In Memoriam Mariana Huerta

Mariana was a nice young Chilean student. She was about to defend her Phd on « Statistical properties of turbulent Fourier modes in turbulence » at the LEGI. Mariana was 27 years old. After bravely fighting against very bad weather conditions for several hours, she died in the night of July 23rd, together with 3 other young students : Jane, Morgane and Mark. They were stopped in the "arête de Bionnassay" at an altitude of 4100m, very close to the Mont Blanc summit they were dreaming to climb up before returning back to their respective countries.

Lagrangian investigations of turbulence :
experimental techniques and statistical results

• P. Gervais (PhD : acoustic tracking & Lagrangian statistics)
• N. Qureshi (PhD : finite size & density effects)
• M. Huerta (PhD : statistics of turbulent Fourier modes)
• M. Bourgoin (Optical PTV & finite size effects, acceleration)
• Y. Gagne
• Christophe Baudet

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE
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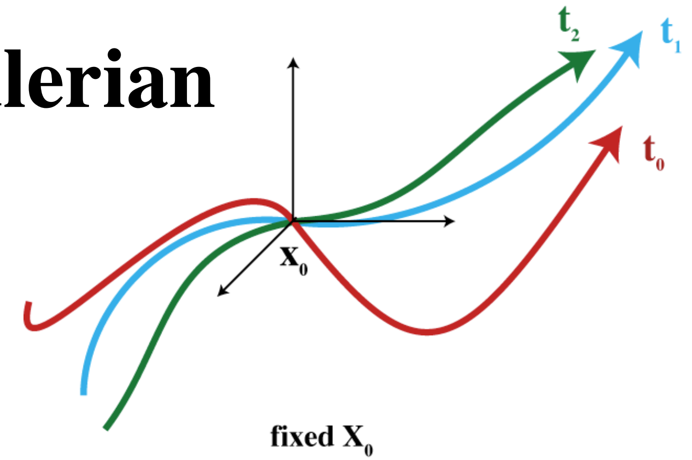
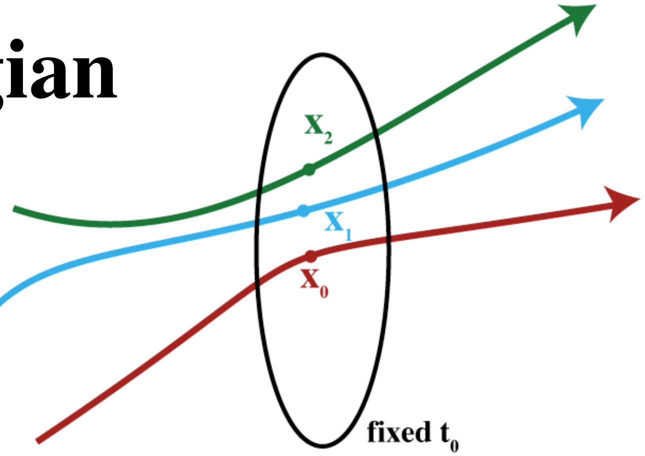
Eulerian & Lagrangian Flow Descriptions

Eulerian



Lagrangian



"Fluid Particles" are indexed by their position at some initial time t_0 : $\vec{x}_i = \vec{X}(\vec{x}_i, t_0)$

- Flow : set of Streamlines

- PIV
- Eulerian field : $\vec{u}\left(\vec{X}, t\right)$

- Flow : set of Pathlines (fluid particle trajectories)
- PTV
- Lagrangian fields :

$$\vec{X}(\vec{x},t) \qquad \vec{V}(\vec{x},t) = \left[\frac{\partial \vec{X}(\vec{x},t)}{\partial t}\right]_t$$

- S.B. Pope, *Turbulent Flows* Cambridge Univ. Press (2000)
- A.S. Monin & A.M. Yaglom, *Statistical Fluid Mechanics*, Vol 1 M.I.T Press (1971)

Why a Lagrangian description of Turbulence ?

- Turbulent Dispersion (pollutants), particle segregation and aggregation (cloud formation)
- Deformation of material lines, surfaces and volumes :
  - ✓ eg : combustion fronts
- Turbulence stochastic modelization may be simpler in the Lagrangian framework (Langevin equation)
- Lagrangian intermittency vs Eulerian intermittency ?

- But :
  - ✓ similar statistical closure problems as Eulerian
  - ✓ a challenging problem for experimentalists (tracers, spatial resolution) and numericians (finite sized particles)

From Lagrangian to Eulerian description (1)

- Lagrangian ⇨Eulerian : "simply" a matter of changing variables from $(\vec{x},t)$ ⇨ $(\vec{X},t)$
- Relation between Eulerian and Lagrangian Velocities :

$$\vec{V}(\vec{x},t) = \frac{\partial \vec{X}(\vec{x},t)}{\partial t} = \vec{u}\left[\vec{X}(\vec{x},t),t\right] \quad \text{with} \quad \vec{x} = \vec{X}(\vec{x},t_0)$$ ⇨DNS

- In the transformation from Lagrangian to Eulerian description a major role is played by the Jacobian of the mapping (cf. A. Pumir)

$$M_{t_0 \to t} : \vec{x} = \vec{X}(\vec{x},t_0) \to \vec{X}(\vec{x},t)$$

- Jacobian of : $M_{ij} = \left(\frac{\partial X_i}{\partial x_j}\right)$ ⇨ $|M| = Det\ M = \frac{\partial(X_1,X_2,X_3)}{\partial(x_1,x_2,x_3)} = |X_1,X_2,X_3|$

- Lagrangian ⇨Eulerian : $\frac{\partial f}{\partial X_i} = \sum_\alpha \frac{\partial f}{\partial x_\alpha}\frac{\partial x_\alpha}{\partial X_i} \equiv \frac{1}{Det\ M}|X_j,X_k,f|$

From Lagrangian to Eulerian description (2)

- Example 1 : incompressibility condition

$$\vec{\nabla}\vec{u} = \frac{\partial u_\alpha}{\partial X_\alpha} = \frac{\partial}{\partial X_\alpha}\frac{\partial X_\alpha}{\partial t} = 0 \quad \Rightarrow \quad \frac{1}{\left|M_{t_0 \to t}\right|}\frac{\partial \left|M_{t_0 \to t}\right|}{\partial t} = 0$$

$$M_{t_0 \to t_0} = \mathbf{Id} \quad \Rightarrow \quad \left|M_{t_0 \to t}\right| = Det\, M = \frac{\partial(X_1, X_2, X_3)}{\partial(x_1, x_2, x_3)} = \left|X_1, X_2, X_3\right| = 1$$

- Actually $M_{t_0 \to t} : \vec{x} = \vec{X}(\vec{x}, t_0) \to \vec{X}(\vec{x}, t)$ not so simple
  - ✓ non-Galilean
  - ✓ stochastic mapping (turbulent flows)

- Example 2 : Navier-Stokes

Euler : $\frac{\partial u_i}{\partial t} + \left(\vec{u}.\vec{\nabla}\right)u_i = -\frac{\partial p}{\partial X_i} + \nu \nabla_X^2 u_i$

⇨ Lagrange : $\frac{\partial^2 X_i}{\partial t^2} = -\frac{1}{\rho}\left|X_j, X_k, p\right| + \nu \left\{ \begin{array}{l} \left\| X_2, X_{3,} \left| X_2, X_3, \frac{\partial X_i}{\partial t} \right\| \right. + \left| X_3, X_1, \left| X_3, X_1, \frac{\partial X_i}{\partial t} \right\| \right. + \\ \left\| X_1, X_{2,} \left| X_1, X_2, \frac{\partial X_i}{\partial t} \right\| \right. \end{array} \right\}$

Eulerian vs Lagrangian One Point Statistics

- In *Homogeneous & Incompressible* turbulent flows : Lagrangian and Eulerian velocity PDF are identical
- Let $\varphi\left(\vec{V}\right)$ be some function of the velocity field
- Use incompressibility condition $\dfrac{\partial(X_1,X_2,X_3)}{\partial(x_1,x_2,x_3)} = \left|X_1,X_2,X_3\right| = 1$

V_{t_0} volume occupied by fluid at time t_0 and $V_t = M_{t_0 \to t}\left(V_{t_0}\right)$

$$\int_{V_{t0}} \varphi\left(V\left(x,t\right)\right) d^3x = \int_{V_t} \varphi\left(u\left(X,t\right)\right) d^3X$$

- Homogeneity (unbounded flow), and $V_{t_0}, V_t \to \infty$ take average

⇨ $V_{t_0} \approx V_t$ ⇨ $\overline{\varphi\left(V\left(x,t\right)\right)} = \overline{\varphi\left(u\left(X,t\right)\right)}$

⇨In particular if $\varphi(V) = e^{iKV}$ then : $PDF(V) = PDF(u)$

- J.L. Lumley (1962)
- see : A.S. Monin & A.M. Yaglom, *Statistical Fluid Mechanics*, Vol 1

Lagrangian 2-points Statistics : velocity correlation

- Velocity Correlation is important :
  - ✓ for one particle dispersion in stationnary & isotropic turbulence :

$$\sigma_X(t) = \left\langle X_i(t,0)^2 \right\rangle = v'^2 \int_0^t \int_0^t R_{ii}^L(t' - t'')dt'dt''$$

  - ✓ statistical models of Lagrangian velocity dynamic (Langevin equation).
- No exact relation between Eulerian et Lagrangian Velocity Correlations : an old and still open problem (Richardson, Taylor, Kraichnan, Pullin).
- No experimental data of both (simultaneous) Eulerian and Lagrangian correlations (i.e. in the same flow).
- DNS (homogeneous and isotropic turbulence) indicate a slower decay of the Eulerian correlation vs the Lagrangian one.

Lagrangian vs Eulerian Scalings

- Eulerian structure functions

$$S_p^E(r) = \left\langle \delta u(r)^p \right\rangle$$

- Statistics of *spatial* velocity increments

$$\left\langle \delta u(r)^p \right\rangle = \left\langle \left(u(X_o + r, t) - u(X_o, t) \right)^p \right\rangle_t$$

$$\simeq \left\langle \left(u(X_o, t) - u(X_o, t - \frac{r}{\bar{u}}) \right)^p \right\rangle_t$$

- *Random Sweeping* : Taylor hypothesis is mandatory
- K41 scaling : $\bar{\varepsilon} \approx \frac{\delta u_r^3}{r} \approx Cte$

$$\delta u_r \triangleq \bar{\varepsilon}^{1/3} r^{1/3}$$

- Kolmogorov 4/5 law : $\delta u_r^3 = -\frac{4}{5} \bar{\varepsilon} r + \ldots$

$$E^E(k) \propto \delta u_k^2 \propto \bar{\varepsilon}^{2/3} k^{-2/3-1} = \bar{\varepsilon}^{2/3} k^{-5/3}$$

$$\eta_K = \left(\frac{\nu^3}{\bar{\varepsilon}} \right)^{1/4} \Rightarrow \Delta k_{inert}^E \propto \mathrm{Re}^{3/4}$$

- Lagrangian structure functions

$$S_p^L(\tau) = \left\langle \delta V(\tau)^p \right\rangle$$

- Statistics of *temporal* velocity increments

$$\left\langle \delta V(\tau)^p \right\rangle = \left\langle \left(V(x_o, t + \tau) - V(x_o, t) \right)^p \right\rangle_{x_0}$$

$$\simeq \left\langle \left(V(x_o, t + \tau) - V(x_o, t) \right)^p \right\rangle_t$$

- *No Random Sweeping* (by construction !)

