Large eddy simulations and subgrid scale modelling of turbulent shear flows

Pierre Comte

Laboratoire d'Etudes Aérodynamiques, Poitiers, France

Small-scale turbulence : Theory, Phenomenology and Applications, Cargèse, 13-25 Aug. 2007

- in applications, current practice differs from classical LES, as defined by A. Leonard
- from isotropic turbulence with spectral methods to industrial applications with robust codes

Outline

- spectral eddy-viscosity and eddy-diffusivity
- some SGS models in the physical space (incl. filtered, selective, and structure-function models)
- mixing-layer and mid-size vortex dynamics
- boundary layer transition
- compressible LES formalism
- assessment of high-order shock-capturing schemes
- implicit time integration in LES
- Application to controlled transonic cavity flow

Textbooks on LES :

- LESIEUR, M., MÉTAIS, O. & C., P., 2005 Large-Eddy Simulation of Turbulence, *Cambridge University Press*, p. 320.
- GEURTS, B.J., 2003, Elements of Direct and Large-Eddy Simulation, *Edwards*.
- SAGAUT P. 1998, Introduction à la simulation des grandes échelles pour les écoulements de fluide incompressible. Series *Mathématiques et applications*, vol. 30, p. 282. *Springer*.

- Navier-Stokes in Fourier space (statistical homogeneity)

$$\hat{u}_i(\underline{k}, t) = \left(\frac{1}{2\pi}\right)^3 \int e^{-i\underline{k}.\underline{x}} \; u_i(\underline{x}, t) d\underline{x}$$

$$\frac{\partial}{\partial t}\hat{u}_i(\underline{k}, t) + \nu k^2 \hat{u}_i(\underline{k}, t) = \\ - ik_m \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \int_{\underline{p}+\underline{q}=\underline{k}} \hat{u}_j(\underline{p}, t)\hat{u}_m(\underline{q}, t) d\underline{p}$$

- Passive scalar

$$\hat{T}(\underline{k},t) = \left(\frac{1}{2\pi}\right)^3 \int e^{-i\underline{k}.\underline{x}} \; T(\underline{x},t) d\underline{x}$$

$$\frac{\partial}{\partial t}\hat{T}(\underline{k},t) + \kappa k^2 \hat{T}(\underline{k},t) = - ik_j \int_{\underline{p}+\underline{q}=\underline{k}} \hat{u}_j(\underline{p},t)\hat{T}(\underline{q},t) d\underline{p}$$

- Low-pass filter (sharp filter) :

$$\bar{\hat{f}} = \hat{f} \ \text{ for } \ |\underline{k}| < k_C = \pi/\Delta x, \bar{\hat{f}} = 0 \ \text{ for } \ |\underline{k}| > k_C$$

- Spectral eddy viscosity (Heisenberg, Kraichnan ...) :

$$\frac{\partial}{\partial t}\hat{u}_i(\underline{k},t) + [\nu + \nu_t(k|k_C)]k^2\hat{u}_i(\underline{k},t) =$$

$$-ik_m\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|,|\underline{q}|<k_C} \hat{u}_j(\underline{p},t)\hat{u}_m(\underline{q},t)d\underline{p}$$

with

- Spectral eddy viscosity $\nu_t(k|k_C)$:

$$\nu_t(k|k_C)\; k^2 \hat{u}_i(\underline{k},t) =$$
$$ik_m(\delta_{ij} - \frac{k_i k_j}{k^2}) \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}| \mathrm{or} |\underline{q}| > k_C} \hat{u}_j(\underline{p},t) \hat{u}_m(\underline{q},t) d\underline{p}$$

- Spectral eddy diffusivity $\kappa_t(k|k_C)$:

$$\kappa_t(k|k_C)\; k^2 \hat{T}(\underline{k},t) =$$
$$ik_j \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}| \mathrm{or} |\underline{q}| > k_C} \hat{u}_j(\underline{p},t) \hat{T}(\underline{q},t) d\underline{p}$$

- Spectral eddy diffusivity $\kappa_t(k|k_C)$ satisfies :

$$\frac{\partial}{\partial t}\hat{T}(\underline{k},t) + [\kappa + \kappa_t(k|k_C)]k^2\hat{T}(\underline{k},t) = - ik_j \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|,|\underline{q}|<k_C} \hat{u}_j(\underline{p},t)\hat{T}(\underline{q},t)d\underline{p}$$

- Two-point stochastic closures (EDQNM, TFM, LHDIA ...) provide model expressions for $\nu_t(k|k_C)$ and $\kappa_t(k|k_C)$

- Spectral-peak eddy coefficients : EDQNM $\longrightarrow$

$$\nu_t(k|k_C) = \left[\frac{E(k_C)}{k_C}\right]^{1/2} \nu_t^+\left(\frac{k}{k_C}\right)$$

assuming $E(k) \sim k^{-5/3}$ for $k \gtrsim k_C$

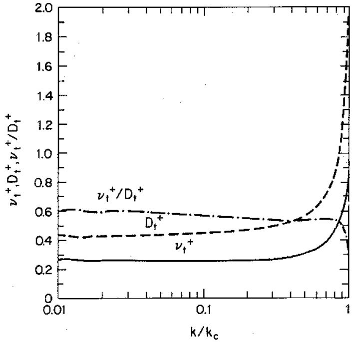
- Asymptotics : $\nu_t^+\left(\frac{k}{k_C}\right) \longrightarrow 0.441\ C_K{}^{-3/2} \sim 0.28$ when $\frac{k}{k_C} \longrightarrow 0$
- reminder : (isotropic) energy spectrum $E(k)$:

$$E(k,t) = 2\pi k^2 \langle \underline{\hat{u}}(\underline{k},t).\underline{\hat{u}}^*(\underline{k},t)\rangle_{||\underline{k}||=k}$$

$$\frac{1}{2}\langle \underline{u}.\underline{u}\rangle = \int E(k)dk$$

- EDQNM non-dimensional eddy coefficients :

$$\nu_t^+ \ , \quad \kappa_t^+ = \frac{\nu_t^+}{Pr_t}$$



- $\longrightarrow Pr_t \approx 0.6$

- Spectral-dynamic model (Lesieur-Métais-Lamballais, 1996) : for $E(k) \sim k^{-m}$ at k_C. The value of the plateau is recomputed using EDQNM non-local expansions, the peak is unchanged $\longrightarrow$

$$\nu_t(k|k_C) = 0.31 \, \frac{5-m}{m+1} \sqrt{3-m} \, C_K{}^{-3/2} \left[\frac{E(k_C)}{k_C}\right]^{1/2} \nu_t^+ \left(\frac{k}{k_C}\right)$$

$$Pr_t = 0.18 \, (5-m)$$

whenever $m \leq 3$, otherwise $\nu_t = 0$.

- Physical space (*finite-differences methods, or finite-volume...*), ρ uniform, grid of mesh Δx
- low-pass spatial filter $G_{\Delta x}$, cut-off scale Δx

$$\bar{f}(\underline{x}, t) = f * G_{\Delta x} = \int f(\underline{y}, t) G_{\Delta x}(\underline{x} - \underline{y}) d\underline{y} \, .$$

- filter commutes with space and time derivatives (if mesh uniform).

- Navier-Stokes equations

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu\, S_{ij})$$

with $S_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, strain-rate tensor

- filtered equations :

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu \bar{S}_{ij} + T_{ij})$$

with $T_{ij} = \bar{u}_i\bar{u}_j - \overline{u_i u_j}$, SubGrid-Stress tensor

- eddy-viscosity assumption (Boussinesq) :

$$T_{ij} = 2\nu_t(\underline{x}, t)\ \bar{S}_{ij} + \frac{1}{3} T_{ll}\ \delta_{ij}$$

- LES momentum equations

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0}\ \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}[2(\nu + \nu_t)\bar{S}_{ij}]$$

- continuity : $\partial \bar{u}_j / \partial x_j = 0$,
- macro pressure $\bar{P} = \bar{p} - (1/3)\rho_0 T_{ll}$.
- models : Smagorinsky, structure-function, dynamic Smagorinsky ...

