Preferential concentration of impurities in turbulent suspensions

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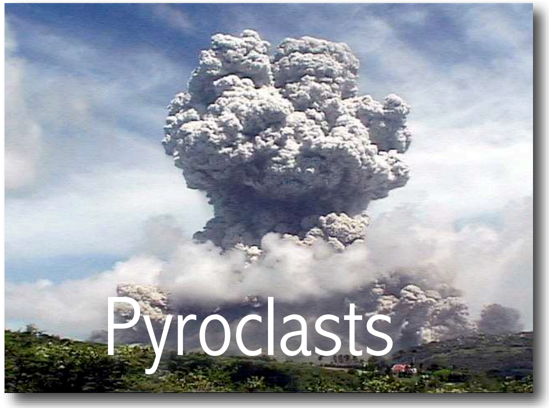
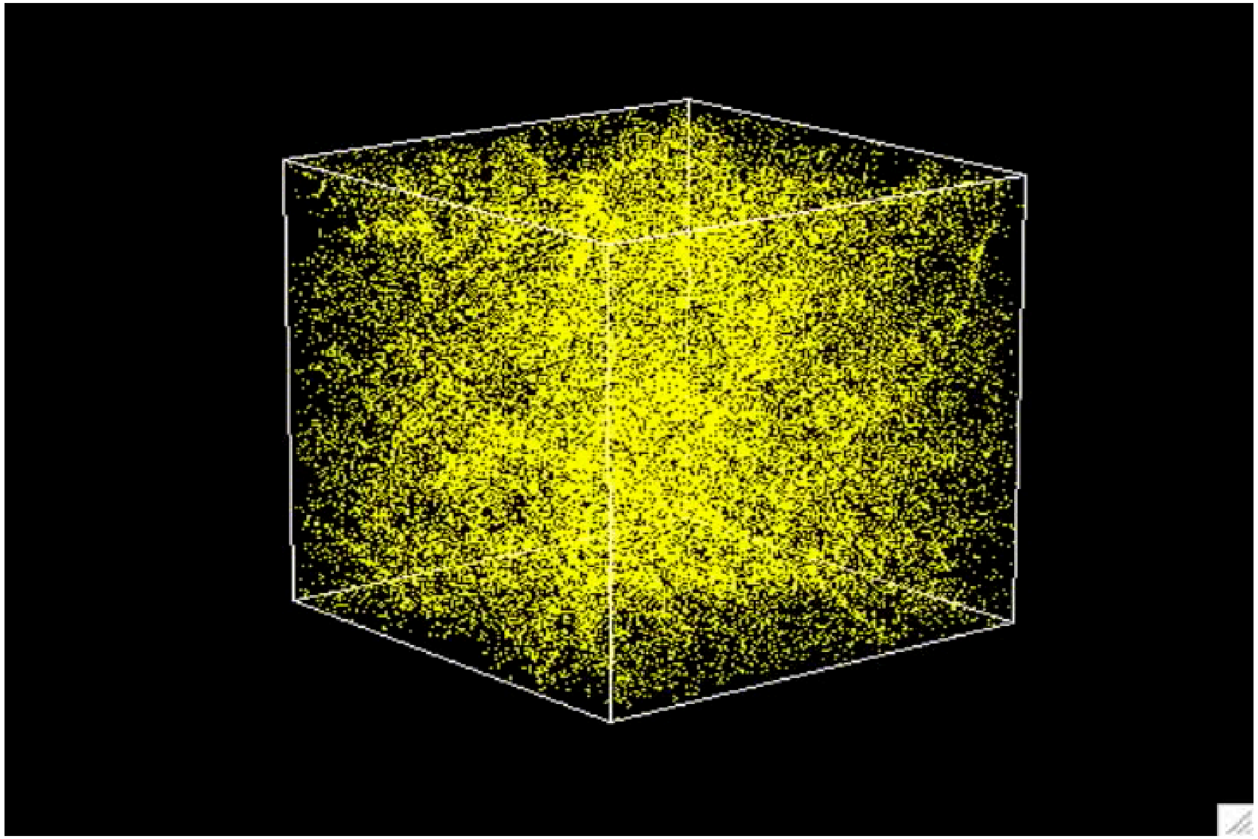
Particle-laden flows



Finite-size and **mass** impurities advected by **turbulent** flow

Preferential concentration



Particles have **inertia** and do **not** follow exactly the fluid flow
$\Longrightarrow$ they distribute **non-homogeneously**

Outline

Lecture 1

Introduction

Model, applications (rain, planets), questions

Small-scale clustering

Relation with dissipative dynamical systems, fractal attractors, etc.

Lecture 2

Inertial range clustering

Scale-dependence of inertia effects

Dispersed suspensions

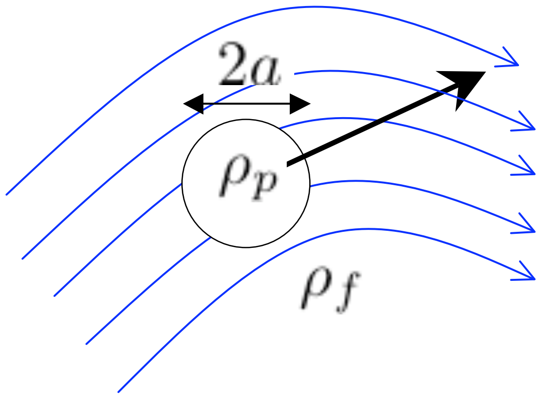
- **Passive suspensions:** no feedback of the particles onto the fluid flow (e.g. very dilute suspensions)
- **Rigid spherical particles** that are assumed to

(i) be much smaller than the smallest active scale of the flow (Kolmogorov scale in turbulence)

(ii) have a very small Reynolds number

$\Rightarrow$ Surrounding flow = Stokes flow

$\Rightarrow$ Maxey & Riley (1983)



$$m_p \ddot{\boldsymbol{X}} = m_f \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}(\boldsymbol{X}, t) - 6\pi a \mu [\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X}, t)] - \frac{m_f}{2}\left[\ddot{\boldsymbol{X}} - \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{u}(\boldsymbol{X}, t))\right]$$

$$- \frac{6\pi a^2 \mu}{\sqrt{\pi\nu}} \int_0^t \frac{\mathrm{d}s}{\sqrt{t-s}} \frac{\mathrm{d}}{\mathrm{d}s}[\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X}, s)].$$

+ Buoyancy

+ Faxén corrections

Very heavy inertial particles

- Impurities with mass density $\rho_p \gg \rho_f$

$$\ddot{\boldsymbol{X}} = -\frac{1}{\tau}\left(\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X},t)\right)$$

Prescribed velocity field
(random or solution to Navier-Stokes)

- Viscous drag ⇒ Response time (Stokes time): $\tau = \frac{2}{9}\frac{\rho_p}{\rho_f}\frac{a^2}{\nu}$

Stokes number $\boxed{\mathrm{St} = \tau/\tau_\eta}$

τ_η turnover time associated to the Kolmogorov dissipative scale

$$\tau_\eta = \eta/\delta_\eta u = \varepsilon^{-1/3}\eta^{2/3}$$

- Minimal model: 2 parameters $\begin{cases} \mathrm{St} & \text{inertia} \\ \mathrm{Re} & \text{turbulence intensity} \end{cases}$

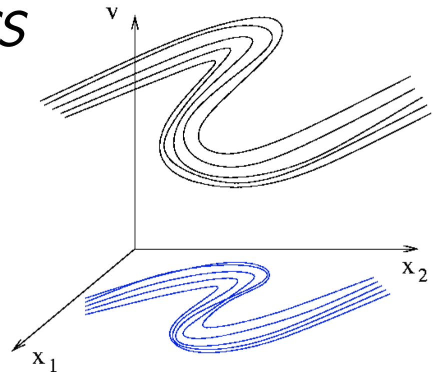
⇒ allows for a systematic investigation

Clustering of heavy particles

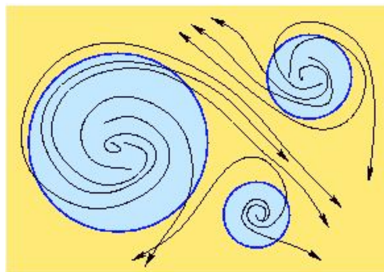
- Important for estimating
  - particle interactions (collisions, chemical reactions, gravitation)
  - fluctuations in the concentration of a pollutant
  - possible feedback of the particles on the fluid

- **Different mechanisms**

Dissipative dynamics
$\Longrightarrow$ ***attractor***



Ejection from ***eddies*** *by centrifugal forces*



Philosophy

- Describe with as much generality as possible clustering in turbulent flows
- Find models to disentangle these effects and understand their physics

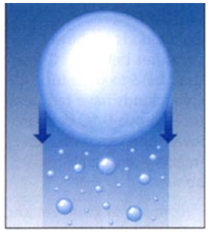
Rain initiation

Warm clouds

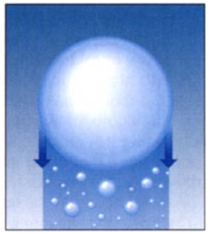
1 raindrop = 10^9 droplets

Growth by continued condensation way

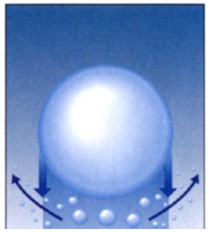
= too slow



(a)



(b)



Collisions

Polydisperse suspensions with a wide range of droplet sizes with different velocities

Larger, faster droplets overtake smaller ones and collide

Droplet growth by **coalescence**

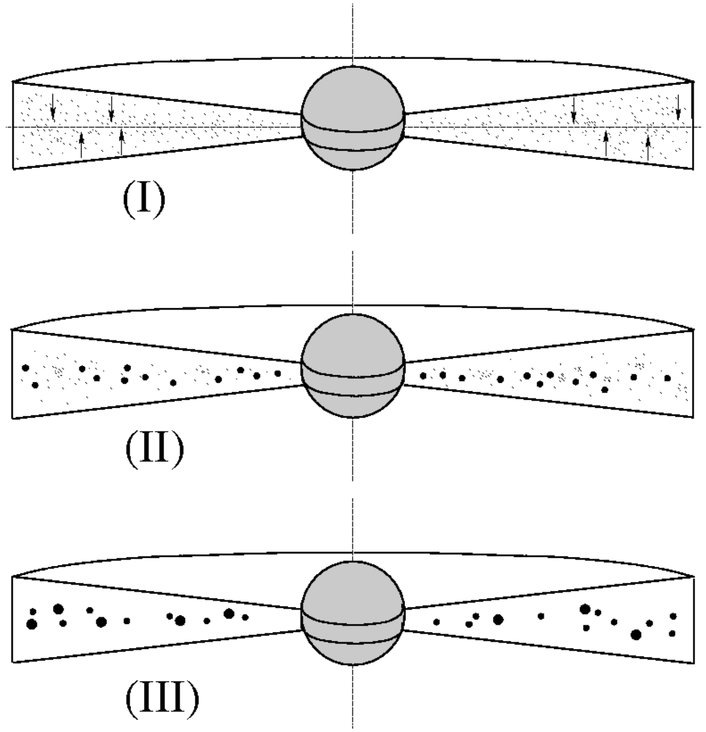
Formation of the Solar system

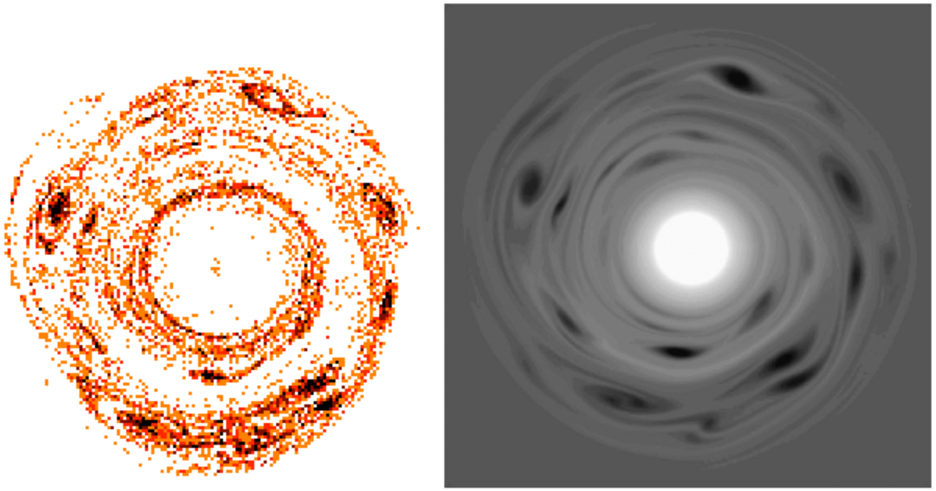
Protoplanetary disk after the collapse of a nebula

(I) Migration of dust toward the equatorial plane of the star

(II) Accretion ⇨ 10^9 ***planetesimals*** from 100m to few km

(III) Merger and growth ⇨ *planetary embryos* ⇨ planets





From Bracco et al. (Phys. Fluids 1999)

Problem =
time scales ?

Dissipative range clustering

- Stokes drag $\Longrightarrow$ **dissipative dynamics**

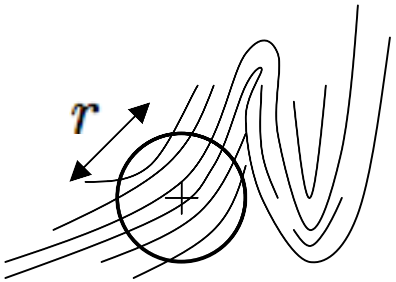
  At large times, particle trajectories converge to a dynamically evolving **attractor**, which is in general multifractal

- In other terms, the phase-space particle density $f(\boldsymbol{x}, \boldsymbol{v}, t)$ becomes singular in the asymptotic stationary regime

- Relevant tools: borrowed from dissipative dynamical systems (Lyapunov exponents, fractal dimensions, etc.)

Attractor and mass distribution

- Dissipative system $\Longrightarrow$ trajectories converge to a fractal attractor



$f(\boldsymbol{x},\boldsymbol{v},t)$ density of particles is singular

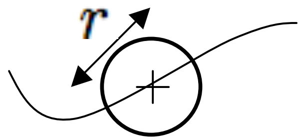
$\Longrightarrow$ need for characterizing the particle distribution in terms of **mass**.

