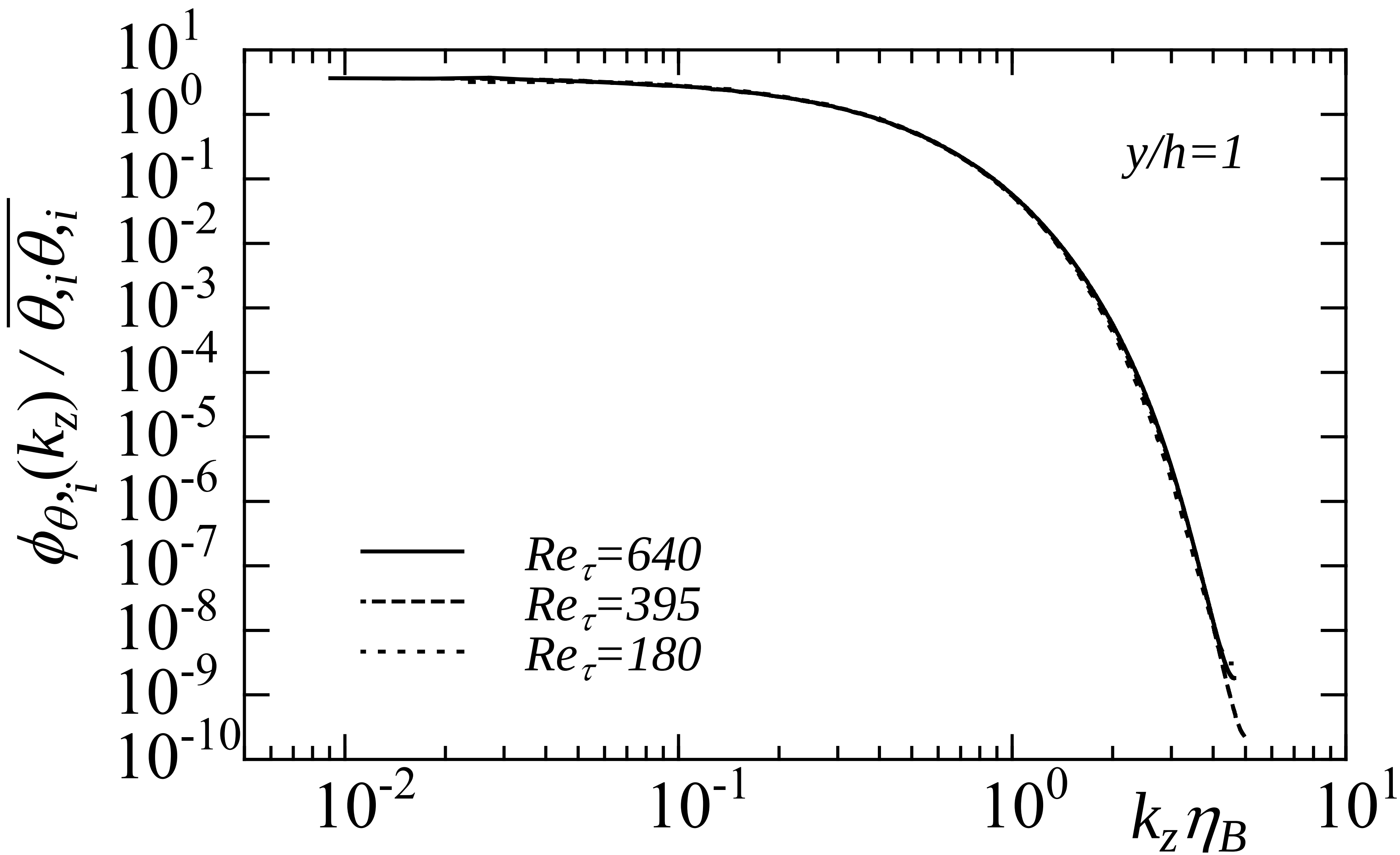


Effect of spatial resolution (and filter cut-off frequency) on $<\varepsilon_\theta>^M / <\varepsilon_\theta>$

axis of circular jet

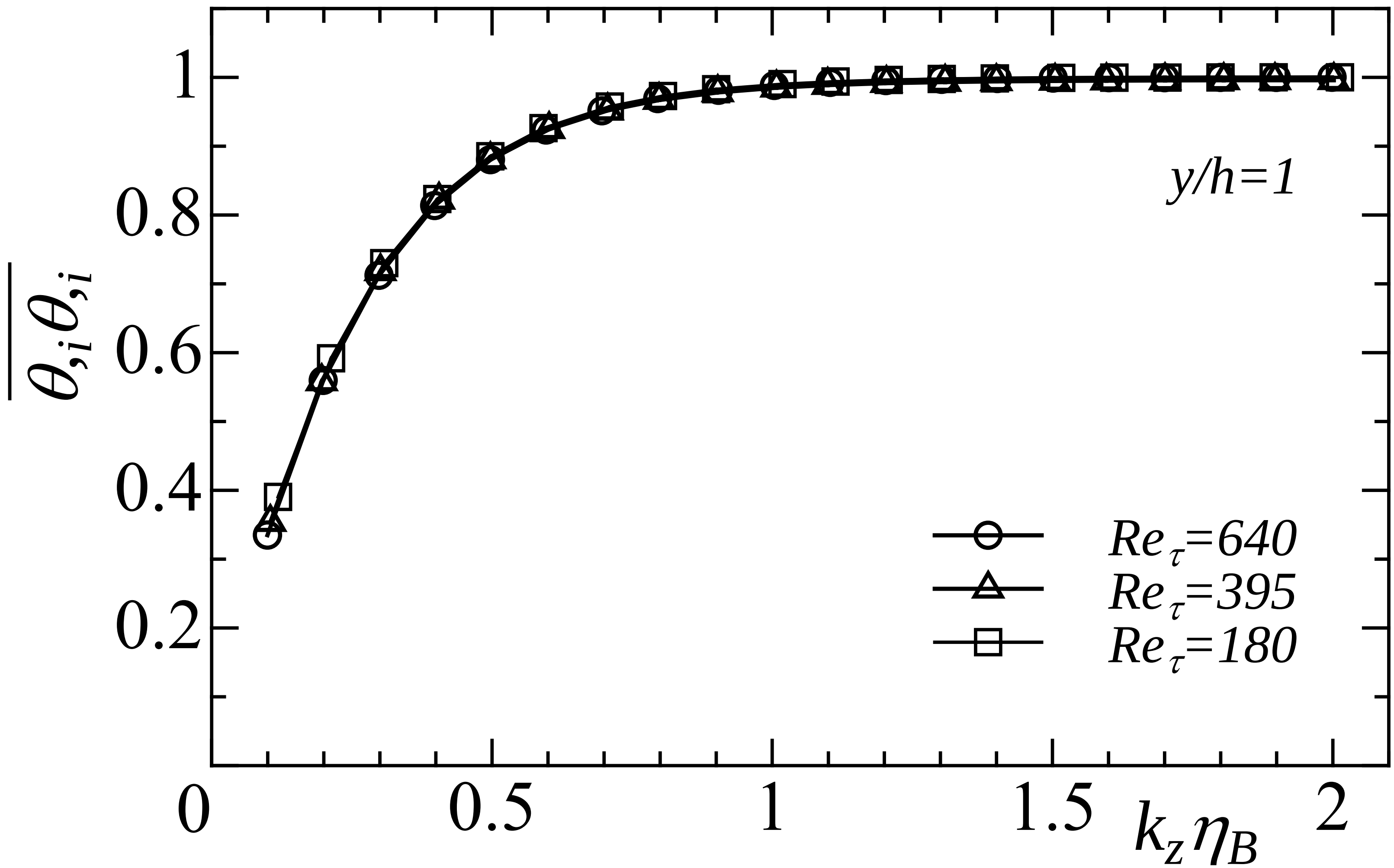


Scalar dissipation spectra (DNS channel flow)



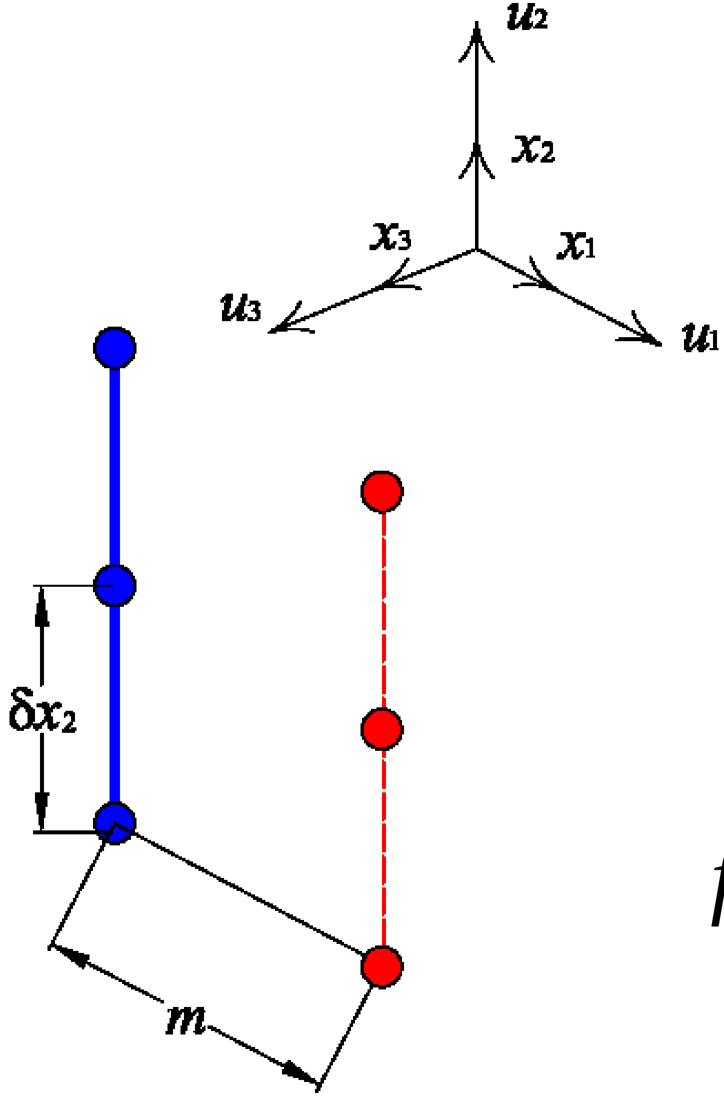
Effect of cutoff on mean scalar dissipation rate

$$\left(\overline{\theta_{,i}\,\theta_{,i}}\right)_{kz_{cutoff}} = \int_0^{kz_{cutoff}} \phi(k_z)\,dk_z$$



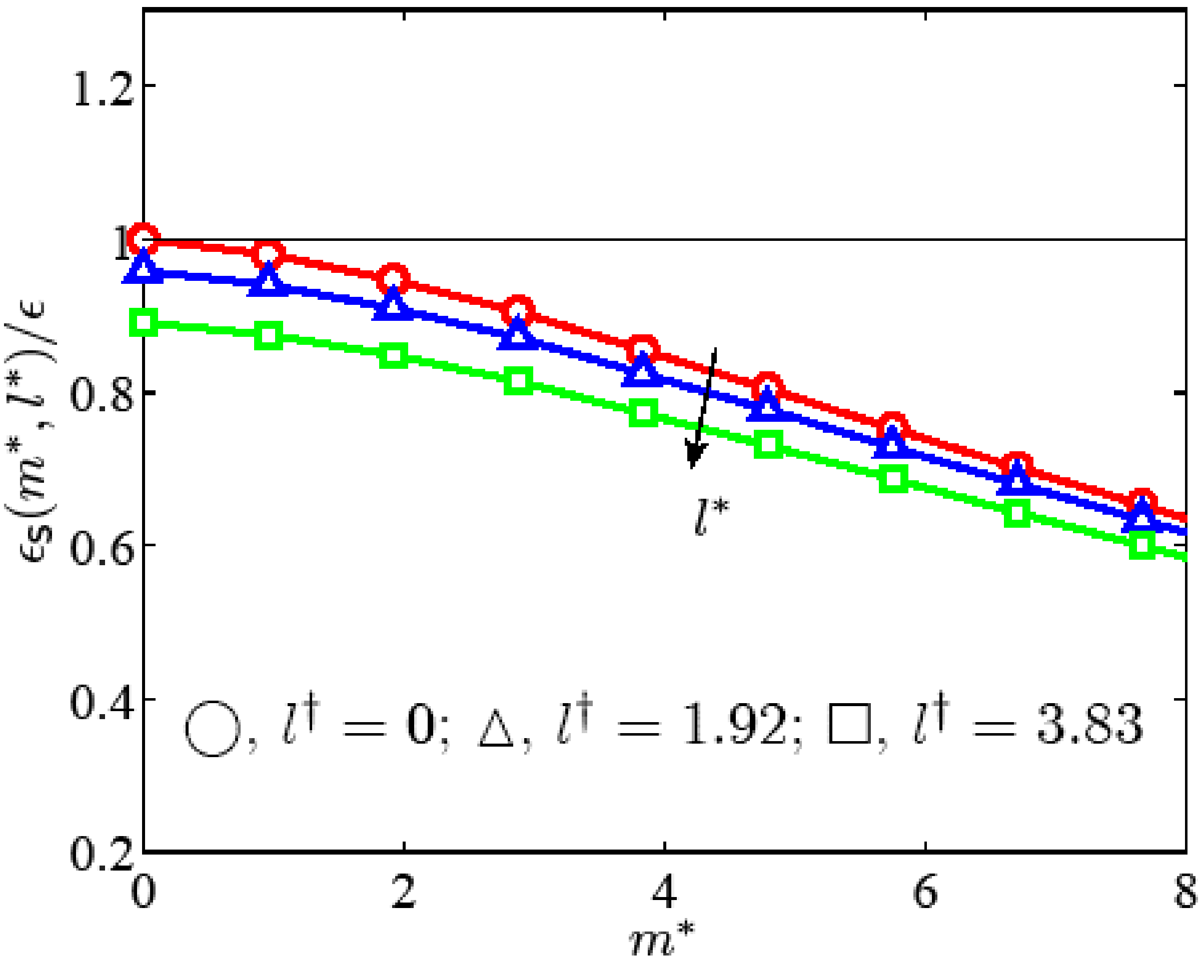
SW

Burattini (2007)