- Smagorinsky model
  - *a la* Prandtl mixing length argument : $\nu_t \sim \Delta x \, v_{\Delta x}$
  - $v_{\Delta x} \sim \dfrac{\partial v}{\partial x} \Delta x$
  - $v_{\Delta x} = \Delta x |\bar{S}|$, with $|\bar{S}| = \sqrt{2 \bar{S}_{ij} \bar{S}_{ij}}$.
  - $\longrightarrow$
$$\nu_t = (C_S \Delta x)^2 |\bar{S}|$$
  - inertial arguments $\longrightarrow C_S \approx \dfrac{1}{\pi} \left(\dfrac{3 C_K}{2} \right)^{-3/4} \longrightarrow C_S \approx 0.18$ for $C_K = 1.4$

- Structure-function model (Métais-Lesieur, *J. Fluid Mech.*, 1992)

$$\nu_t^{SF}(\underline{x},\Delta x,t) = 0.105\ C_K^{-3/2}\ \Delta x\ [\bar{F}_2(\underline{x},\Delta x,t)]^{1/2}$$

with the 2nd-order velocity structure-function at scale Δx

$$\bar{F}_2(\underline{x},\Delta x,t) = \left\langle \|\underline{\overline{u}}(\underline{x},t) - \underline{\overline{u}}(\underline{x}+\underline{r},t)\|^2 \right\rangle_{\|\underline{r}\|=\Delta x}$$

  - consistent with the spectral peak model thru (Batchelor)

$$\langle \bar{F}_2(\underline{x},\Delta x,t)\rangle_{\underline{x}} = 4\int_0^{k_c} E(k,t)\left(1-\frac{\sin(k\Delta x)}{k\Delta x}\right)dk\ .$$

  - In the limit of $\Delta x \to 0$

$$\nu_t^{SF} \approx 0.777\ (C_S\Delta x)^2\sqrt{2\bar{S}_{ij}\bar{S}_{ij}+\bar{\omega}_i\bar{\omega}_i}$$

- Filtered Structure Function model (Ducros *et al.*, *J. Fluid Mech.*, **326**, 1-36, 1996)

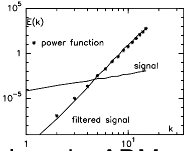
$$\nu_t^{FSF} = 0.0014\ C_K^{-\frac{3}{2}} \Delta x \left[\breve{F}_{2_{\Delta x}}(\underline{x}, \Delta x) \right]^{\frac{1}{2}}$$

  - density-weighted filtered variables : $\widetilde{\underline{u}} = \dfrac{\overline{\rho \underline{u}}}{\overline{\rho}}$

$$\breve{F}_{2_{\Delta x}}(\underline{x}, t) = \left\langle \| \breve{\underline{u}}(\underline{x} + \underline{r}, t) - \breve{\underline{u}}(\underline{x}, t) \|^2 \right\rangle_{\|\underline{r}\| = \Delta x}$$

  - $\breve{\underline{u}}$: convolution of $\widetilde{\underline{u}}$ by 2nd-order centered finite-difference Laplacian filter, iterated 3 times

$$\frac{\tilde{E}(k)}{E(k)} \approx 40^3 \left(\frac{k}{k_C} \right)^9$$



  - $\sim$ hyperviscosity ; spectral-peak ; ADM

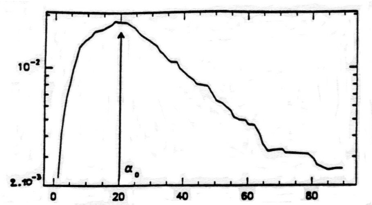
- Selective Structure-Function model David, 1992)

$$\nu_t^{SSF} = 0.16\ \Phi_{20^\circ}(\underline{x}, t)\ C_K^{-\frac{3}{2}} \Delta x \left[\overline{F}_{2_{\Delta x}}(\underline{x}, \Delta x)\right]^{\frac{1}{2}}$$

$$\Phi_{\alpha_0}(\underline{x}, t) = \begin{cases} 1 & \text{if} \quad (\underline{\omega}, \underline{\breve{\omega}}) \geq 20^\circ \\ 0 & \text{otherwise} \end{cases} .$$

with

$$\underline{\breve{\omega}}(\underline{x}, t) = \langle \underline{\omega}(\underline{x} + \underline{r}, t) \rangle_{||\underline{r}|| \leq \Delta x} \quad .$$



- Mixed Scale model (Sagaut, Ta Phuoc)

$$\nu_t^{MS} = C_m(\alpha)|\widetilde{S}|^{\alpha}\left(q_c^2\right)^{\frac{(1-\alpha)}{2}}\Delta^{(1+\alpha)} \tag{1}$$

$$q_c{}^2 = \frac{1}{2}(\widetilde{u}_k - \widehat{\widetilde{u}}_k)^2 \tag{2}$$

  - Gaussian test filter

$$\widehat{\widetilde{u}}_i = \frac{1}{4}\left[\widetilde{u}_{i-1} + 2\widetilde{u}_i + \widetilde{u}_{i+1}\right] \tag{3}$$

  - $\alpha = 1 \longrightarrow$ Smagorinsky's model
  - $\alpha = 0 \longrightarrow$ Bardina's TKE model
  - inertial arguments yield $C_m(\alpha) = 0.06$ for $\alpha = 1/2$.

- Selective Mixed Scale model (Sagaut et al.)

$$\nu_t^{SMS} = 0.06 f_{\theta_0} |\widetilde{S}|^{1/2} \left(q_c^2 \right)^{1/4} \Delta^{3/2} \quad , \tag{4}$$

  - Mixed Scale model with the selection function

$$f_{\theta_0}(\theta) = \begin{cases} 1 & \text{if} \quad \theta \geq \theta_0 \\ r(\theta)^n & \text{otherwise} \end{cases} \tag{5}$$

in which

$$r(\theta) = \frac{\tan^2(\theta/2)}{\tan^2(\theta_0/2)} \quad ; \quad n = 2 \tag{6}$$

instead of David's

$$f_{\theta_0}(\theta) = \begin{cases} 1 & \text{if} \quad \theta \geq \theta_0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

still with $\theta_0 = 20^\circ$.

- Hybrid models (Sagaut et al.)
  - Scale similarity model

$$\frac{\mathcal{T}_{ij}}{\overline{\rho}} = \widetilde{u}_i\widetilde{u}_j - \widetilde{u_i u_j} \equiv \mathcal{L}_{ij} = \widehat{\widetilde{u}}_i\widehat{\widetilde{u}}_j - \widehat{\widetilde{u}_i\widetilde{u}_j} \qquad (8)$$

  - $\mathcal{L}_{ij}$: *Resolved Subgrid Stress Tensor*
  - a la Bardina if filters $\widetilde{\ }$ and $\widehat{\ }$ are both defined at grid level Δ
    (even if the Gaussian filter $\widehat{\widetilde{u}}_i = \dfrac{1}{4}\left[\widetilde{u}_{i-1} + 2\widetilde{u}_i + \widetilde{u}_{i+1}\right]$ is wider than the grid filter (box filter)
  - a la Germano/Liu-Meneveau-Katz, if $\widehat{\ }$ is at scale 2Δ
- Hybridation with an eddy-viscosity model

$$\mathcal{T}_{ij} = \frac{1}{2}\,\overline{\rho}\,\left(\mathcal{L}_{ij} + \nu_t \widetilde{S_{ij}}\right) \qquad (9)$$

- See Lenormand *et al.*, *AIAAJ*, **38**, 8, pp. 1340-1350 for assessement in channel flow at Mach 0.5 and 1.5.

Dynamic model I

- double filtering :
  - resolved fields $\overline{f}$ at grid level Δx
  - filtered by a “test filter" $\hat{.}$ of larger width $\alpha\Delta x$ (for instance $\alpha = 2$) yielding $\widehat{\overline{f}}$.
- apply the double filter to the Navier-Stokes equation (with constant density)

Dynamic model II

- yields subgrid-scale tensor of field $\widehat{\bar{u}}$:

$$\mathbb{T}_{ij} = \widehat{\bar{u}}_i \widehat{\bar{u}}_j - \widehat{\overline{u_i u_j}} \qquad . \tag{10}$$

in the same way as SGS tensor

$$T_{ij} = \bar{u}_i \bar{u}_j - \overline{u_i u_j} \tag{11}$$

was obtained by “bar" filtering the NS equations.

- Resolved turbulent stress tensor (corresponds to test-filter applied to $\bar{u}$) :

$$\mathcal{L}_{ij} = \widehat{\bar{u}}_i \widehat{\bar{u}}_j - \widehat{\bar{u}_i \bar{u}_j} \quad . \tag{12}$$

Dynamic model III

- applying filter “hat" to Eq. (11) $T_{ij} = \bar{u}_i \bar{u}_j - \overline{u_i u_j}$. yields

$$\widehat{T_{ij}} = \widehat{\bar{u}_i \bar{u}_j} - \widehat{\overline{u_i u_j}} \quad . \tag{13}$$

- add Eqs (12) and (13), using (10) yields the Germano identity

$$\mathcal{L}_{ij} = \mathbb{T}_{ij} - \widehat{T_{ij}} \quad , \tag{14}$$

  that can be expressed in terms of Poisson brackets.
- $\mathbb{T}_{ij}$ and $\widehat{T_{ij}}$ have to be modelled, while $\mathcal{L}_{ij}$ can be explicitly calculated by applying the test filter to the base LES results.