  cf. NS Equation in Lagrangian form

- K41 scaling : $\bar{\varepsilon} \approx \frac{\delta V_\tau^2}{\tau} \approx Cte$

$$\delta V_\tau \triangleq \bar{\varepsilon}^{1/2} \tau^{1/2}$$

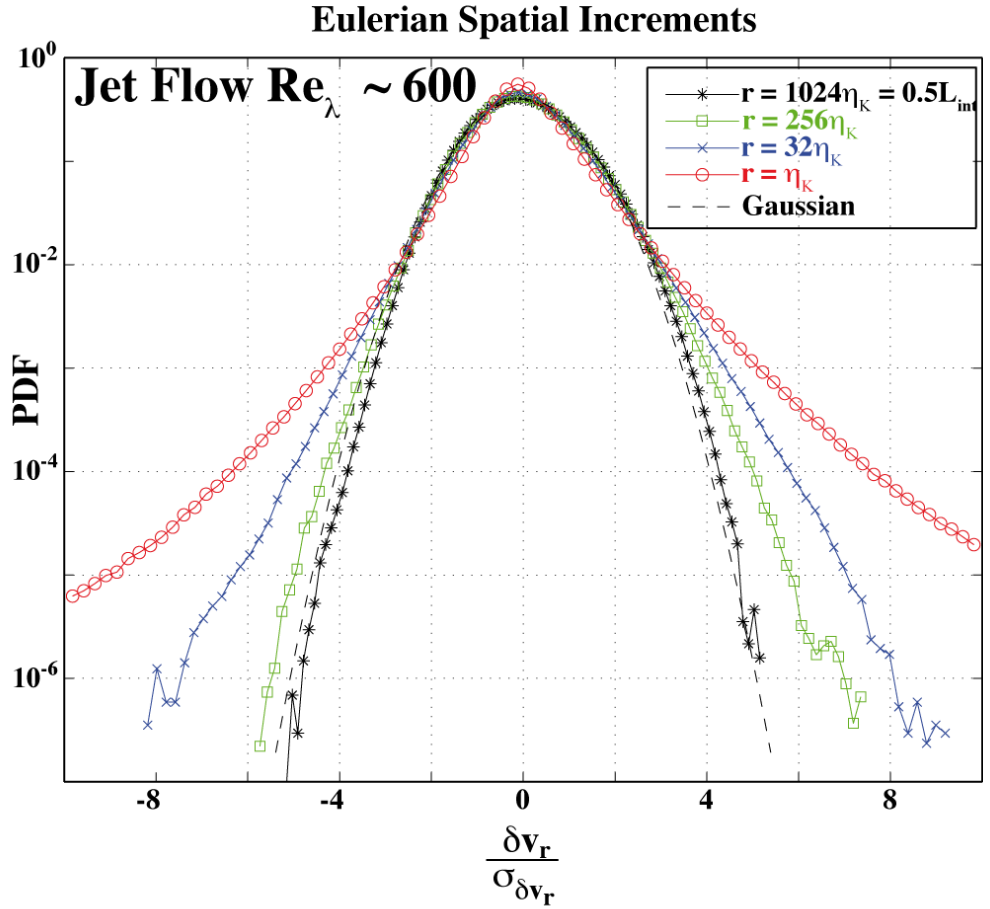
$$E^L(\omega) \propto \delta V_\omega^2 \propto \bar{\varepsilon} \omega^{-1-1} = \bar{\varepsilon} \omega^{-2}$$

$$E_{Acc}^L(\omega) \propto \omega^2 E_V^L = \bar{\varepsilon} \omega^0 \Rightarrow \text{ White Noise}$$

$$\tau_K = \left(\frac{\nu}{\bar{\varepsilon}} \right)^{1/2} \Rightarrow \Delta \omega_{inert}^L \propto \mathrm{Re}^{1/2}$$

Lagrangian vs Eulerian Intermittency

- Intermittency : evolution of the shape of the PDF of the velocity increments with respect to spatial increment.
- Origin : spatial fluctuations of the dissipation field.



- What about intermittency of Lagrangian velocity (time) increments ?
- Some DNS and experiments show an enhancement of intermittency in the Lagrangian case.

Lagrangian Tracers : finite size effects

In the limit of small relative velocity flow : $R_e = \frac{|V-u|a}{\nu} \simeq 1$

Basset (1886), Bousinesq (1903), Faxen (1922), Oseen (1927).

Maxey & Riley (Phys. Fluids. Vol. 26, No 4, 1983) :

- Equation of motion of a rigid sphere with radius a, mass m_p, located at Y(t) with velocity V(t) in a non uniform, non stationnary flow with velocity **u**(**x**,t) :

$$m_p \frac{dV_i}{dt} = \left(m_p - m_f\right) g_i + m_f \left.\frac{Du_i}{Dt}\right|_{Y(t)} - \frac{1}{2} m_f \frac{d}{dt}\left\{V_i(t) - u_i\left[Y(t),t\right]\right\} - 6\pi a_p \mu_f \left\{V_i(t) - u_i\left[Y(t),t\right]\right\}$$

$$-6\pi a_p^2 \mu_f \int_0^t \left(\frac{d/d\tau \left\{V_i(\tau) - u_i\left[Y(\tau),\tau\right]\right\}}{\left[\pi \nu_f \left(t-\tau\right)\right]^{1/2}} \right) d\tau \qquad + \text{... Faxen's terms}$$

- Response time : $\tau_{resp} \simeq \frac{m_p}{6\pi a \mu} = \frac{2}{9} \frac{\rho_p}{\rho_f} \frac{a^2}{\nu}$

- Stokes number : $S_t = \frac{\tau_{resp}}{\tau_\eta} = \frac{2}{9} \frac{m_p}{m_f} \frac{a^2}{\eta_K^2}$

Lagrangian Tracers : density effects

- Minimize particle sedimendation under gravity $\rho_p \approx \rho_f$

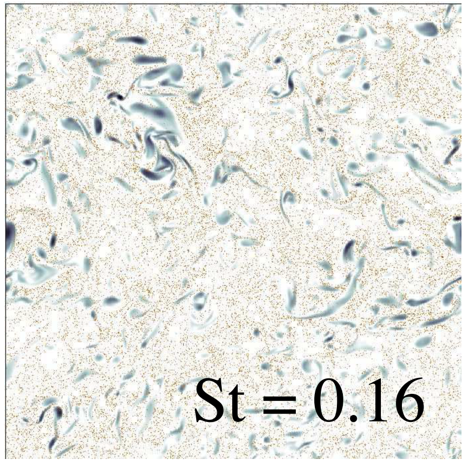
- Preferential concentration : $\rho_p \neq \rho_f$

✓ light particles (bubbles in liquids) tend to cluster inside low pressure regions (vortex filaments)

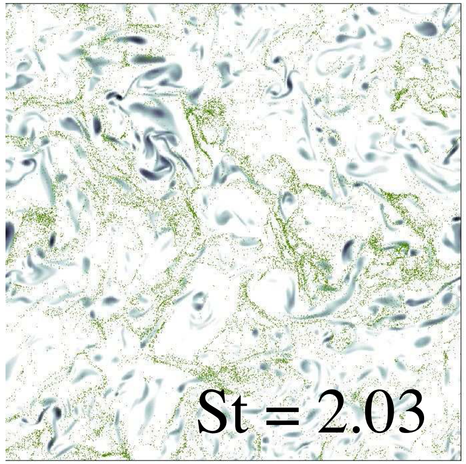
conversely

✓ heavy particles (solid particles) tend to be expulsed from low pressure regions

- Preferential concentration :
  - ⇨ non uniform spatial sampling of the flow
    - ⇨ biased statistics



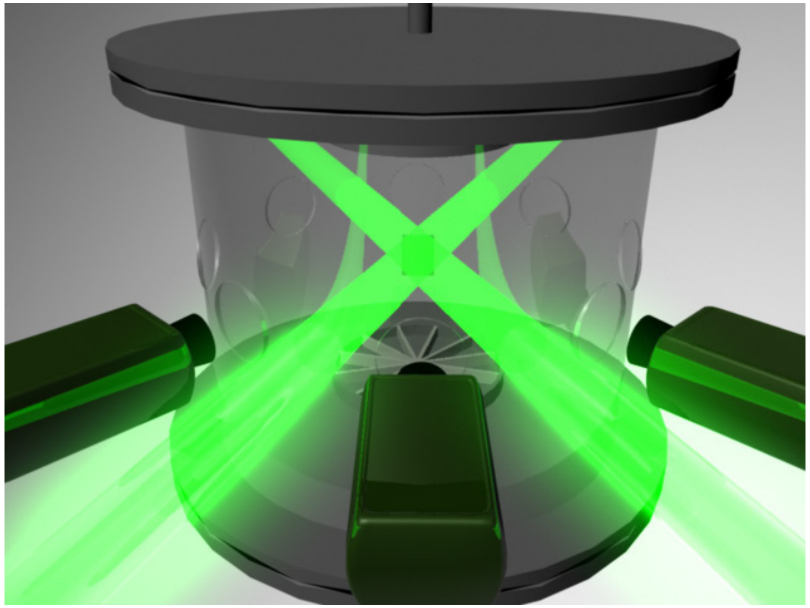
$DNS : R_\lambda \approx 185 \; \rho_p / \rho_f \sim 1500$



M Cencini, J Bec, L Biferale, G Boffetta, A Celani, A. S. Lanotte, S. Musacchio, F. Toschi
Journal of Turbulence Volume 7, No. 36, 2006

Particle Tracking with Optical Techniques

- Stereo PTV : simultaneous tracking of up to 300 particles
- 2 cameras => position measurements in a 3D volume
- 3 cameras => resolve position ambiguities
- Position measurements => Lagangian dispersion

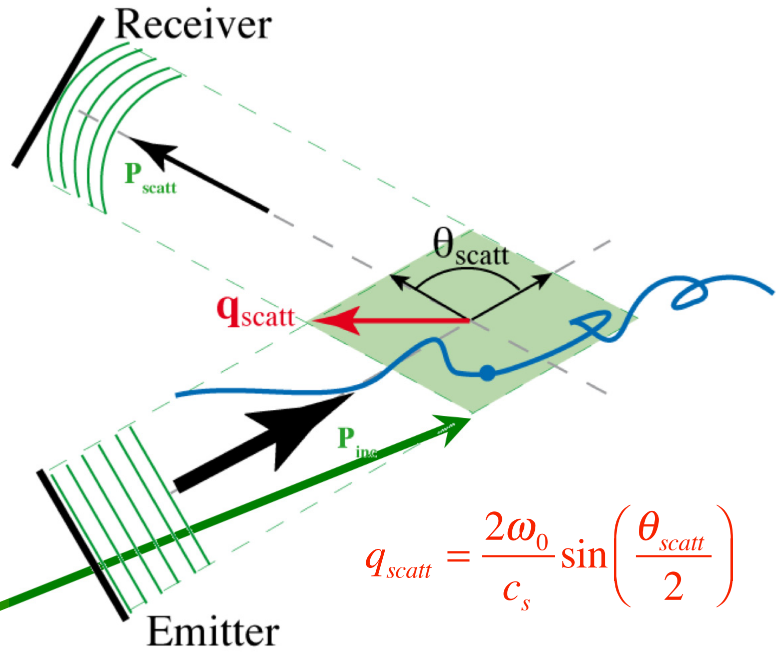


N T Ouellette, H Xu, M. Bourgoin & E. Bodenschatz
New Jour. Phys. **8** (2006)

But :

- Necessary compromise between :
  - ✓ time resolution (frame rate) (velocity increments)
  - ✓ spatial resolution (# pixels)
  - ✓ spatial extension volume (correlation estimates)
- Sub-Pixel resolution : need for interpolation (gaussian) => additional spatial filtering
- Lagrangian Velocity measurements
  - ✓ Time derivative : noisy