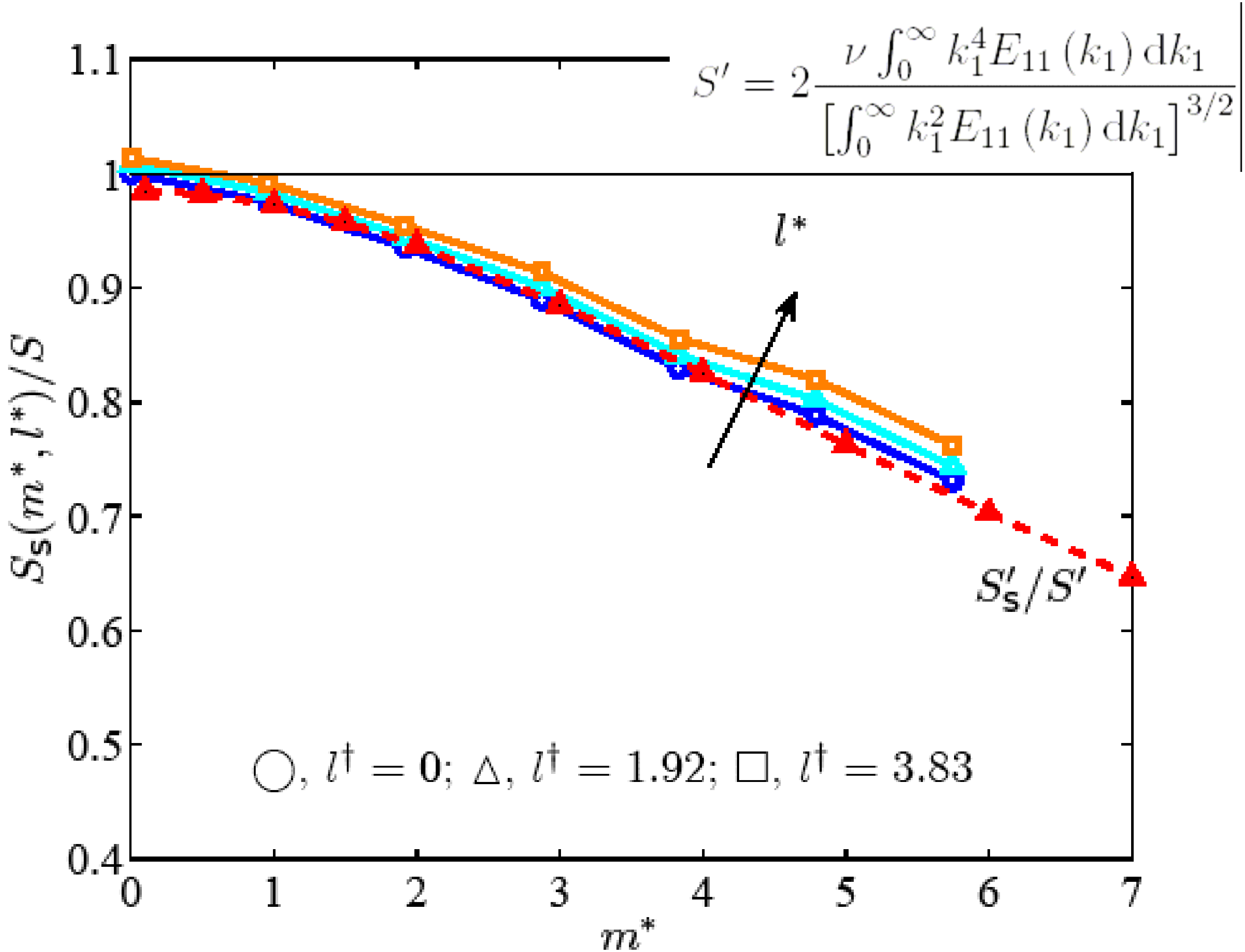


$$f = U_1 / m$$

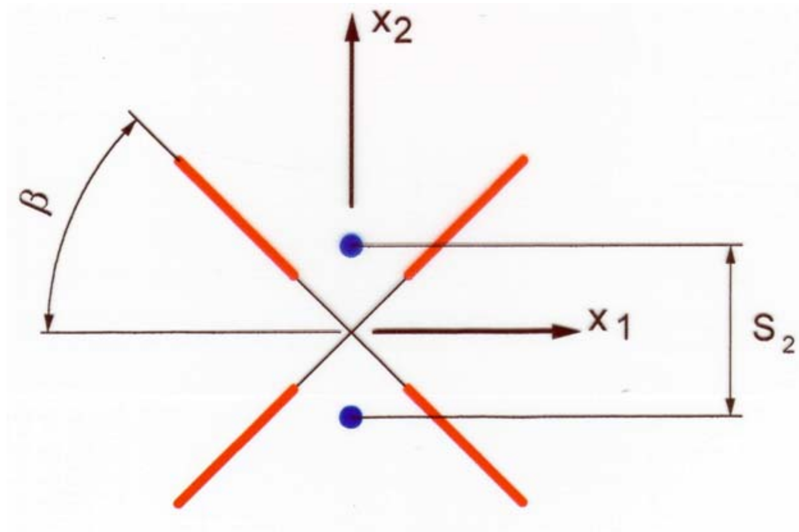
Effect of m and l on the mean isotropic dissipation rate



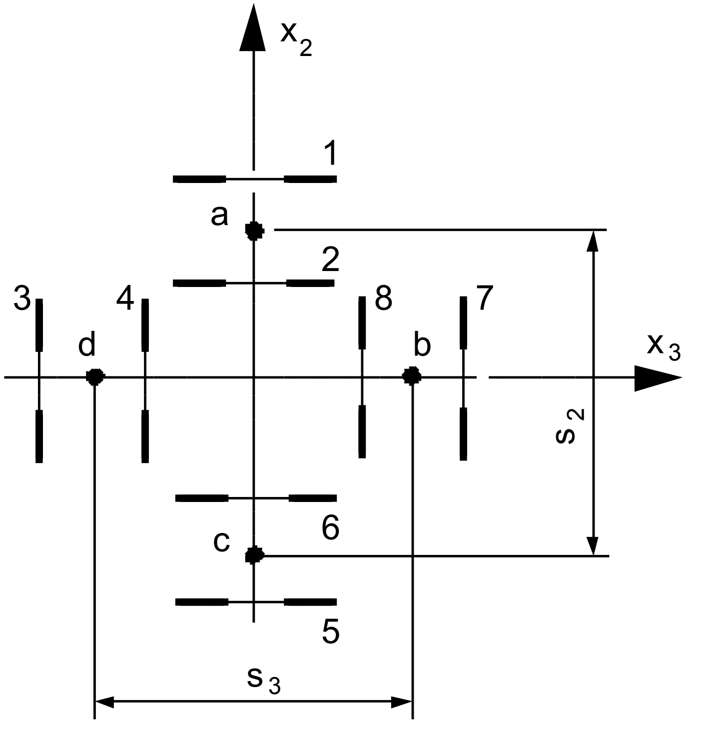
Burattini Antonia Lavoie (work in progress)



- **parallel hot wires + X-wire (one component vorticity probe)**



- **four X-wires (3-component vorticity probe)**



$$u_{2,2} + u_{3,3} = -u_{1,1}$$

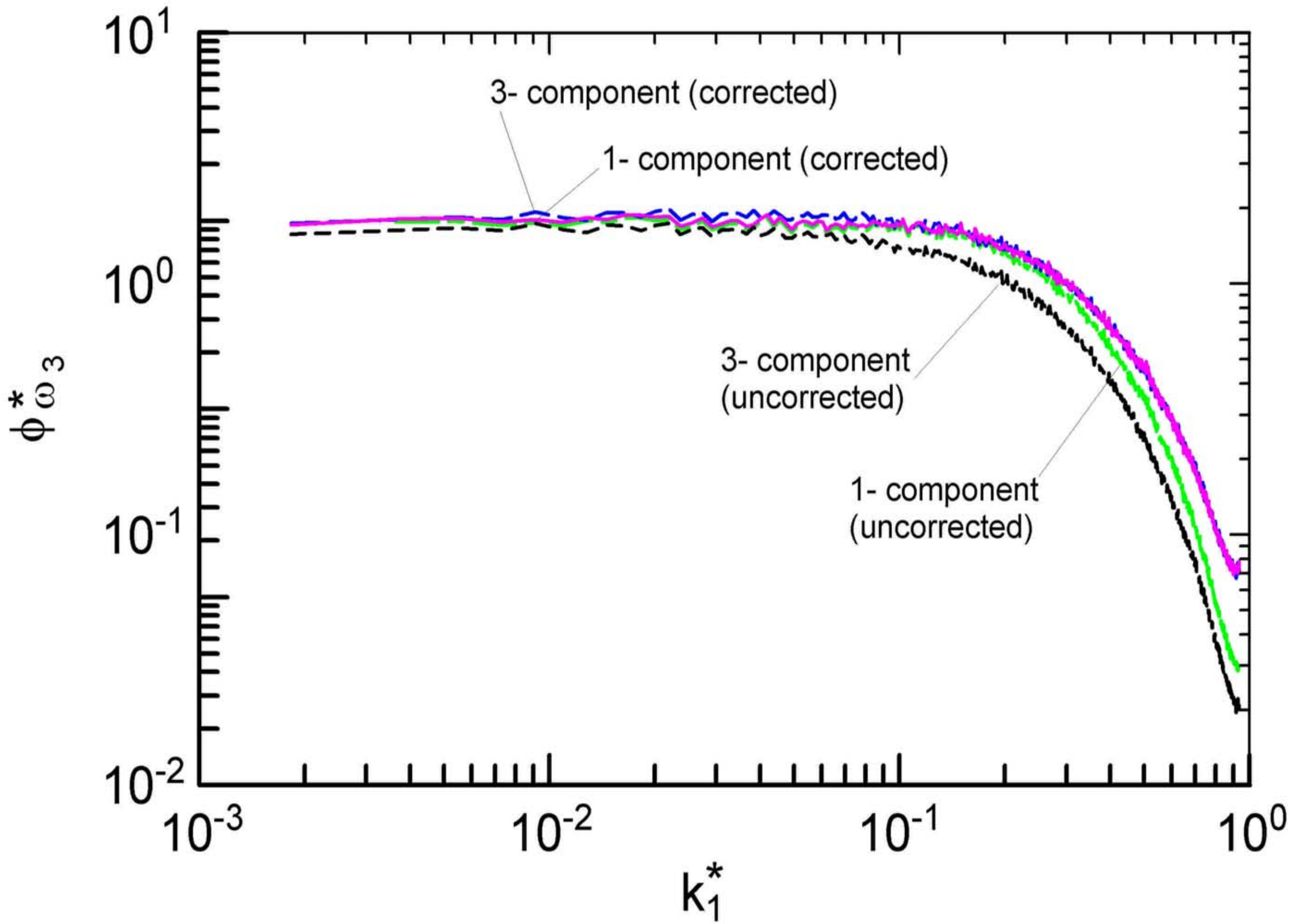
$$2\left\langle u_{2,2}^2 \right\rangle + 2\left\langle u_{3,3}^2 \right\rangle =$$

$$2\left\langle u_{1,1}^2 \right\rangle - 4\left\langle u_{2,2} u_{3,3} \right\rangle$$

$$\left\langle u_{2,2} u_{3,3} \right\rangle = \left\langle u_{2,3} u_{3,2} \right\rangle$$