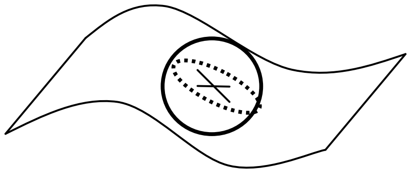
$$m_r(\boldsymbol{x},t) = \int_{|\boldsymbol{y}|<r} f(\boldsymbol{x}+\boldsymbol{y},t)\,dy \equiv$$ probability to have a particle in the ball of size r

x chosen as the position of a given particle

$$m_r(\boldsymbol{X}(t),t) = m_r(t)$$

Fractal dimensions

- Scale-dependence of the moments of mass $\langle m_r^p(t) \rangle$
  $\langle \cdot \rangle =$ average over all trajectories (i.e. w.r.t. the density f)
- Examples: particles uniformly distributed
  - on a **curve** 

    $m_r \propto r \quad \Rightarrow \quad \langle m_r^p \rangle \sim r^p$
  - on a **surface** 
    $m_r \propto r^2 \quad \Rightarrow \quad \langle m_r^p \rangle \sim r^{2p}$
  - on a **fractal set** $m_r \propto r^{\mathcal{D}} \quad \Rightarrow \quad \langle m_r^p \rangle \sim r^{p\mathcal{D}}$
- Generically, mass is not uniformly distributed on the set but there are fluctuations $\Rightarrow \quad \langle m_r^p \rangle \sim r^{p\mathcal{D}_{p+1}}$

$\mathcal{D}_p$ = **spectrum of dimension**
(Grassberger, Hentschel-Procaccia 1983)

$\mathcal{D}_1$ = information dimension $\qquad$ $\mathcal{D}_2$ = correlation dimension

Tangent system

- Linearization $\dot{X} = F(X,t) \;\Rightarrow\; \delta\dot{X} = D_X F(X(t),t)\delta X$

- **1D:** $\delta X(t) = \delta X(0)\,\mathrm{e}^{\int_0^t D_X F(s)\mathrm{d}s}$

$$\frac{1}{t}\ln\frac{|\delta X(t)|}{|\delta X(0)|} = \frac{1}{t}\int_0^t D_X F(s)\,\mathrm{d}s \xrightarrow[t\to\infty]{\text{Ergodicity}} \langle D_X F\rangle$$

- Generalization to **multi-dimensional** systems:

  Linearized system: $\delta\boldsymbol{X}(t) = \mathcal{W}_{0,t}\,\delta\boldsymbol{X}(0)$

  $\mathcal{W}_{0,t}^{\mathsf{T}}\,\mathcal{W}_{0,t}$ symmetric positive matrix diagonalizes in $\mathcal{Q}_t^{\mathsf{T}}\,\Lambda_t\,\mathcal{Q}_t$

  with $\Lambda_t = \mathrm{diag}[\mathrm{e}^{t\rho_1(t)},\ldots,\mathrm{e}^{t\rho_d(t)}]$ $\quad \rho_1(t) > \cdots > \rho_d(t)$

  $\rho_i(t)$ = stretching rates (or finite-time Lyapunov exponents)

Lyapunov exponents

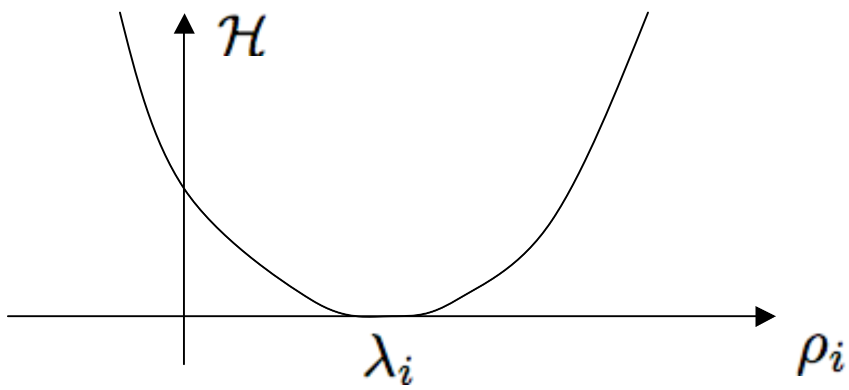
- **Oseledets** ergodic theorem: $\rho_i(t) \to \lambda_i$ as $t \to \infty$
  $\lambda_1, \ldots, \lambda_d$ = Lyapunov exponents
  ($\approx$ law of large numbers)

- **Large deviations** of the stretching rates

$$p_t(\rho_1, \ldots, \rho_d) \propto \mathrm{e}^{-t\mathcal{H}(\rho_1-\lambda_1,\ldots,\rho_d-\lambda_d)}$$

  $\mathcal{H}$ = **rate function**

  convex, attaining its minimum (equal to 0) at 0



Lyapunov exponents and mass distribution

- λ_1 = growth rate of an infinitesimal segment

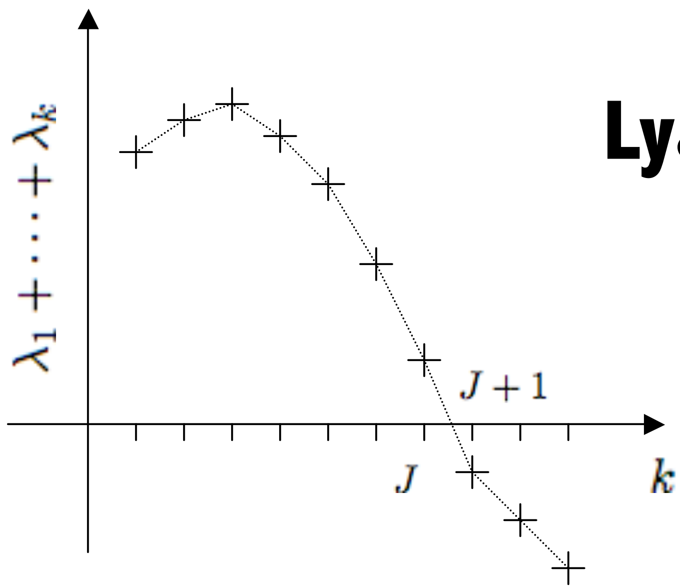
  $\lambda_1 + \lambda_2$ = growth rate on an infinitesimal surface

  $\lambda_1 + \lambda_2 + \lambda_3$ = growth rate on an infinitesimal volume

  $\ldots$

  $\lambda_1 + \cdots + \lambda_d$ = growth rate of phase-space volumes

- Chaotic dissipative systems: $\begin{cases} \lambda_1 > 0 \\ \lambda_1 + \cdots + \lambda_d < 0 \end{cases}$



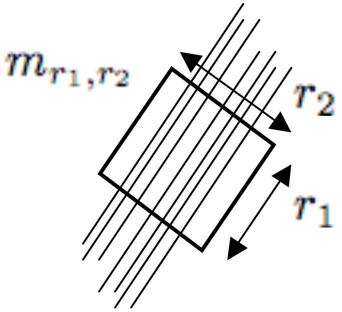
Lyapunov dimension (Kaplan & Yorke, 1979)

$$\mathcal{D}_{\mathrm{KY}} = J + \frac{\lambda_1 + \cdots + \lambda_J}{|\lambda_{J+1}|}$$

Under some hypotheses: $\mathcal{D}_{\mathrm{KY}} = \mathcal{D}_1$

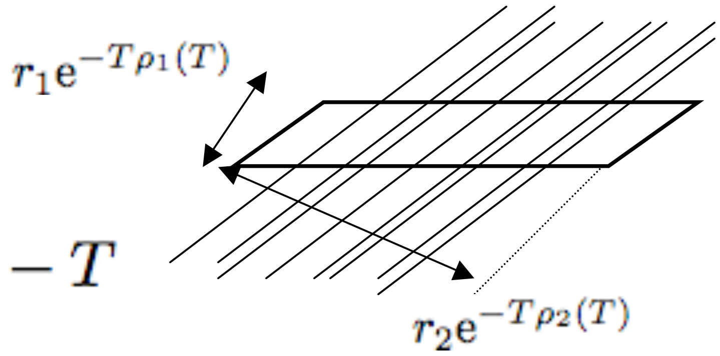
(Ledrappier & Young, 1988)

Generalization to the dimension spectrum



$$\langle m^p_{r_1,r_2}\rangle \propto r_1^p r_2^{p(\mathcal{D}_{p+1}-1)}$$

$$t \longleftarrow t-T$$



- Mass conservation: $m_{r_1,r_2} = m_{r_1\mathrm{e}^{-T\rho_1},r_2\mathrm{e}^{-T\rho_2}}$

  "Markovianity" $\langle m^p_{r_1,r_2}\rangle = \langle\langle m^p_{r_1\mathrm{e}^{-T\rho_1},r_2\mathrm{e}^{-T\rho_2}}\rangle_{]-\infty,t-T]}\rangle_{[t-T,t]}$

  $r_1^p r_2^{p(\mathcal{D}_{p+1}-1)} \sim \langle r_1^p \mathrm{e}^{-pT\rho_1} r_2^{p(\mathcal{D}_{p+1}-1)} \mathrm{e}^{-p(\mathcal{D}_{p+1}-1)T\rho_2}\rangle_{[t-T,t]}$

  $\Rightarrow \langle \mathrm{e}^{-pT[\rho_1+(\mathcal{D}_{p+1}-1)\rho_2]}\rangle \sim \mathrm{const}$

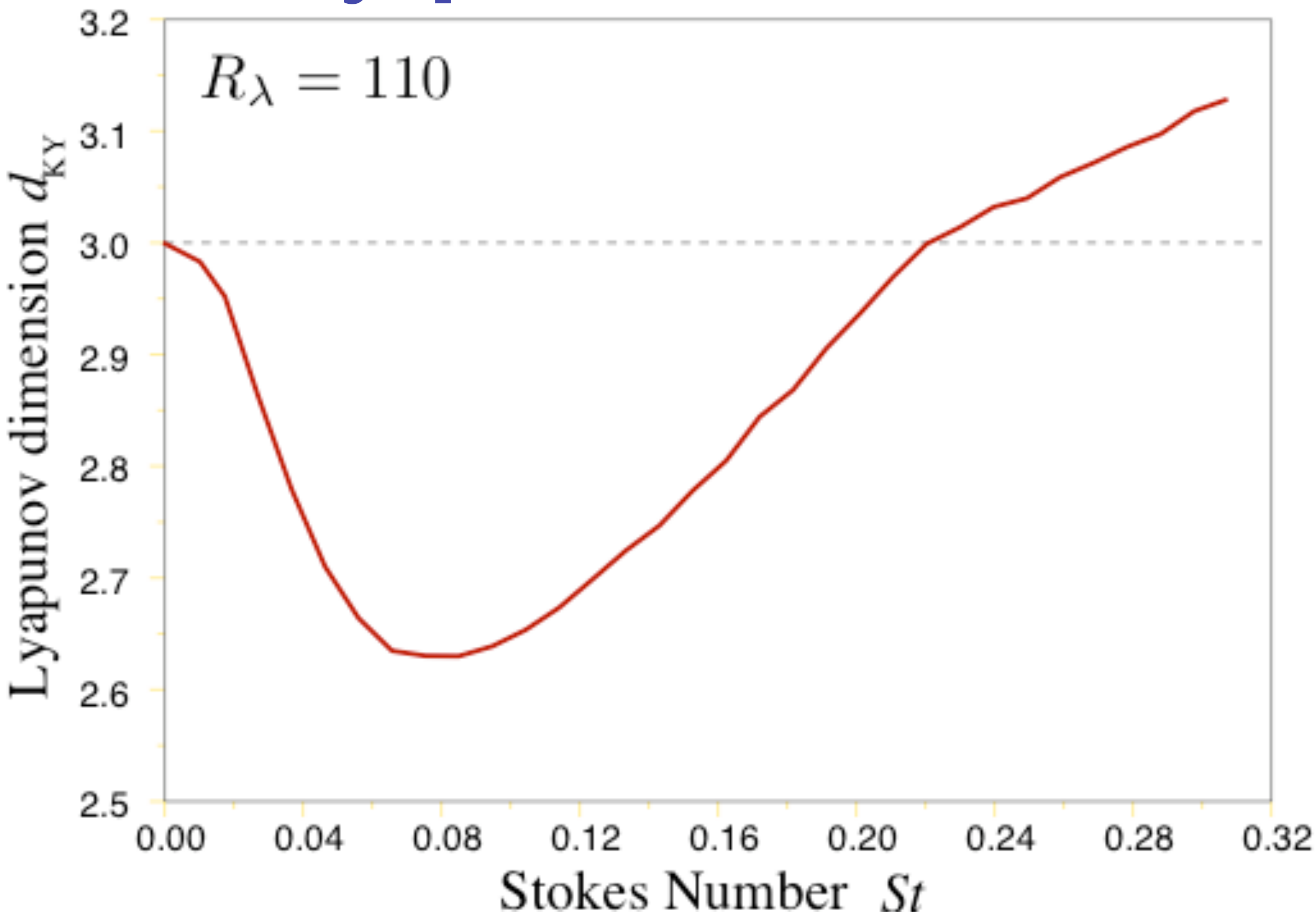
- Large deviations: T large $\Rightarrow$ $p_T(\rho_1,\rho_2) \propto \mathrm{e}^{-T\mathcal{H}(\rho_1-\lambda_1,\rho_2-\lambda_2)}$

$$\int \mathrm{e}^{-T[p\rho_1+p(\mathcal{D}_{p+1}-1)\rho_2+\mathcal{H}(\rho_1-\lambda_1,\rho_2-\lambda_2)]}\mathrm{d}\rho_1\mathrm{d}\rho_2 \sim \mathrm{const}$$

Saddle point: $\min\limits_{\rho_1,\rho_2}[p\rho_1 + p(\mathcal{D}_{p+1}-1)\rho_2 + \mathcal{H}(\rho_1-\lambda_1,\rho_2-\lambda_2)] = 0$

(JB, Gawedzki & Horvai, 2004)