Particle Tracking with Acoustic Scattering (1)
Bistatic configuration



$$q_{scatt} = \frac{2\omega_0}{c_s}\sin\left(\frac{\theta_{scatt}}{2}\right)$$

- Bubbles or Rigid Sphere (radius a) ⇨acoustic impedance inhomogeneity $\delta z = \rho_t . c_t - \rho_f . c_f$
- Scattering of acoustic waves by spherical inhomogenities ⇨ Mie scattering (Dipolar)
- Bistatic Transducers configuration

$$p_{scatt}(\theta_{Scatt},t) \propto F(q_{Scatt}a,\theta_{Scatt}).p_{inc}(t).\iiint_{\mathbb{R}^3} n(\vec{r},t)e^{i\vec{q}_{Scatt}\vec{r}}d^3r$$

- Finite Size Transducers

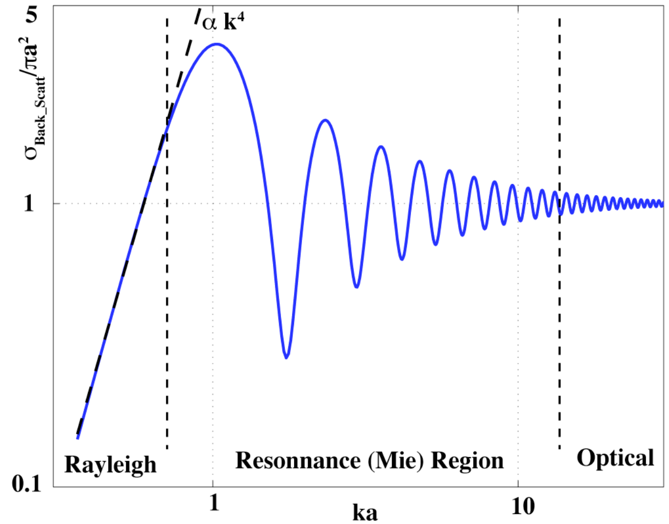
$$p_{scatt}(\theta_{Scatt},t) \propto F(q_{Scatt}a,\theta_{Scatt}).p_{inc}(t).\iiint_{V_{Scatt}} n(\vec{r},t)e^{i\vec{q}_{Scatt}\vec{r}}d^3r$$

- Doppler Shift

Single particle moving with velocity $\vec{V}$ $\quad n(\vec{r},t) = \delta(\vec{r} - \vec{V}t)$

$$p_{scatt}(\theta_{Scatt},t) \propto F(q_{Scatt}a,\theta_{Scatt}).p_{inc}(t).e^{i\vec{q}_{Scatt}\vec{V}t}$$

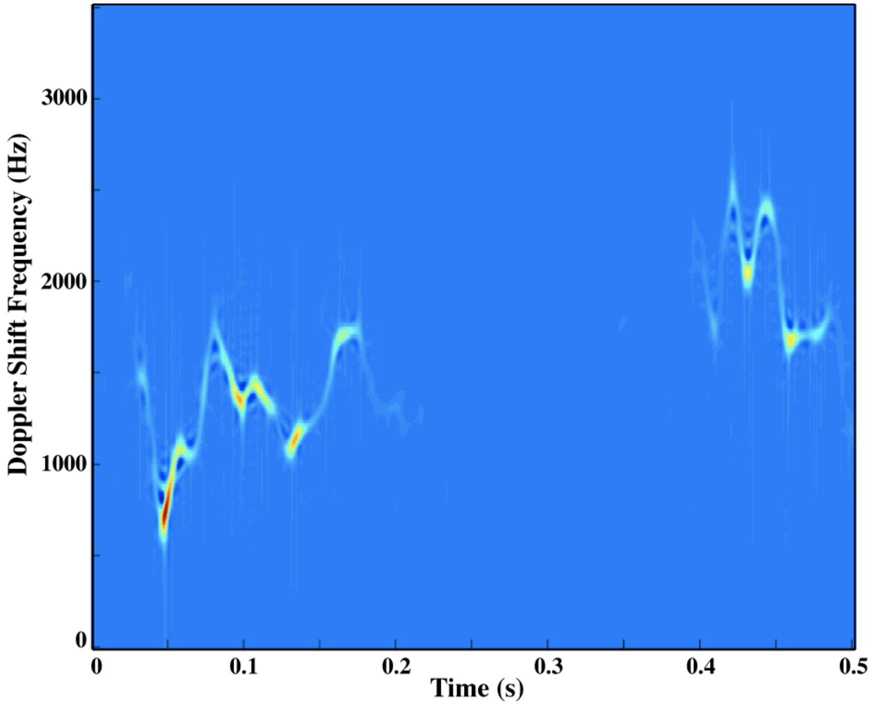
$$\omega = \omega_0 + \vec{q}_{scatt}.\vec{V}$$



Particle Tracking with Acoustic Scattering (2)

Signal conditionning @ LEGI

- Random Doppler Shift : frequency modulation (and amplitude modulation)
- Non-Stationary Analytic Signals



Energetic Time-Frequency Distribution (Cohen's Class)

- Non parametric estimation :

$$W_x(t,\nu) = \iint_{\mathbb{R}^2} K\left(s-t+\frac{\tau}{2}, s-t-\frac{\tau}{2}\right) x\left(s+\frac{\tau}{2}\right) x^*\left(s-\frac{\tau}{2}\right) e^{-i2\pi\nu\tau} ds d\tau$$

- Marginal preservation properties

$$\int_{\mathbb{R}} W_x(t,\nu)dt = |X(\nu)|^2$$

$$\int_{\mathbb{R}} W_x(t,\nu)d\nu = |x(t)|^2$$

- Choï-Williams Distribution Apodization Kernel :

$$K_{Choï-Williams}(t,\tau) = e^{-(\pi t\tau/\sigma)^2/2}$$

Particle Tracking with Acoustic Scattering (3)
Doppler Shifts estimation

- Doppler shift $\Delta\nu(t)$: 1st ν-moment of the time-frequency distribution

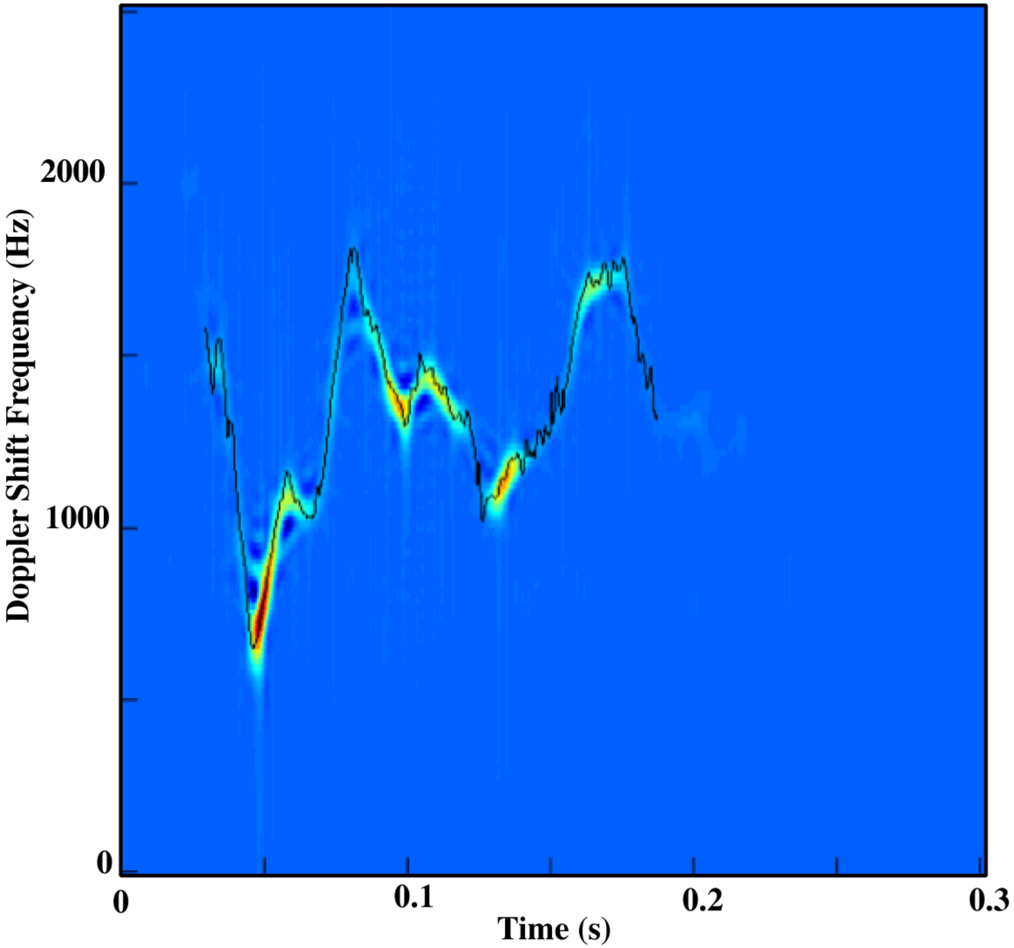
$$\Delta\nu(t) = \frac{\int \nu W_x(t,\nu)\,d\nu}{\int W_x(t,\nu)\,d\nu}$$

- Instantaneous Velocity 1-component :

$$\vec{q}_{Scatt}.\vec{V}(t) = 2\pi\Delta\nu(t)$$

$$V(t) = \frac{\Delta\nu(t)}{2\nu_{inc}\sin\left(\frac{\theta_{Scatt}}{2}\right)}c_s$$

- Direct velocity measurement : no need for differentiation (less noisy)



Particle Tracking with Acoustic Scattering (4)
Signal conditionning @ ENS-Lyon

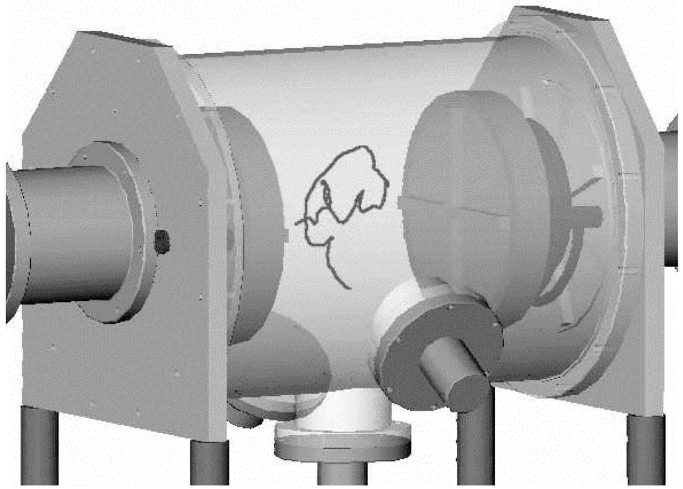
- Close flow : Von Karman flow
- PMMA solid spheres in water
- A few small particles in the flow volume (<10 !)
- Acoustic tracking using tranducers arrays : $\vec{r}_i$
- Acoustic scattering by spherical particles :