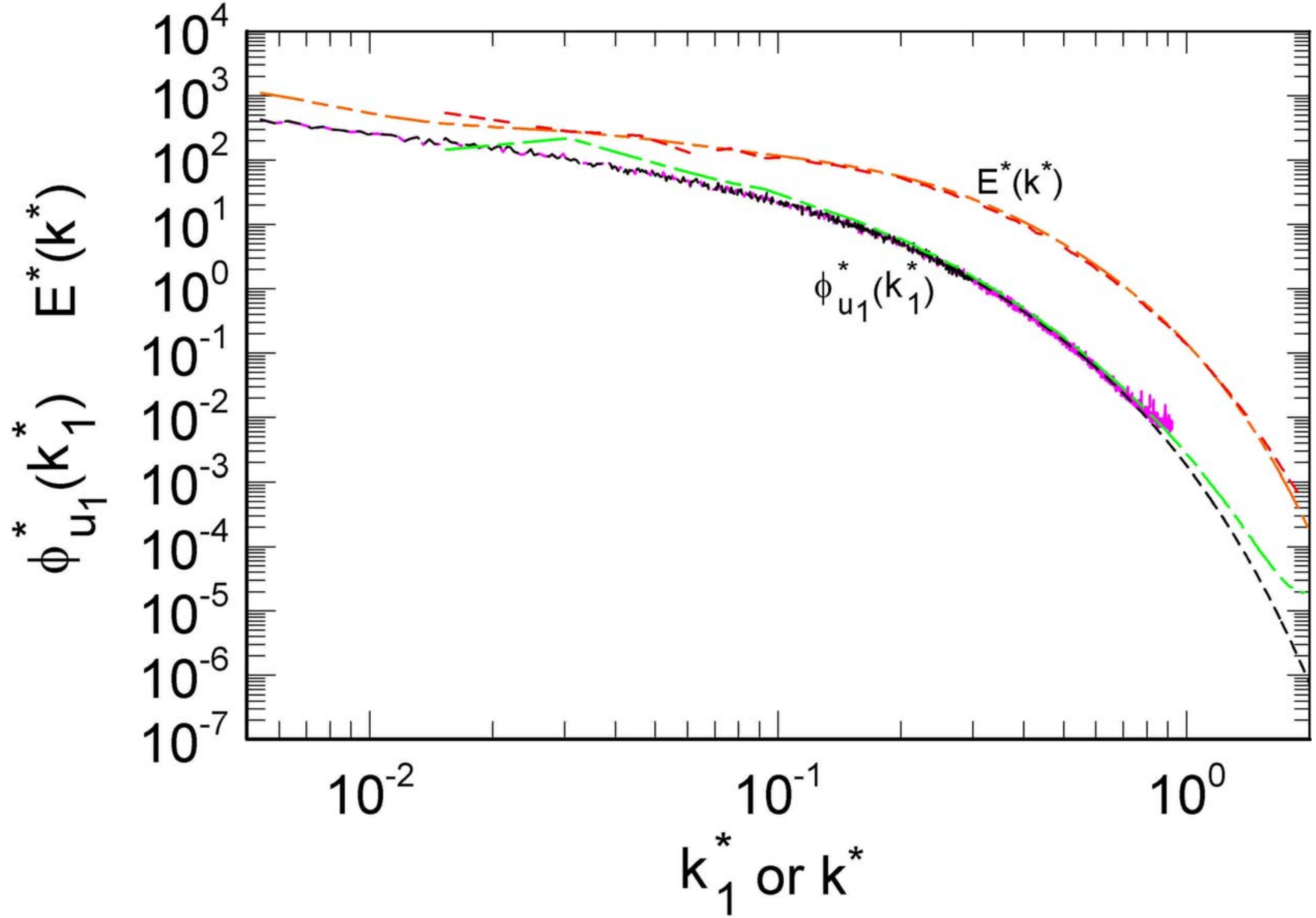
homogeneous turbulence

Spanwise vorticity spectrum obtained with both probes

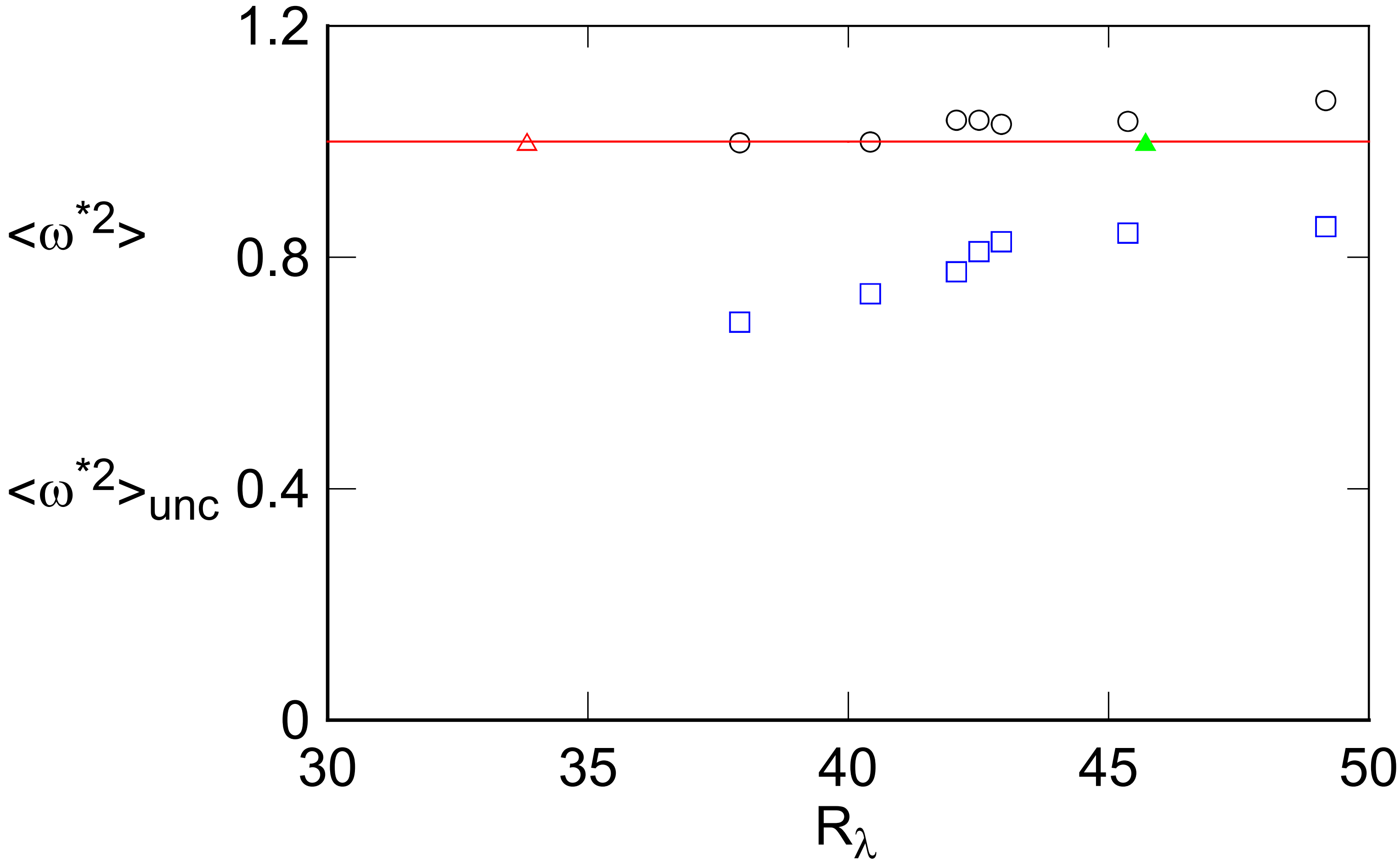


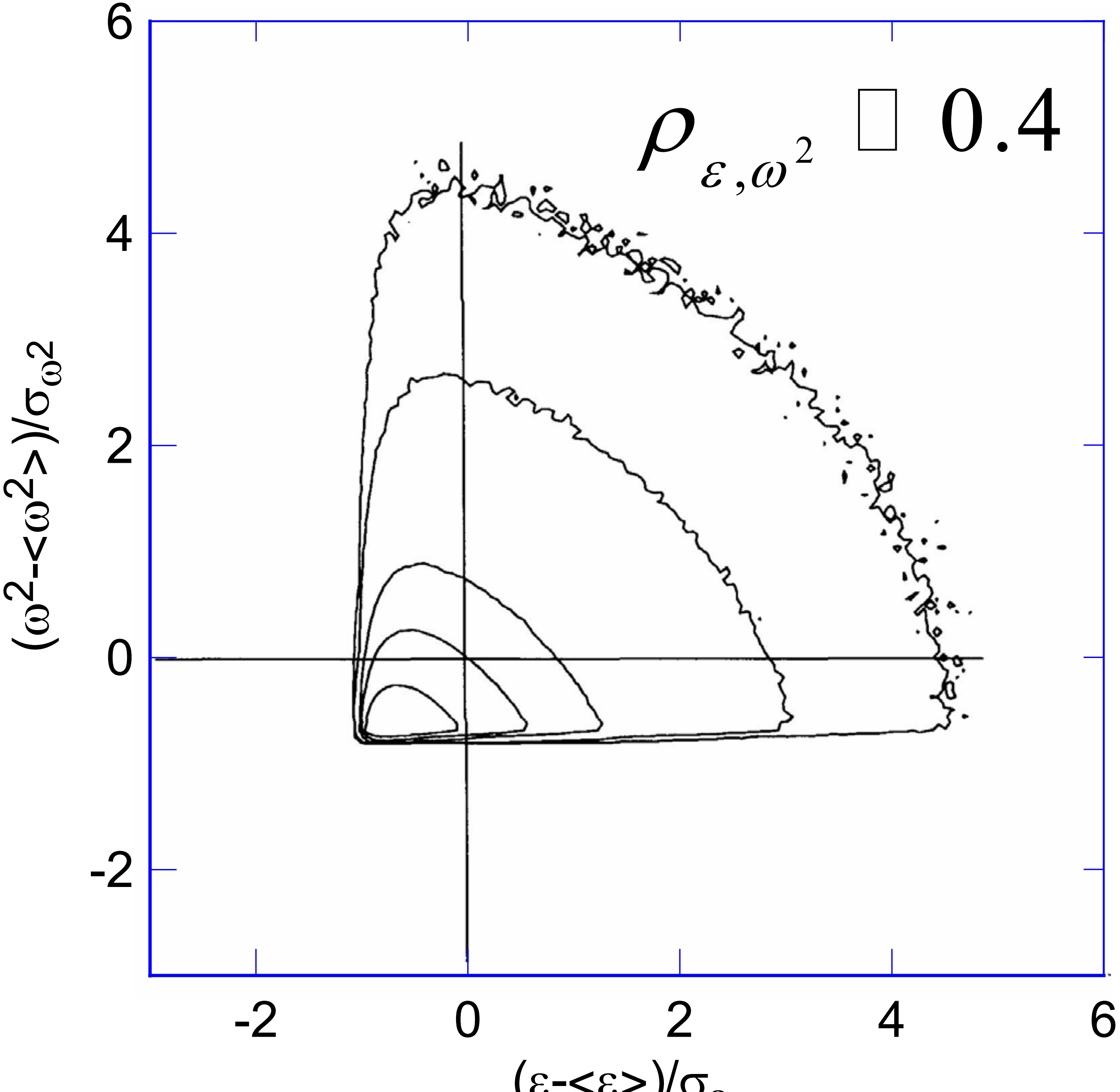
3D energy spectrum- comparison with DNS (box turbulence)

Antonia Orlandi Zhou (2002)

