A two-point closure based on a Lagrangian timescale

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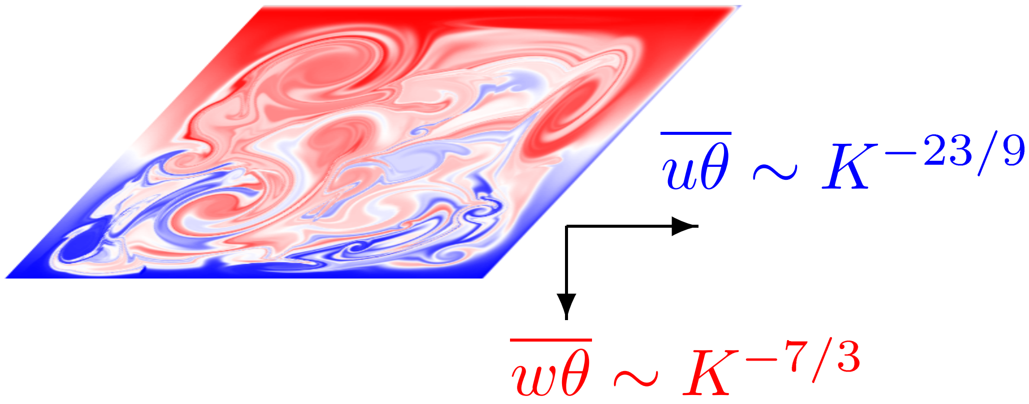
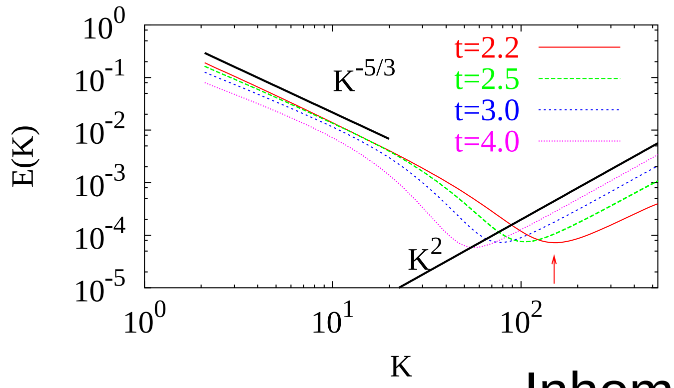
Applications of two-point closures

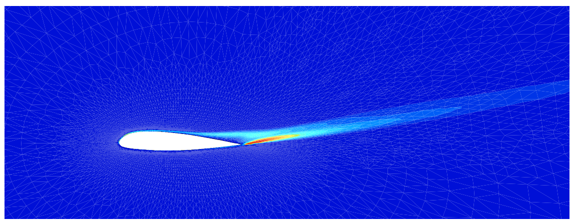
Mixing in shear flow (B&B PoF'07)



Truncated inviscid flow (B&B PoF'06)