$$\varphi_{scatt} = \varphi_{scatt}\left(r_i, t\right)$$

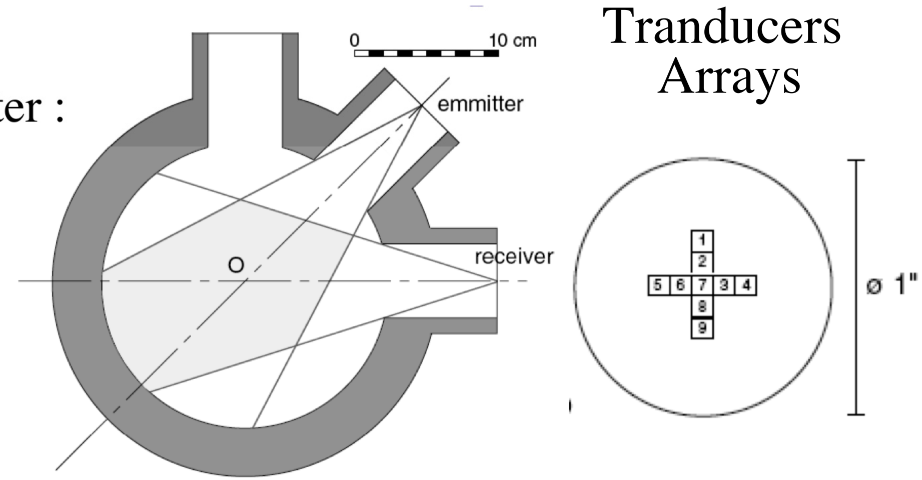
- Echo-location ($\vec{\nabla}_{\vec{r}_i} \varphi_{scatt}$) ⇨3D positions
- Doppler Shifts ($\frac{\partial \varphi_{scatt}}{\partial t}$)
  - Parametric estimation using Kalman Filter :

$$p_{Scatt}(t) = \sum_{m=1}^{M} a_m(t) e^{i2\pi\delta\nu_m t} + noise(t)$$

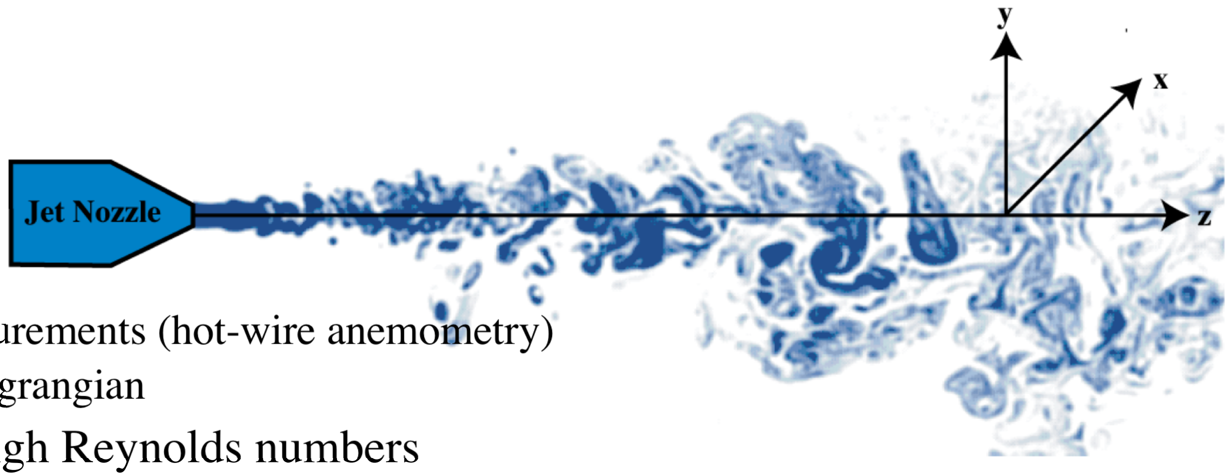
  - ⇨3D velocities
  - Tranducers array ⇨ redundancy



New Jour. Phys. **6** (2004)
N. Mordant, E. Lévêque & J-F. Pinton



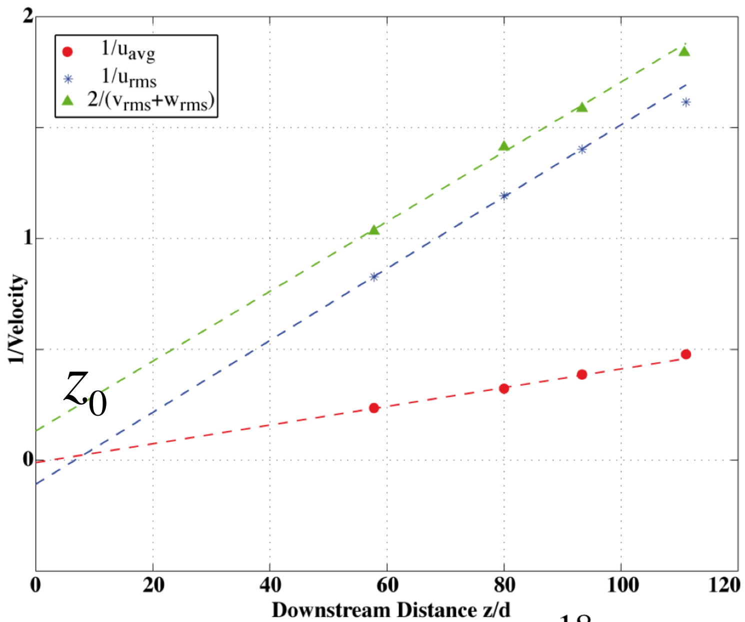
3D Lagrangian measurements in a Turbulent Air Jet Flow



- Motivations :
  - ✓ Open flow : mean velocity
    - ➪ Simultaneous Euler measurements (hot-wire anemometry)
    - ➪ Comparisons Euler vs Lagrangian
  - ✓ High turbulence level ➪ High Reynolds numbers
  - ✓ Self-preservation : $U, u', v', w' \propto (z - z_0)^{-1}$
  - ✓ Length-scales : $L_{\text{int}}, \lambda, \eta \propto z - z_0$
  - ✓ Constant Reynolds number : $R_\lambda \simeq 320$

- But :
  - ✓ Non homogeneous : along fluid particle trajectories the mean and standard deviation of $\vec{V}(\vec{x}, t)$ evolve
  - ➪ Non-stationarity

P. Gervais, C. Baudet & Y. Gagne
Experiments in Fluids (2007)



Particle Tracking with Acoustic Scattering (5)

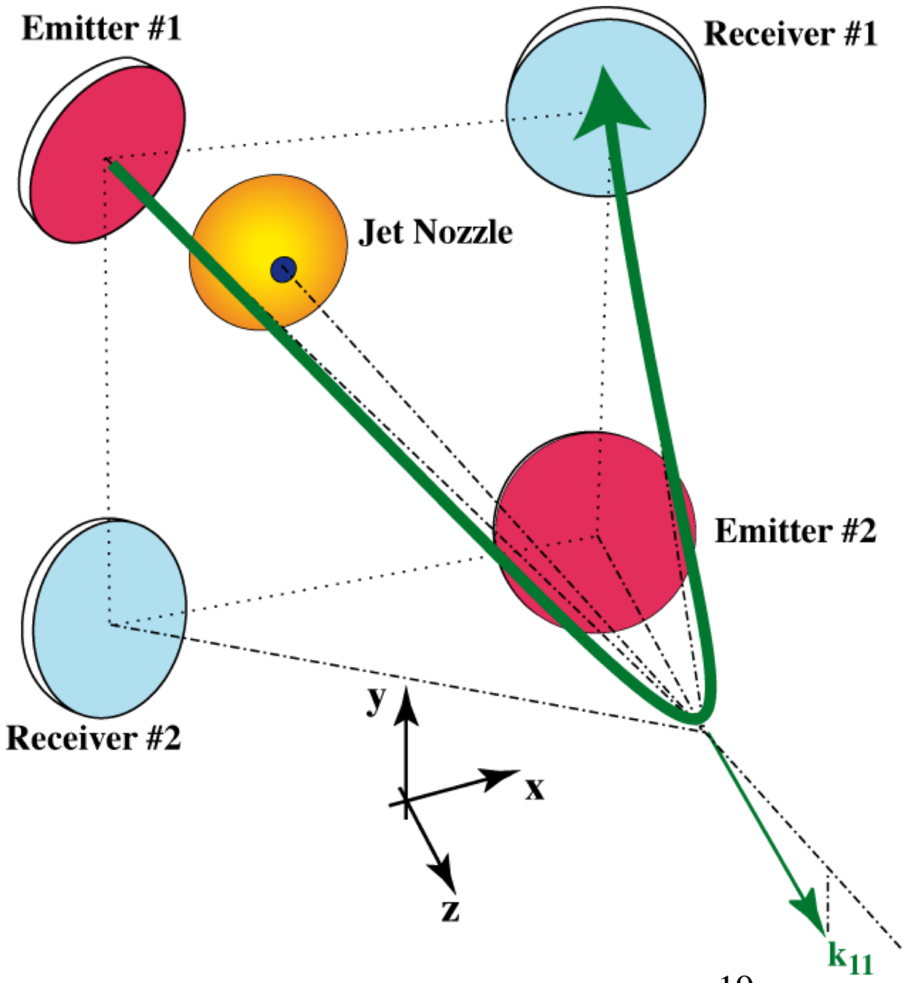
3D Lagrangian velocity measurements

- One pair Emitter/Receiver

& One incoming frequency :

✓ one projection of the velocity along the direction of the scattering wave-vector $\vec{k}_{11}$

$$\frac{\vec{k}_{11}}{k_{11}}.\vec{V}(t) = \frac{\Delta\nu(t)}{2\nu_1 \sin\left(\frac{\theta_{Scatt}}{2}\right)} c_s$$

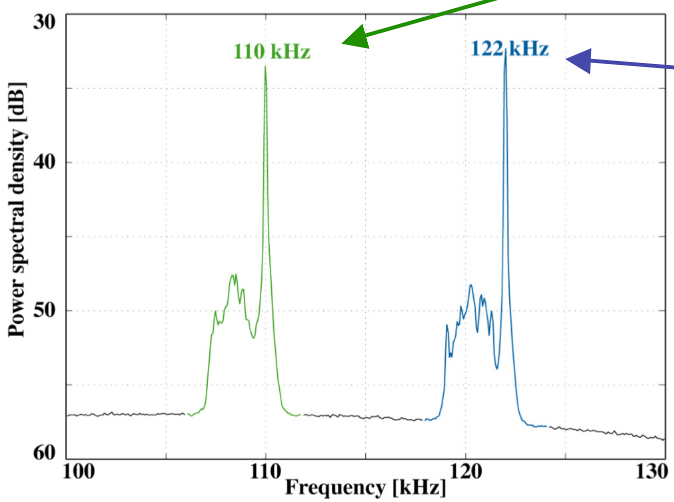


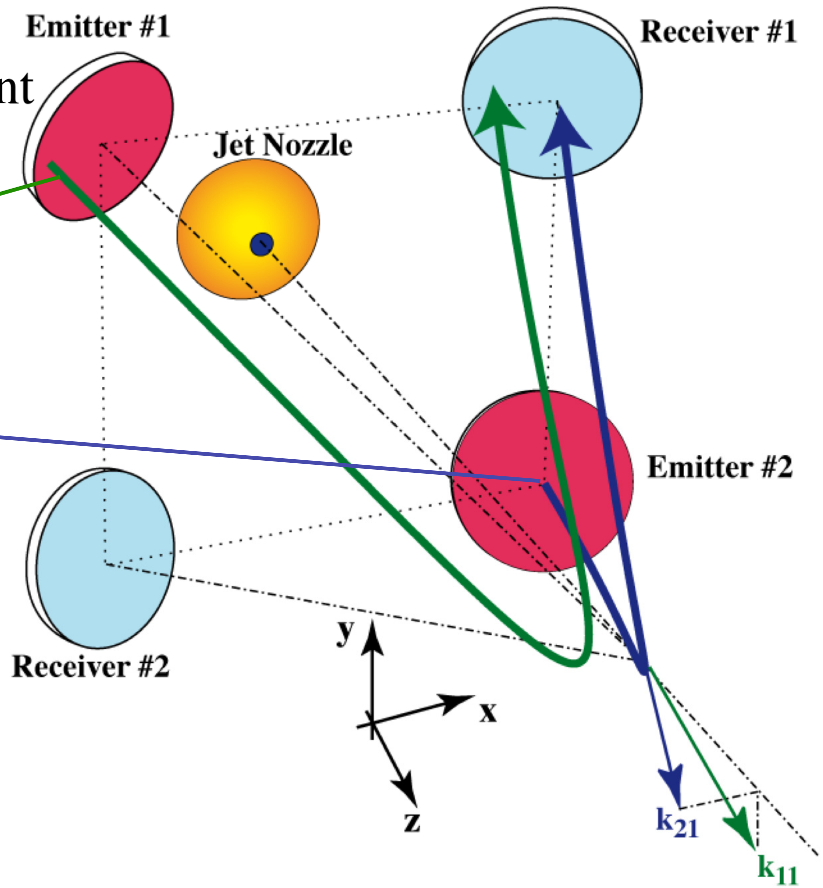
Particle Tracking with Acoustic Scattering (5)
3D velocity measurements