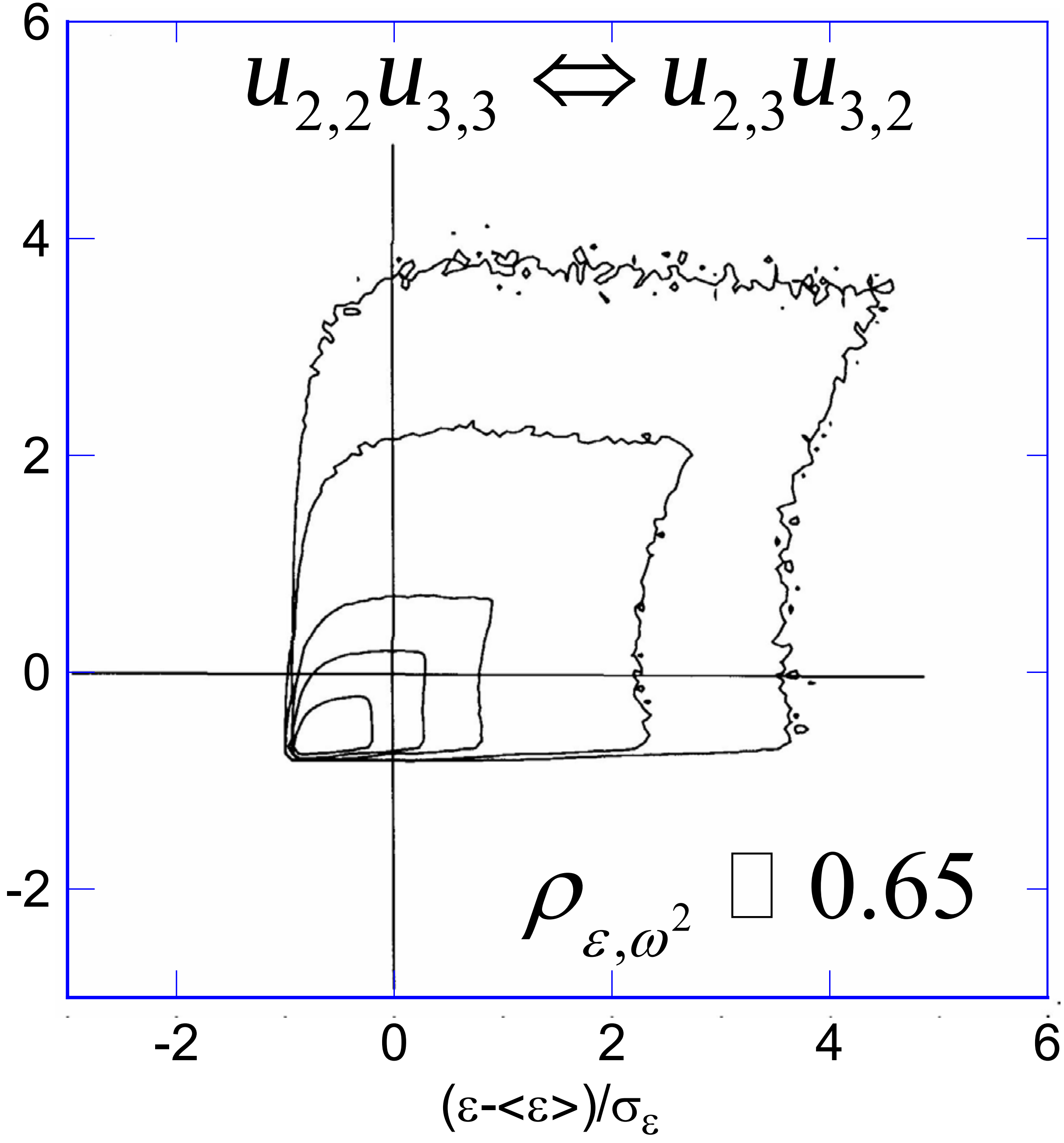


Kolmogorov-normalized mean enstrophy

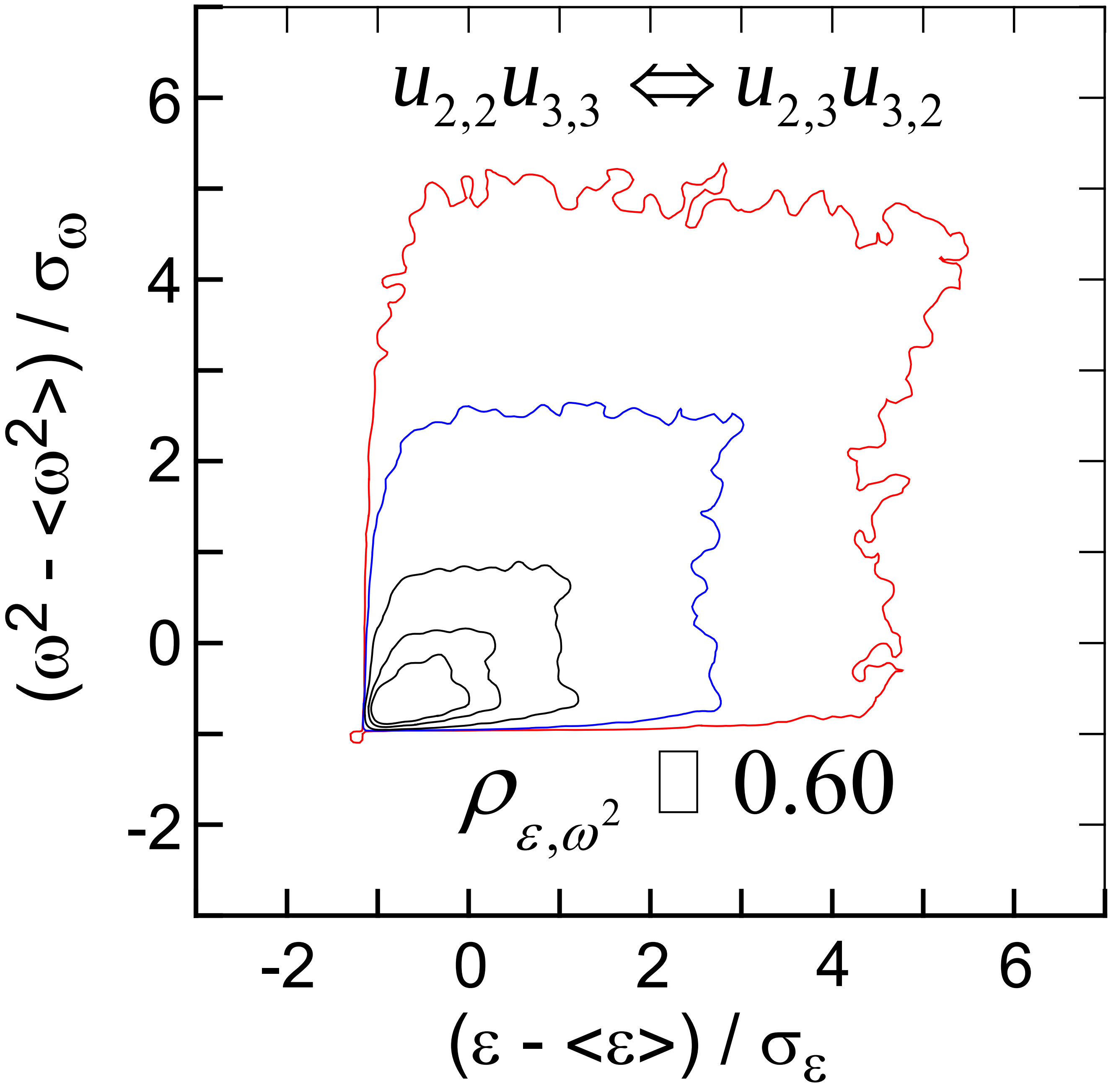




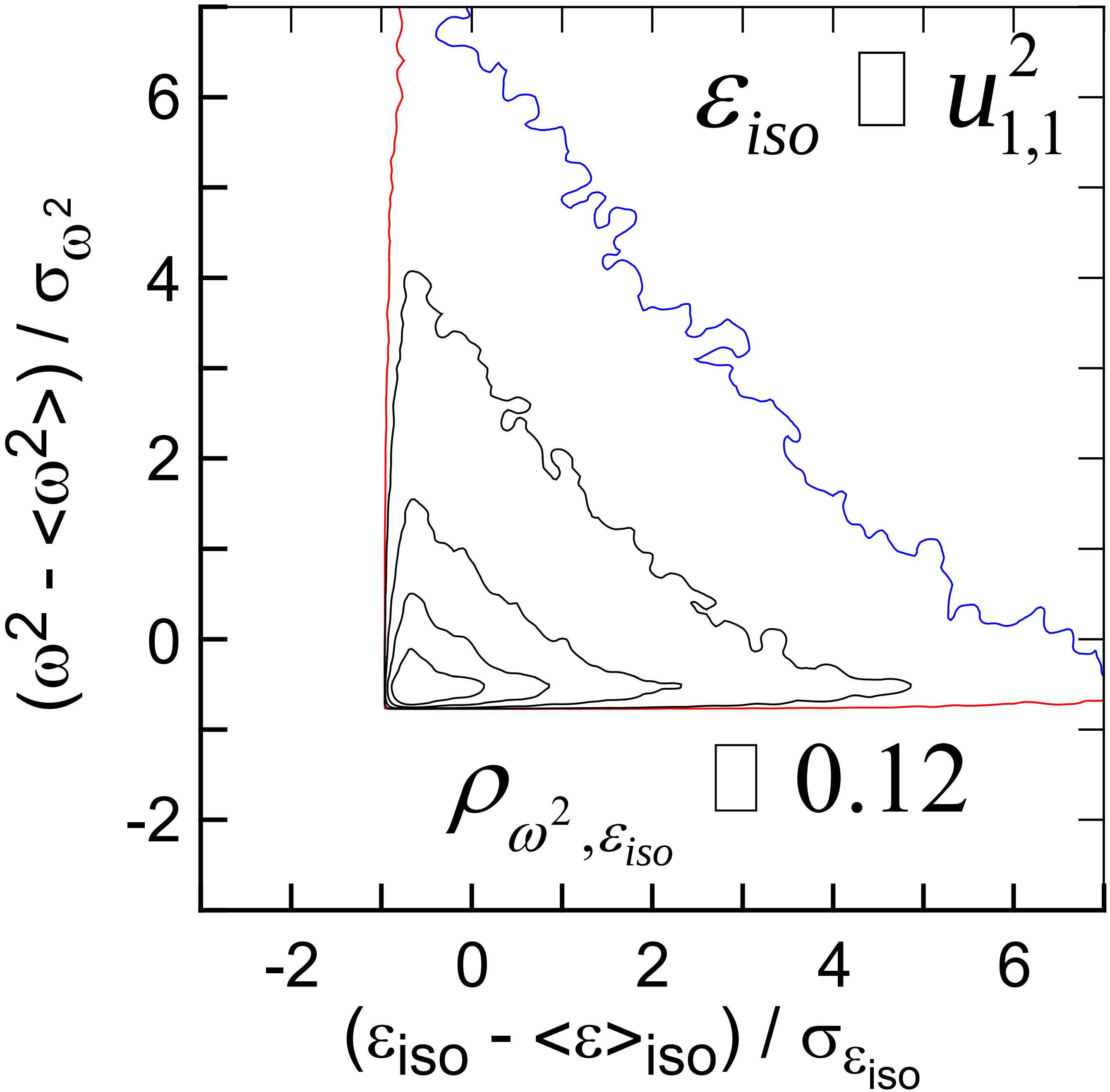
$\rho_{\varepsilon,\omega^2}$ □ 0.4
6
4
2
0
-2
-2
0
2
4
6
$(\omega^2\text{-}\langle\omega^2\rangle)/\sigma_{\omega^2}$
$(\varepsilon\text{-}\langle\varepsilon\rangle)/\sigma_\varepsilon$



$u_{2,2}u_{3,3} \Leftrightarrow u_{2,3}u_{3,2}$
$\rho_{\varepsilon,\omega^2}$ 0.65
$(\omega^2-\langle\omega^2\rangle)/\sigma_{\omega^2}$
$(\varepsilon-\langle\varepsilon\rangle)/\sigma_\varepsilon$
6
4
2
0
-2
-2
0
2
4
6

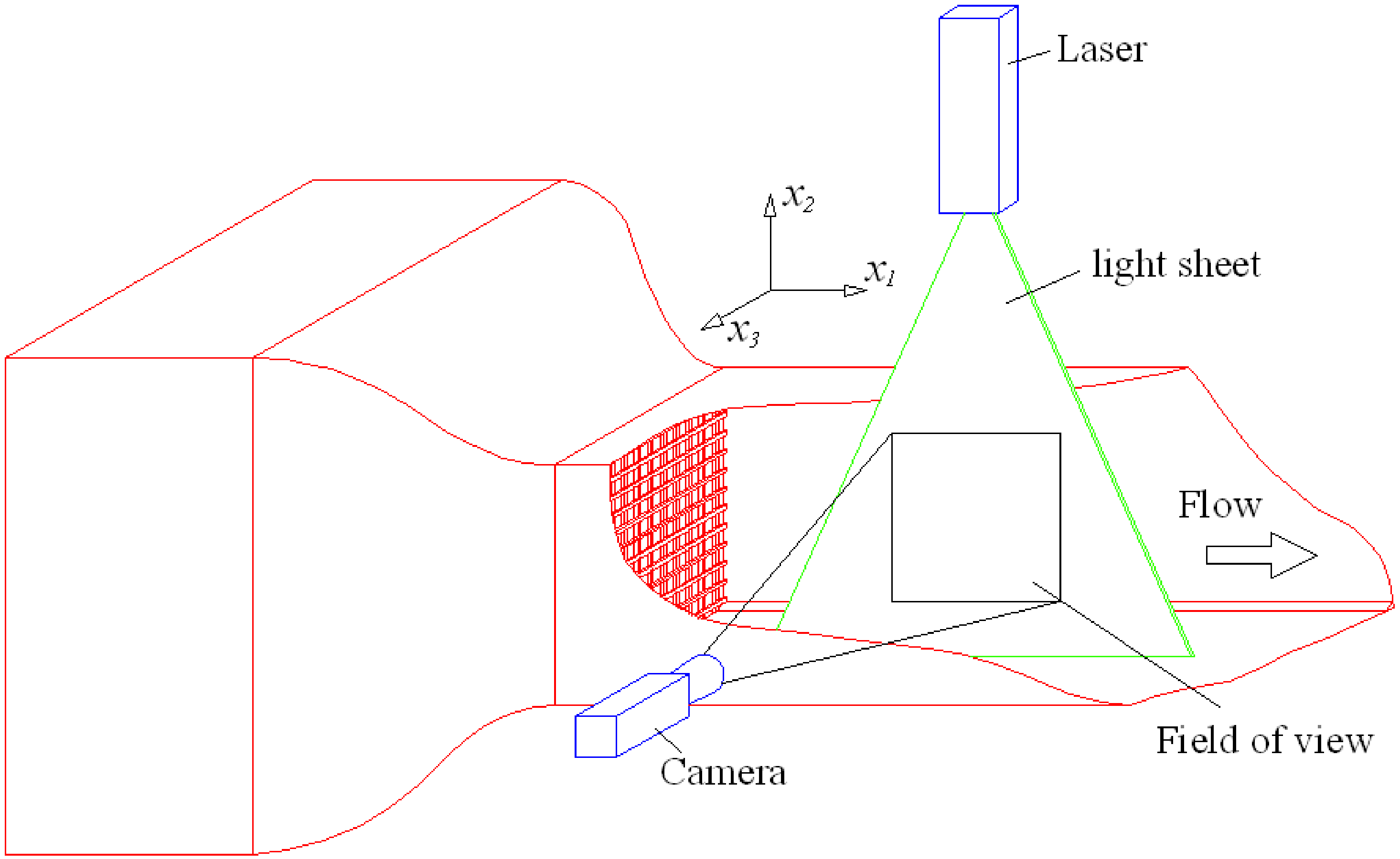


$u_{2,2}u_{3,3} \Leftrightarrow u_{2,3}u_{3,2}$
$\rho_{\varepsilon,\omega^2}$ 0.60
6
4
2
0
-2
-2
0
2
4
6
$(\omega^2 - \langle\omega^2\rangle) / \sigma_\omega$
$(\varepsilon - \langle\varepsilon\rangle) / \sigma_\varepsilon$

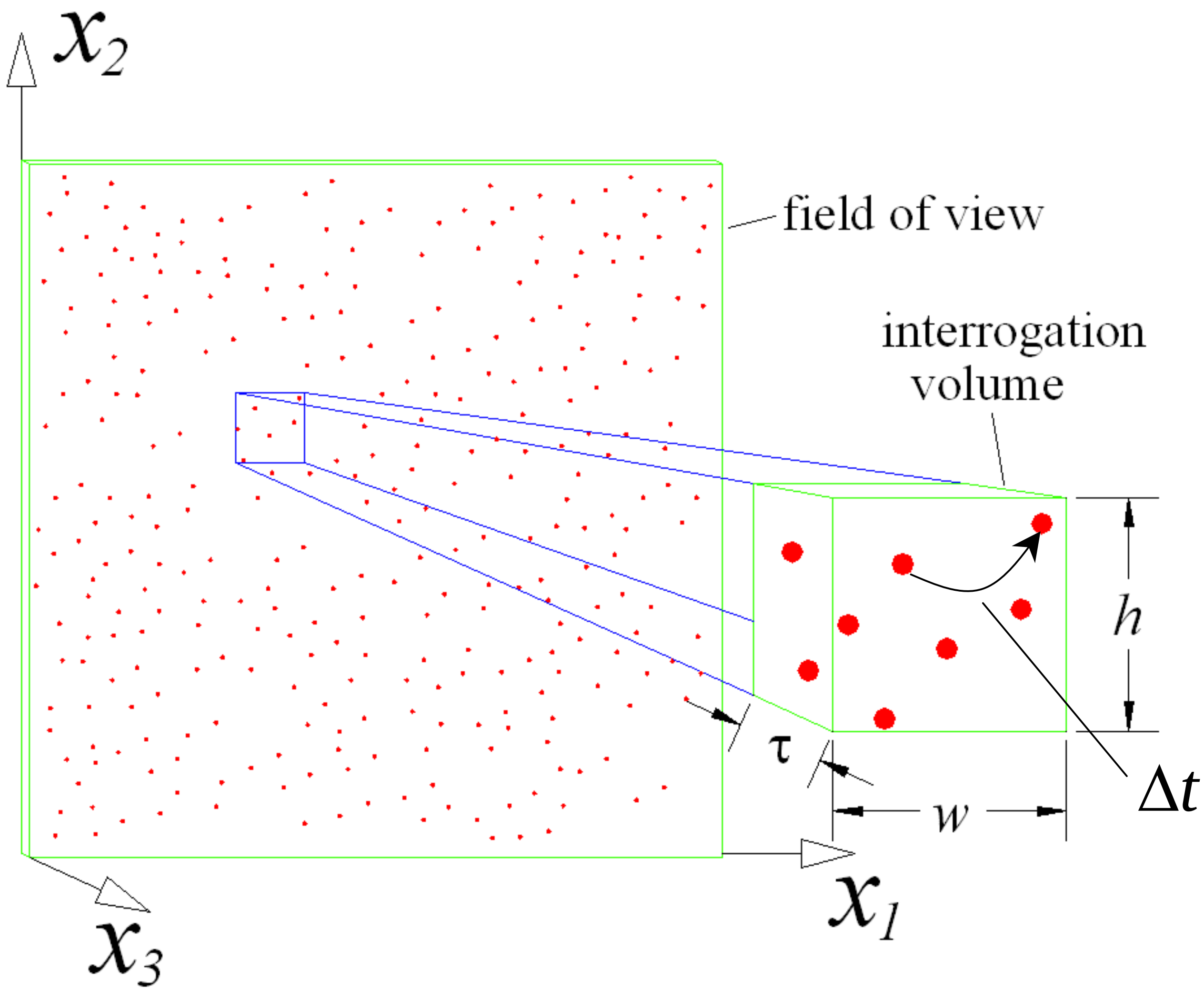


ε_{iso} □ $u_{1,1}^2$
$\rho_{\omega^2,\varepsilon_{iso}}$ □ 0.12
6
4
2
0
-2
-2
0
2
4
6
$(\omega^2 - \langle\omega^2\rangle) / \sigma_{\omega^2}$
$(\varepsilon_{iso} - \langle\varepsilon\rangle_{iso}) / \sigma_{\varepsilon_{iso}}$