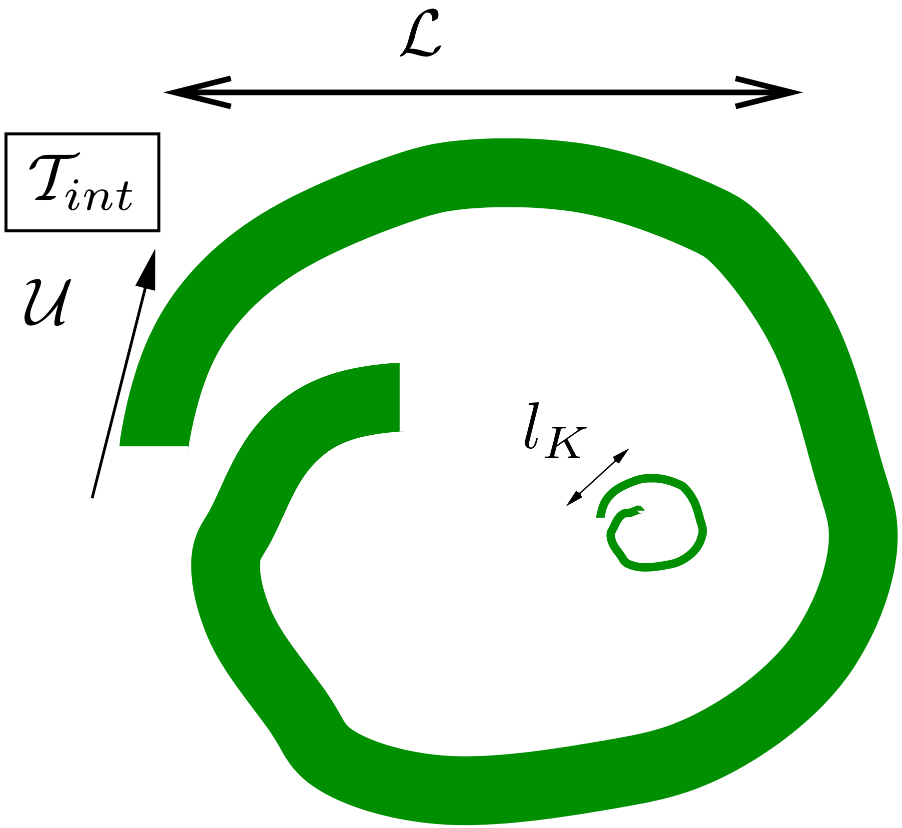
Inhomogeneous flows



H. Touil 2002

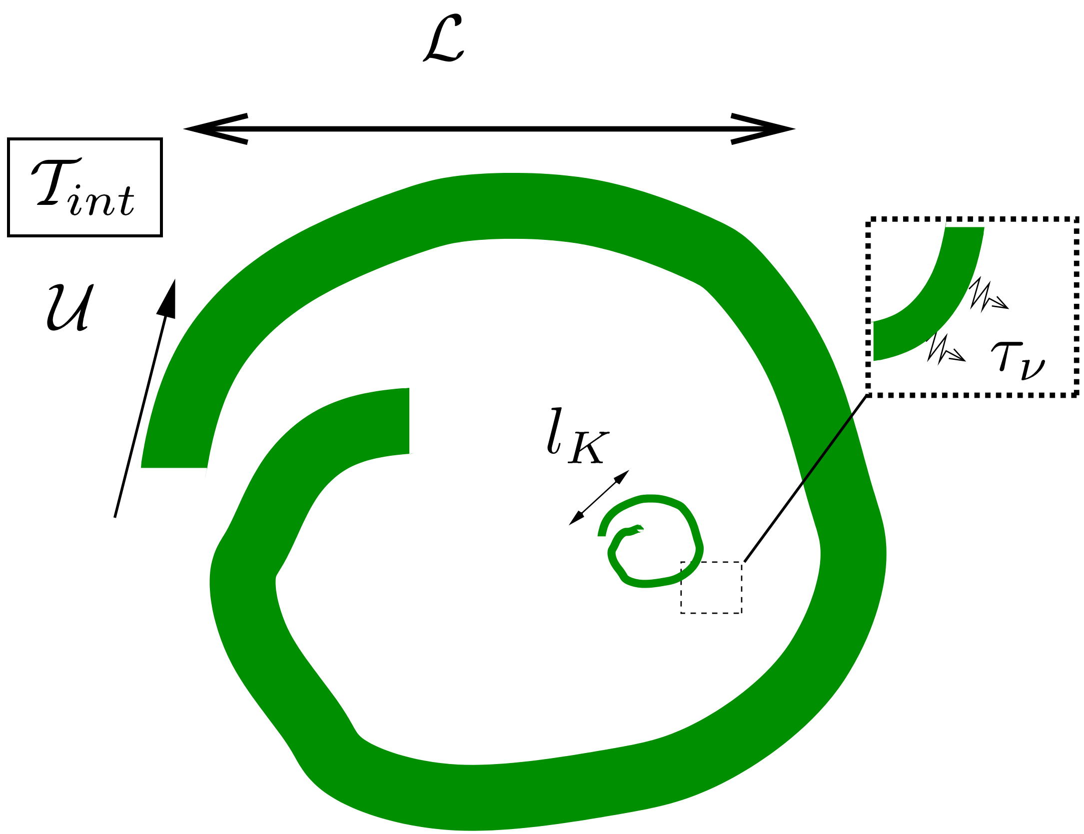
Single-time two-point closures (like EDQNM) need specification of a timescale $\tau(K)$