Dynamic model IV

- Using Smagorinsky's model, we have

$$\widehat{T_{ij}} - \frac{1}{3}\widehat{T_{ll}}\,\delta_{ij} = 2\widehat{\mathcal{A}_{ij}C} \quad , \qquad (15)$$

with $C = C_S^2$ and

$$\mathcal{A}_{ij} = (\Delta x)^2\,|\bar{S}|\bar{S}_{ij} \quad .$$

- Still using Smagorinsky, we have

$$\mathcal{T}_{ij} - \frac{1}{3}\mathcal{T}_{ll}\,\delta_{ij} = 2\mathcal{B}_{ij}C\ , \qquad (16)$$

with

$$\mathcal{B}_{ij} = \alpha^2(\Delta x)^2\,|\widehat{\bar{S}}|\ \widehat{\bar{S}}_{ij} \quad .$$

Dynamic model V

- $|\widehat{\bar{S}}|$ and $\widehat{\bar{S}}_{ij}$ are the quantities analogous to $|\bar{S}|$ and $\bar{S}_{ij}$ built with the doubly-filtered field $\hat{\bar{u}}$.
- Substracting Eq. (15) from Eq. (16) yields with the aid of Eq. (14)

$$\mathcal{L}_{ij} - \frac{1}{3}\mathcal{L}_{ll}\,\delta_{ij} = 2\mathcal{B}_{ij}C - 2\widehat{\mathcal{A}_{ij}C} \quad .$$

- In order to obtain C, many people remove it from the filtering as if it were constant, leading to

$$\mathcal{L}_{ij} - \frac{1}{3}\mathcal{L}_{ll}\,\delta_{ij} = 2CM_{ij} \quad , \tag{17}$$

with

$$M_{ij} = \mathcal{B}_{ij} - \widehat{\mathcal{A}}_{ij} \quad .$$

Dynamic model VI

- Now, all the terms of Eq. (17) can be determined with the aid of $\bar{u}$. There are however five independent equations for only one variable C, and the problem is overdetermined.
- Two alternatives have been proposed to deal with this undeterminacy.
- A first solution (Germano *et al.*, 1991) is to contract Eq. (17) by $\bar{S}_{ij}$ to obtain

$$C = \frac{1}{2} \frac{\mathcal{L}_{ij} \bar{S}_{ij}}{M_{ij} \bar{S}_{ij}} \quad , \tag{18}$$

since, due to incompressibility, $\bar{S}_{ij}$ is traceless. This permits in principle to “dynamically" determine the “constant" C as

Dynamic model VII

a function of space and time, to be used in the LES of the base field $\bar{u}$.

- In tests using channel flow data obtained from DNS, it was however shown in (Gemano *et al.*, 1991) that the denominator in Eq. (18) could locally vanish or become sufficiently small to yield computational instabilities.
- To get rid of this problem, Lilly (see e.g. Lilly, 1993, Les Houches, session *LIX*) chose to determine the value of C which "best satisfies" the system Eq. (17) by minimizing the error using a least squares approach. It yields

$$C = \frac{1}{2} \frac{\mathcal{L}_{ij} M_{ij}}{M_{ij}^2} \quad . \tag{19}$$

Dynamic model VIII

This removes the undeterminacy of Eq. (17).

- The analysis of DNS data revealed, however, that the C field predicted by the models (18) or (19) varies strongly in space and contains a significant fraction of negative values, with a variance which may be ten times higher than the square mean.
- So, the removal of C from the filtering operation is not really justified and the model exhibits some mathematical inconsistencies.

Dynamic model IX

- The possibility of negative C is an advantage of the model since it allows a sort of backscatter in physical space, but very large negative values of the eddy viscosity is a destabilizing process in a numerical simulation, yielding a non-physical growth of the resolved scale energy.
- The cure which is often adopted to avoid excessively large values of C consists in averaging the numerators and denominators of (18) and (19) over space and/or time, thereby losing some of the conceptual advantages of the “dynamic" local formulation.

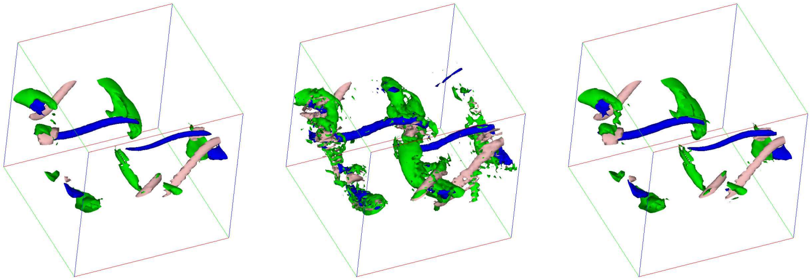
Dynamic model X

- Averaging over direction of flow homogeneity has been a popular choice, and good results have been obtained in (Germano *et al.*, 1991) and (Piomelli *et al.*, 1993), who took averages in planes parallel to the walls in their channel flow simulation.
- Remark that the same thing has been done, with success, when averaging the dynamic spectral eddy viscosity in the channel-flow LES presented before.
- It can be shown that the dynamic model gives a zero subgrid-scale stress at the wall, where L_{ij} vanishes, which is a great advantage with respect to the original Smagorinsky model ; it gives also the correct asymptotic behavior near the wall.

Dynamic model XI

- the use of Smagorinsky's model for the dynamic procedure is not compulsory
- As an example, (El-Hady *et al.*, Theor. Comp. Fluid Dyn., 1995) have applied the dynamic procedure to the structure-function model applied to a compressible boundary layer above a long cylinder.
- Compressible extensions do exist, with dynamic turbulent Prandtl number (Moin *et al.*, 1993). A third level of filtering is needed.

SGS models assessment :

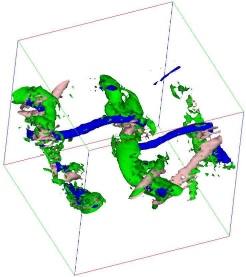


Smagorinsky model : max $|\omega_1| = 2.92\ \omega_i$

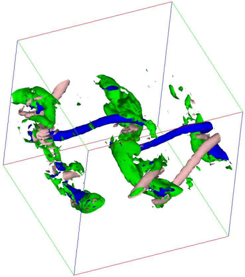
Spectral-Cusp model : max $|\omega_1| = 4.75\ \omega_i$

Structure-Function model : max $|\omega_1| = 2.86\ \omega_i$

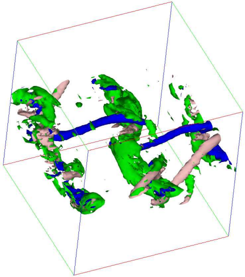
"smarter" SGS models



Spectral-Cusp model : max $|\omega_1| = 4.75\ \omega_i$

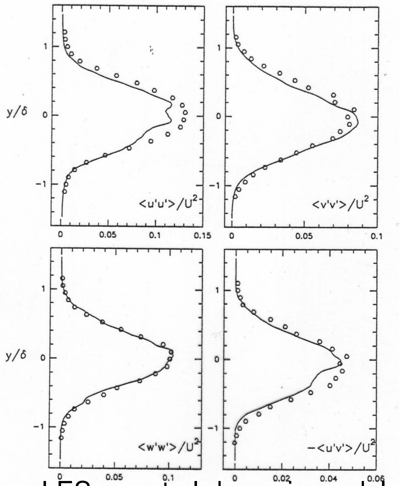


Filtered Structure-Function model : max $|\omega_1| = 4.83\ \omega_i$



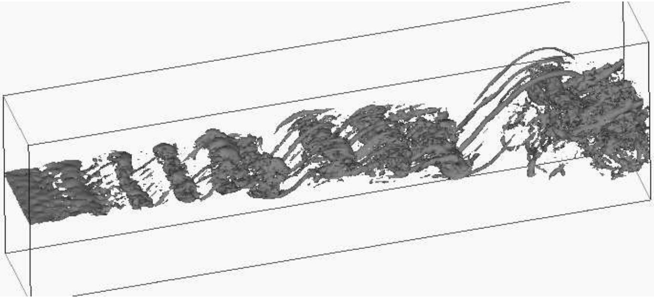
Selective Structure-Function model : max $|\omega_1| = 5.42\ \omega_i$

- Reynolds stresses



Lines : LES, spectral-dynamic model.