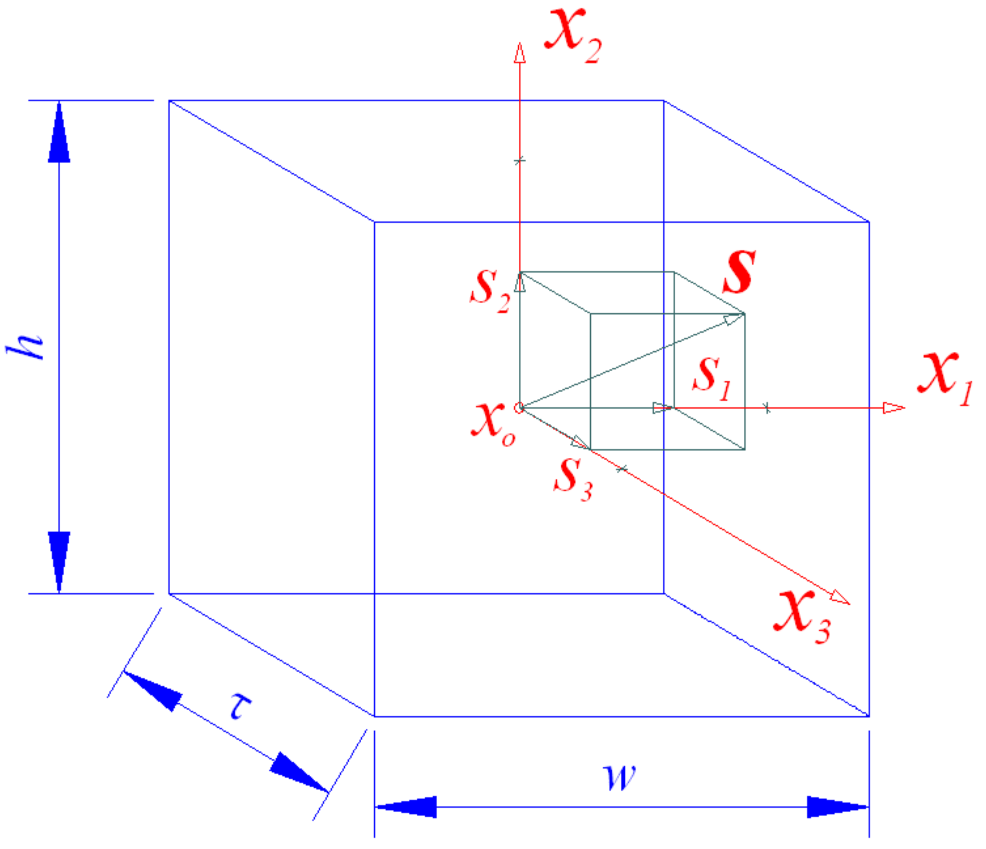
PIV: validation in grid turbulence



Interrogation Volume



Interrogation Volume



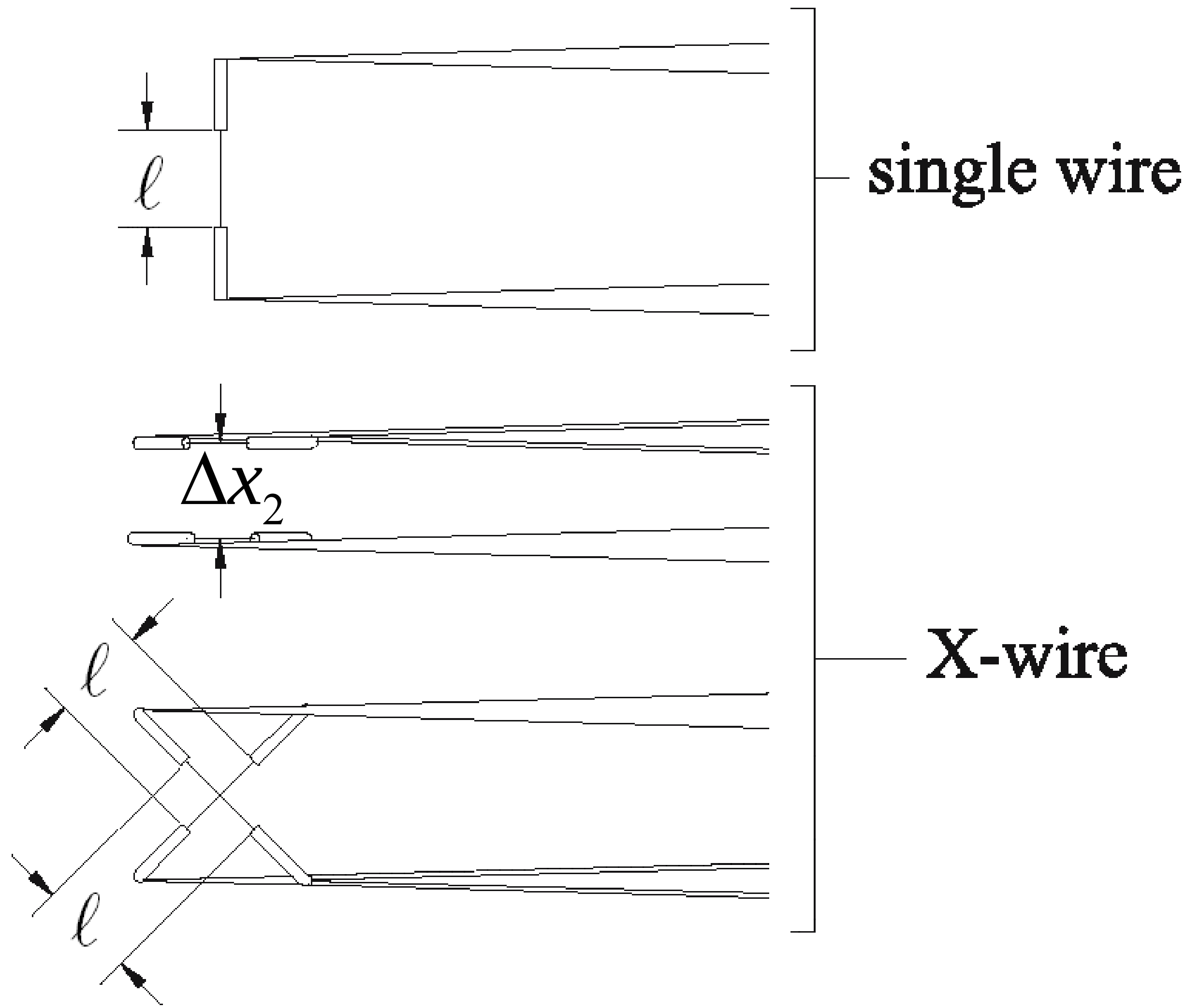
$$u_i^M(\boldsymbol{x}_o, t_o) = \frac{1}{V\Delta t}\int_{-\Delta t/2}^{\Delta t/2} \iiint_V u_i(\boldsymbol{x}, t)\, \mathrm{d}\boldsymbol{s}\, \mathrm{d}t$$

Filter Transfer Function

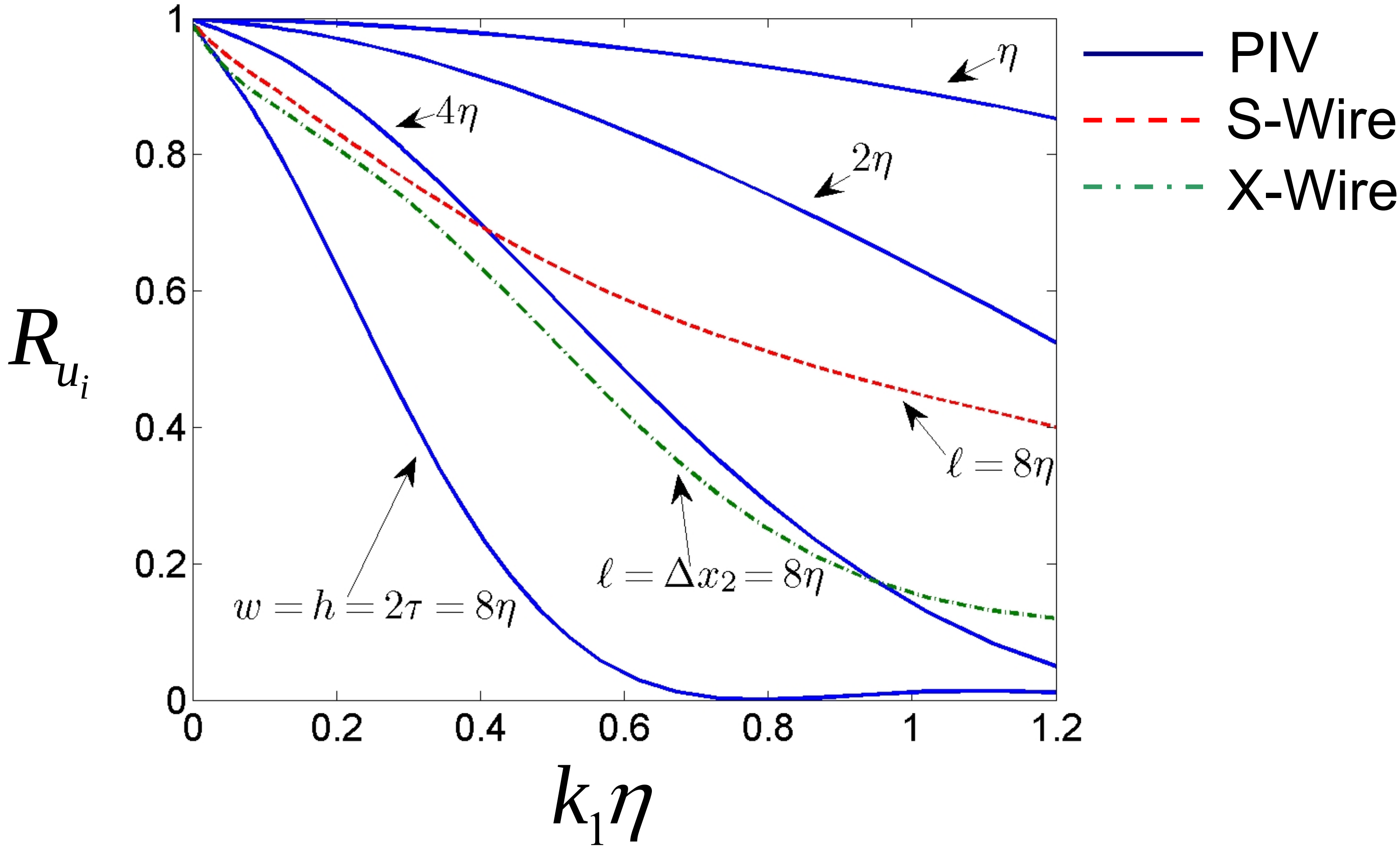
$$A = \mathrm{sinc}\left(\frac{k_1 w}{2}\right)\mathrm{sinc}\left(\frac{k_2 h}{2}\right)\mathrm{sinc}\left(\frac{k_3 \tau}{2}\right)\mathrm{sinc}\left(\frac{k_1 w}{2}\right)$$

$$\text{where}\quad \mathrm{sinc}(x) = \sin(x)/x$$

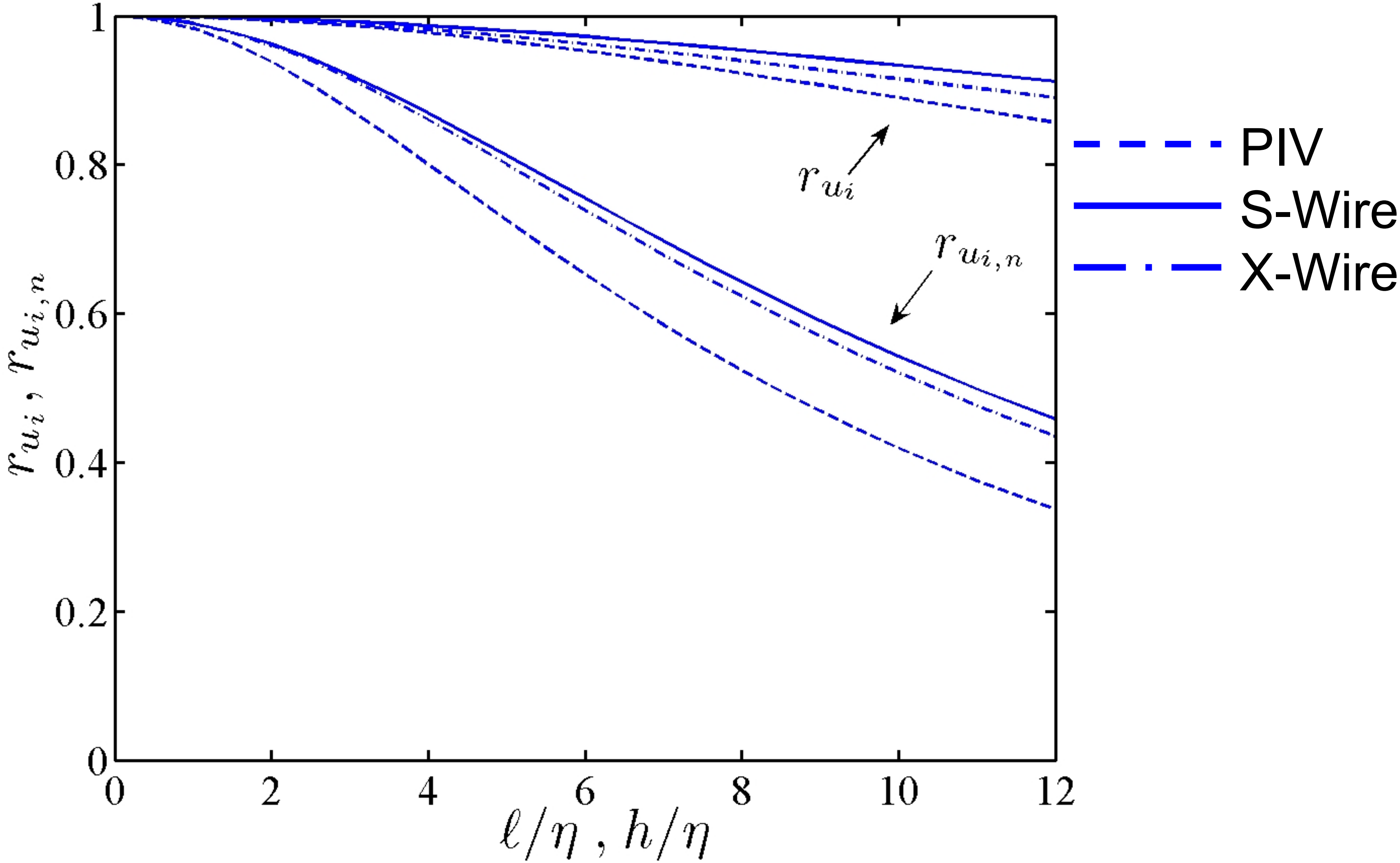
For Comparison…



Spectral Correction Ratio



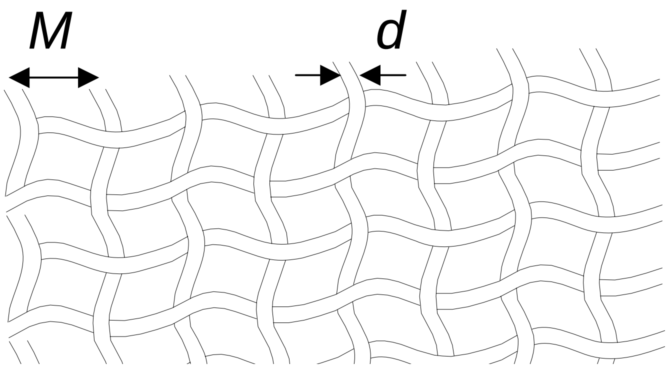
Correction coefficients



Lavoie Avallone De Gregorio Romano Antonia (2007)