Timescales in turbulence



1. $\mathcal{T}_{int} = \mathcal{L}\mathcal{U}^{-1}$

Timescales in turbulence

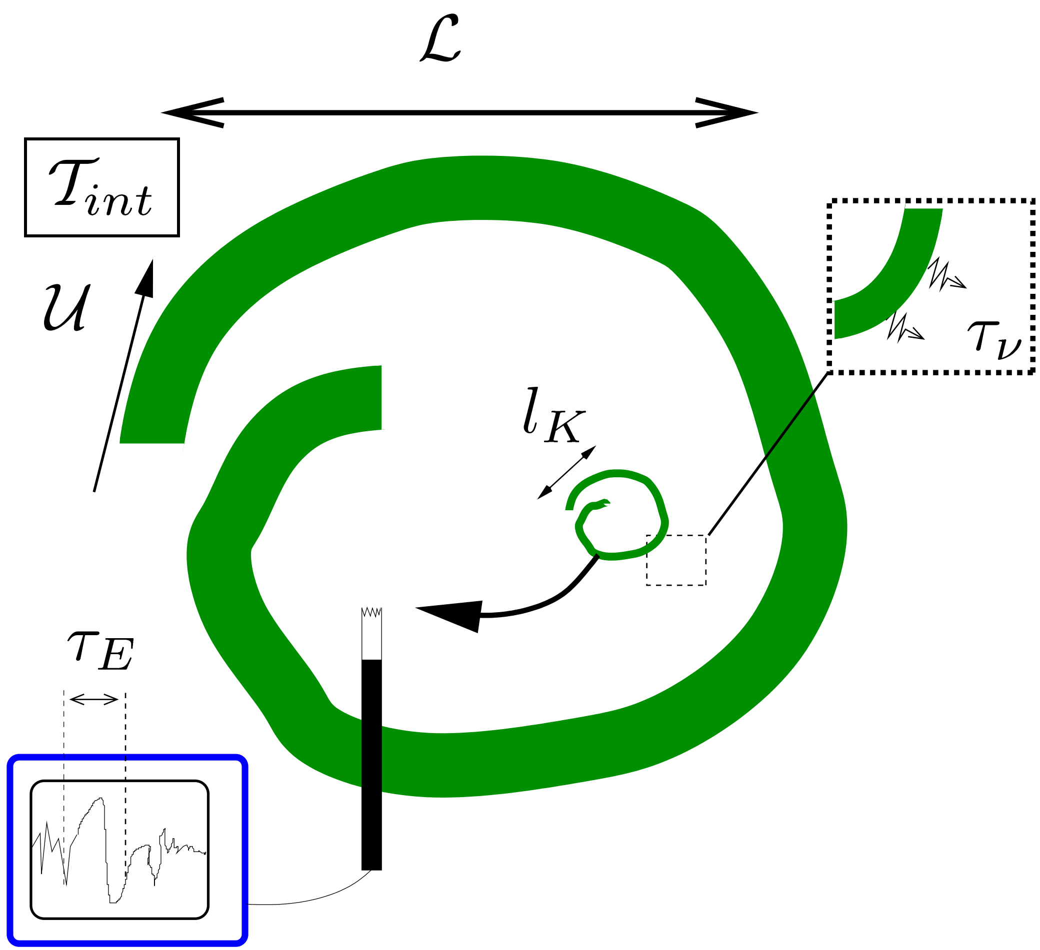


1. $\mathcal{T}_{int} = \mathcal{L}\mathcal{U}^{-1}$

2. $\nu\partial^2_{x_j} \rightarrow \nu K^2$

   $\tau_\nu = (\nu K^2)^{-1}$

Timescales in turbulence

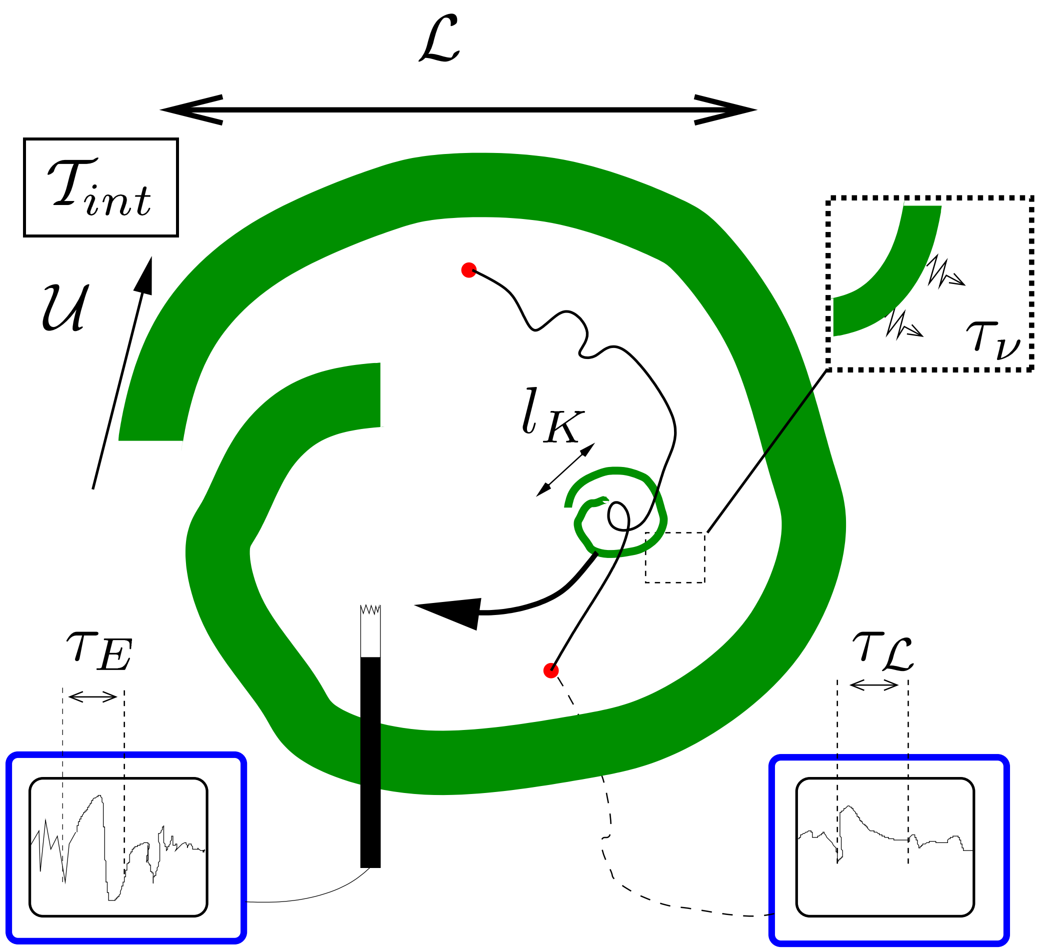


1. $\mathcal{T}_{int} = \mathcal{L}\mathcal{U}^{-1}$
2. $\nu\partial^2_{x_j} \rightarrow \nu K^2$