Symbols : exp. Bell & Mehta (1990)

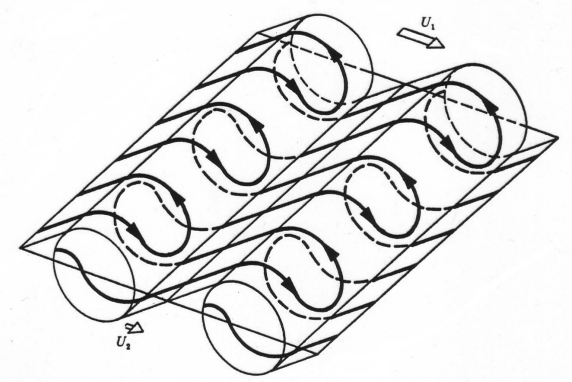


$\|\vec{\omega}\| = 2/3\ \omega_i$, in LES at $\nu = 0$, with $\varepsilon_{2D} = 10^{-5}$ and $\varepsilon_{1D} = 10^{-4}$.

LES $(L_x, L_y) = (16\lambda_i, 4\lambda_i),\ (N_x, N_y) = (384, 96)$

- narrow domain : $L_z = 2\ \lambda_i$, $N_z = 48$
  - side view
  - pressure
  - vorticity
- wider domain : $L_z = 4\ \lambda_i$, $N_z = 96$
  - pressure
  - vorticity

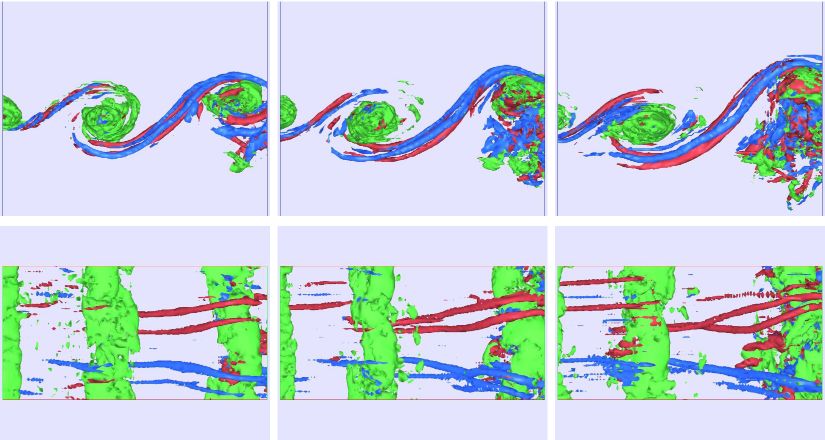
- Instead of



(Bernal and Roshko, 1986)

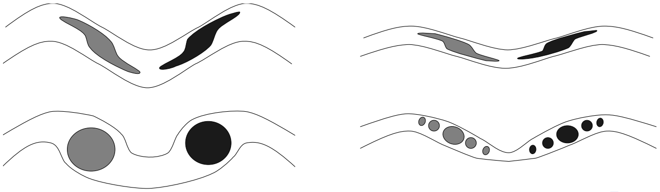
we see . . .

- Multiple-stage roll-up & pairing

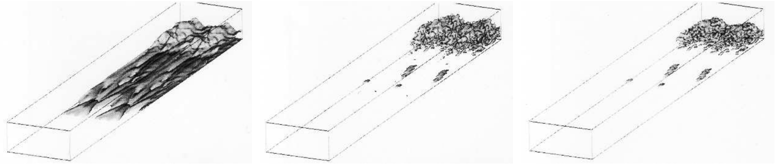


- as conjectured by Lin & Corcos, *J. Fluid Mech.*, **141**, 139–178 (1978).

. . .In a layer where the sign of the vorticity alternates (in the direction along which strain is absent), each portion of the layer that contains vorticity of a given sign eventually contributes that vorticity to a single vortex. This may occur in a single stage if the initial layer thickness is not excessively small next to the spanwise extent of vorticity of a given sign or, otherwise, in a succession of stages involving local roll-up and pairing.

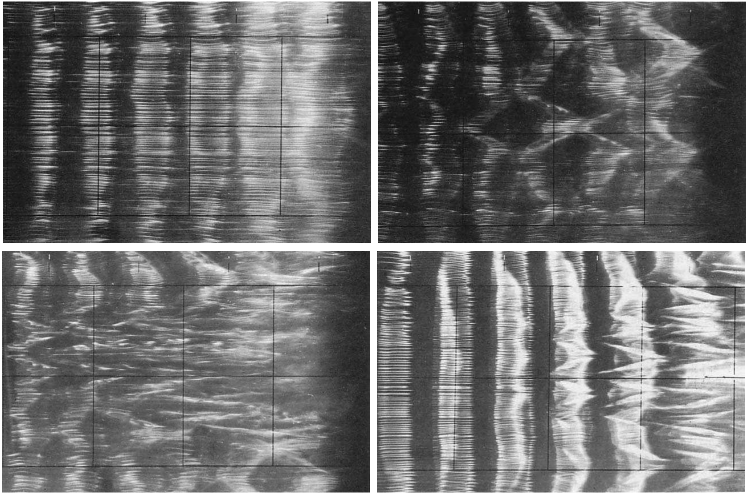


- Transitional boundary layer (simulated with FSF model (Ducros *et al.J. Fluid Mech.*, **336**, *1996)*) : $\nu_t = 2/3\ \nu$



SF FSF SSF

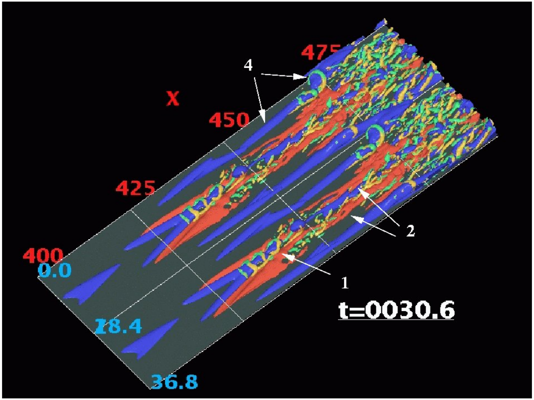
Boundary layer : Forced transition (Saric)



(a) (b)

(c) (d)

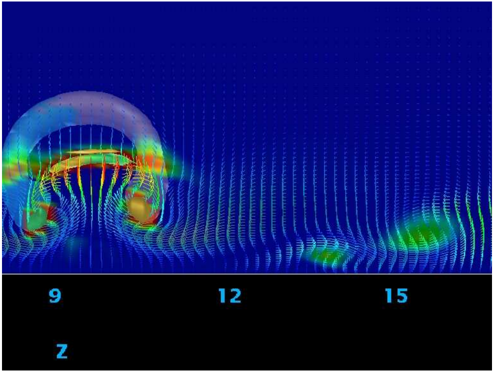
- K-type transition, grid 2



$$u' = +0.18\ U_\infty \text{ (red)},\ u' = -0.18\ U_\infty \text{ (blue)},$$

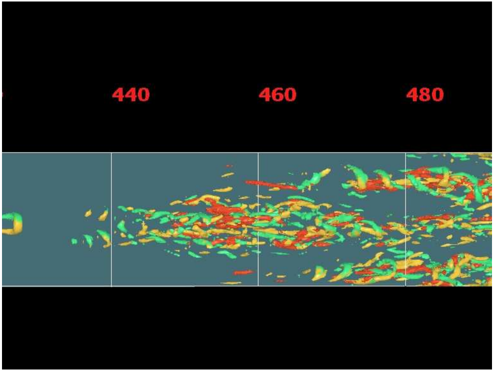
$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = \frac{1}{2\rho}\nabla^2 P = 0.1\ U_\infty^2/\delta_1^2\ (\omega_x > 0,\ \omega_x < 0)$$

- H-type transition, grid 1



$u' = +0.18\ U_\infty$ (red), $u' = -0.18\ U_\infty$ (blue),
$Q = 0.1\ U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$).

- Is the SGS model intelligent ?

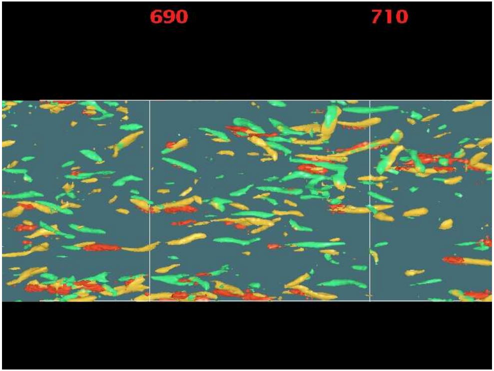


Transitional portion :

$\nu_t = 0.5\nu$ (red) ;

$Q = 0.1\ U_\infty^2/\delta_1^2$ ($\omega_x > 0$ yellow, $\omega_x < 0$ green).

- Is the SGS model intelligent ?



Turbulent portion :

$\nu_t = 0.5\nu$ (red) ;

$Q = 0.1\ U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$).