Validation: Grid Turbulence

	M (mm)	d (mm)	σ	C_D^*	R_M
PIV	4.5	1.2	0.45	5.12	3000
HWA	5.1	1.2	0.42	4.70	3000



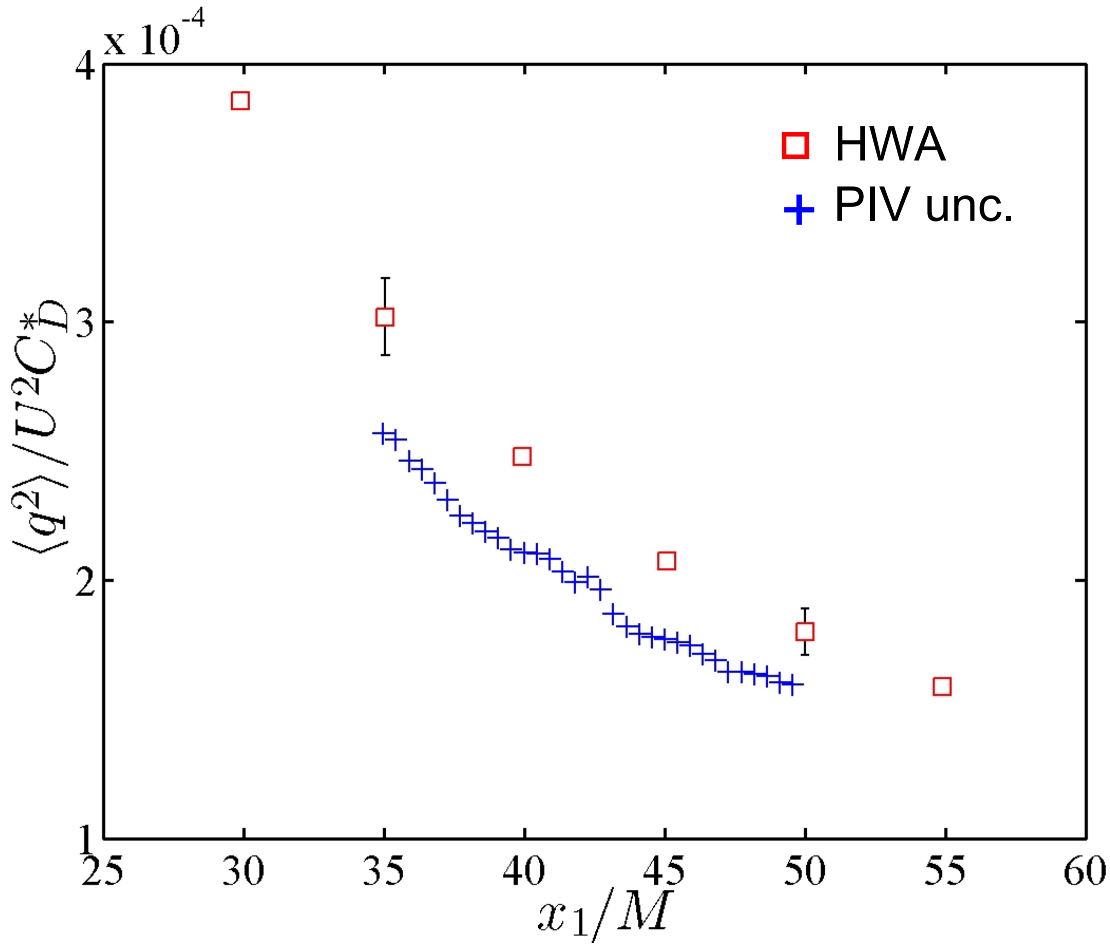
$$\sigma \equiv \frac{d}{M}\left(2-\frac{d}{M}\right)$$

$$C_D^* \equiv \frac{1-\left(1-\sigma^2\right)}{\sigma\left(1-\sigma^2\right)}$$

Resolution

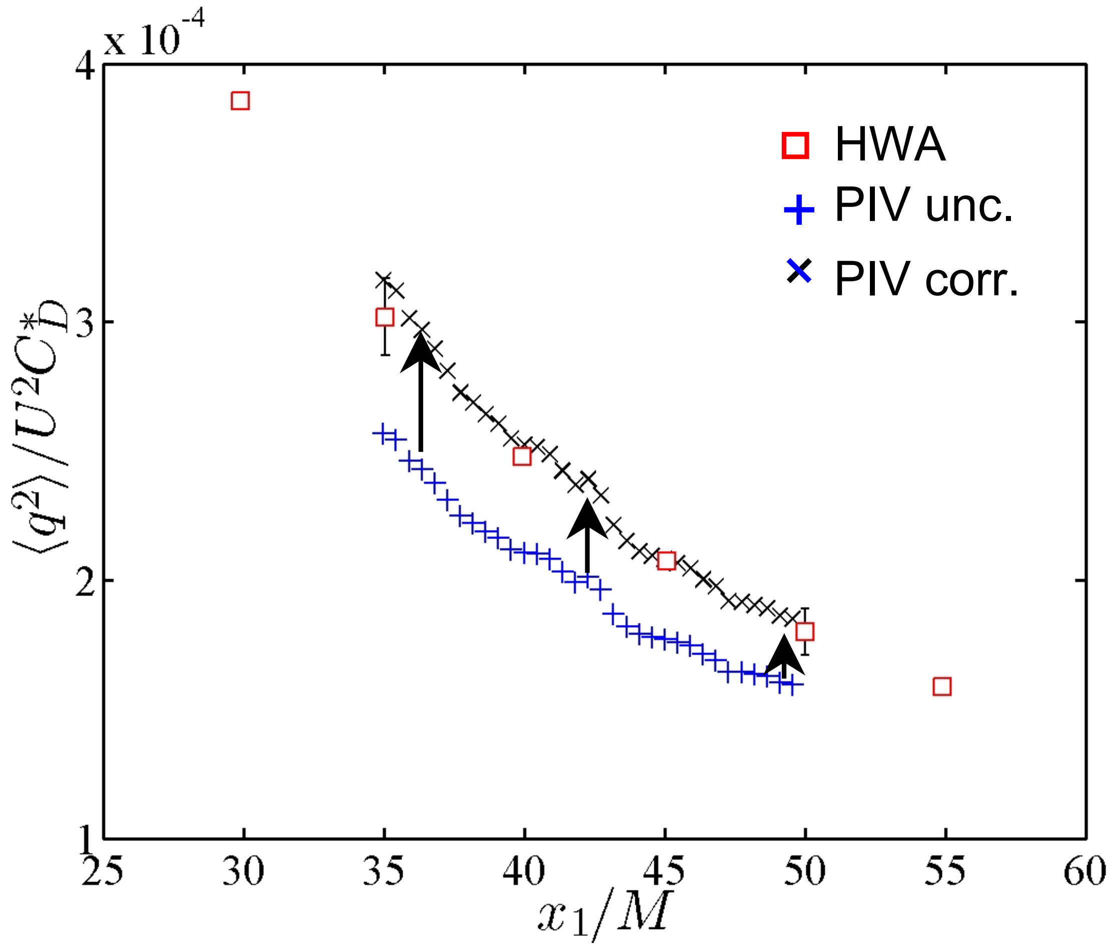
	η (mm)	Resolution
PIV	0.15-0.19	$11 \le (h/\eta = w/\eta) \le 14$ $5 \le \tau/\eta \le 7$ $5 \le U\Delta t/\eta \le 6$
HWA	0.16-0.21	$2.4 \le \ell/\eta \le 3$ $4.3 \le \Delta x_3/\eta \le 5.6$

Results



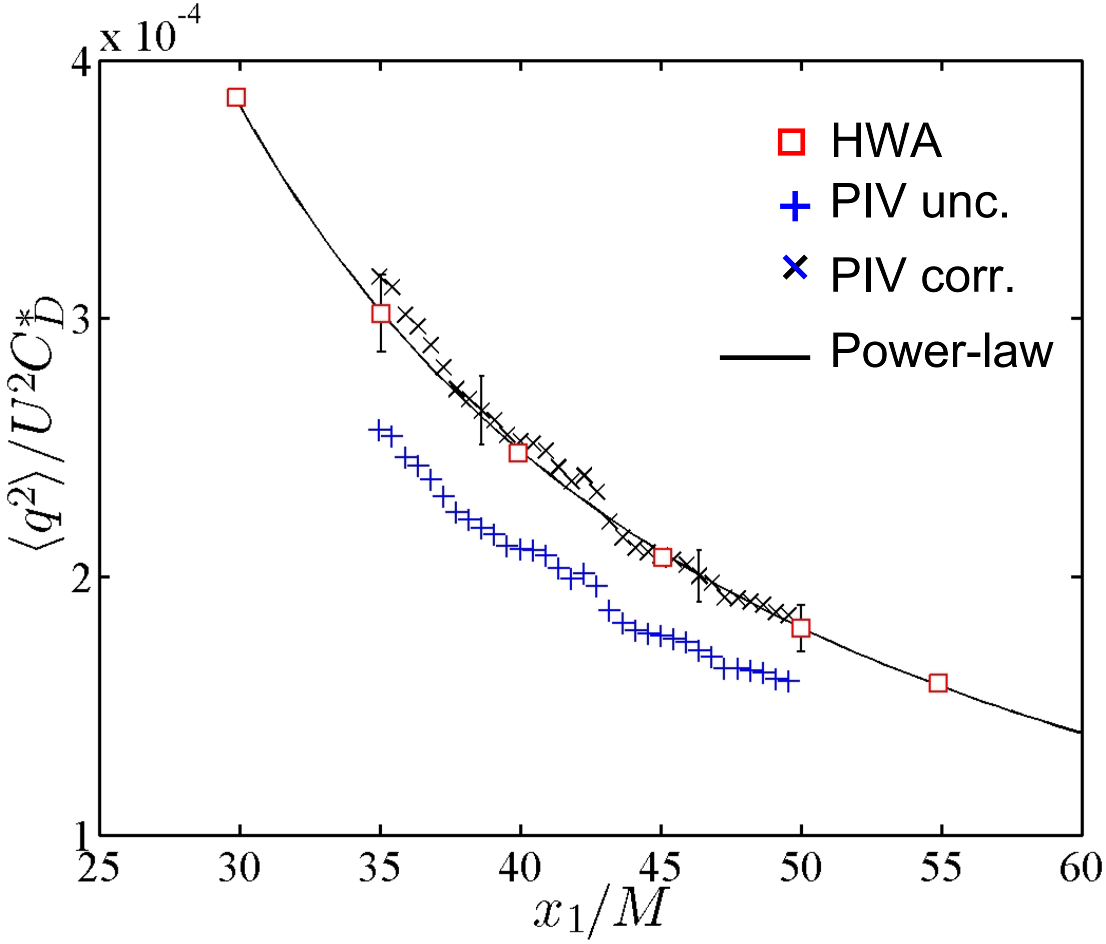
x 10^-4
4
3
2
1
25
30
35
40
45
50
55
60
x_1/M
$\langle q^2\rangle/U^2C_D^*$
HWA
PIV unc.

Results



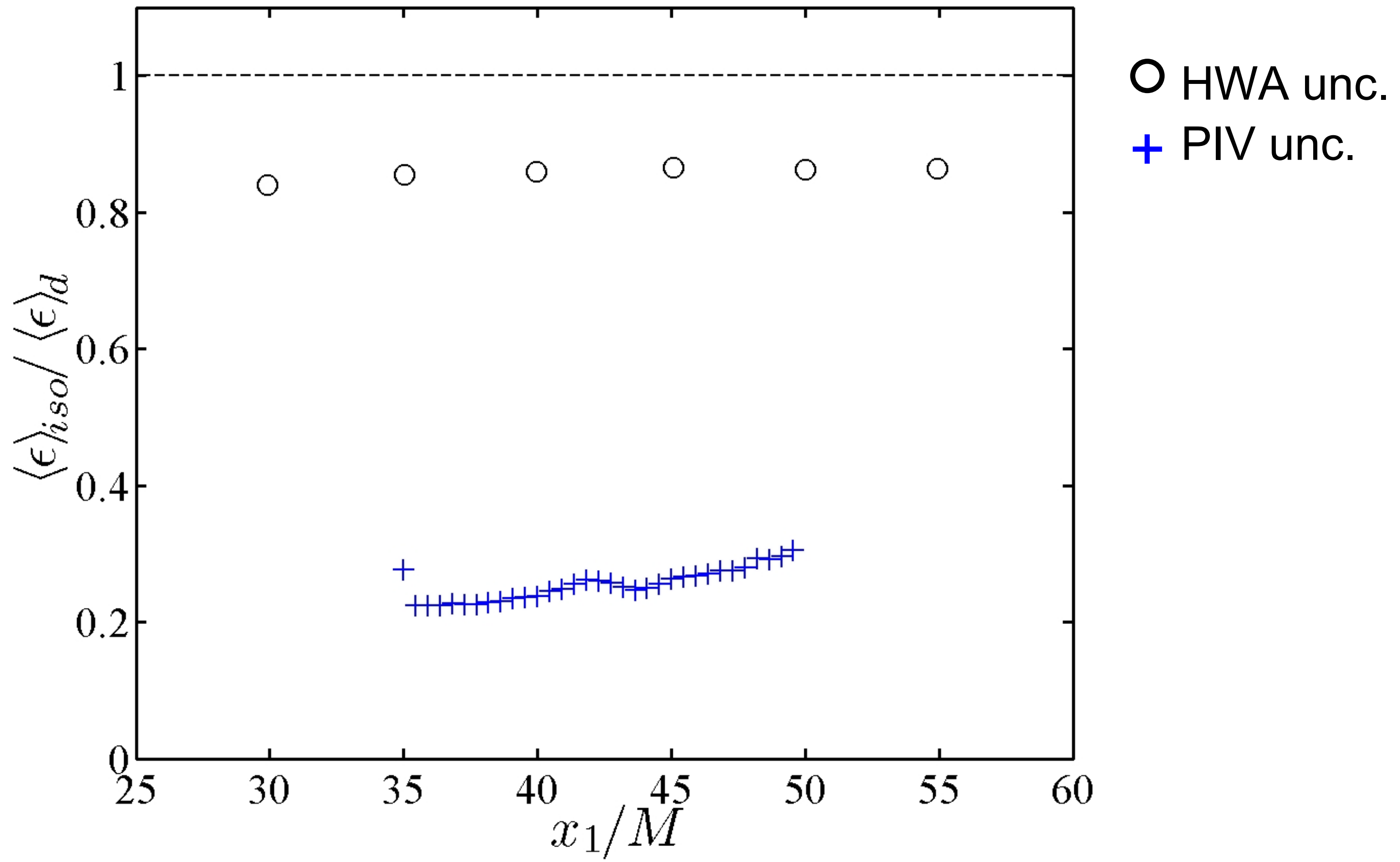
x 10^-4
4
3
2
1
25
30
35
40
45
50
55
60
$\langle q^2 \rangle / U^2 C_D^*$
x_1/M
HWA
PIV unc.
PIV corr.