Lyapunov dimension

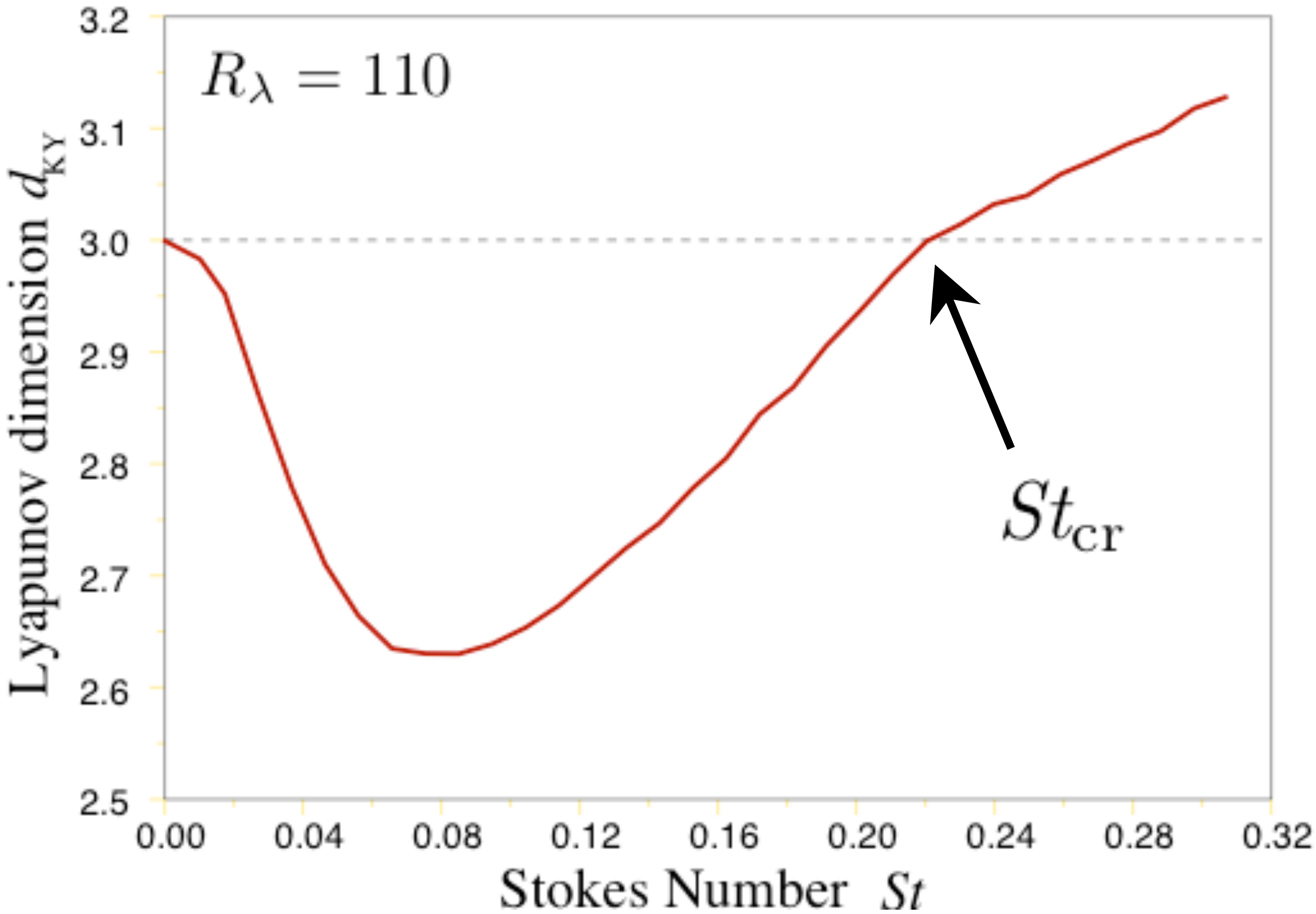


Asymptotics: $\mathrm{St} \ll 1 \qquad \mathcal{D}_{KY} \simeq d - C\,\mathrm{St}^2$

$\mathrm{St} \gg 1 \qquad \mathcal{D}_{KY} \to 2d$

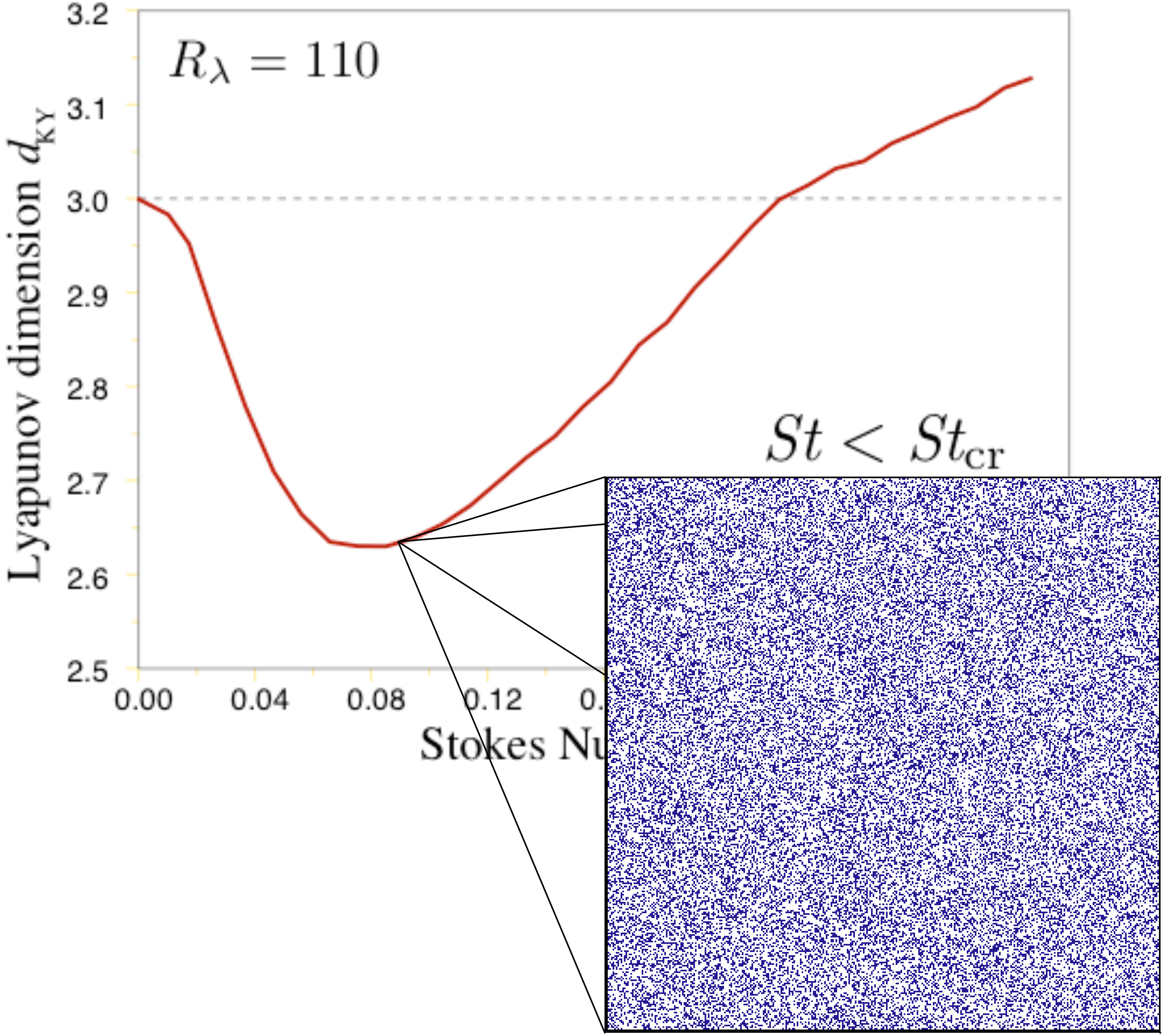
Falkovich *et al.* (2001)

Lyapunov dimension

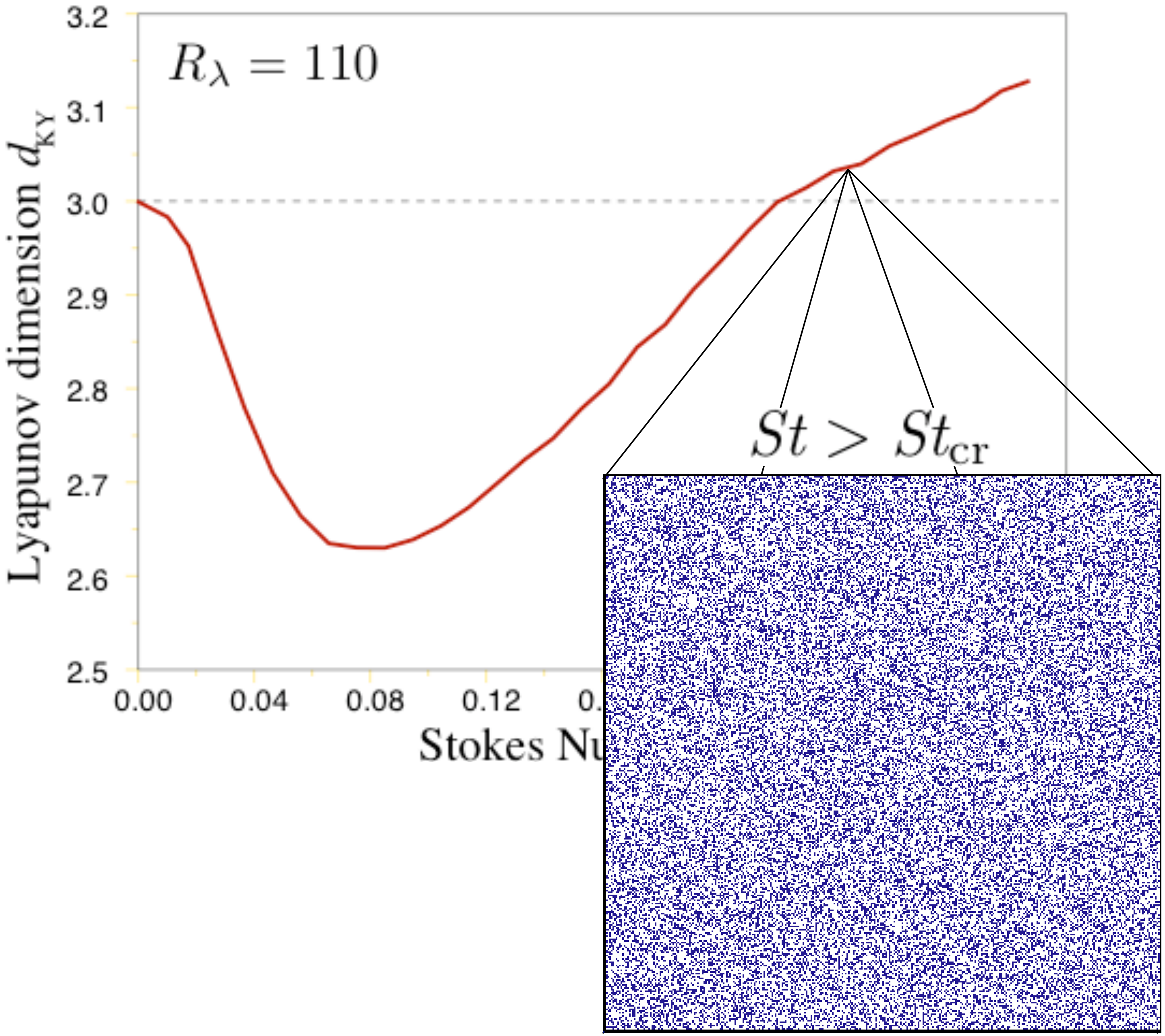


Threshold in Stokes number for the presence of fractal clusters in physical space (JB 2003)