   $\tau_\nu = (\nu K^2)^{-1}$
3. $\tau_E = l_K \mathcal{U}^{-1}$

   $\tau_E \sim (K\mathcal{U})^{-1}$

Timescales in turbulence



1. $\mathcal{T}_{int} = \mathcal{L}\mathcal{U}^{-1} \quad \sim K^0$

2. $\nu \partial^2_{x_j} \rightarrow \nu K^2$

$$\tau_\nu = (\nu K^2)^{-1} \quad \sim K^{-2}$$

3. $\tau_E = l_K \mathcal{U}^{-1}$

$$\tau_E \sim (K\mathcal{U})^{-1} \quad \sim K^{-1}$$

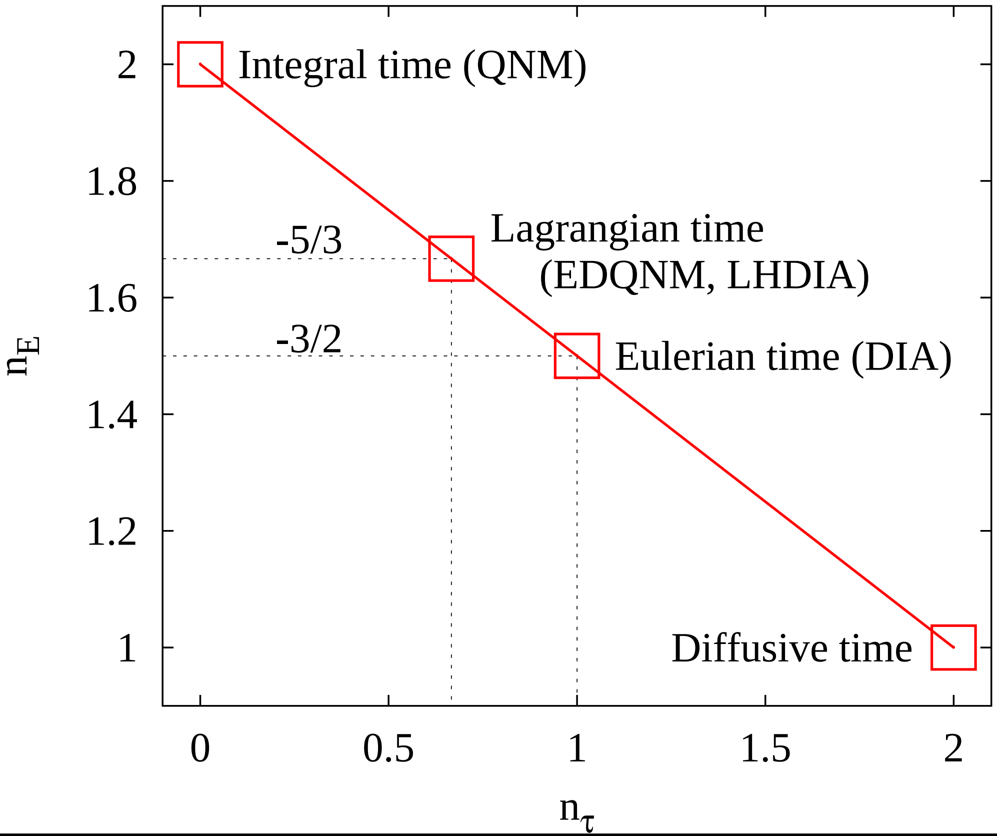
4. $\tau_{\mathcal{L}} = l_K u(K)^{-1}$

$$u(K) \sim \sqrt{KE(K)}$$

$$\tau_{\mathcal{L}} \sim \left(K^3 E(K)\right)^{-1/2} \sim K^{-2/3}$$

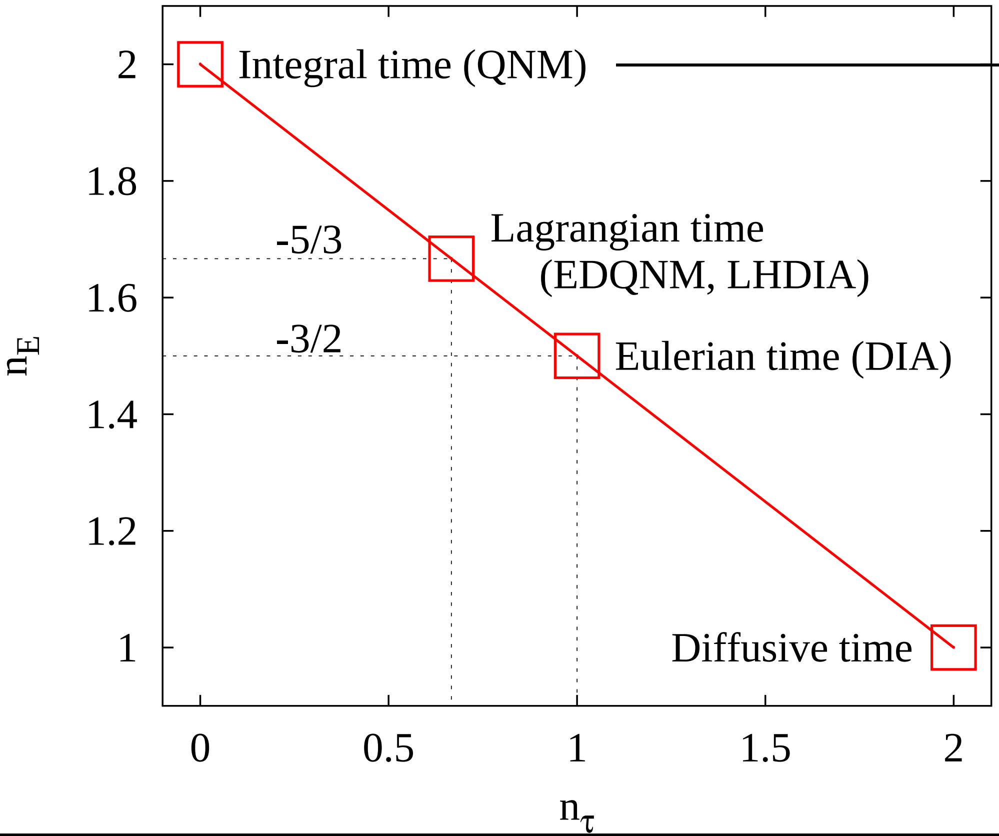
Dependence two-point closure on timescale