- 2-components measurements :
  - ✓ 2 emitters
  - ✓ 2 incoming frequencies
  - ✓ 1 receiver

⇨2 non-coplanar projections

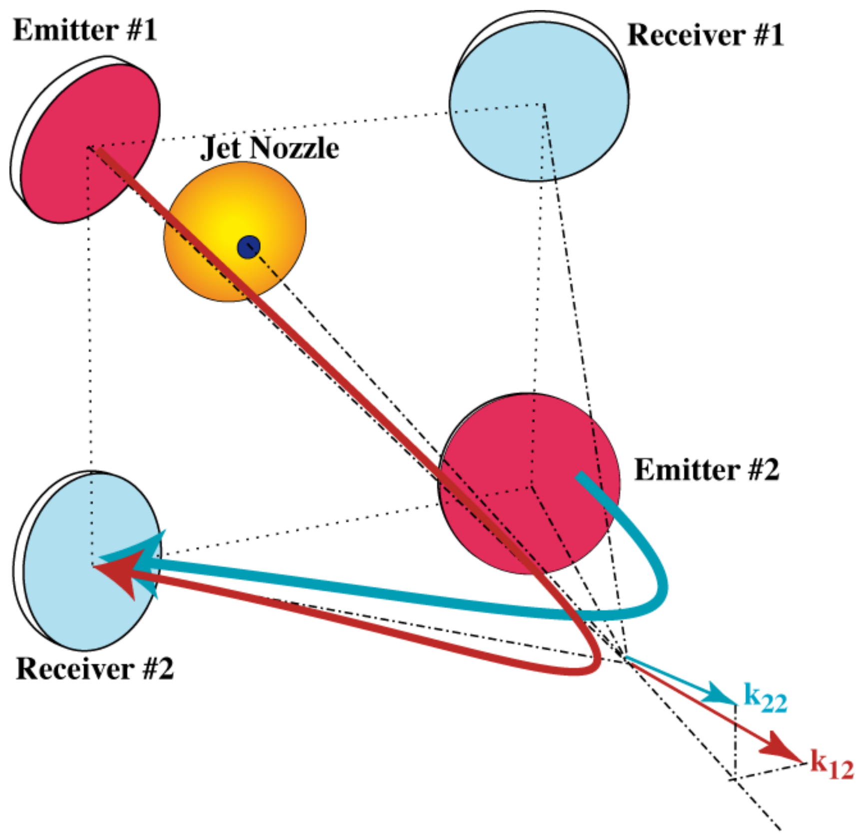
- Heterodyne demodulation around the mid-point frequency
- Channel separation : band-pass filtering





Particle Tracking with Acoustic Scattering (5)
3D velocity measurements

- One more receiver
  - ✓ 2 other non-coplanar projections

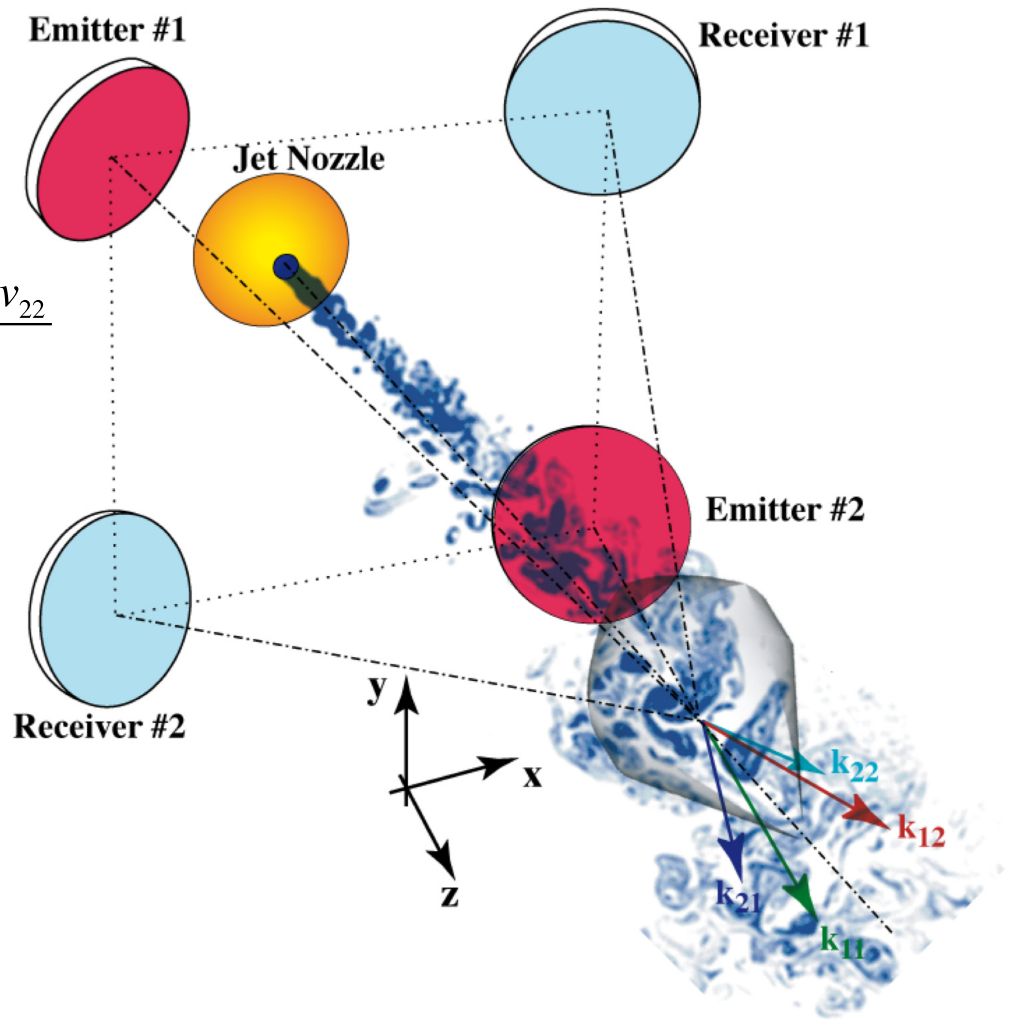


Particle Tracking with Acoustic Scattering (5)
3D velocity measurements

- 4 non-coplanar projections
  ⇨ redundancy

$$V_x = \frac{v_{12} - v_{21}}{2\sin\alpha}, V_y = \frac{v_{22} - v_{11}}{2\sin\alpha}, V_z = \frac{v_{11} + v_{12} + v_{21} + v_{22}}{4\cos\alpha}$$

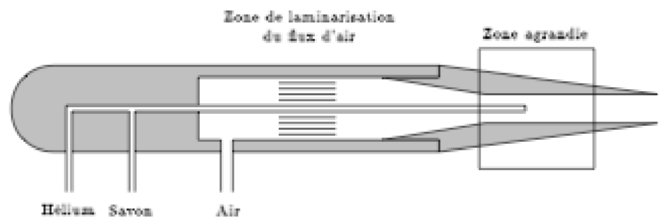
- Size of the measurement volume :
  ~50cm (z), ~30cm (x,y)
- Noise contamination :
  - ✓ echoes
  - ✓ scattering by thermal fluctuations
  - ✓ scattering by vorticity fluctuations



Lagrangian tracers in air flow measurements

- Soap Bubbles inflated with He
- Neutrally buoyant (or heavier CO_2 or lighter H_2)
- Small size : 1 mm ➔ 6mm
- Rigid ➔ $We = \dfrac{\rho_{air}.u_{rms}^2 a}{\sigma_{soap}} \ll 1$
- Reproducible size and shape
- Injected upstream the jet nozzle
- Low injection rate : ~ 1 single bubble in the scattering volume
- Size and density controlable and reproducible (St = cte)

Versatile Bubble Generator

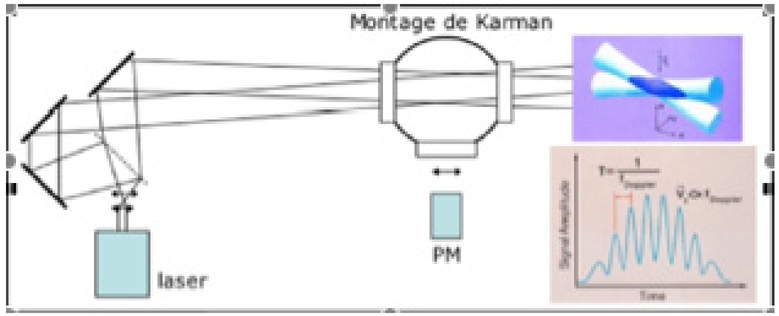


For 1 mm bubbles

x [m]	1.3 m	1.8 m	2.1 m	2.5 m
St	17.7	8.8	6.4	4.4
λ/a_p	4.4	6.2	7.3	8.8
η/a_p	0.12	0.17	0.20	0.24

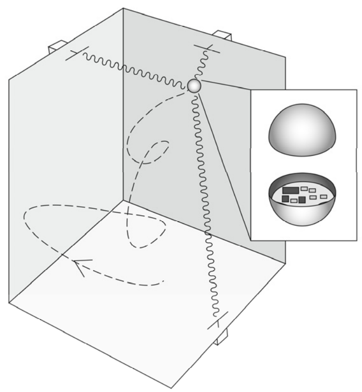
Other Lagrangian Turbulence Probing under development @ENS-Lyon

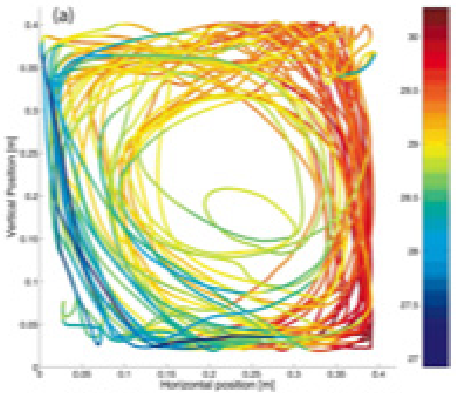
- Extended Laser Doppler Anemometry
  - ✓ very small tracer particles tracked in a large volume (a few cm)
  - ✓ Lagrangian acceleration : 1 & 2-points statistics
  - ✓ Ask Romain Volk for further details



- Smart particles
  - ✓ "small" (~2 cm, presently) particles, equipped with sensor (eg temperature, acceleration) and RF emitter (localization "GPS", transmission of measures)
  - ✓ Lagrangian temperature field in Rayleigh-Benard turbulence

Y. Gasteuil, W.L. Shew, M.Gibert, F. Chillá, B. Castaing and J.-F. Pinton. arXiv:0706.0594v1





Turbulent Jet Flow : 1-point Statistics (1)

- Four measurement positions in the far-field region (fully developped turbulence) :