Results

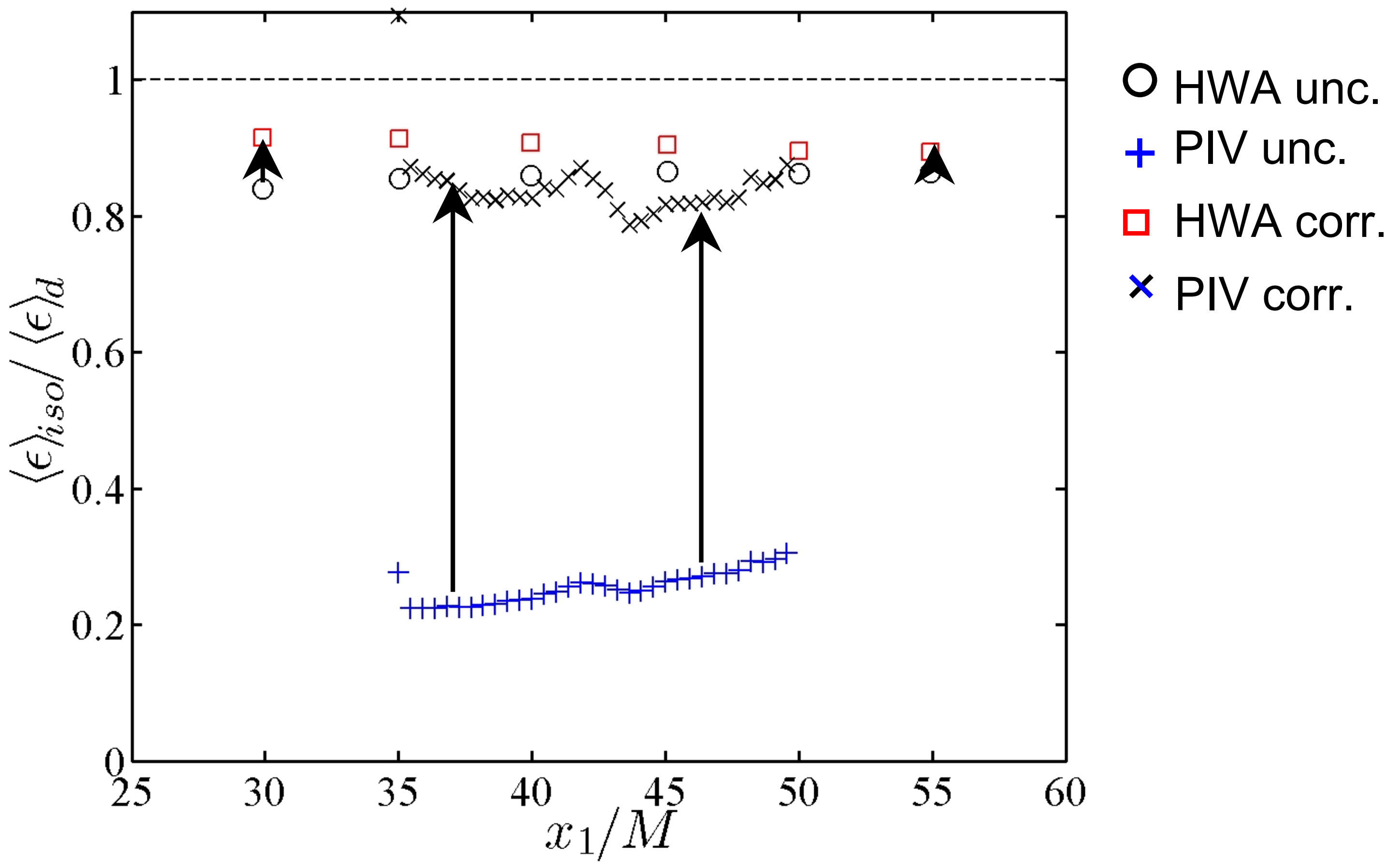


x 10^-4
4
3
2
1
25
30
35
40
45
50
55
60
x_1/M
$\langle q^2 \rangle / U^2 C_D^*$
HWA
PIV unc.
PIV corr.
Power-law

Results

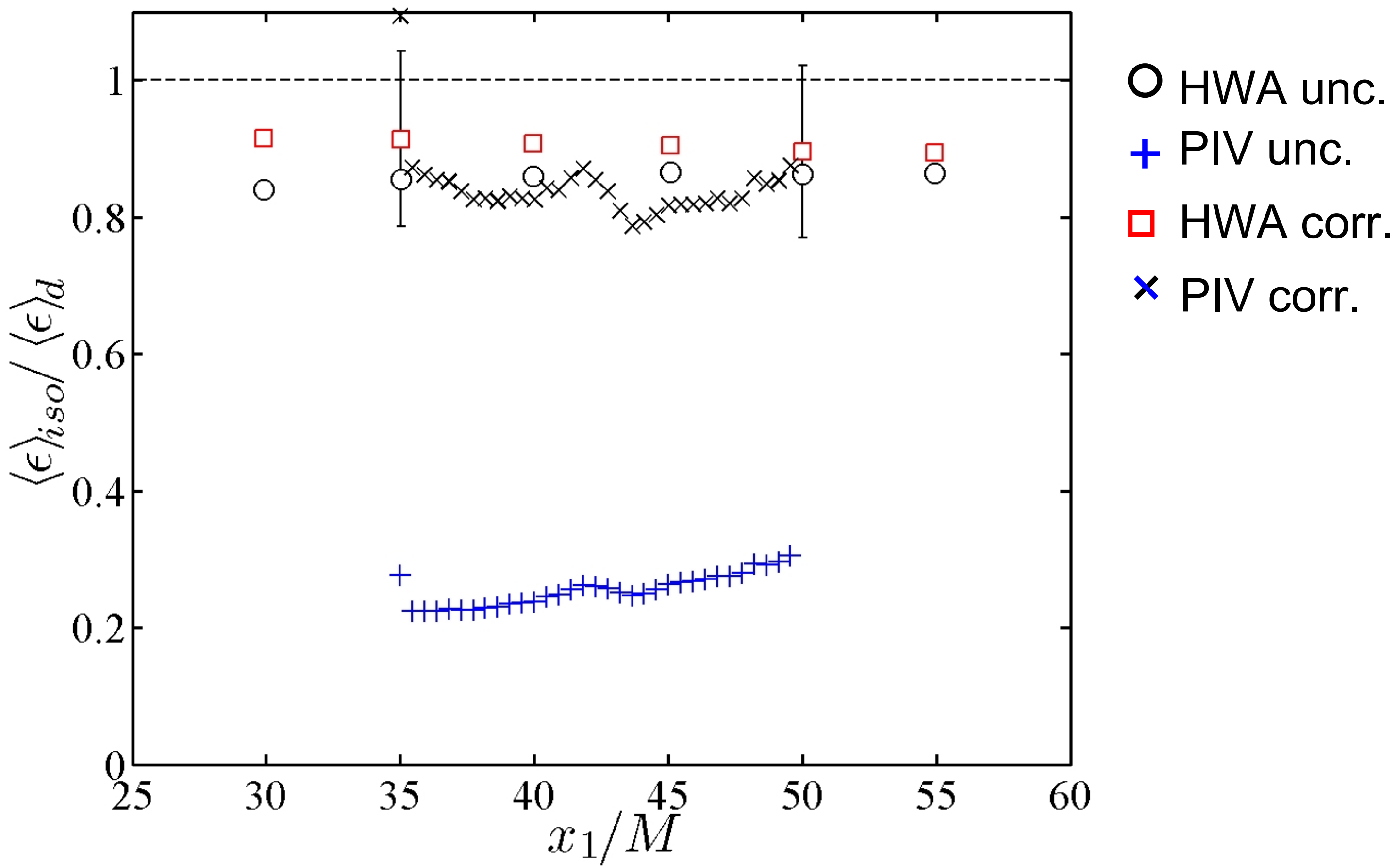

1
0.8
0.6
0.4
0.2
0
25
30
35
40
45
50
55
60
x_1/M
$\langle\epsilon\rangle_{iso}/\langle\epsilon\rangle_d$
HWA unc.
PIV unc.

Results



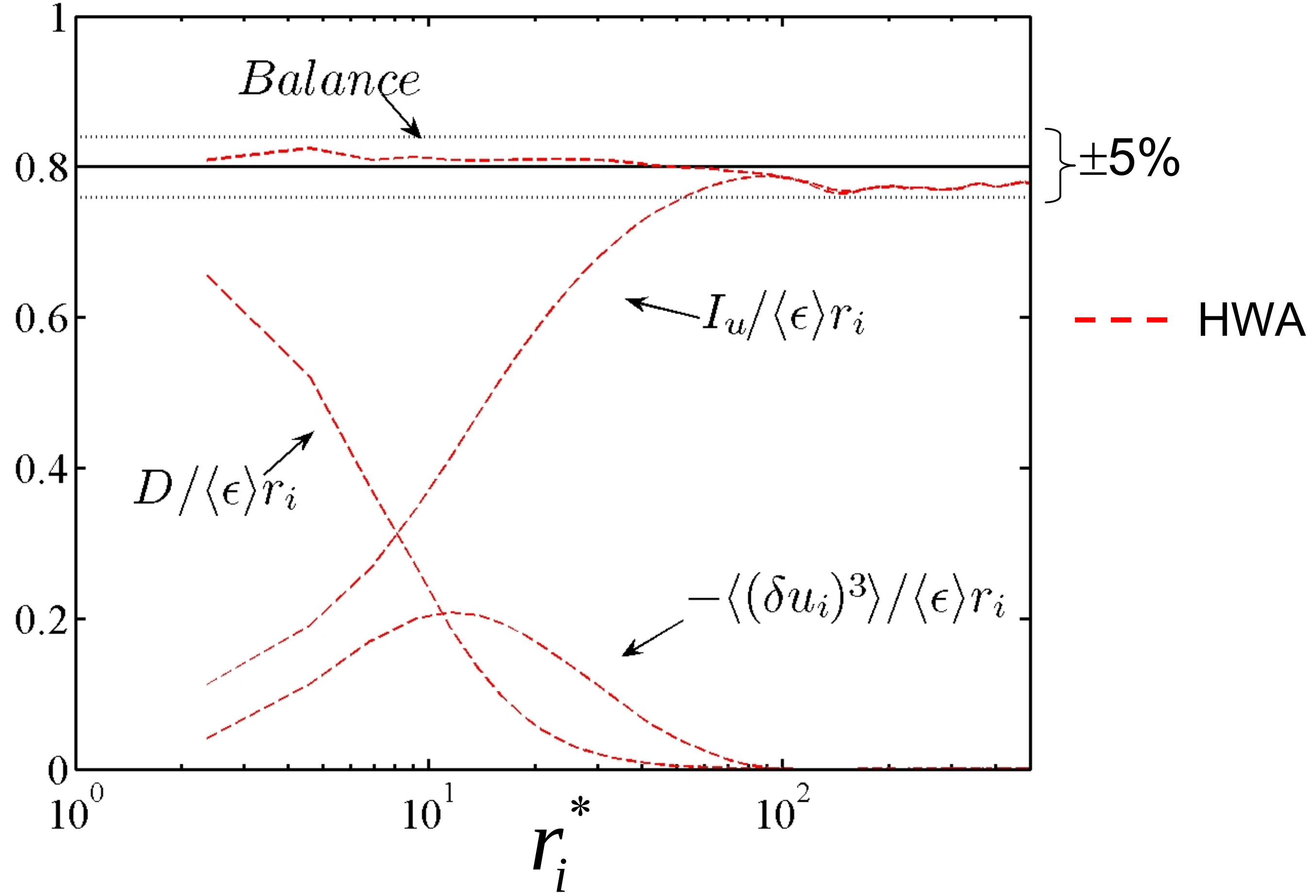
1
0.8
0.6
0.4
0.2
0
25
30
35
40
45
50
55
60
x_1/M
$\langle\epsilon\rangle_{iso}/\langle\epsilon\rangle_d$
HWA unc.
PIV unc.
HWA corr.
PIV corr.