Lyapunov dimension

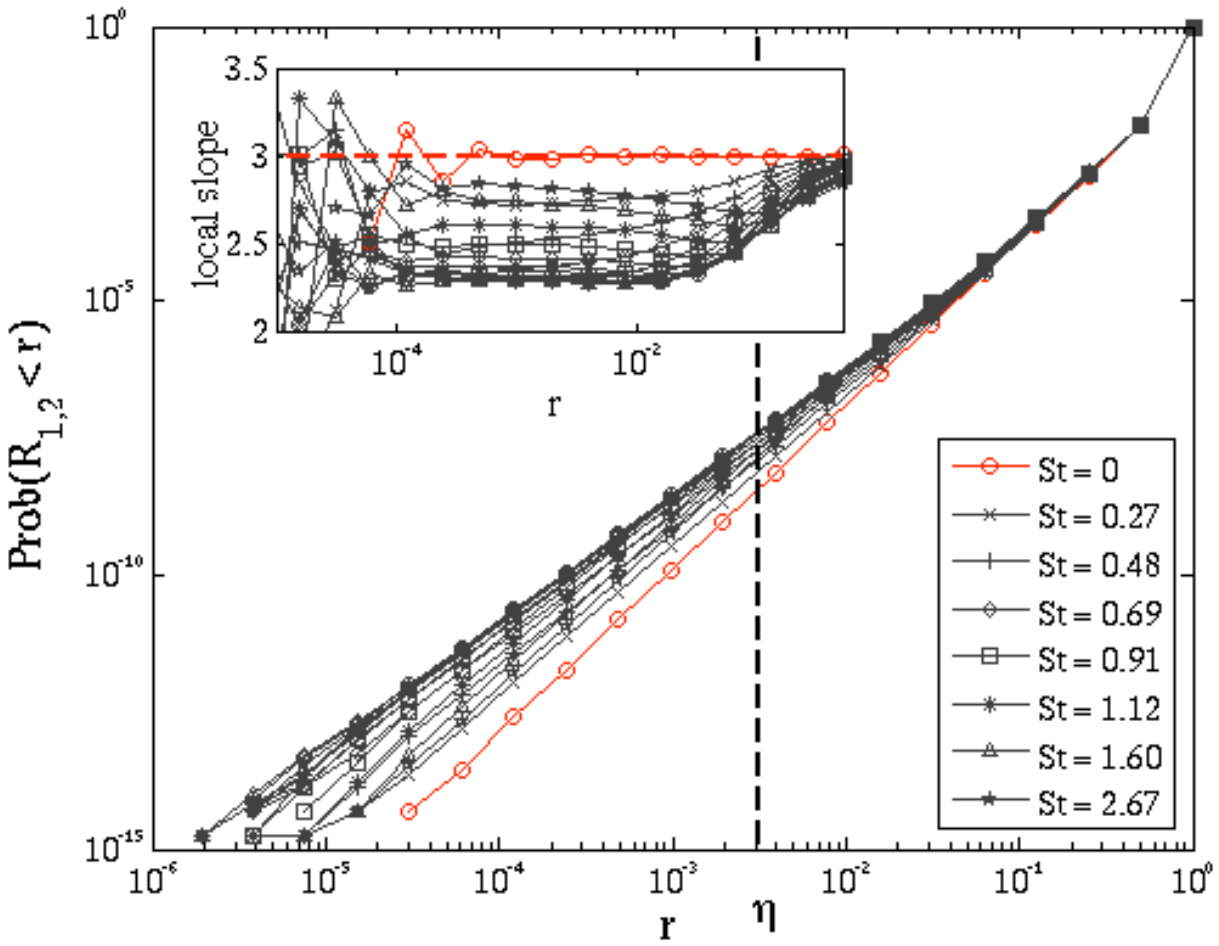


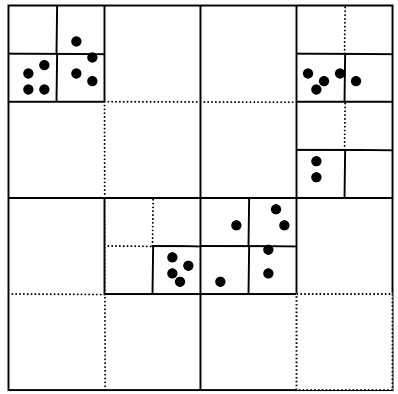
Lyapunov dimension



Correlation dimension

- Estimated from $\mathrm{Prob}(|\boldsymbol{R}| < r) \propto r^{\mathcal{D}_2}$

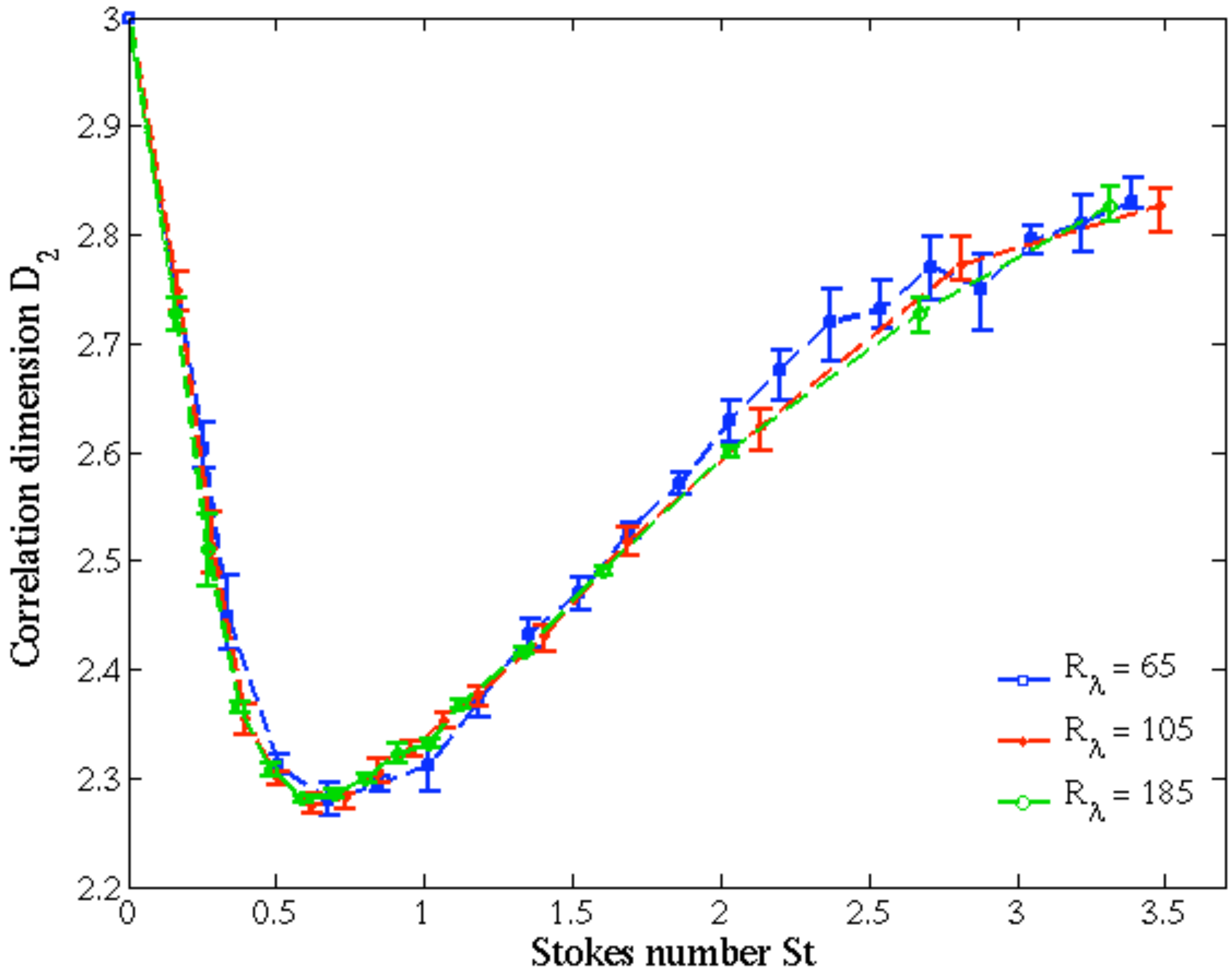




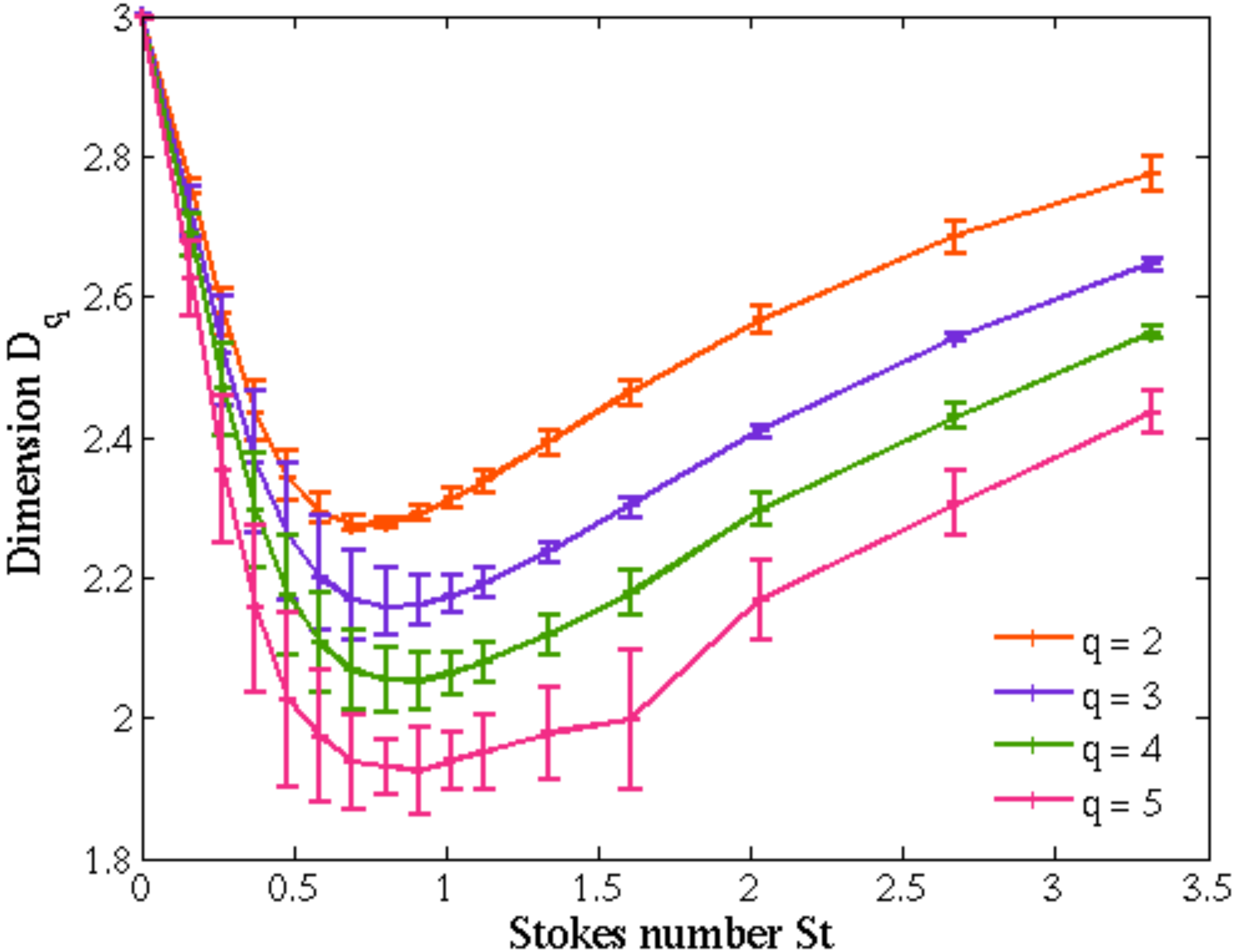
Use of a tree algorithm to measure dimensions at scales $\ll \eta$

DNS (JB, Biferale, Cencini, Lanotte, Musacchio & Toschi, 2007)

Correlation dimension



Multifractal distribution



$$\mathcal{D}_p \neq \mathcal{D}_q$$

$$\Downarrow$$

Intermittency in the mass distribution

Kraichnan flow

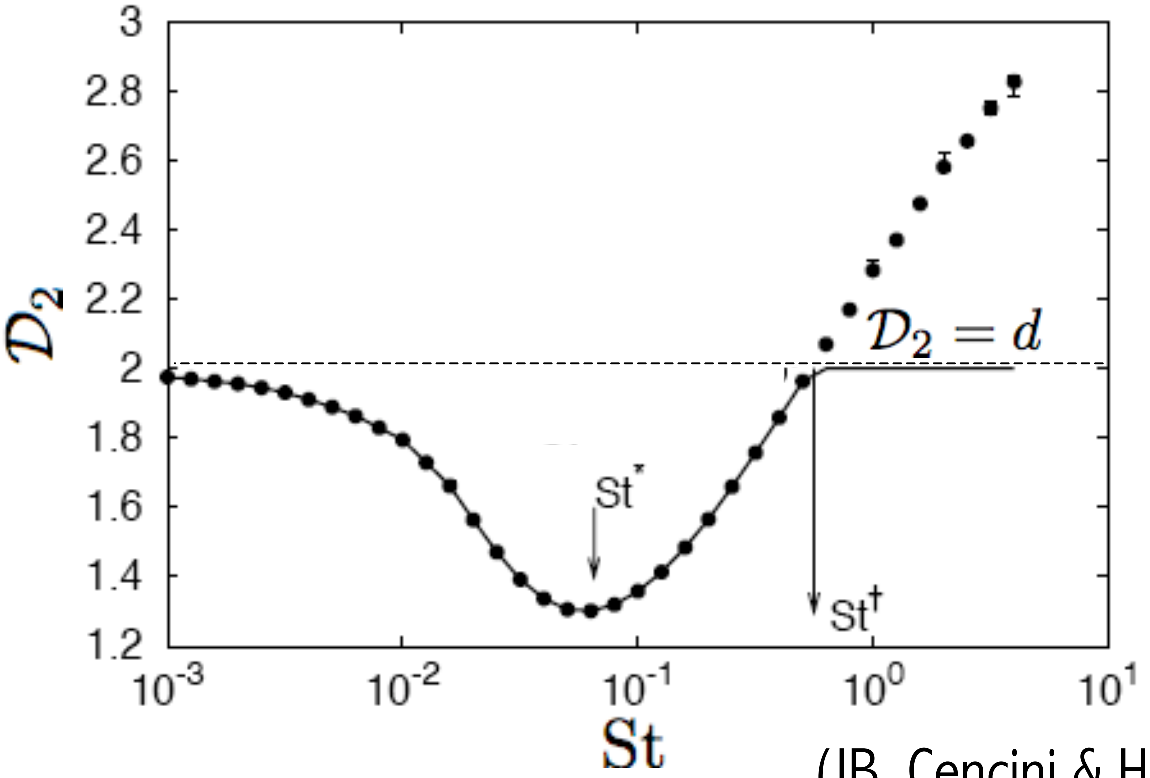
- Gaussian carrier flow with no time correlation

  Incompressible, homogeneous, isotropic

$$\langle u_i(\boldsymbol{x},t)\, u_j(\boldsymbol{x}',t')\rangle = [2D_0\delta_{ij} - B_{ij}(\boldsymbol{x}-\boldsymbol{x}')]\,\delta(t-t')$$

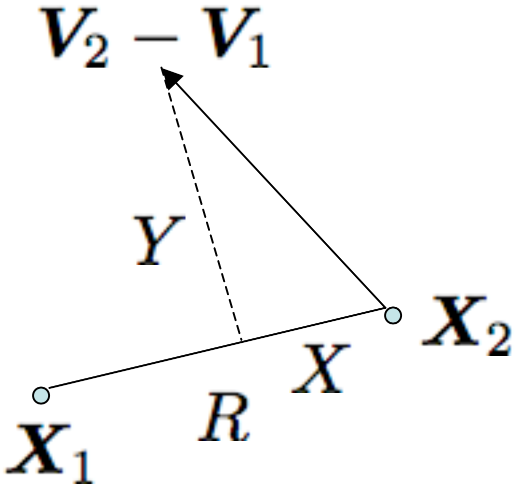
$$B_{ij}(\boldsymbol{r}) \simeq D_1\ \left[(d+1)\,r^2\delta_{ij} - 2\,r_i r_j\right]$$

- No structure $\Longrightarrow$ effect of dissipative dynamics isolated



(JB, Cencini & Hillerbrand, 2007)