- Filtering of direct application of conservation principles :

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div}\ (\overline{\rho \underline{u}}) = 0 \tag{20a}$$

$$\frac{\partial \overline{\rho \underline{u}}}{\partial t} + \underline{\text{div}}\ (\overline{\rho \underline{u} \otimes \underline{u}} + \overline{p}\underline{\underline{I}} - \underline{\underline{\overline{\sigma}}}) = 0 \tag{20b}$$

$$\frac{\partial \overline{\rho E}}{\partial t} + \text{div}\ \left[\overline{(\rho E + p)\underline{u}} + \underline{\overline{q}} - \overline{\underline{\underline{\sigma}}.\underline{u}}\right] = 0 \tag{20c}$$

- Newton and Fourier laws :

$$\underline{\underline{\sigma}} = 2\mu(T)\underline{\underline{S_0}} + \mu_v \text{div } \underline{u} \quad , \quad \underline{q} = -k(T) \, \underline{\text{grad}} \; T$$

$$\underline{\underline{S_0}} = \frac{1}{2}\left(\underline{\underline{\text{grad}}} \; \underline{u} + {}^t \, \underline{\underline{\text{grad}}} \; \underline{u}\right) - \frac{1}{3} \text{ div } \underline{u}$$

- Filtered ideal-gas equations of state

$$\overline{p} = R \, \overline{\rho \; T} \; , \quad \overline{\rho E} = C_v \overline{\rho T} + \frac{1}{2}\overline{\rho \underline{u}.\underline{u}} = \overline{p}/(\gamma - 1) + \frac{1}{2}\overline{\rho \underline{u}.\underline{u}} \; , \tag{21}$$

correct up to about $600K$ in air, with $\gamma = C_p/C_v = 1.4$.

- Beyond $600K$, $\gamma \nearrow$ (vibrations of polyatomic molecules).
- μ_v never zero in polyatomic molecules, and can be $\gg \mu$ across shocks (Smits & Dussauge, 1996).
- $\longrightarrow$ Stokes hypothesis ($\underline{\underline{\sigma}}$ trace-free) also excludes shocks.

- in monoatomic gases, helium or argon (no vibration nor rotation), $\gamma = 5/3$ until ionization and $\mu_v = 0$.
- Sutherland's law for μ valid between $100K$ and $1900K$. Constant $Pr = 0.7$ valid in air, even beyond 600K.

- specificity of filtered conservative equations : triple correlation ($\frac{1}{2}\overline{\rho \underline{u}.\underline{u}}$) involved in a time derivative.
- 2 approaches :
  - Reynolds filtering
  - Favre filtering

- Non-density-weighted variables (e.g. Boersma & Lele, 1999, CTR Briefs, 365-377).
  - resolved variables $(\overline{\rho}, \underline{\overline{u}}, \overline{p}, \overline{T})$
  - continuity equation (20a) becomes

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div}\ (\overline{\rho}\ \underline{\overline{u}}) = -\text{div}\ (\overline{\rho \underline{u}} - \overline{\rho}\ \underline{\overline{u}}) \tag{22}$$

  - exact pointwise mass preservation lost, but r.h.s. is conservative and $\int_{\Omega} r.h.s$ can be zero, with appropriate flux corrections (in 3D FV or conservative FD).
  - weakly-dissipative model of r.h.s. could increase robustness drastically (as in A.D.M., Leonard, Adams, Stoltz).
  - closure of $\frac{1}{2}\overline{\rho \underline{u}.\underline{u}}$: secondary issue

- Density-weighted variables : $\widetilde{\phi} = \dfrac{\overline{\rho\,\phi}}{\overline{\rho}}$,

  $\forall \phi \notin [\rho, p]$

  - resolved variables $(\overline{\rho}, \underline{\widetilde{u}}, \overline{p}, \widetilde{T})$
  - $\underline{\overline{u}}$, $\overline{T}$ not computable (but molecular terms $\overline{\underline{\underline{\sigma}}}$, $\underline{\overline{q}}$ and $\overline{\underline{\underline{\sigma}}.\underline{u}}$ are non-linear and thus non-computable anyway).

- Density-weighted variables (cont'd)
  - (pointwise) exact mass preservation ensured : continuity equation (20a) becomes

$$\frac{\partial \overline{\rho}}{\partial t} + \mathrm{div}\ \left(\overline{\rho}\ \widetilde{\underline{u}}\right) = 0 \tag{23}$$

  - subgrid-stress tensor

$$\begin{aligned} \underline{\underline{\tau}} &= -\overline{\rho \underline{u} \otimes \underline{u}} + \overline{\rho}\ \widetilde{\underline{u}} \otimes \widetilde{\underline{u}} \\ &= -\overline{\rho}\ \left(\widetilde{\underline{u} \otimes \underline{u}} - \widetilde{\underline{u}} \otimes \widetilde{\underline{u}}\right) \end{aligned} \tag{24}$$

- Density-weighted variables (cont'd)
  - filtered total (or stagnation) energy

$$\begin{aligned} \overline{\rho E} &= \overline{\rho} C_v \widetilde{T} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} - \tfrac{1}{2} tr(\underline{\tau}) \\ &= \frac{\overline{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} - \tfrac{1}{2} tr(\underline{\tau}) \end{aligned} \tag{25}$$

  - weakly-compressible two-scale DIA expansions (Yoshizawa, 1986, *Phys. Fluids*, **29**, 2152.) suggest model for $-\frac{1}{2} tr(\underline{\tau})$ (which doesn't act in the incompressible regime)
  - adequation to more compressible situations questioned by Speziale *et al.* (1988, *Phys Fluids*, **31** (4), 940-942.).
  - Three ways out

- Density-weighted variables, 3 ways out (cont'd)

1. Replace (20c) by non-conservative

$$\frac{\partial \overline{\rho e}}{\partial t} + \mathrm{div}\ \left[\overline{\rho e \underline{u}} + \overline{\underline{q}}\right] \quad = \quad -\overline{p\ \mathrm{div}\ \underline{u}} + \overline{\underline{\underline{\sigma}} : \underline{\underline{\mathrm{grad}\ \underline{u}}}} \tag{26}$$

(Moin *et al.*, 1991 *Phys. Fluids A*, **3** (11)) or (Erlebacher *et al.*, 1992, *J. Fluid Mech.*, **238**)

$$\frac{\partial \overline{\rho h}}{\partial t} + \mathrm{div}\ \left[\overline{\rho e \underline{u}} + \overline{\underline{q}}\right] = \frac{\partial \overline{p}}{\partial t} - \overline{p\ \mathrm{div}\ \underline{u}} + \overline{\underline{\underline{\sigma}} : \underline{\underline{\mathrm{grad}\ \underline{u}}}} \tag{27}$$

- Density-weighted variables, 3 ways out (cont'd)

1. (cont'd), with internal energy
$$\rho e = C_v \rho T = \frac{p}{\gamma - 1} \text{ or (static) enthalpy}$$
$$\rho h = \rho e + p = C_p \rho T = \frac{\gamma p}{\gamma - 1}.$$

2. add transport equation of resolved kinetic energy (RKE), i.e. $\frac{1}{2}\,\overline{\rho}\,\underline{\widetilde{u}}.\underline{\widetilde{u}}$ to (26) or (27) (Lee, 1992, Kuerten *et al.*, 1992, Vreman *et al.*, 1995, System I)
   - non-conservative terms $-\overline{p\,\mathrm{div}\,\underline{u}}$ and $+\overline{\underline{\underline{\sigma}} : \underline{\underline{\mathrm{grad}\,\underline{u}}}}$ remain, along with RKE's contribution $\underline{\widetilde{u}}.\underline{\mathrm{div}}\,(\underline{\underline{\tau}})$
   - succesful, e.g. in channel flow $M = 1.5$, $Re_\tau = 222$ (Lenormand *et al.*, 2000, *AIAA J.*, **38**, 8)

- Density-weighted variables, 3 ways out (cont'd) :

  3. keep fully conservative (20c) and lump $tr(\underline{\underline{\tau}})$ with the filtered internal energy. $\longrightarrow$ modified (and computable) pressure and temperature $\check{p}$ and $\check{T}$: (Vreman *et al.*, 1995, System II)

$$\overline{\rho E} = \overline{\rho} C_v \check{T} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} = \frac{\check{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} \quad , \qquad (28)$$

$$\check{p} = \overline{p} - \frac{\gamma - 1}{2} tr(\underline{\underline{\tau}}) \quad , \quad \check{T} = \check{p}/(\overline{\rho} R) \quad . \qquad (29)$$

- counterpart of *macro-pressure* $\overline{p} - \frac{1}{3} tr(\underline{\underline{\tau}})$ in incompressible LES with eddy-viscosity assumption $\underline{\underline{\tau_D}} \simeq 2\overline{\rho}\nu_t \underline{\underline{S_0}}$, with $\underline{\underline{\tau_D}} = \underline{\underline{\tau}} - \frac{1}{3} tr(\underline{\underline{\tau}})$.