Results



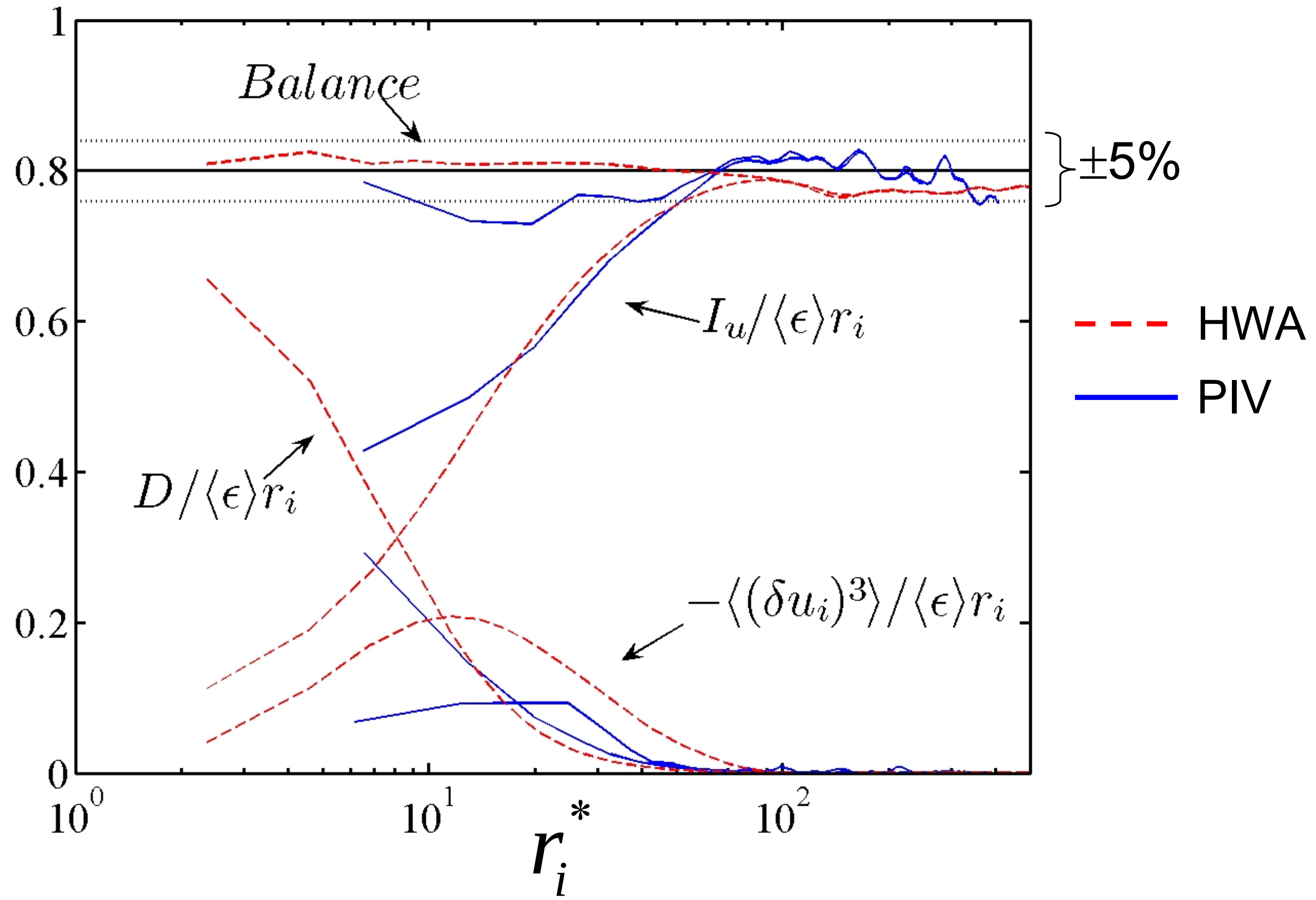
1
0.8
0.6
0.4
0.2
0
25
30
35
40
45
50
55
60
x_1/M
$\langle\epsilon\rangle_{iso}/\langle\epsilon\rangle_d$
HWA unc.
PIV unc.
HWA corr.
PIV corr.

Results



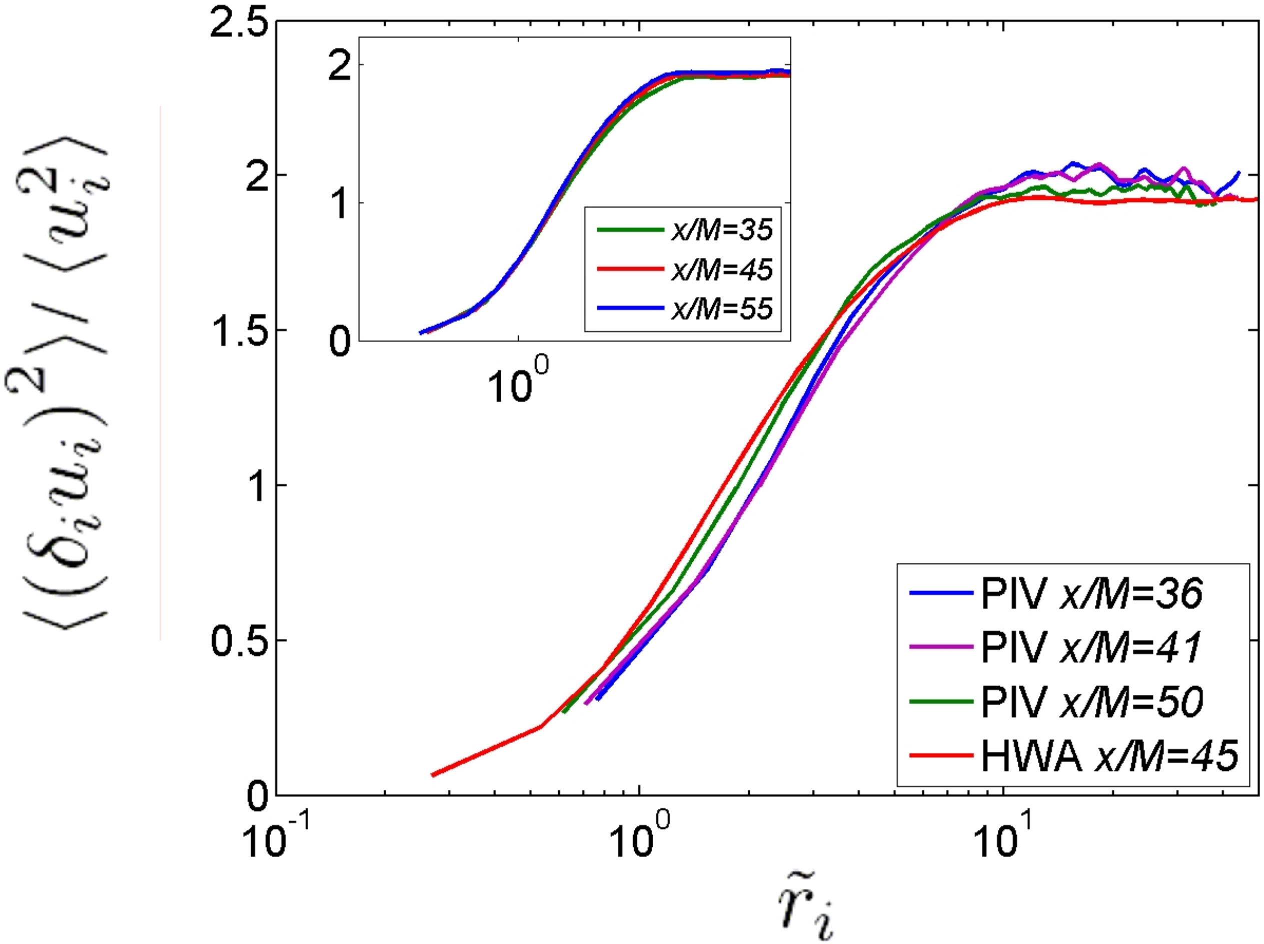
Balance
±5%
$I_u/\langle\epsilon\rangle r_i$
HWA
$D/\langle\epsilon\rangle r_i$
$-\langle(\delta u_i)^3\rangle/\langle\epsilon\rangle r_i$
1
0.8
0.6
0.4
0.2
0
10^0
10^1
10^2
r_i^*

Results

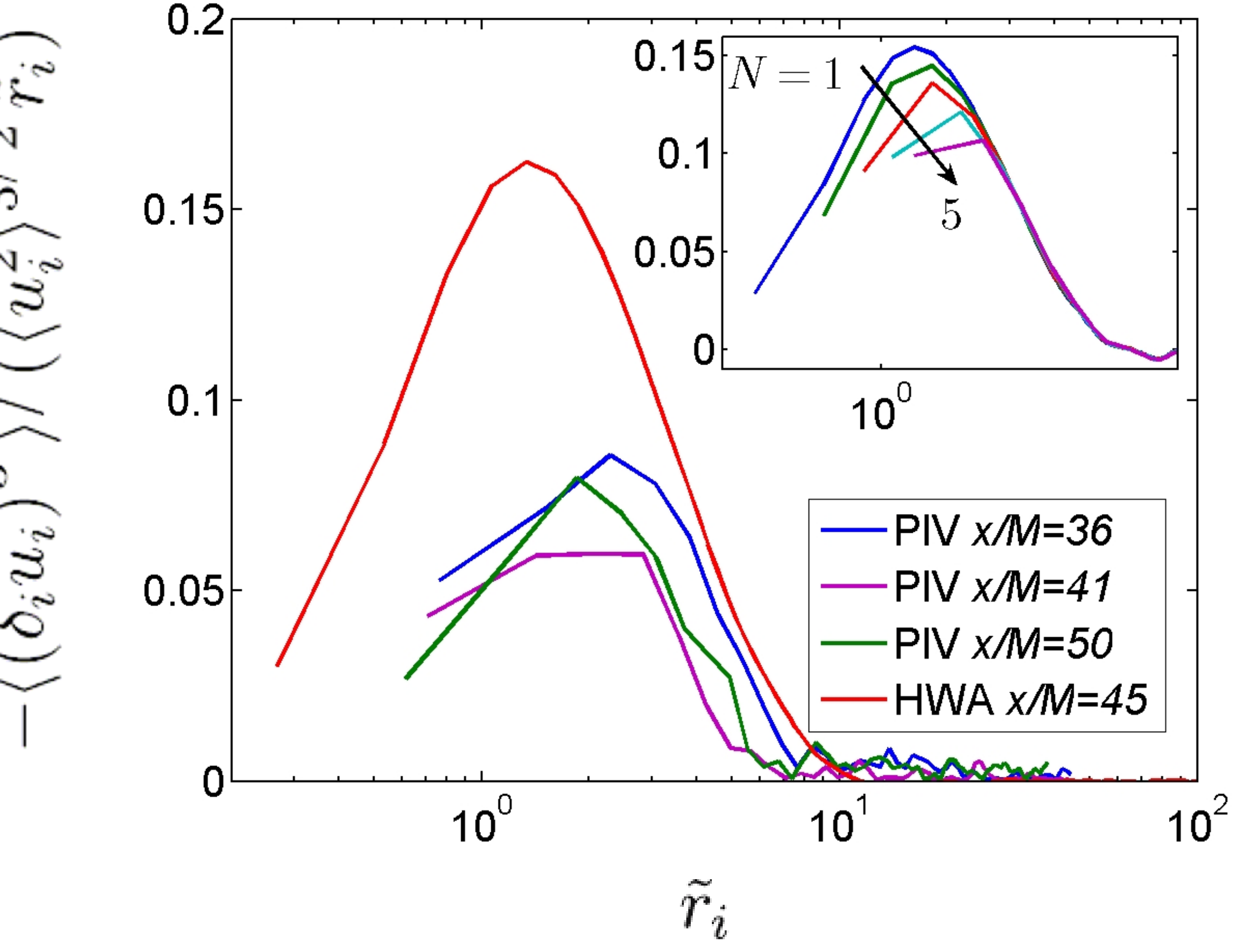


1
0.8
0.6
0.4
0.2
0
10^0
10^1
10^2
r_i^*
$Balance$
±5%
$I_u/\langle\epsilon\rangle r_i$
$D/\langle\epsilon\rangle r_i$
$-\langle(\delta u_i)^3\rangle/\langle\epsilon\rangle r_i$
HWA
PIV

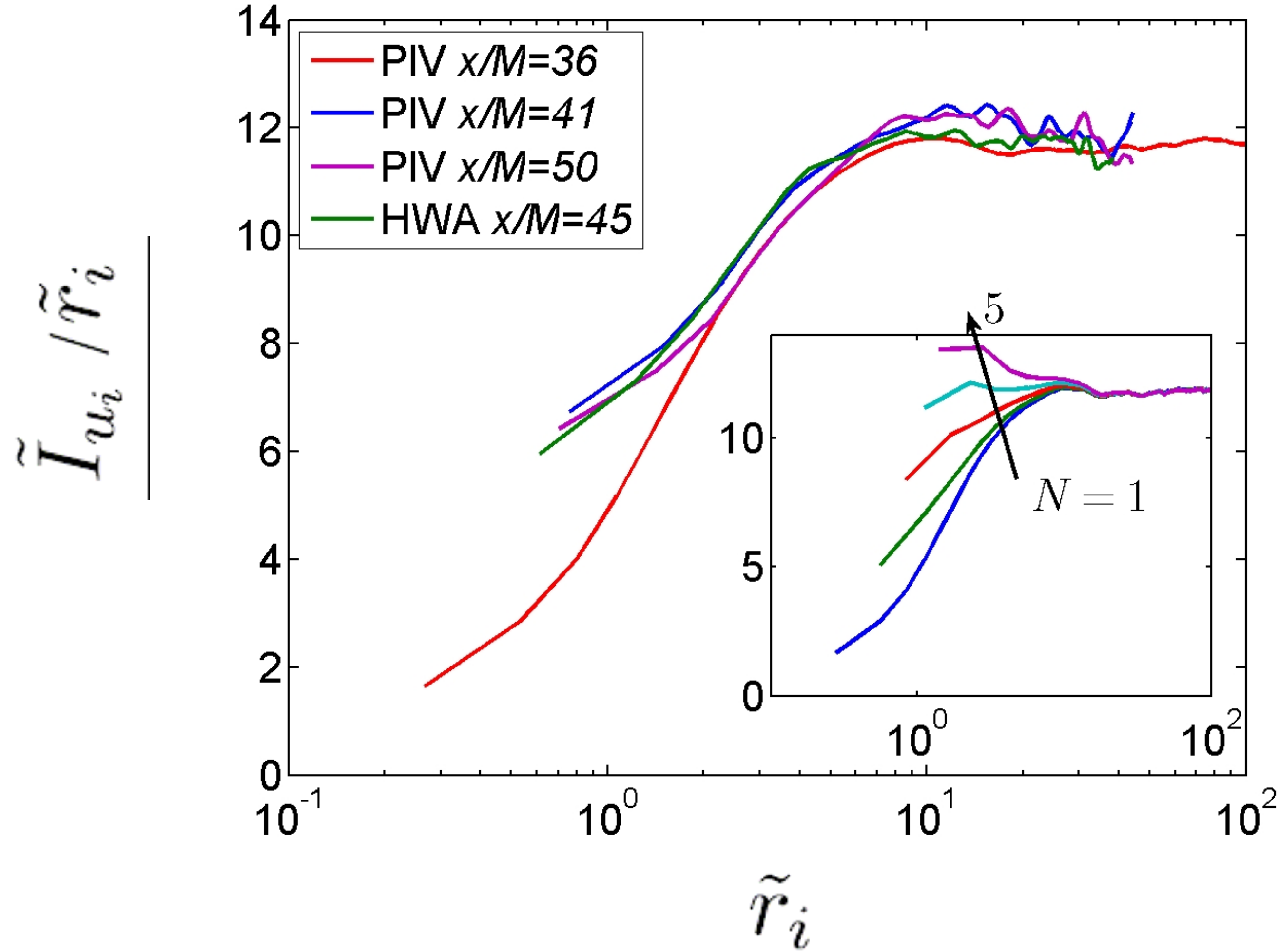
Second-order structure functions



Third-order structure functions



Non-stationary term $\tilde{I}_{u_i}\Big|$



Summary

- it is essential that resolution errors are quantified correctly

- the assumptions on which the spectral correction approach is based seem adequate when local isotropy applies. The most compelling support for the approach has come from DNS data bases.

- the approach is generic in nature –it is applicable to flow imaging techniques as well as HWA

- major limitation of the approach is that it addresses only the spectrum

- relaxation of local isotropy assumption ?...to local axisymmetry ?