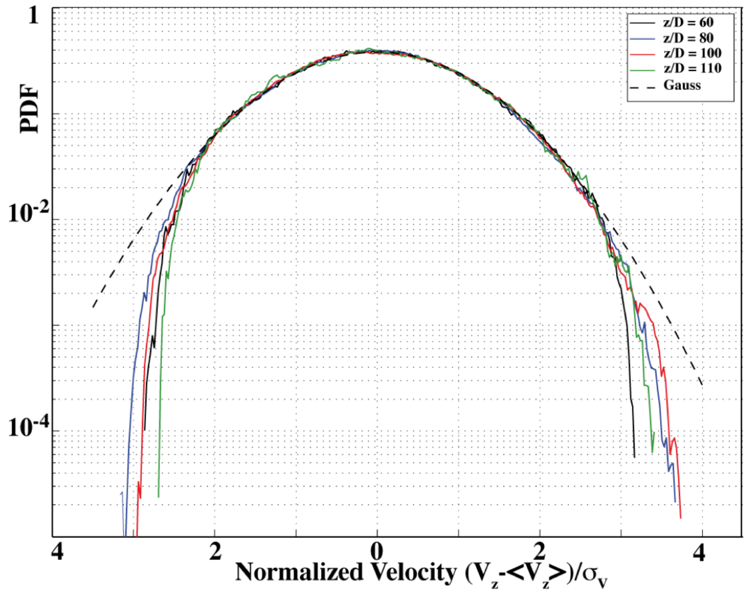
$$60 \le z / D_{Nozzle} \le 110$$

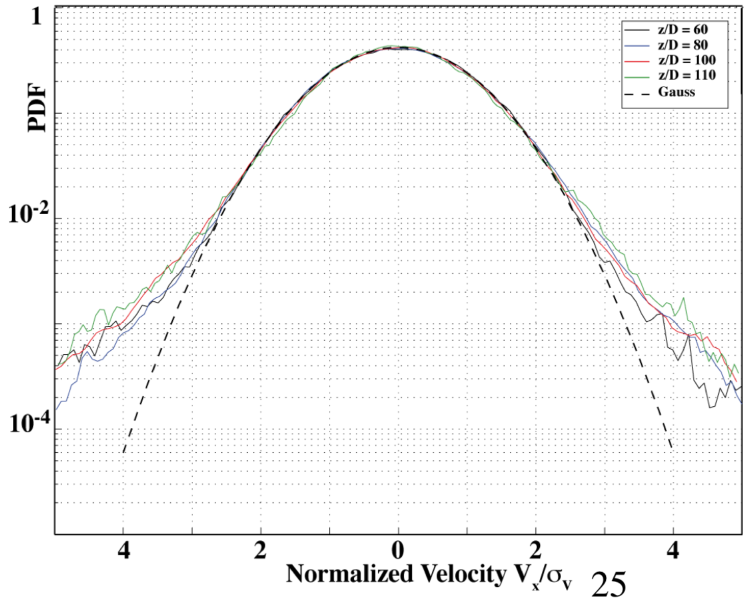
- Constant Nozzle velocity i.e. constant Re : $R_\lambda \approx 300$
- Varying length scales

$$L_{\text{int}}, \lambda_T \text{ and } \eta_K$$

- Lagrangian PDFs are close to Gaussian like Eulerian ones
- Lagrangian turbulence level close to Eulerian one

$$v'_{x,y,z\, rms} / \langle V_z \rangle \approx 28\%$$





Turbulent Jet Flow : 1-point Statistics (2)

- Flow inhomogeneity :

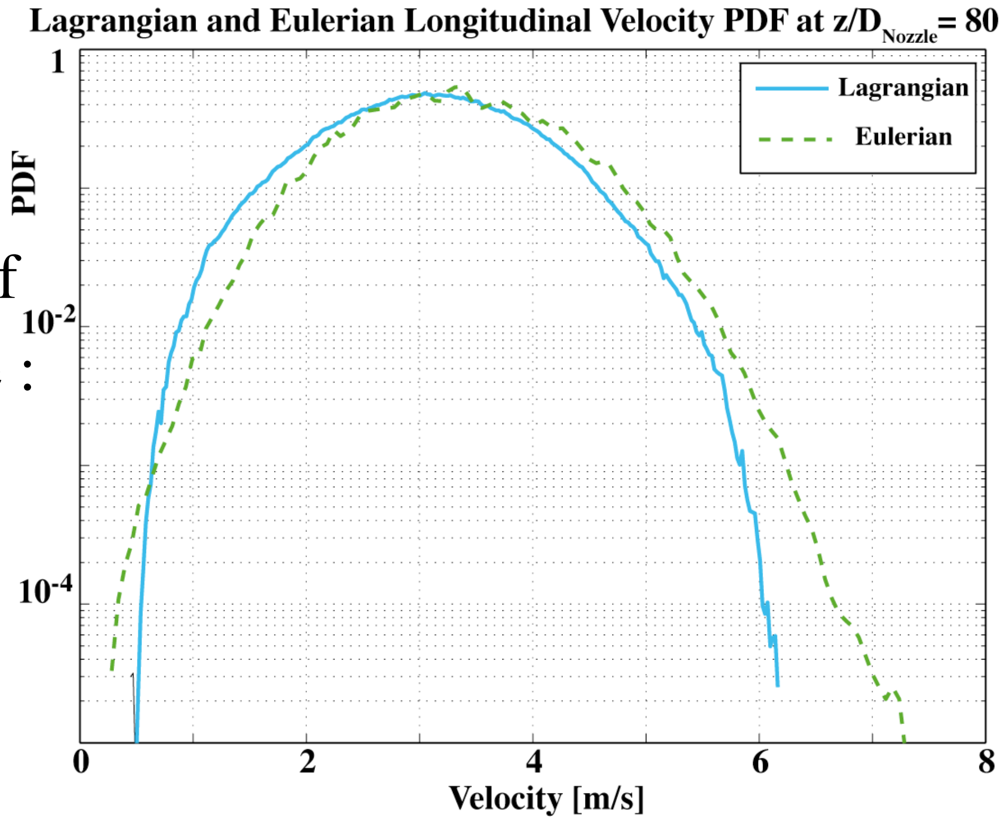
$$\langle V_z^L \rangle \leq \langle u_z^E \rangle$$

$$\langle v'^L_{z,rms} \rangle \leq \langle u'^E_{z,rms} \rangle$$

- Accounting for the finite size of the Lagrangian probing volume :

$$\langle V_z^L \rangle_x \approx \langle u_z^E(X) \rangle_{V_{meas}}$$

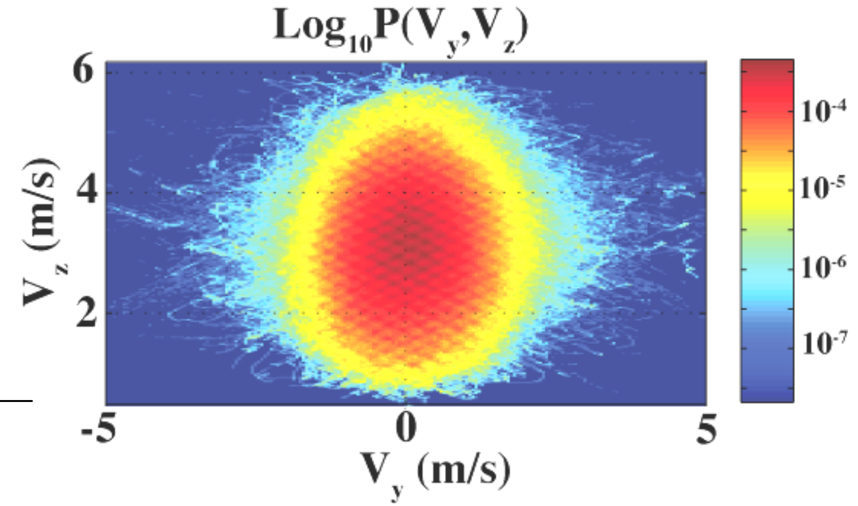
$$\langle v'^L_{z,rms} \rangle_x \approx \langle u'^E_{z,rms}(X) \rangle_{V_{meas}}$$



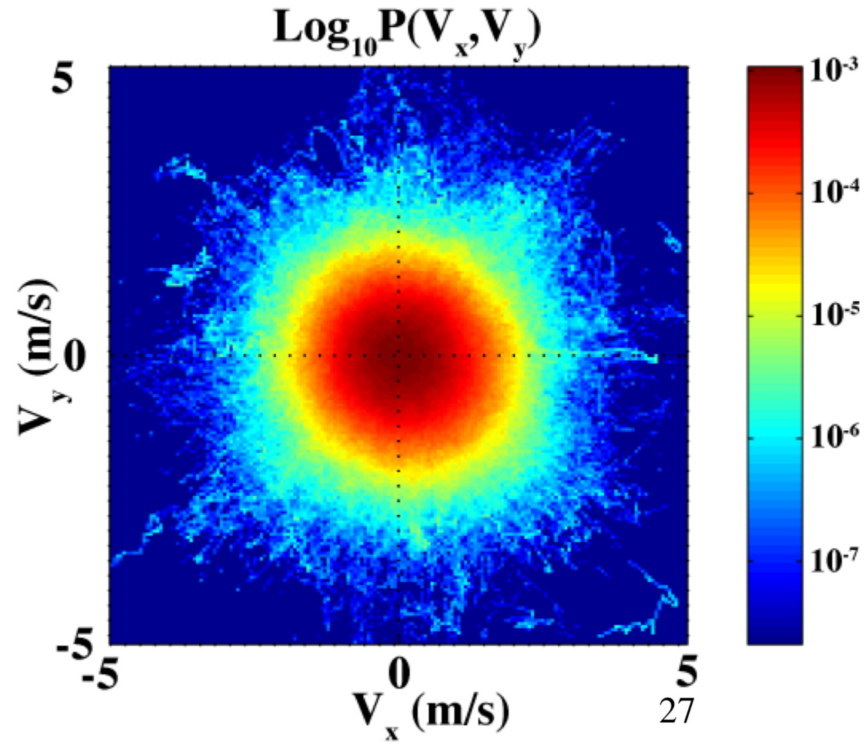
Turbulent Jet Flow : Joint Statistics

- V_x, V_y, V_z are statistically mutually independant
  - ✓ Slightly elliptical shape of $P(V_y, V_z)$, related to the fact that :

$$\sigma_{V_x}, \sigma_{V_y} \leq \sigma_{V_z}$$



- Eulerian velocity components statistically dependant (at least in turbulent jet flows)



Turbulent Jet Flow : 2-points Statistics (1)
Lagrangian Velocity Covariance

- Velocity Covariance estimation :

$$R^L_{\alpha\beta}(t_1,t_2) = \frac{\left\langle \left(V_\alpha(x,t_1) - \overline{V_\alpha(x,t_1)}\right)\left(V_\beta(x,t_2) - \overline{V_\beta(x,t_2)}\right)\right\rangle_x}{\sigma_{V_\alpha}(t_1)\sigma_{V_\alpha}(t2)}$$

- If stationarity (1st & 2nd order) :

$$R^L_{\alpha\beta}(\tau) = \frac{\left\langle \left(V_\alpha(x,t) - \overline{V_\alpha}\right)\left(V_\beta(x,t+\tau) - \overline{V_\beta}\right)\right\rangle_{x,t}}{\sigma_{V_\alpha}\sigma_{V_\beta}} \qquad \tau = t_2 - t_1$$