- 3rd way out : macro-temperature closure (cont'd) :
  - Filtered momentum eq. (20b) becomes
  $$\frac{\partial \overline{\rho}\underline{\widetilde{u}}}{\partial t} + \underline{\text{div}} \left[\overline{\rho}\underline{\widetilde{u}} \otimes \underline{\widetilde{u}} + \left(\check{p} - \frac{5-3\gamma}{6} tr(\underline{\underline{\tau}}) \right) \underline{\underline{I}} - \underline{\underline{\tau_D}} - \underline{\underline{\overline{\sigma}}} \right] = 0$$
  - $\frac{5-3\gamma}{6} tr(\underline{\underline{\tau}}) = 0$ in monoatomic gases ($\gamma = 5/3$).
  - $\frac{5-3\gamma}{6} tr(\underline{\underline{\tau}})/\check{p} = \frac{5-3\gamma}{3} \gamma M_{sgs}^2$. with
  $$M_{sgs}^2 = \frac{1}{2}|tr(\underline{\underline{\tau}})|/\overline{\rho}\check{c}^2 = \frac{1}{2}|tr(\underline{\underline{\tau}})|/(\gamma\check{p}).$$
  - neglecting it in air is 3.75 less stringent than approximation $\gamma M_{sgs}^2 \ll 1$ required to neglect $-\frac{1}{2} tr(\underline{\underline{\tau}})$ with respect to $\overline{p}$ (see Erlebacher *et al.*, 1992, in a non-conservative context).

- Density-weighted variables (cont'd) :
  closure of total enthalpy flux $\overline{(\rho E + p)\underline{u}}$

  - resolved pressure : $\breve{p} = \check{p}$ or $\overline{p}$
  - at least three levels of decomposition are possible :

$$\overline{(\rho E + p)\underline{u}} = (\overline{\rho E} + \breve{p})\underline{\widetilde{u}} - \underline{\mathcal{Q}_H} \tag{30}$$

  with

$$\underline{\mathcal{Q}_H} = \left[-\overline{(\rho E + p)\underline{u}} + (\overline{\rho E} + \check{p})\underline{\widetilde{u}}\right] \tag{31a}$$

$$= \underbrace{\left[-\overline{(\rho e + p)\underline{u}} + (\overline{\rho e} + \check{p})\underline{\widetilde{u}}\right]}_{\underline{\mathcal{Q}_h}} + \underbrace{\left[-\tfrac{1}{2}\overline{\rho(\underline{u}.\underline{u})\underline{u}} + \tfrac{1}{2}(\overline{\rho\underline{u}.\underline{u}})\underline{\widetilde{u}}\right]}_{\underline{\mathcal{W}}} \tag{31b}$$

- Density-weighted variables (cont'd) : closure of total enthalpy flux $\overline{(\rho E + p)\underline{u}}$ (cont'd)

$$\underline{\mathcal{Q}_h} = \underbrace{\left[-\overline{(\rho e)\underline{u}} + (\overline{\rho e})\underline{\widetilde{u}}\right]}_{\underline{\mathcal{Q}_e}} + \left[-\overline{p\underline{u}} + \check{p}\underline{\widetilde{u}}\right] \qquad (32)$$

$\rho e = \rho C_v T = p/(\gamma - 1)$: internal energy.

- $\underline{\mathcal{Q}_h}$ and $\underline{\mathcal{Q}_e} \propto \underline{\text{grad}}\ \widetilde{T}$ (in Erlebacher *et al.*, 1992, and Moin *et al.*, 1991, resp.)

- $\underline{\mathcal{Q}_H} \simeq \overline{\rho} C_p(\nu_t/Pr_t)\underline{\text{grad}}\ \check{T}$ with $\check{p} = \check{p}$ yields Normand and Lesieur (1992) :

- Normand and Lesieur (1992) heuristic form

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div}\ \left(\overline{\rho}\underline{\widetilde{u}}\right) = 0 \tag{33a}$$

$$\frac{\partial \overline{\rho}\underline{\widetilde{u}}}{\partial t} + \underline{\text{div}}\ \left(\overline{\rho}\underline{\widetilde{u}} \otimes \underline{\widetilde{u}} + \check{p}\underline{\underline{I}} - 2\left[\mu(\check{T}) + \overline{\rho}\nu_t(\underline{\widetilde{u}})\right] \underline{\underline{S_0}}(\underline{\widetilde{u}})\right) = 0 \tag{33b}$$

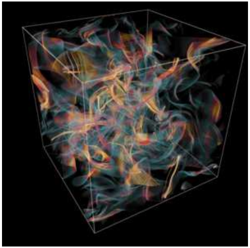
$$\frac{\partial}{\partial t}\left(\frac{\check{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}}\right) + \text{div}\left[\left(\tfrac{\gamma}{\gamma-1}\check{p} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}}\right)\underline{\widetilde{u}} - C_p\left(\frac{\mu(\check{T})}{Pr} + \frac{\overline{\rho}\nu_t(\underline{\widetilde{u}})}{Pr_t}\right)\underline{\text{grad}}\ \check{T} - 2\mu(\check{T})\underline{\underline{S_0}}(\underline{\widetilde{u}}).\underline{\widetilde{u}}\right] = 0 \tag{33c}$$

still with $\check{T} = \check{p}/(\overline{\rho}R)$.

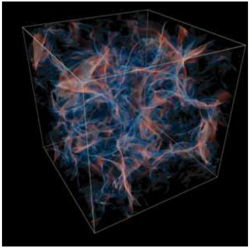
- Normand and Lesieur (1992) heuristic form (cont'd)
  - amounts to adding $\overline{\rho}\nu_t$ and $\overline{\rho}C_p(\nu_t/Pr_t)$ to their molecular counterpart in (33c) except in the last term of the energy equation.
  - This exception disappears when option (32) is taken, with $\underline{\mathcal{Q}_h} \simeq \overline{\rho}C_p\ (\nu_t/Pr_t)\underline{\text{grad}}\ \check{T}$ and the RANS type model $\underline{\mathcal{W}} \simeq \underline{\underline{\tau}}.\underline{\widetilde{u}}$.
  - used succesfully by Knight *et al.* (1998, see also Okong'o & Knight, 1998) on unstructured grids.
  - $|\underline{\mathcal{W}} - \underline{\underline{\tau}}.\underline{\widetilde{u}}|$ small in constant-density RANS filtering.

- MILES : Monotone Integrated Large-Eddy Simulation :
  - Shock-capturing schemes, dissipative (upwind, limiters . . .)
  - Euler equations
- Quasi-incompressible isotropic turbulence.

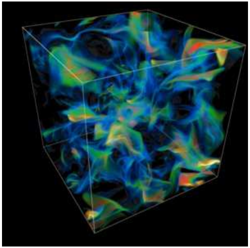
- PPM : Piecewise Parabolic Method



vorticity



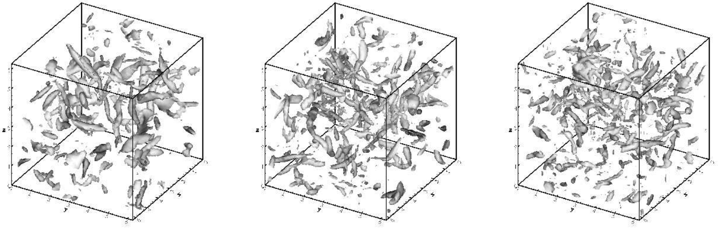
dilatation



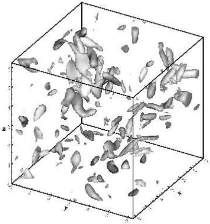
entropy

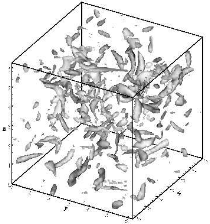
2048^3 (Porter, Woodward, Pouquet 1997)

- Assessment of numerical dissipation of high-order shock-capturing schemes : visualizations



ENO WENO MENO





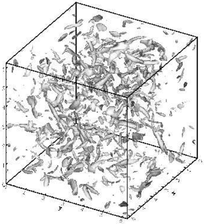
- vorticity magnitude
- resolution 64^3

Garnier *et al.*(JCP, 1999)

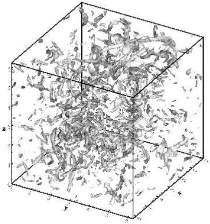
Jameson MUSCL4

vorticity magnitude, 64^3 (Garnier *et al.*, J.C.P., 1999) :

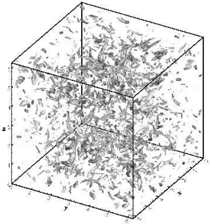
- Assessment of numerical dissipation of high-order shock-capturing schemes : visualizations