Reduction of the dynamics

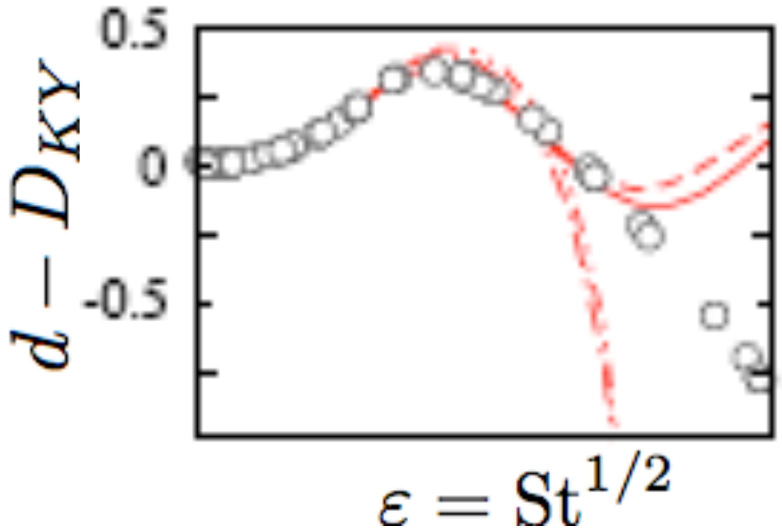


$$
\begin{aligned}
\mathrm{d}X &= -\left[X + X^2 - Y^2\right]\mathrm{d}s + \sqrt{2\mathrm{St}}\,\mathrm{d}B_1 \\
\mathrm{d}Y &= -\left[Y + 2X\,Y\right]\mathrm{d}s + \sqrt{6\mathrm{St}}\,\mathrm{d}B_2 \\
\mathrm{d}R &= X\,R\,\mathrm{d}s
\end{aligned}
$$

with $\mathrm{St} = D_1\tau$

- Lyapunov exponent $\lambda_1 = \langle X \rangle / \tau$

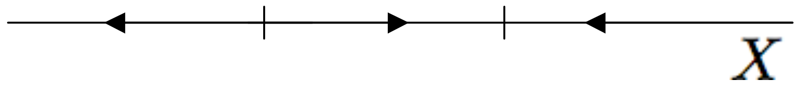
Expansion in powers of the Stokes number
$=$ diverging series $\Longrightarrow$ Borel resummation
Duncan, Mehlig, Östlund & Wilkinson (2005)



"Solvable" cases

- **One dimension** (Wilkinson & Mehlig 2004, Derevyanko et al. 2006)

  Potential $U(X) = \dfrac{X^2}{2} + \dfrac{X^3}{3}$



  Constant flux solution $p_t(\rho_1) \propto \mathrm{e}^{-tD_1\mathrm{St}^{-2/3}h(\mathrm{St}^{2/3}\rho_1/D_1)}$

  Lyapunov exponent: $\lambda_1 = \dfrac{D_1}{2}\left[-1 + \dfrac{\mathrm{St}}{\alpha}\dfrac{\mathrm{Ai}'(\alpha^2/\mathrm{St}^2)}{\mathrm{Ai}(\alpha^2/\mathrm{St}^2)}\right]$

- **Large-Stokes asymptotics**

  (Horvai nlin.CD/0511023)

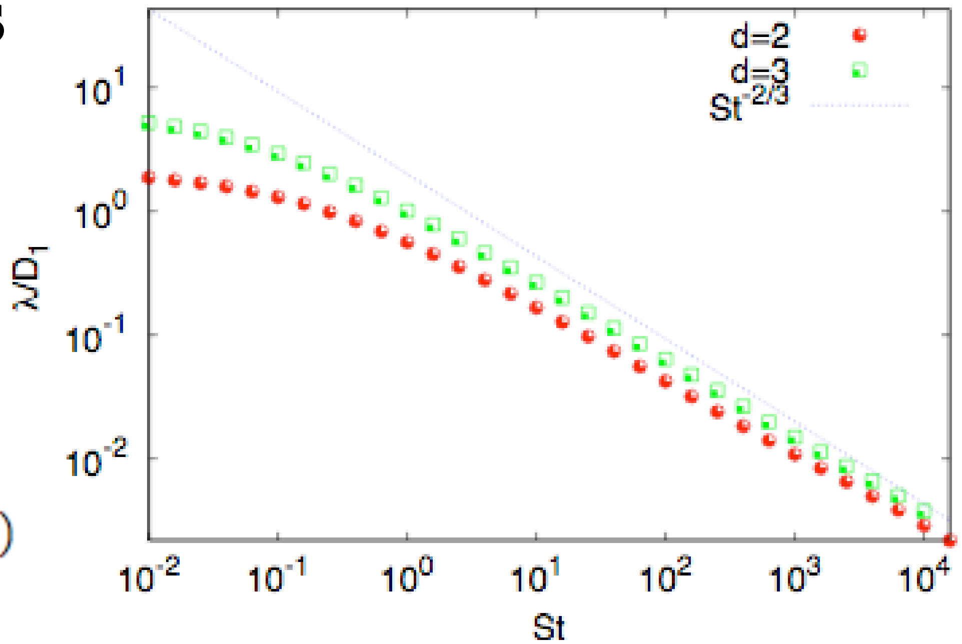
$$\lambda_1 \propto D_1\mathrm{St}^{-2/3}$$

\+ same for stretching rate

$$\rho_1(t) = \frac{1}{t}\ln[|\boldsymbol{R}(t)|/|\boldsymbol{R}(0)|]$$

$$p_t(\rho_1) \propto \mathrm{e}^{-tD_1\mathrm{St}^{-2/3}h(\mathrm{St}^{2/3}\rho_1/D_1)}$$

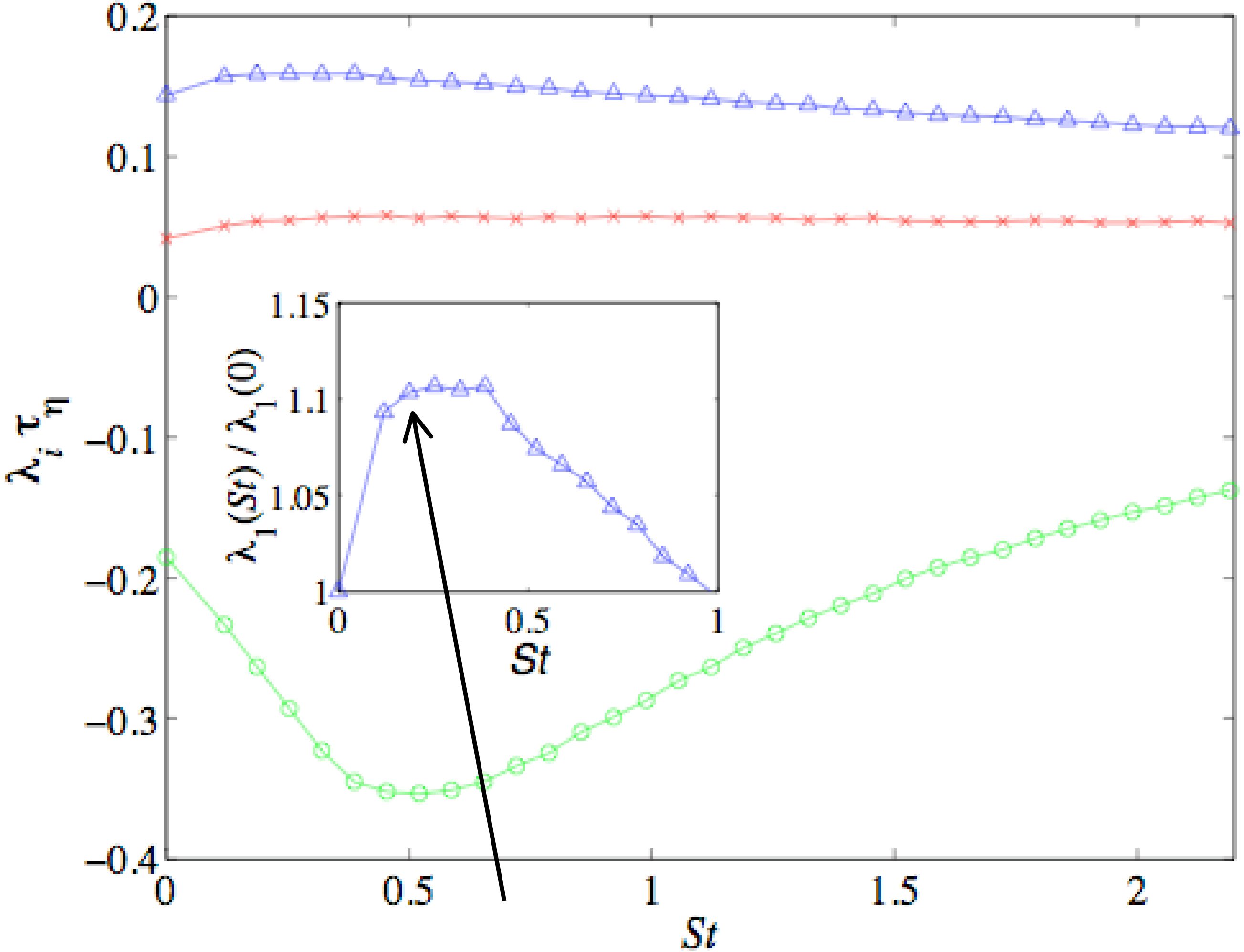
(JB, Cencini & Hillerbrand, 2007)



Lyapunov exponent in DNS

$R_\lambda \approx 185$ JB, Biferale, Boffetta, Cencini, Musacchio & Toschi (2006)

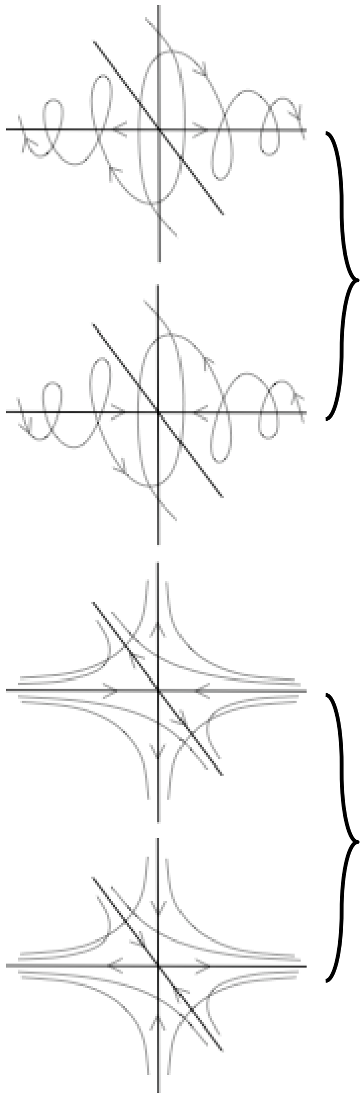
Same qualitative picture



Increase of chaoticity not reproduced in δ-correlated flows

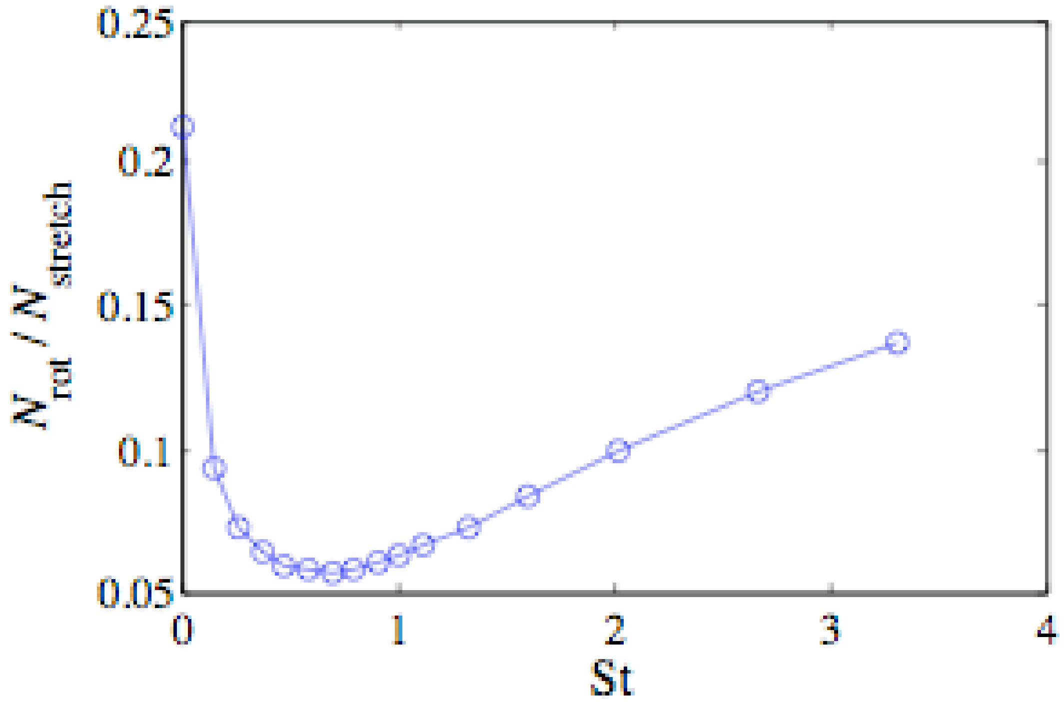
Correlation with the local flow structure

- Depending whether the eigenvalues of the strain matrix are real or complex conjuguate: different local structure