$$\tau(K) \sim K^{-n_\tau} \rightarrow E(K) \sim K^{-n_E}$$



Dependence two-point closure on timescale

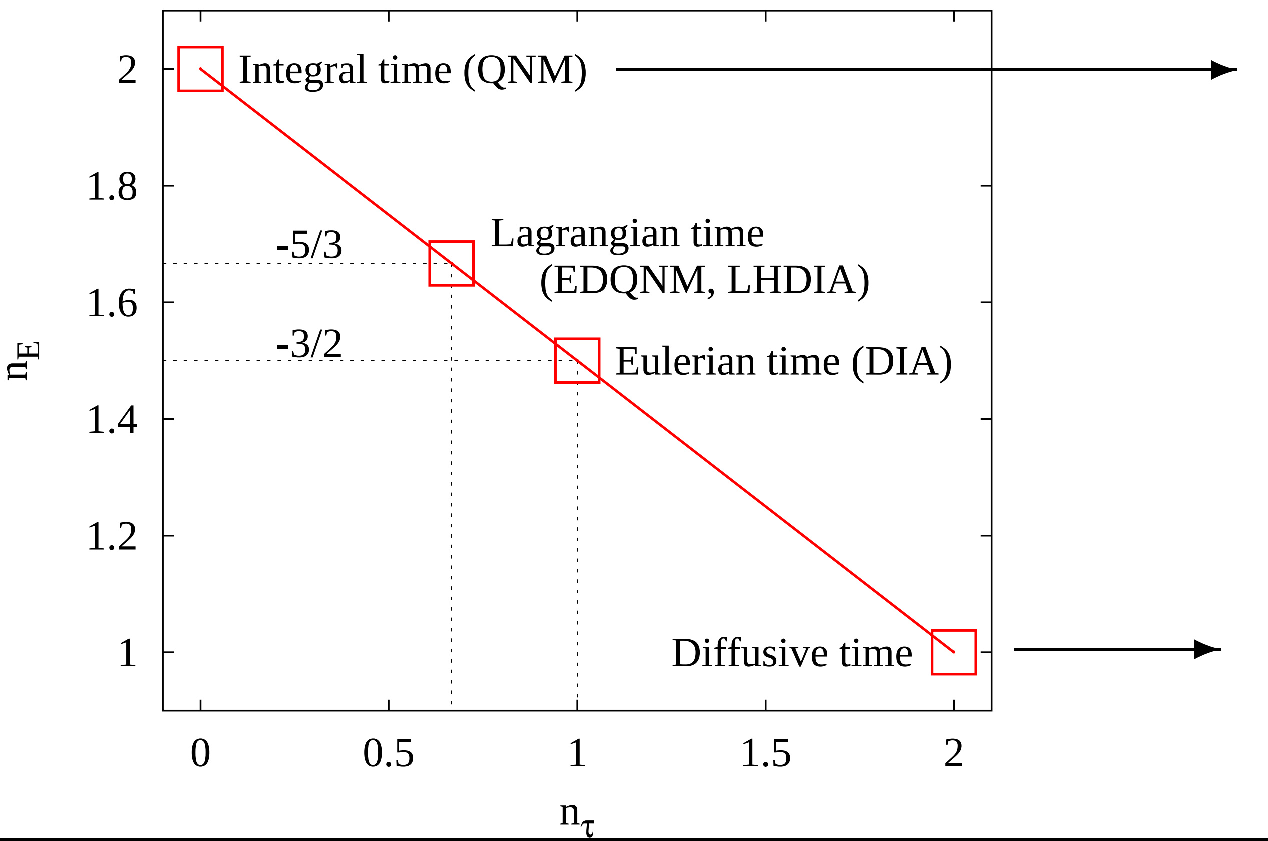
$$\tau(K) \sim K^{-n_\tau} \rightarrow E(K) \sim K^{-n_E}$$