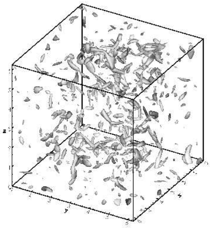
ENO



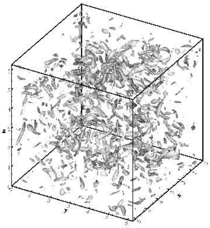
WENO



MENO



Jameson

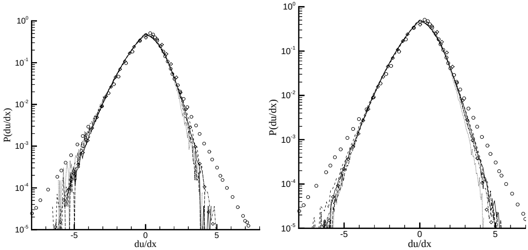


MUSCL4

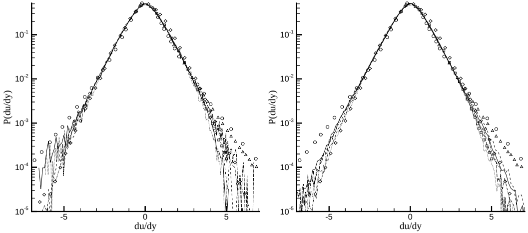
- vorticity magnitude
- résolution 128^3

Garnier *et al.*(JCP, 1999)

- pdf's of velocity derivatives

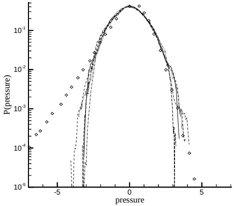


$\partial u/\partial x$ 64^3 128^3

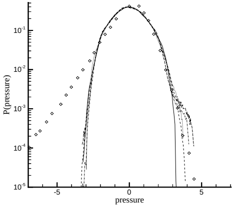


ENO ———
WENO – – – – –
MENO –·–·–·–·–
Jameson ··········
MUSCL4 — — — — —
Métais and Lesieur 1992 $(Re_\lambda \approx 20)$ $\Diamond$
She 1991 $(Re_\lambda \approx 24)$ $\Box$
She 1991 $(Re_\lambda \approx 77)$ $\triangle$
Vincent and Meneguzzi 1991 $(Re_\lambda \approx 150)$ $\bigcirc$

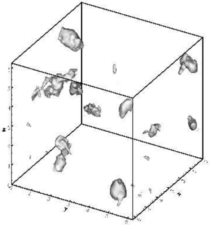
- pdf's of pressure fluctuations



64^3



128^3



128^3

ENO ———

WENO – – – – –

MENO –·–·–·–·–

Jameson ··············

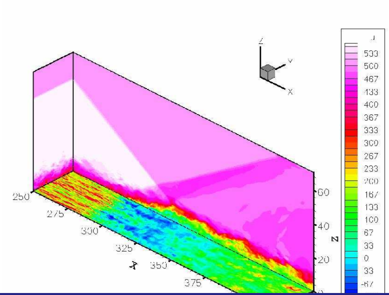
MUSCL4 –– –– ––

Métais & Lesieur 1992 ($Re_{\bar{\lambda}} \approx 20$) ◊

- Summary (Garnier, 1999)
  - all tested schemes show excessive numerical dissipation :
    - equivalent Re_λ
    - equivalent C_S
    - Gaussianization of pdf's
  - need of :
    - marginally-stable centered schemes
    - SGS models based on physical considerations
    - numerical dissipation only around shocks

Shock Wave / Boundary Layer Interaction (1 of 1)

- Exp : Dussauge *et al.*, $M_\infty = 2.4$
- LES : (Garnier, 2002) :
  - 4th-order centered conservative (skew-symmetric FV, Ducros)
  - Selective Mixed-Scale Model
  - with local ENO filtering (with Ducros sensor)

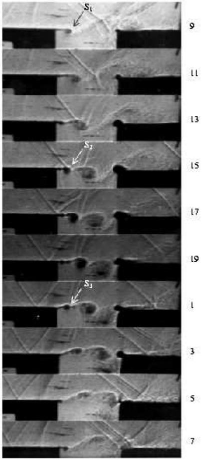


Motivations for open transonic cavity flows :

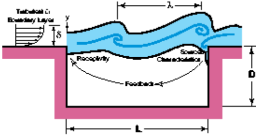
- Self-sustained oscillations in open cavity flows remain mysterious
- fluid-acoustic coupling (Rossiter, 1964)
- fluid-fluid coupling (Gharib & Roshko, JFM, 1987)
- somewhat easy to mitigate in transonic regime
- Complex system $\longrightarrow$ control $\longrightarrow$ understanding ?

1. LES in Fourier space
2. LES in physical space
3. Boundary Layers
4. Compressible LES formalism
5. Assessment of high-order shock-capturing schemes
6. **Natural compressible cavity flows**
7. Controlled compressible cavity flows

- planar open cavities $\longrightarrow$ sustained oscillations

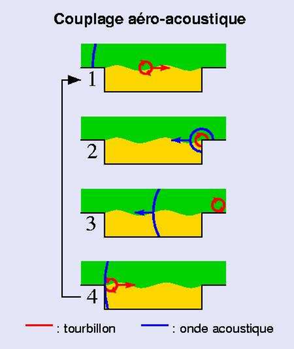


Forestier, 2003



- geometrical parameters
  - L/D
  - L/W
  - L/δ
- flow parameters
  - Re_δ
  - M_∞
  - $H = \delta/\theta$
  - p_{rms}/p_{t_∞}

- Rossiter (R.A.E Tech. Rep. 64037, 1964)



$$f_m = \frac{U_c}{L}\,(m-\gamma)\ ; \quad U_c = \frac{U_\infty}{\left(\frac{1}{K} + M_\infty\right)}$$

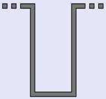
avec e.g. $\gamma = 0.25$ et $K = 0.57$ pour $L/D = 4$.

Baseline configurations (Larchevêque *et al.*, 2003, 2004)

Cavités étudiées

L/D=0,41
Forestier *et al.* (2002)
(ONERA / DAFE)

- M=0,8
- $\mathrm{Re_L} = 6,5.10^5$
- $1,8.10^6$ mailles
- données expérimentales :
  - strioscopies,
  - pression,
  - vitesses.



L/D=2
Forestier *et al.* (2000)
(ONERA / DAFE)

- M=0,8
- $\mathrm{Re_L} = 6,5.10^5$
- $6,1.10^6$ mailles
- données expérimentales :
  - strioscopies,
  - pression,
  - vitesses.

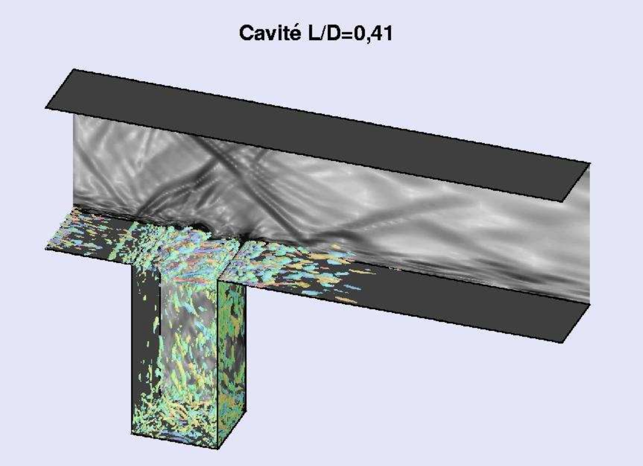
L/D=5
Henshaw (2000)
(Quinetiq)

- M=0,85
- $\mathrm{Re_L} = 7,2.10^6$
- $3,5.10^6$ mailles
- données expérimentales :
  - pression

- hybrid LES/URANS or LES/DES ONERA code
- Space integration : AUSM+(P) scheme, simplified (Mary and Sagaut)
- Time integration : explicit RK3 or Gear scheme (BDF2, A-stable, approximate Newton method, LU-SGS (Jameson & Yoon, Coackley),
- SA model in URANS or DES
- Selective Mixed-Scale SGS model in LES, in Density-weighted filtered variables (Lenormand *et al.*, AIAA J. 2000)

- LU-SGS :
  - Weber & Ducros (IJCFD, 2000) : transition on ONERA A airfoil
  - van Buuren, Kuerten & Geurts (JCP, 1997)
- DP-LUR (parallel variant of LU-SGS)
  - Martin & Candler (JCP 2006)