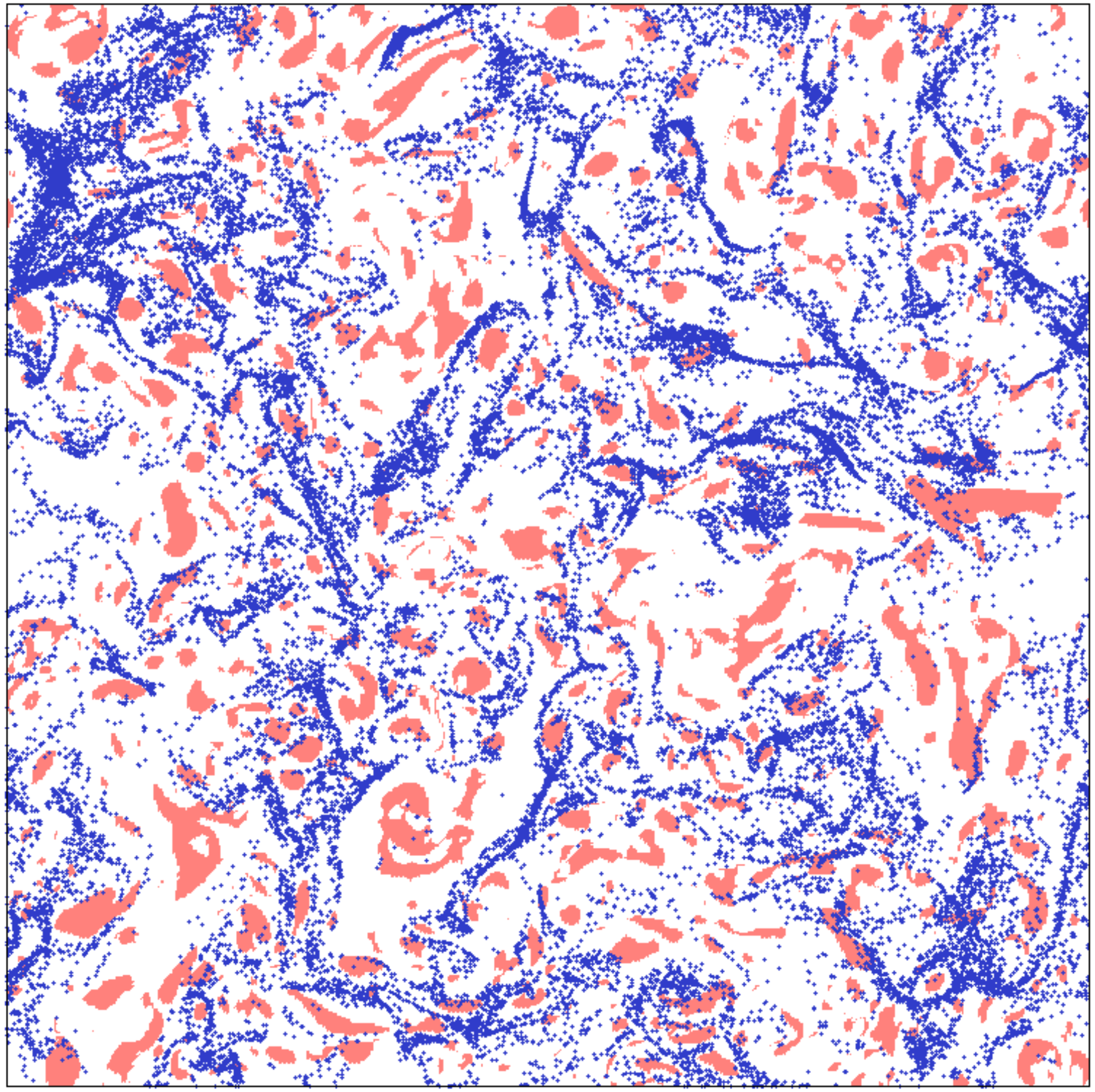
Rotating regions

Strain-dominated regions



Inertial range clustering

$\approx 30\eta$



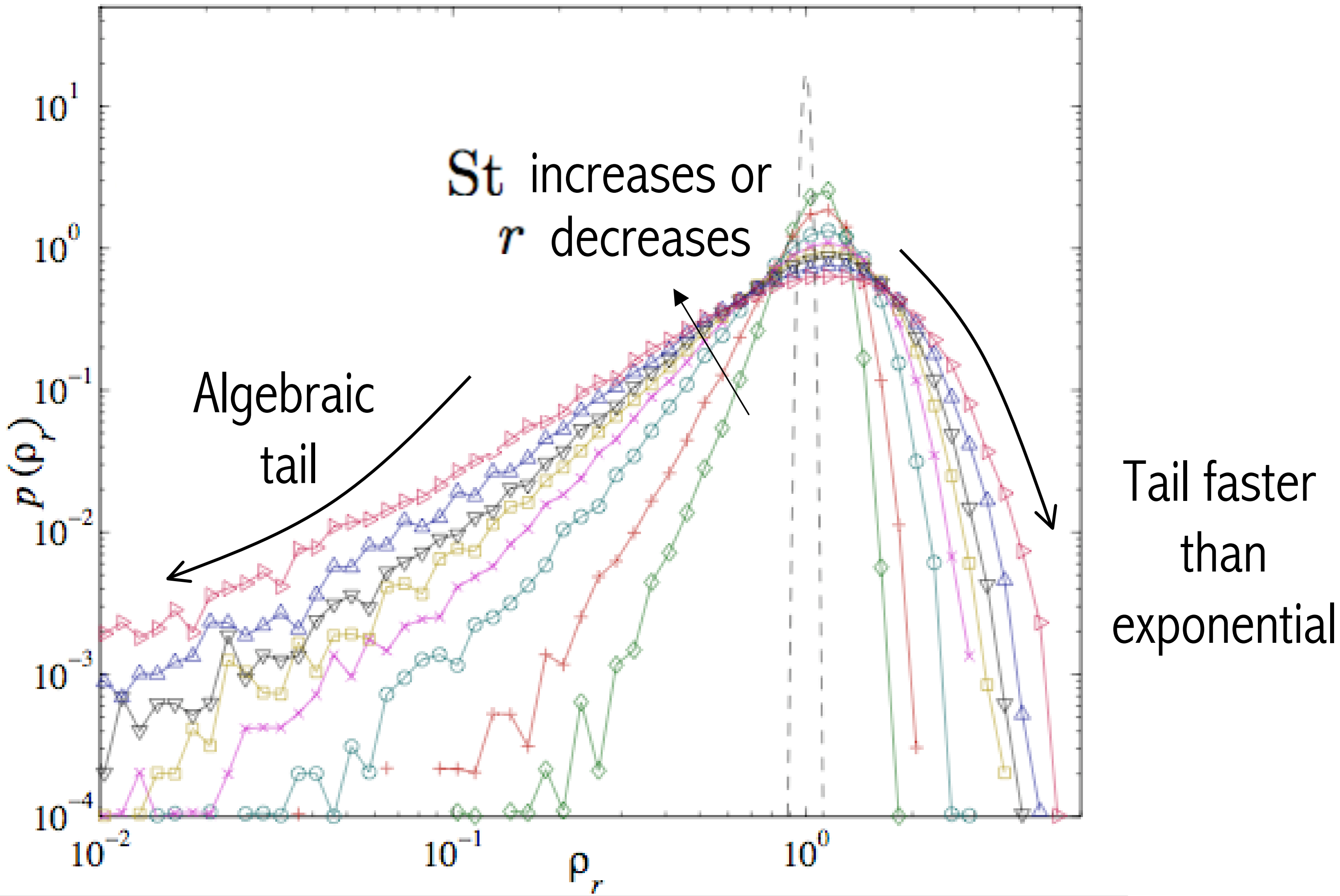
$\mathrm{St} = 1.34$

Slice width
$\approx 2.5\eta$

Red = rotating regions

Coarse-grained mass

- $\rho_r = m_r / r^3$ = density averaged in a box of size r



Small Stokes / Large box scaling

- The two limits $\tau_s \to 0$ and $r \to \infty$ are equivalent
- Naïve idea: **Local Stokes number**

$$St(r) = \frac{\tau_s}{\varepsilon^{-1/3} r^{2/3}}$$

works in random self-similar flows

- Actually, scaling determined by the **increments of pressure**:
Small inertia: Maxey's approximation

$$\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X}, t) = \tau \ddot{\boldsymbol{X}} \approx \boldsymbol{a}(\boldsymbol{X}, t)$$

$\Longrightarrow$ synthetic compressible flow:

$$\dot{\boldsymbol{X}} \approx \boldsymbol{v}(\boldsymbol{X}, t) \qquad\qquad \boldsymbol{v} = \boldsymbol{u} - \tau_s(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u})$$

Small Stokes / Large box scaling

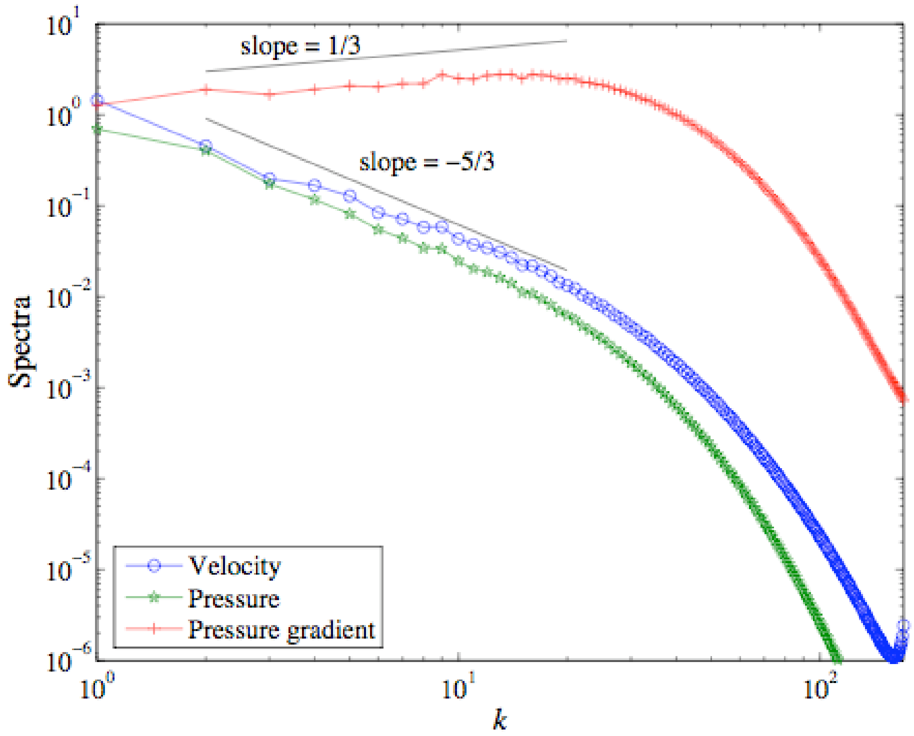
- Relevant time scale for the time evolution of a blob of particles

$$\Gamma = \frac{1}{r^3}\int_{B_r} \nabla\cdot\boldsymbol{v}\, d^3x \sim -\frac{\tau_s}{r}\Delta_r \nabla p$$

- Dimensional analysis: $\Delta_r \nabla p \sim \varepsilon^{2/3} r^{-1/3}$

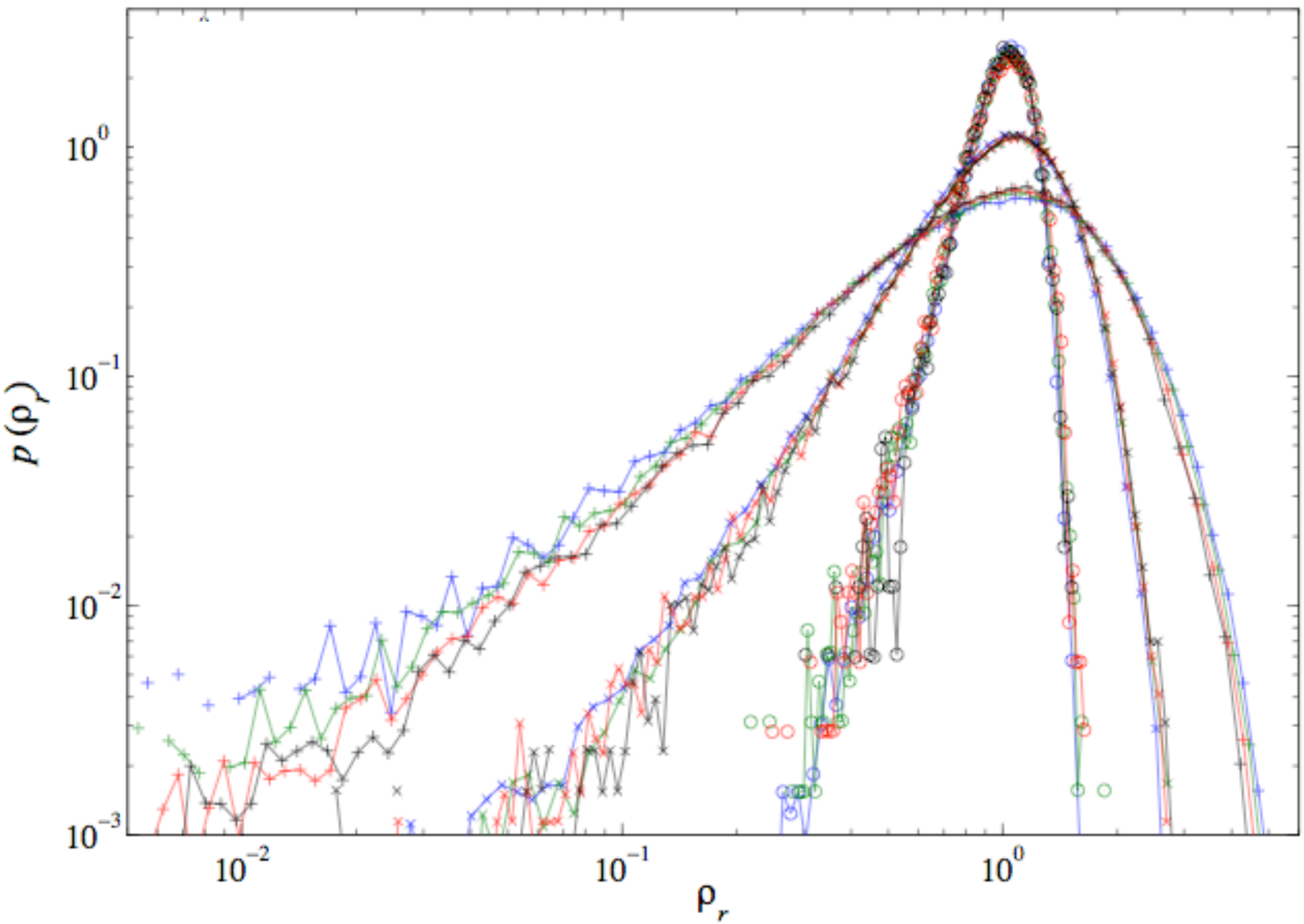
  Observed: scaling dominated by sweeping

$$\Delta_r \nabla p \sim U\varepsilon^{1/3} r^{-2/3} \quad \text{so that} \quad \boxed{\Gamma \propto \tau_s r^{-5/3}}$$