$$E(K) \sim K^{-2}$$

Dependence two-point closure on timescale

$$\tau(K) \sim K^{-n_\tau} \rightarrow E(K) \sim K^{-n_E}$$

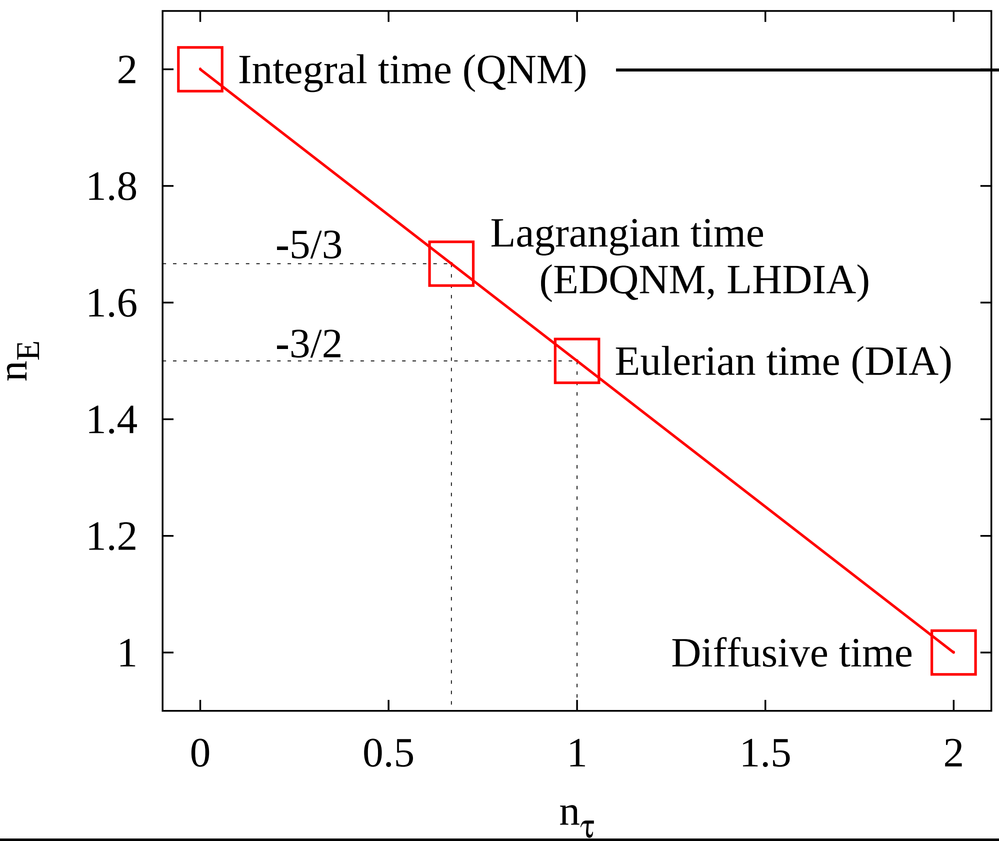


$$E(K) \sim K^{-2}$$

$$E(K) \sim K^{-1}$$

Dependence two-point closure on timescale

$$\tau(K) \sim K^{-n_\tau} \rightarrow E(K) \sim K^{-n_E}$$



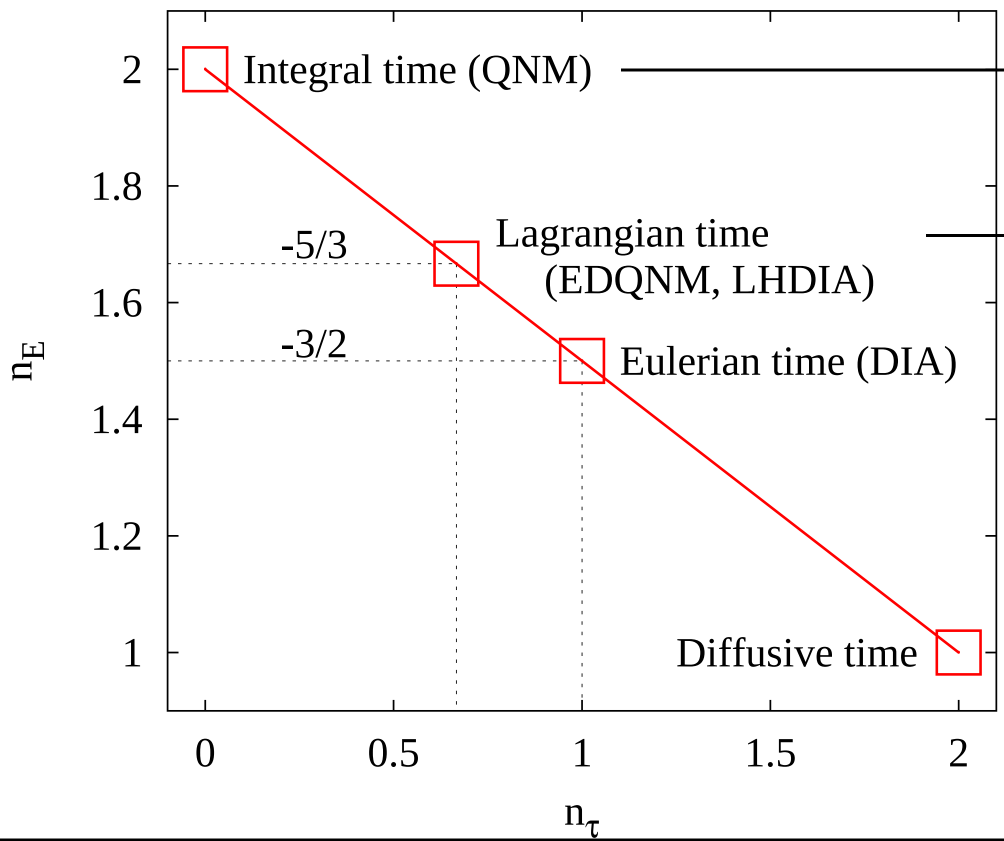
$E(K) \sim K^{-2}$

$E(K) \sim K^{-3/2}$

$E(K) \sim K^{-1}$

Dependence two-point closure on timescale

$$\tau(K) \sim K^{-n_\tau} \rightarrow E(K) \sim K^{-n_E}$$



$E(K) \sim K^{-2}$

$E(K) \sim K^{-5/3}$

$E(K) \sim K^{-3/2}$

$E(K) \sim K^{-1}$

Determine Lagrangian timescale

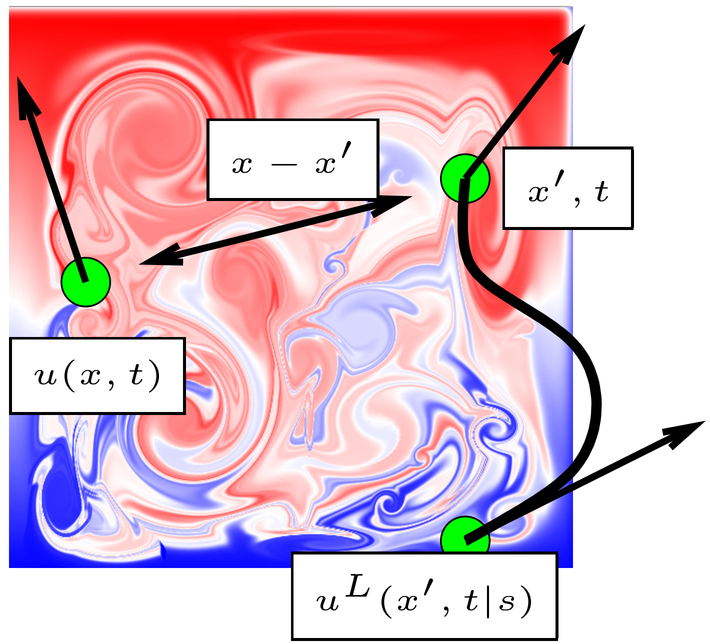
Two-point closures need specification of a Lagrangian timescale :