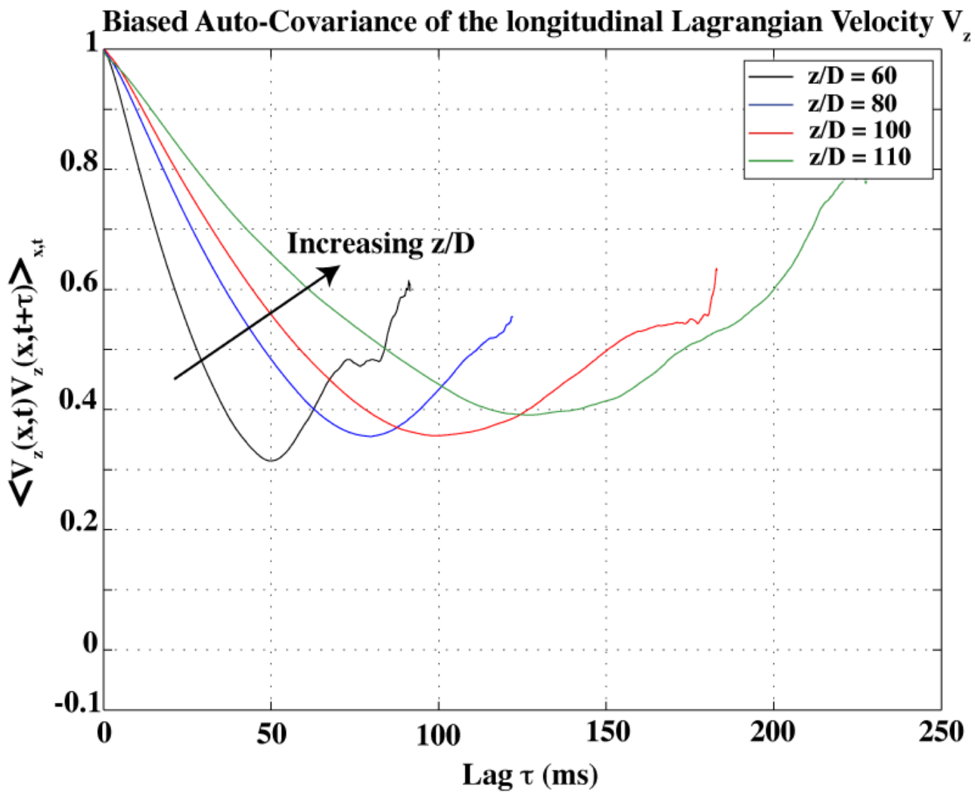
- Practically :
  - ✓ One bubble trajectory (#i duration L_i) ≡ one statistical realization

    $V(x,t) \Rightarrow V_{i,j}$ with $i \equiv x$ and $t_j = j \times T_{sampling}$
  - ✓ Then compute :

$$R^L_{\alpha\beta}(\tau = k \times T_{sampling}) = \frac{1}{\sigma_{V_\alpha}\sigma_{V_\beta}} \sum_{i=1}^{N_b} \left(\frac{1}{L_i - k}\right) \sum_{j=1}^{L_i - k} \left(V_{i,j} - \left\langle V_{i,j}\right\rangle_{i,j}\right)\left(V_{i,j+k} - \left\langle V_{i,j}\right\rangle_{i,j}\right)$$

Turbulent Jet Flow : 2-points Statistics (2)
Lagrangian Velocity Covariance

- No convergency towards 0 at large time lags
- Lack of stationarity : the mean and standard deviation of velocity depend on z $\left(\propto (z - z_0)^{-1}\right)$
- Practically : one needs a preliminary statistical conditioning in order to stationarize the velocity signals



Turbulent Jet Flow : 2-points Statistics (3)

- Signals stationarization

$$V_{i,j} \Rightarrow V^s_{i,j} = \frac{V_{i,j} - \langle V_{i,j} \rangle_i}{\sigma_j(V_{i,j})}$$

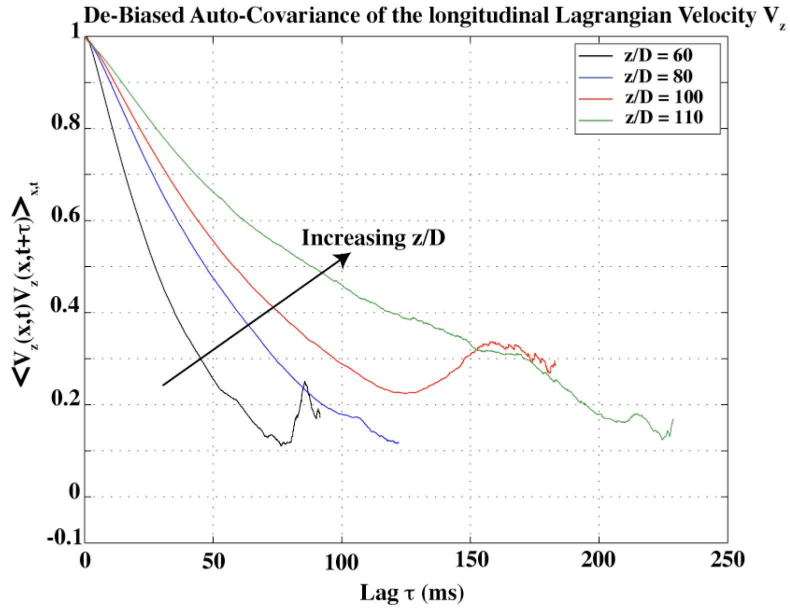
- Where :

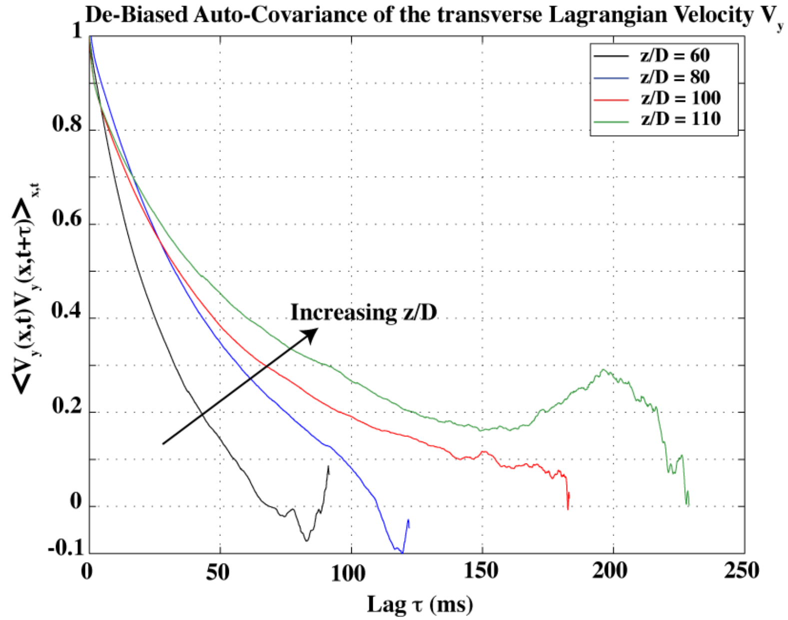
$$\langle V_{i,j} \rangle_i \;\; \& \;\; \sigma_j(V_{i,j}) = \sqrt{\left\langle \left(V_{i,j} - \langle V_{i,j} \rangle_i \right)^2 \right\rangle_i}$$

✓ *ensemble averages* (over bubble index i)

✓ *local* (depend on $j = t - t_0$)

Yeung PK (2002) Ann Rev Fluid Mech 34:115–142





Turbulent Jet Flow : 2-points Statistics (4)
Eulerian vs Lagrangian

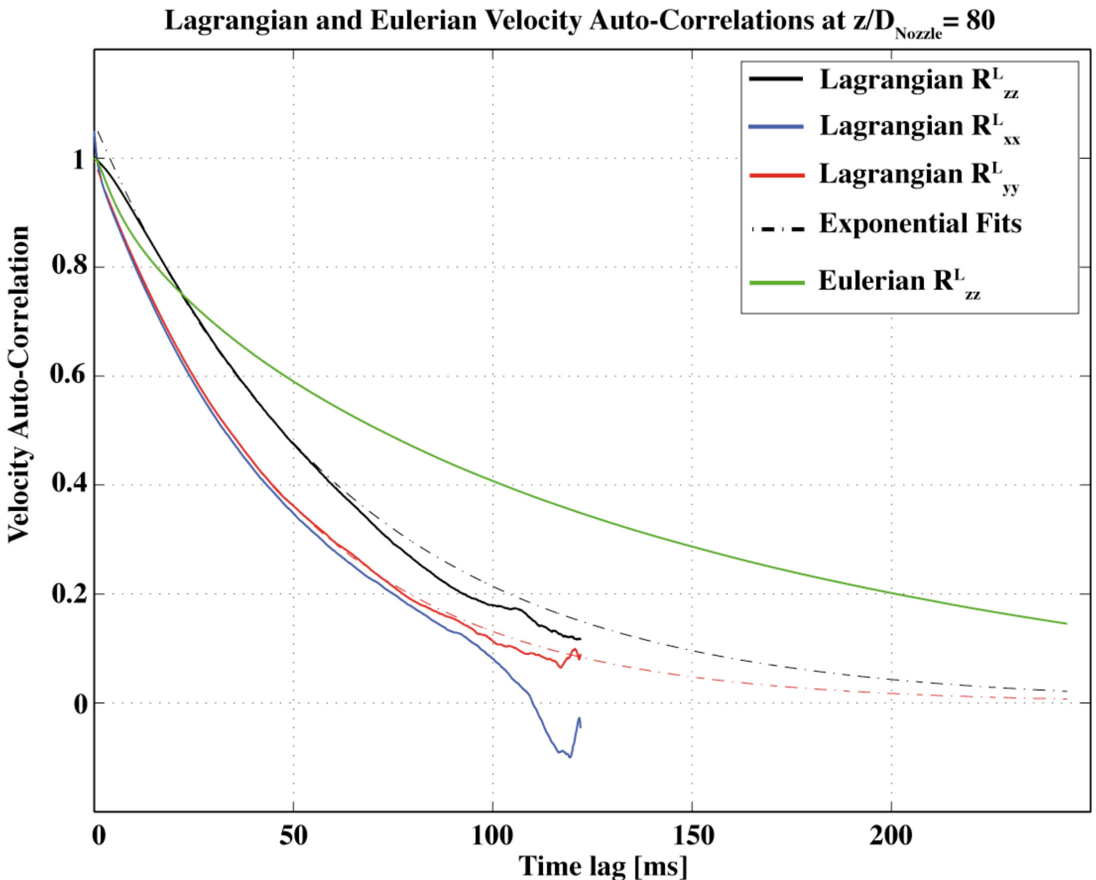
- Integral Time Scales :

$$T_{\text{int}}^{L,E} = \int_0^{+\infty} R_{\alpha\alpha}^{L,E}(\tau)d\tau$$

- $$T_{\text{int}\,xx}^{L} \approx T_{\text{int}\,yy}^{L} < T_{\text{int}\,zz}^{L}$$
- $$T_{\text{int}\,zz}^{L} < T_{\text{int}\,zz}^{E}$$

For 60 < z/D < 110 : $\dfrac{T_{\text{int}\,zz}^{E}}{T_{\text{int}\,zz}^{L}} \approx 1.35$

- Origin ?
  - ✓ effect of random sweeping on Eulerian statistics
  - ✓ P A O'Gorman & D I Pullin Jour. of Turb. 5 (2004) 035