Small Stokes / Large box scaling

- The density distribution depends only on $\Gamma \propto \tau_s \, r^{-5/3}$



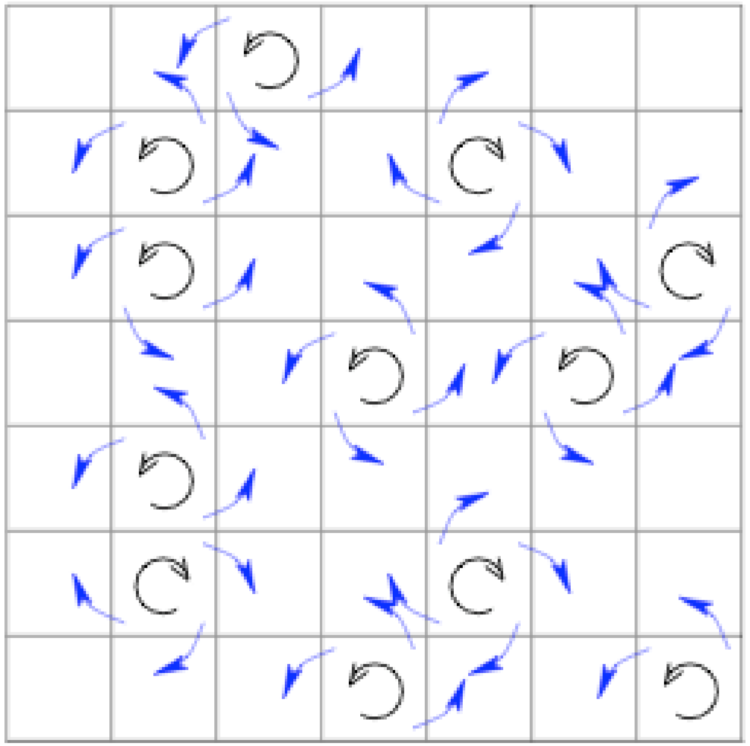
Modeling mass dynamics

(JB & R. Chétrite, 2007)

- **Motivation:** understand how universal is the shape of the mass distribution observed

- Flow divided in cells. With a probability p the cells are rotating and eject particles to their non-rotating neighbors

- Each cell contains a continuous mass m_j of particles



Motivations for such a model

- Two dimensions: approximation = piecewise linear strain
  Particle dynamics in a constant vorticity ω

$$\frac{d^2\boldsymbol{X}}{dt^2} = -\frac{1}{\tau}\frac{d\boldsymbol{X}}{dt} + \frac{\omega}{\tau}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\boldsymbol{X}$$

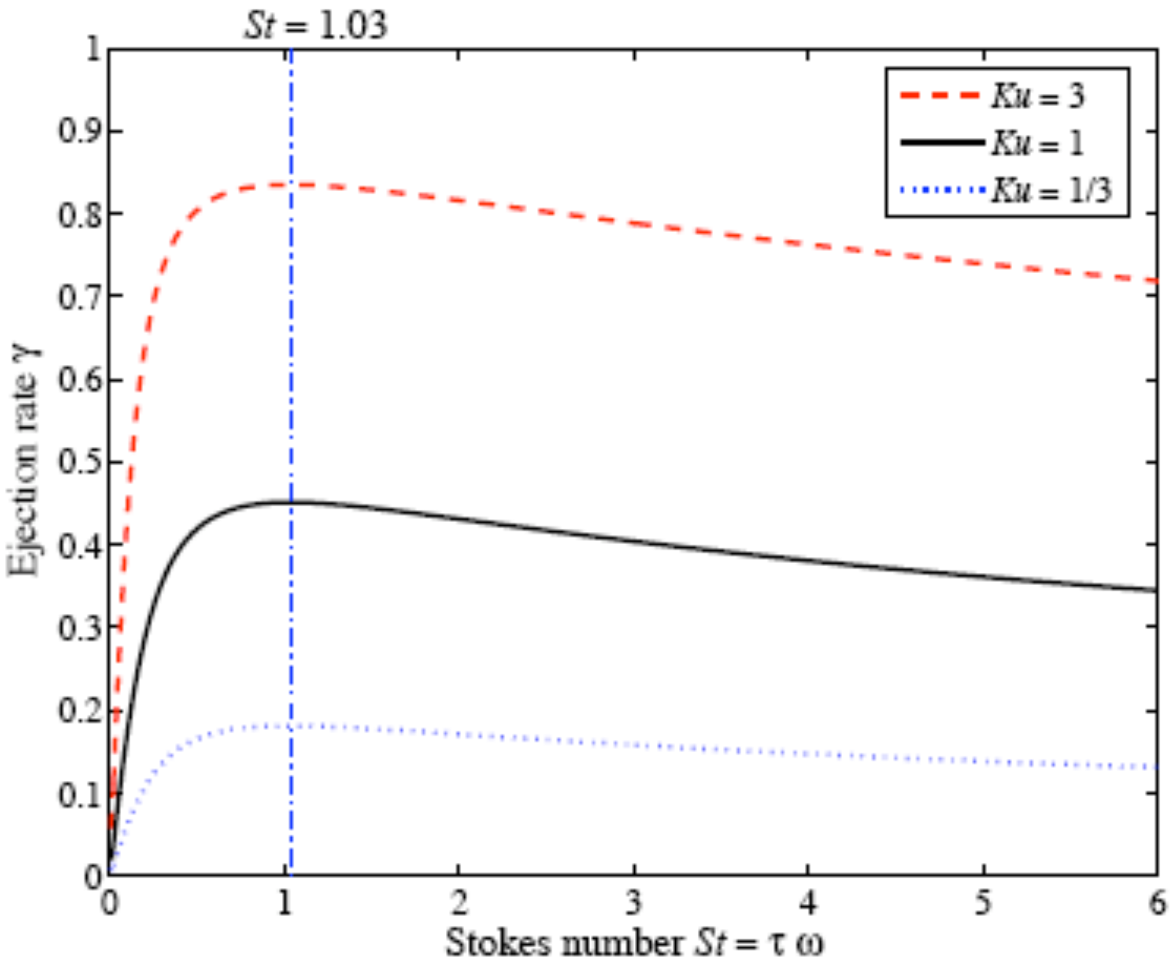
- The distance from the center of the cell increases exponentially with rate $\mu = \dfrac{-1 + \frac{1}{2}\sqrt{2\sqrt{1+16\tau^2\omega^2}+2}}{2\tau}$
- The mass in the cell of size decreases exponentially in time

$$m(T) = m(0)\,(1-\gamma) = m(0)\,\exp\left[-\frac{T}{\tau}\left(-1+\frac{1}{2}\sqrt{2\sqrt{1+16\tau^2\omega^2}+2}\right)\right]$$

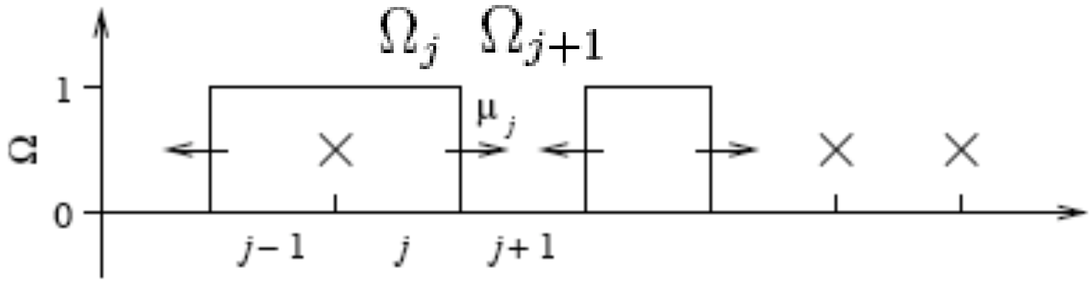
Ejection rate vs. Stokes number

$$\boxed{\gamma = 1 - \exp\left[-\frac{Ku}{St}\left(-1 + \frac{1}{2}\sqrt{2\sqrt{1+16St^2}+2}\right)\right]}$$

$$\begin{cases} \text{Stokes } St = \tau\omega \\ \text{Kubo } Ku = T\omega \end{cases}$$



One-dimensional version of the model



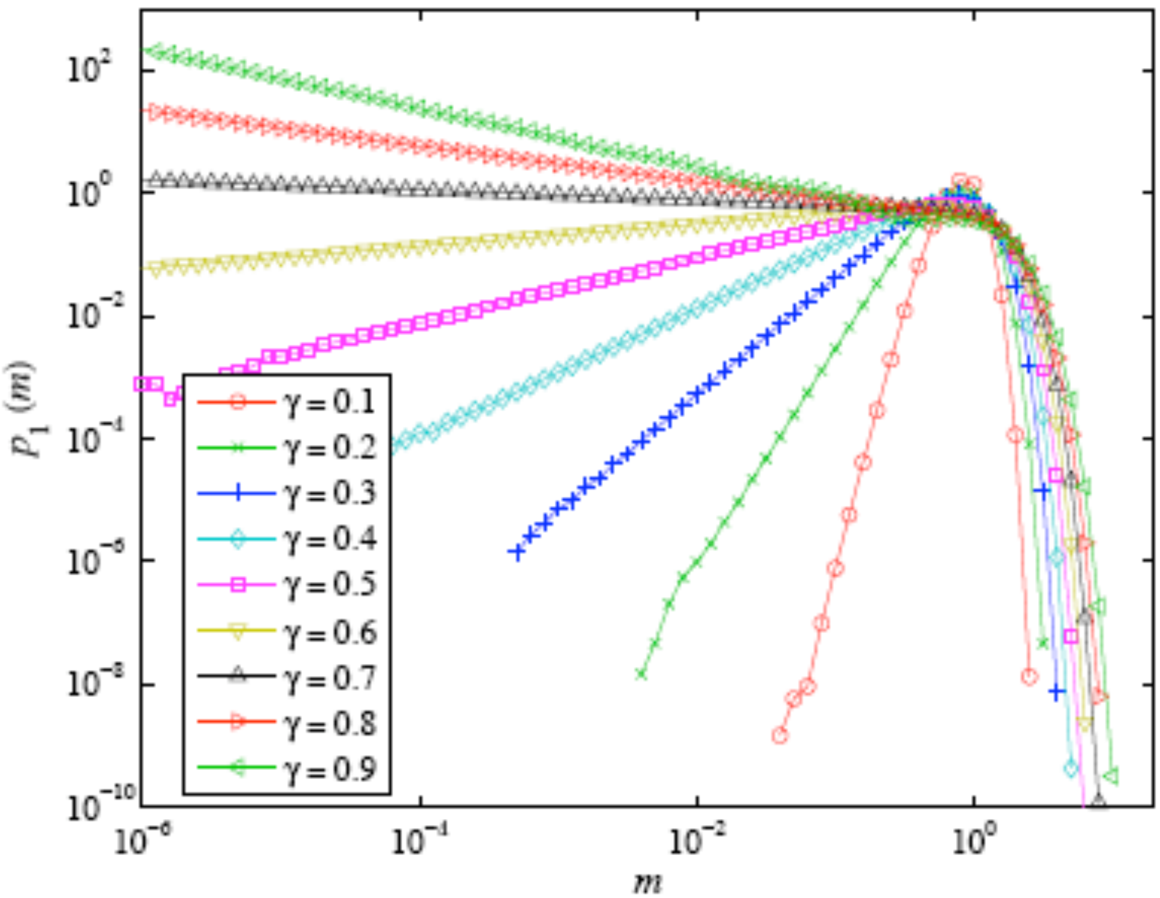
$$m_j(n+1) = \begin{cases} m_j(n) - \frac{\gamma}{2}\,[2 - \Omega_{j-1} - \Omega_{j+1}]\, m_j(n) & \text{if } \Omega_j = 1 \\ m_j(n) + \frac{\gamma}{2}\,[\Omega_{j-1}\, m_{j-1}(n) + \Omega_{j+1}\, m_{j+1}(n)] & \text{if } \Omega_j = 0 \end{cases}$$

Steady state given by stationary solutions to the Markov equation

$$\begin{aligned} p_1(m) = {} & \left[p^3 + (1-p)^3\right] p_1(m) + \frac{2p^2(1-p)}{1-\gamma/2}\, p_1\left(\frac{m}{1-\gamma/2}\right) \\ & + \frac{p(1-p)^2}{1-\gamma}\, p_1\left(\frac{m}{1-\gamma}\right) + 2p(1-p)^2 \int_0^{2m/\gamma} dm'\, p_2\left(m', m - \frac{\gamma}{2} m'\right) \\ & + p^2(1-p) \int_0^{2m/\gamma} dm' \int_0^{2m/\gamma - m'} dm''\, p_3\left(m', m - \frac{\gamma}{2}(m' + m''), m''\right) \end{aligned}$$

One-cell mass distribution

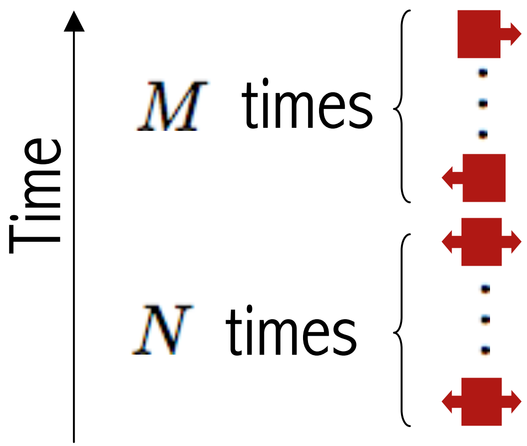
PDF of m_j very similar to that obtained in DNS (same tails)