$$\tau(K) = \int \frac{E(K,t|s)}{E(K,t)} ds \sim \left(K^3 E(K,t)\right)^{-1/2}$$

$$\int E(K,t|s)ds = \frac{1}{2}\iint \left(\widehat{\overline{u_i(\boldsymbol{x},t)u_i^L(\boldsymbol{x}',t|s)}}\right) d\Sigma(K)ds$$

$$= \frac{1}{2}\int \left(\widehat{\overline{u_i(\boldsymbol{x},t)X_i(\boldsymbol{x}',t)}}\right) d\Sigma(K)$$

$$= F_{uX}(K,t)$$



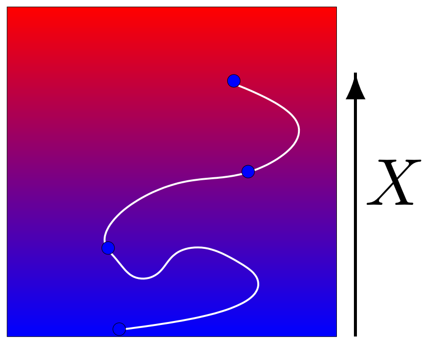
The integral of the two-time quantity $E(K,t|s)$ is expressed as a one-time quantity $F_{uX}(K,t)$!

Link Scalar and displacement

Displacement $\sim$ non-diffusive scalar :

$$\begin{cases} \frac{dX}{dt} = u, \\ \frac{d\theta}{dt} = -\Gamma u \end{cases} \Leftrightarrow F_{uX}(K,t) = -\Gamma^{-1} F_{u\theta}(K,t)$$

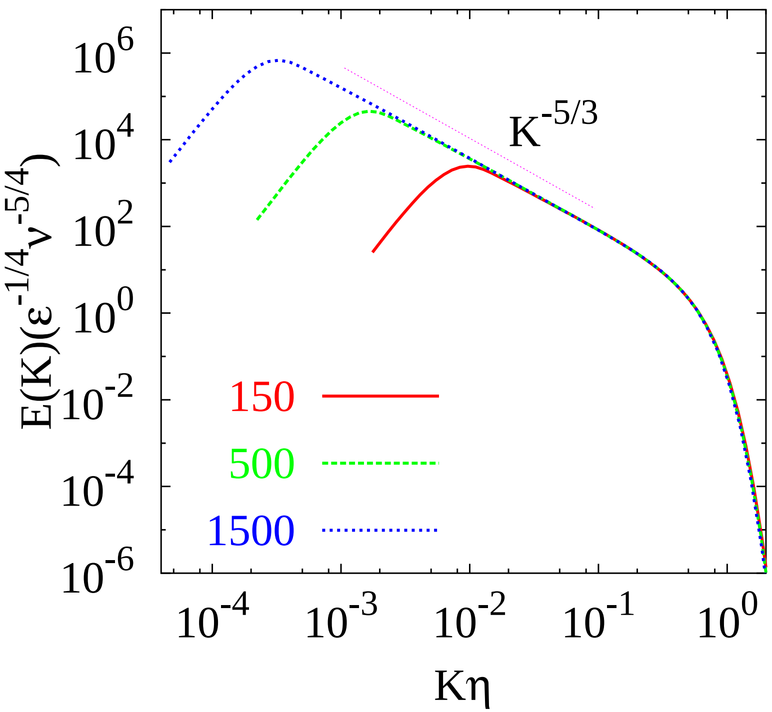
with $\Gamma = \partial\overline{\Theta}/\partial x$ and $\overline{U} = 0$