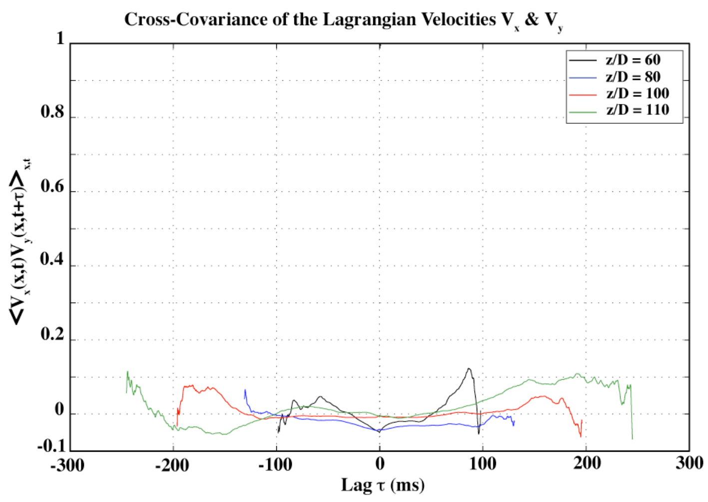
$$R^{swept}(k,\tau) = \left(\int_{-\infty}^{+\infty} p(U) e^{-ikU\tau} dU \right) \tilde{R}^{stat}(k,\tau)$$

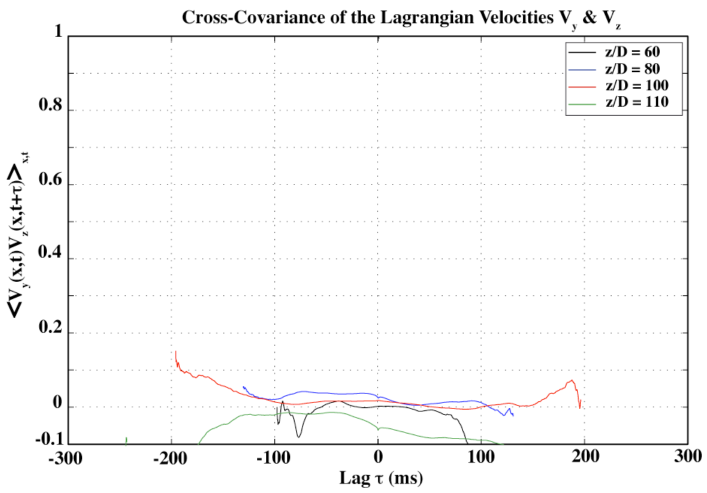
Turbulent Jet Flow : 2-points Statistics (5) : Velocity Cross-Correlations

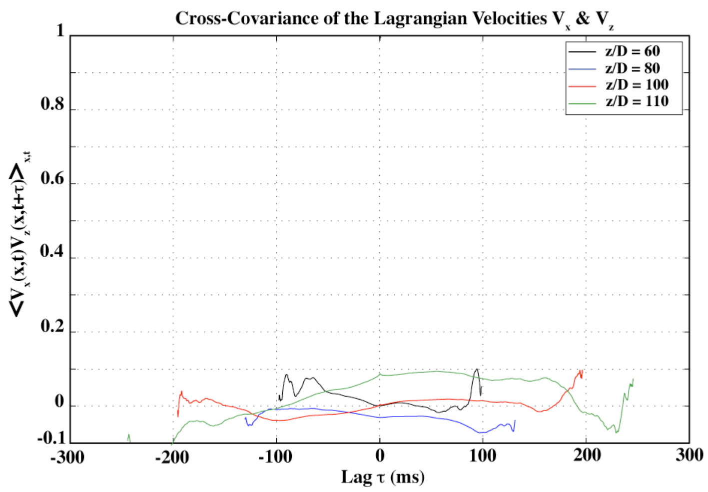
- V_x, V_y, V_z Stat. Indep

⇨ Decorrelated

- Not true for Eulerian velocity components
- Validation of the acoustic set-up (e.g. transducers misalignments)
- ⇨ Confidence level : ~0.1

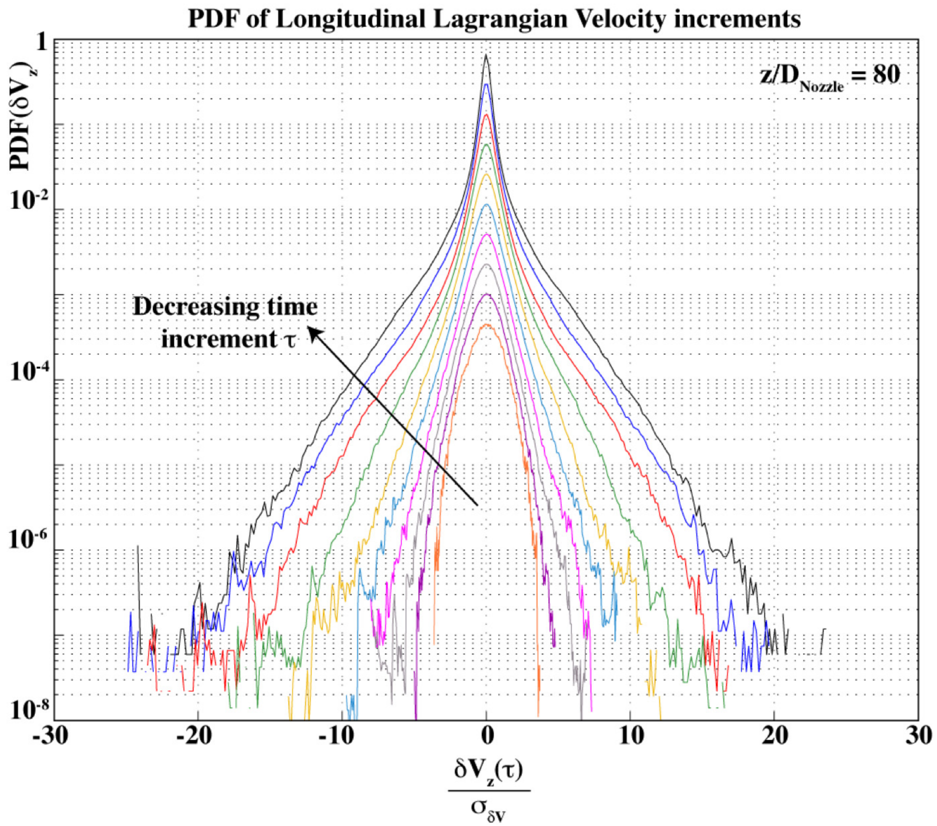






Turbulent Jet Flow : intermittency (1)
PDF of Lagrangian velocity increments

- The shape of the PDF of the Lagrangian velocity time increments
  $$\delta V_{x,y,z}(x,t,\tau) = V_{x,y,z}\left(x,t+\tau\right) - V_{x,y,z}\left(x,t\right)$$
  evolve continuously towards a non gaussian shape with fat tails
- Lagrangian velocity is intermittent too !
- "How much intermittent" as compared to Eulerian spatial velocity increments ?



Turbulent Jet Flow : intermittency (2)
Eulerian vs Lagrangian intermittencies

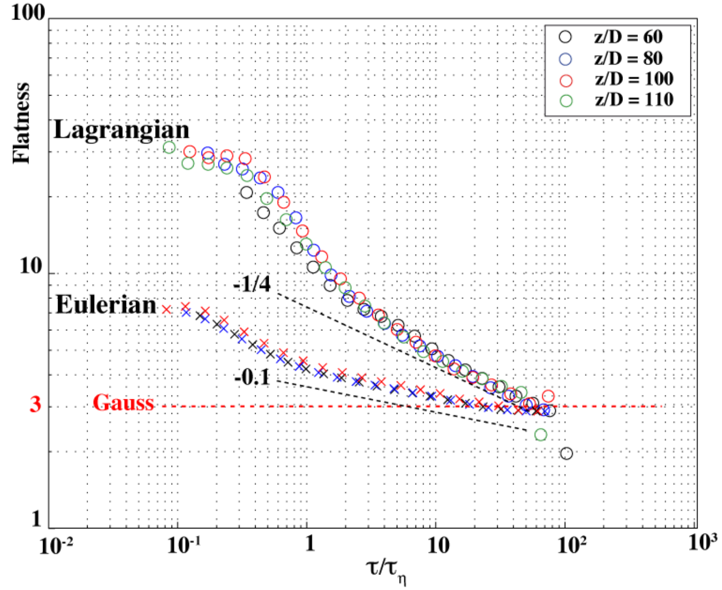
- Flatness Factor : $F^L(\tau) = \dfrac{\left\langle \delta V(x,\tau)^4 \right\rangle_{x,t}}{\left\langle \delta V(x,\tau)^2 \right\rangle_{x,t}^2}$

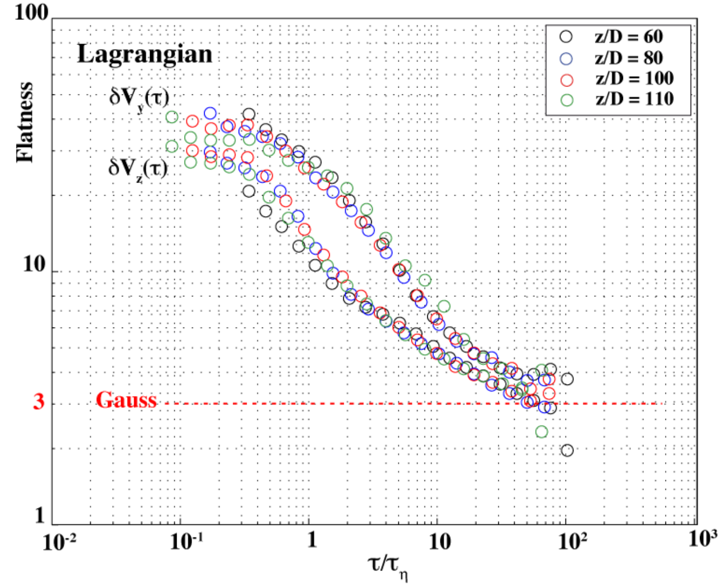
- K41 scalings (Eulerian) :

$$S_p^E(r) = \left\langle \delta u(r)^p \right\rangle \propto \bar{\varepsilon}^{p/3} . \left(\frac{r}{L}\right)^{p/3} \quad \Rightarrow \quad F^E(r) \propto \left(\frac{r}{L}\right)^{4/3-2*2/3} = \left(\frac{r}{L}\right)^0$$

- K62 scalings (Eulerian) :
  - ✓ ε is stochastic (log-normal stat) : $\left\langle \varepsilon_r^{\,p/3} \right\rangle \neq \left\langle \varepsilon_r \right\rangle^{p/3}$
  - ✓ $\left\langle \varepsilon_r^{\,p/3} \right\rangle \propto \left(\frac{r}{L}\right)^{\tau_{p/3}} \quad S_p^E(r) \propto \left(\frac{r}{L}\right)^{\varsigma_p} \quad \Rightarrow \quad \varsigma_p = \frac{p}{3} + \tau_{p/3}$
  - ✓ K62 $\quad \varsigma_4 - 2\varsigma_2 = -\frac{4}{9}\tau_2 \approx -0.1$

- Lagrangian intermittency :
  - ✓ more intermittent : like transverse eulerian velocity increments (vorticity fluctuations ?)
  - ✓ influence of coherent structures on Lagrangian dynamics ?





Lagrangian probing of turbulence

- Tracers and Seeding : main concern
- Correlation estimations are challenging : need for long trajectories, while preserving spatial resolution.
- Scattering (light or sound) probes are valuable
  - ✓ decoupling of the spatial resolution (choice of λ) from the size of samples
  - ✓ direct velocity measurements (less noisy)
- Need for proper statistical conditioning (signals stationarization)
- Open flows ⇨ direct comparison of Lagrangian and Eulerian dynamics

- Origin of larger Eulerian Integral Time Scale : Random sweeping ?
- Origin of Lagrangian intermittency anistropy : effect of the large scale pressure gradients.
- Origin of increased intermittency of Lagrangian fields : vorticity dynamics and coherent structures ?

- Prospective : conditional statistics of the Lagrangian velocity on vorticity