Left tail

Algebraic behavior $p(m) \propto m^{\alpha(\gamma)}$ when $m \ll 1$

Mass-ejecting realization



Final mass $m_j \approx (1-\gamma)^N (1-\gamma/2)^M$

Initial mass $m_j \approx 1$

$$Prob = \left[p(1-p)^2\right]^N \left[p(1-p)\right]^M$$

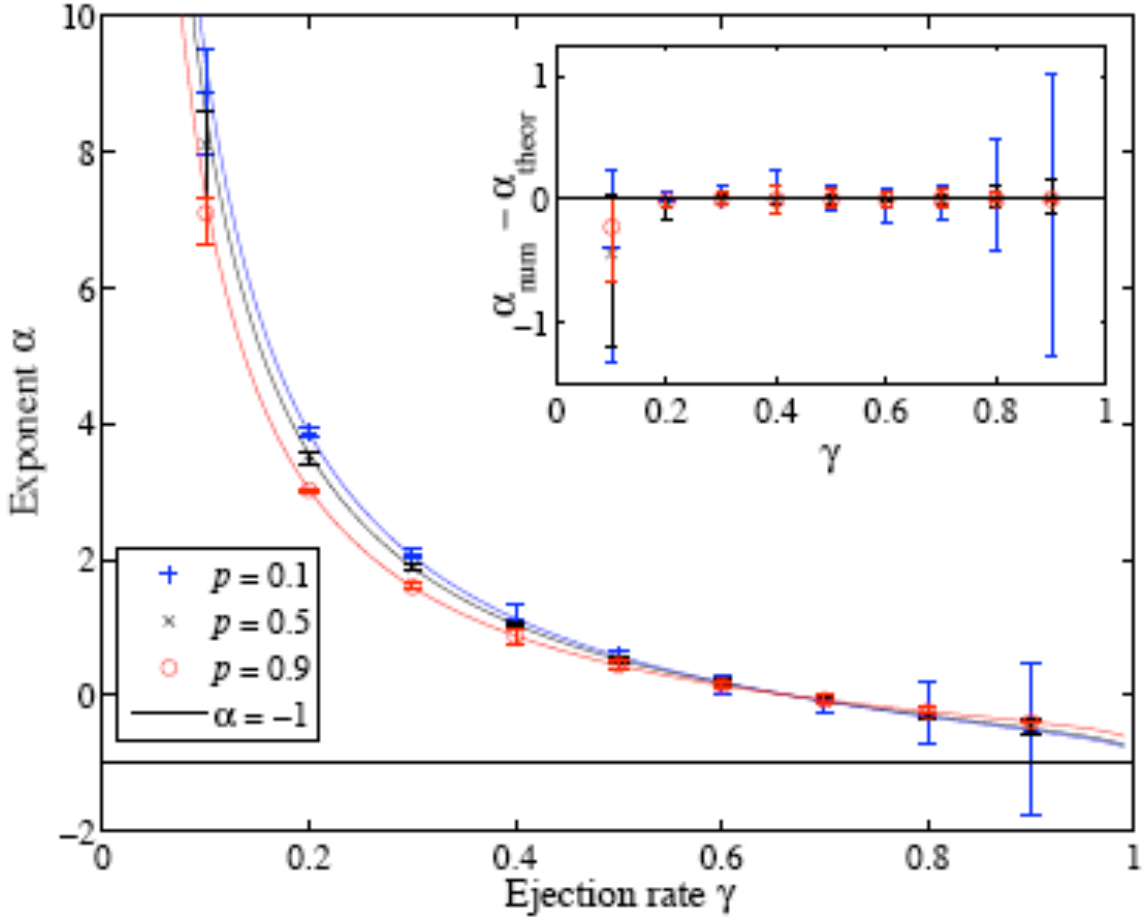
Requirements that mass $m_j = m$ and that $Prob$ is maximal

$\Longrightarrow$ Leading behavior at small masses

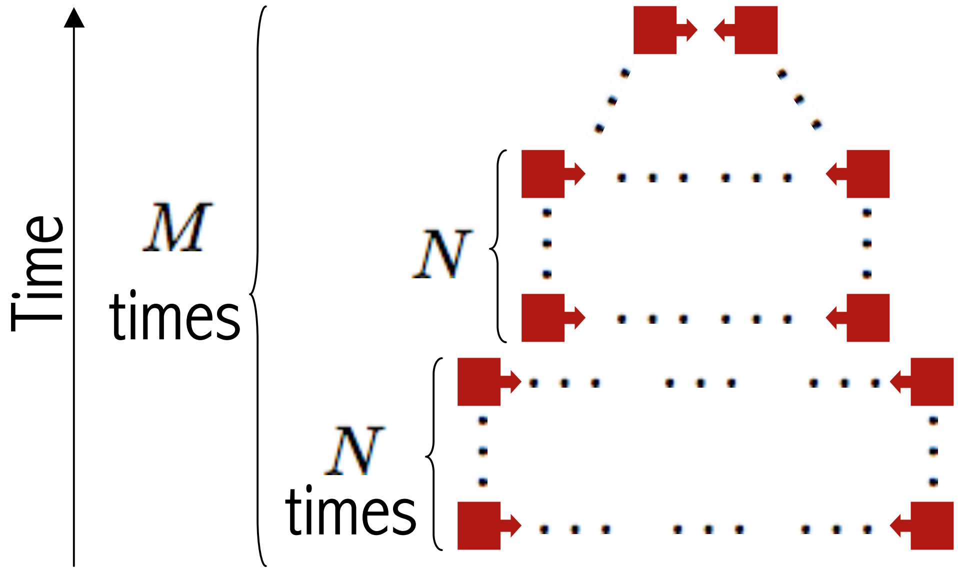
Left tail

$\alpha(\gamma, p)$ solution of

$$\frac{2p}{(1-\gamma/2)^{\alpha+1}} + \frac{(1-p)}{(1-\gamma)^{\alpha+1}} = 3$$



Right tail



Transferred mass

$$m = \frac{1 - \left[1 - (1-\gamma/2)^N\right]^M}{(1-\gamma/2)^N}$$

$$M = M(m, N) = \frac{\log\left[1 - m(1-\gamma/2)^N\right]}{\log\left[1 - (1-\gamma/2)^N\right]}$$

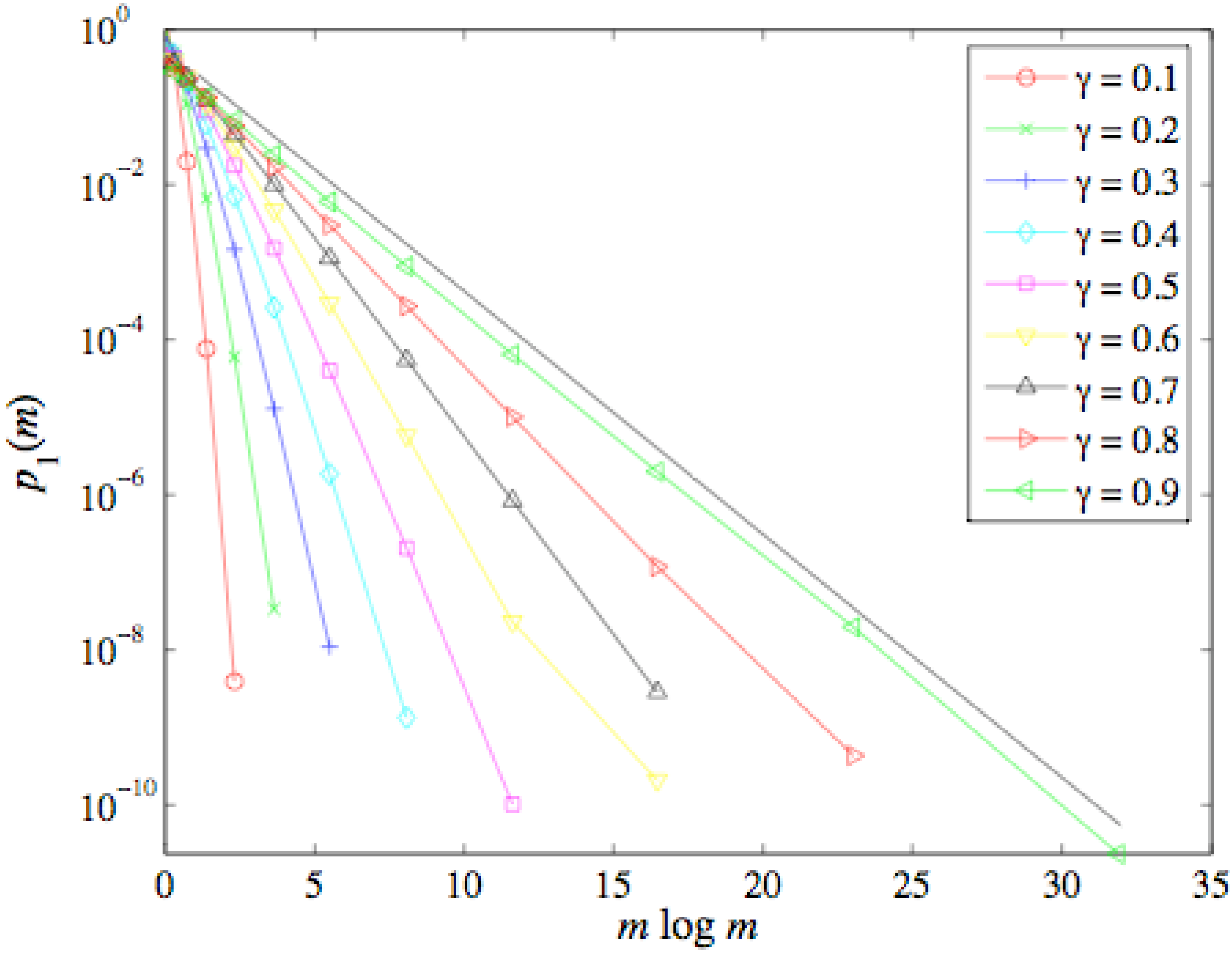
Probability of such configurations

$$\mathcal{P} = \left[p^2(1-p)\right]^{N\,M} = \exp\left[\log(p^2(1-p))\,N\,M(m,N)\right]$$

Dominant contribution given by choosing N such that $\mathcal{P}$ is maximal

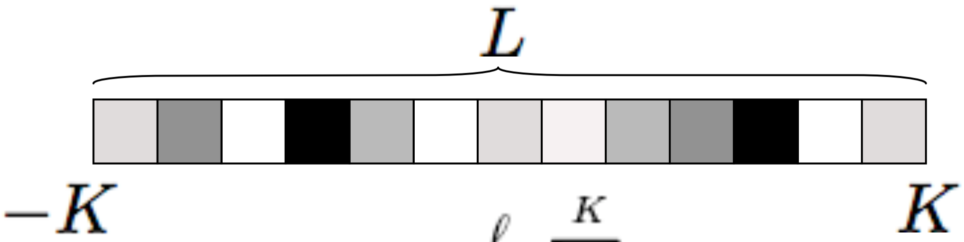
$$N^\star \propto \log m \quad \Rightarrow \quad \boxed{p(m) \propto \exp(-C\,m\,\log m)}$$

Right tail



10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$p_1(m)$
0
5
10
15
20
25
30
35
$m \log m$
$\gamma = 0.1$
$\gamma = 0.2$
$\gamma = 0.3$
$\gamma = 0.4$
$\gamma = 0.5$
$\gamma = 0.6$
$\gamma = 0.7$
$\gamma = 0.8$
$\gamma = 0.9$

Coarse graining



$$\bar{m}_L = \frac{\ell}{L} \sum_{j=-K}^{K} m_j$$

Right tail

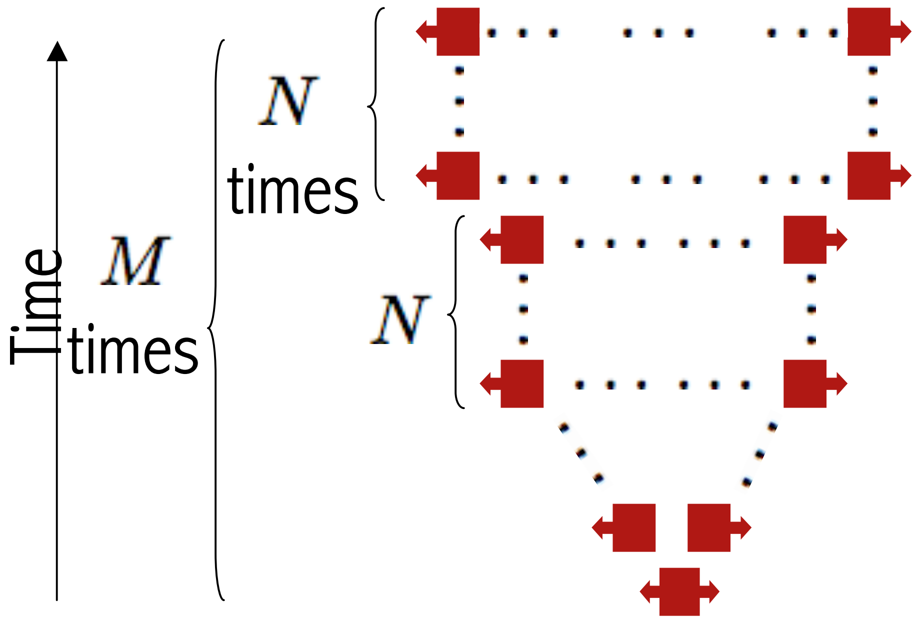
$$p(\bar{m}_L) \propto \exp(-C\bar{m}_L \log \bar{m}_L)$$

Left tail

$$p(\bar{m}_L) \propto \bar{m}_L^{\alpha_L(\gamma)}$$

In both limits $\gamma \to 0$ and $L \to \infty$ we recover uniformity

Is there a rescaling in this limit?



$\Rightarrow$ Prediction for the exponent of the left tail when $L \gg \ell$

$$\alpha_L \approx \frac{1}{2} \frac{L}{\log(1-\gamma/2)} \log\left[p^4(1-p)^2\right]$$

$\Rightarrow$ Picture repeated for

$$L/\log(1-\gamma/2) = const$$

Summary

Different mechanisms for clustering:

- Dissipative dynamics in the viscous range

  scale invariance

  relevant fluid time scale = Kolmogorov time

  insensitivity to flow intermittency

  use of tools borrowed from dynamical systems, model flows

- Ejection from vortices in the inertial range

  scale invariance broken

  relevant fluid time scale given by acceleration

  particle mass distribution has a universal shape

  need for better quantifying correlations with the flow structures and with the acceleration field

Open questions

Collision / reaction rates (important for applications)

requires to quantify not only the distribution but also the velocity difference between particles

Correlation with the flow structure

Inertial-range distribution of acceleration (pressure gradient) plays a crucial rôle

Quantify inertial biases in particle tracking experiments

for e.g. acceleration, Lagrangian structure functions, etc.