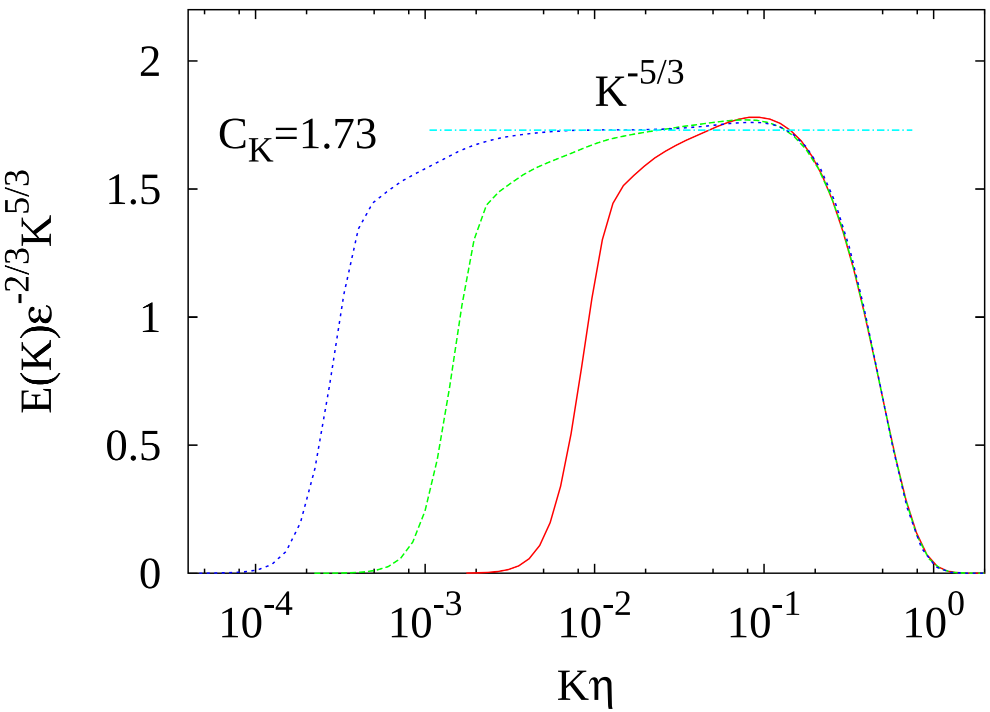


$$Closure : \begin{cases} \partial_t E(K) = f_1(\tau(K)) \\ \partial_t F_{uX}(K) = f_2(\tau(K)) \\ \tau(K) = F_{uX}(K)/E(K) \end{cases}$$

Estimating the Kolmogorov Constant

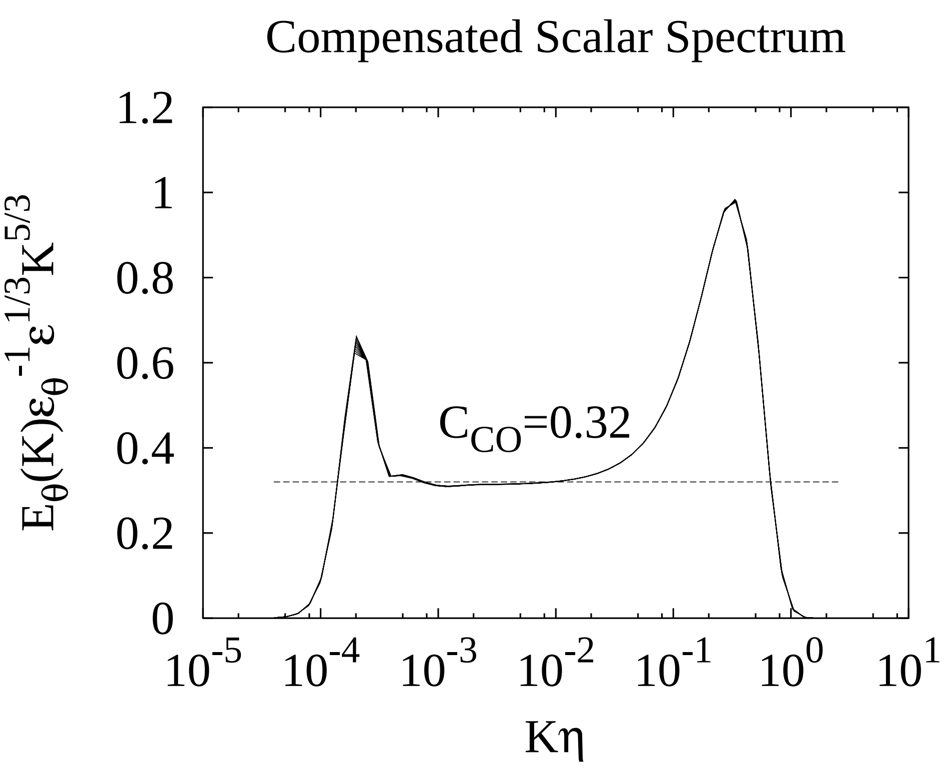
Two-point closure without heuristic time scale (*Phys. Fluids 2006*).





... and the Corrsin-Obukhov Constant

Apply closure to the scalar spectrum $E_\theta(K)$



$R_\lambda \approx 1400, \quad Pr = 0.71$

Conclusions & Perspectives

- A two-point closure without constant was derived
- Numerically integrated using analogy with scalar
- Results for $E(K)$ and $E_\theta(K)$
- Extension to MHD...