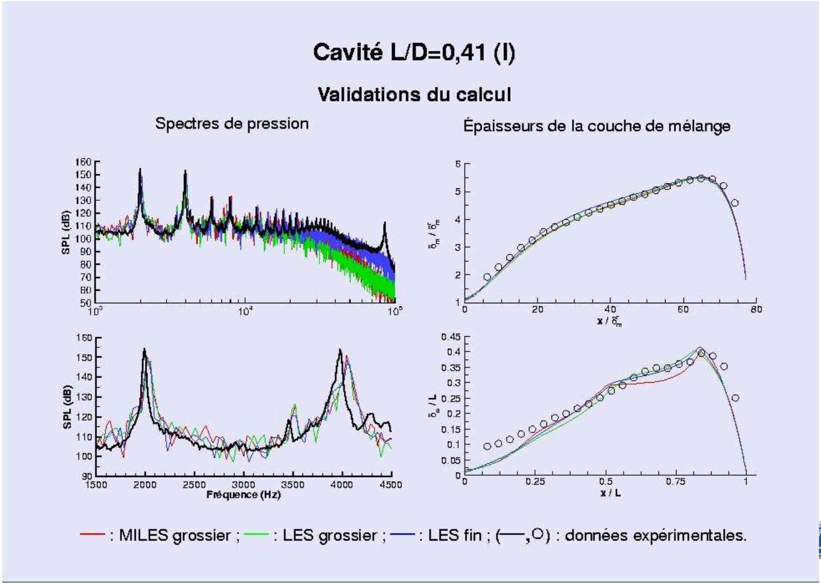
Natural cavity flow, $L/D = 0.41$



Movies : 1 : Schlieren 2 : $Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = \frac{1}{2\rho}\nabla^2 P > 0$

3 : Phase averages

Natural cavity flow, $L/D = 0.41$



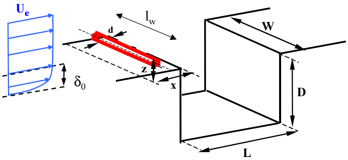
Controlled compressible cavity flows

- passive devices known to reduce oscillations : e.g. spanwise cylinders (Mc Grath & Shaw, AIAA Paper 96-1949)
- physical explanation of efficiency remains open
- need for to investigate the simplest possible case, viz. deep cavity

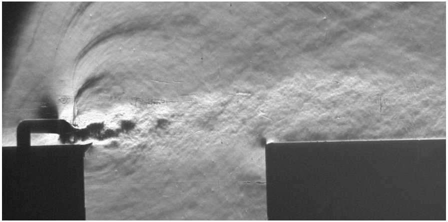
Existing conjectures

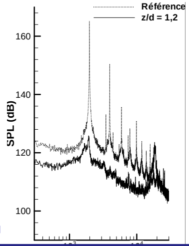
- H1 (Stanek *et al.*, AIAA Paper 2000-1905) :
  - High frequency forcing injects energy at small scale
  - More energy is extracted from the large scales by the Kolmogorov cascade
  - Requires $f_{wake} > 10 f_{Rossiter_1}$
- H2 (Stanek *et al.*, AIAA Paper 2003-2003) :
  - High frequency forcing increases momentum diffusion
  - Modification of the mean flow in the mixing layer region
  - Increased stability
- H3 (Ukeiley *et al.*, AIAA Paper 2002-0661) :
  - The wake lifts up the mixing layer
  - Mitigated vortex impingement onto aft edge of cavity
  - increase of the mixing-layer thickness
  - reduced shear
  - Reduced pressure tones

- $\longrightarrow > 30dB$ peak & 6dB background pressure level reduction



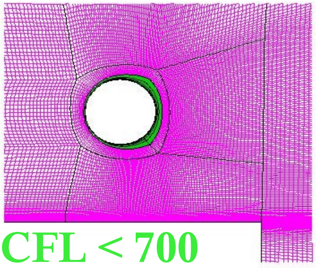
- $W = l_w = 120mm$
- $D = 120mm$, $L = 50mm \longrightarrow L/D = 0.41$
- $L/W < 1 \longrightarrow$ 2D cavity
- $d = 2.5mm$, optimum at $z = 3mm \longrightarrow z/d = 1.2$





- flow parameters
  - $U_\infty = 260ms^{-1} \longrightarrow M = 0.8, Re_L = 8.6\ 10^5$.
  - $Re_d = \sim 3\ 10^4 \quad - \quad 4\ 10^4 \longrightarrow$ subcritical wake
  - $f_{wake} = 20\ 000Hz \longrightarrow St_{wake} = fd/U_\infty \sim 0.2$
  - $f_{Rossiter_1} = 2\ 000Hz$
- requirements
  - $\Delta t = 0.25\mu s \longrightarrow$ 1 wake shedding period $= 200\Delta t$
  - $\Delta t \sim (1/6)\Delta t_{baseflow}$
  - integration over 50 periods of Rossiter $\longrightarrow 100000\Delta t$
    $\Delta x^+ \sim 50 \quad , \qquad \Delta y+_{min} \sim 2 \quad , \qquad \Delta z^+ \sim 20$
  - upstream boundary layer resolved over 80 cells
  - 250 cells in spanwise direction, periodic bc's
  - $W_{num} = 20d$
  - precursor recycled LES : span $W_{num}/5$ replicated 5 times

- Numerically more challenging than natural cavity flow
  - need to simulate the wake created by the rod



CFL < 700

CFL < 20

"Generalized" CFL : 1D expression :

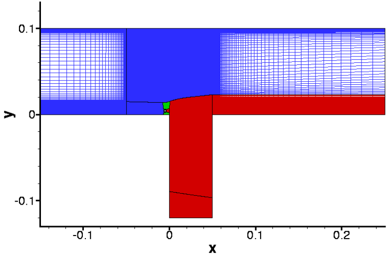
$$CFL = \Delta t \max\left(\frac{|\tilde{u}| + c}{\Delta x}, 2\gamma \frac{\frac{\mu}{Pr} + \rho \frac{\nu_t}{Pr_t}}{\rho \Delta x^2}\right)$$

$CFL < 20$
$20\ 10^6$ grid points

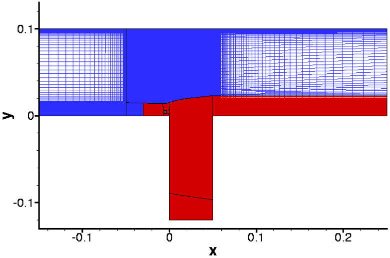
$CFL < 700$
$5\ 10^5$ grid points

  - need for realistic upstream turbulence ?

- block upstream of cylinder :
  - 2D URANS in DES vs.
  - spanwise-replicated LES with Lundt's recycling method



DES grid



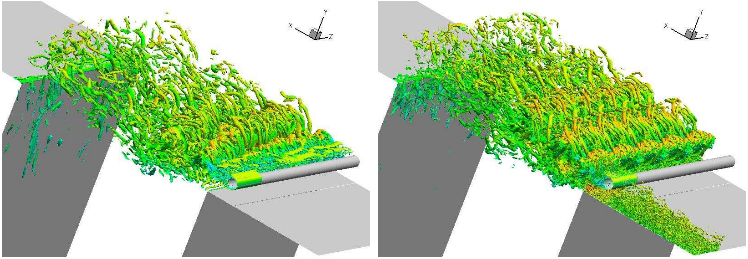
LES grid

for both cases : URANS, LES

- $N = 80$ Newton iterations needed when CFL ~ 700
- $N = 4$ (as in Larchevêque *et al.* (2003, 2004) enough when $CFL \leq 16$
- factor of 10 in CPU gained thanks to Block-LOcal Convergence

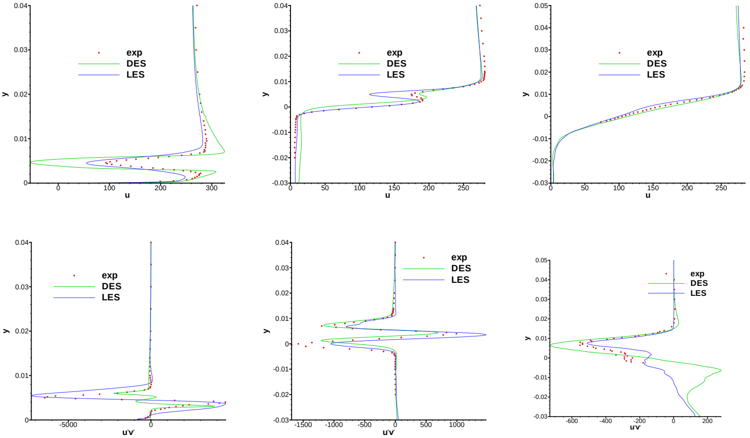
	N		CPU time
	CFL < 16	16 < CFL < 700	
Natural, Fixed N	4		1
Controlled, Fixed N	80	80	556
Controlled, Local N	4	80	56

Isosurface $Q = (1/2)(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = 1/2\rho\nabla^2 P > 0$



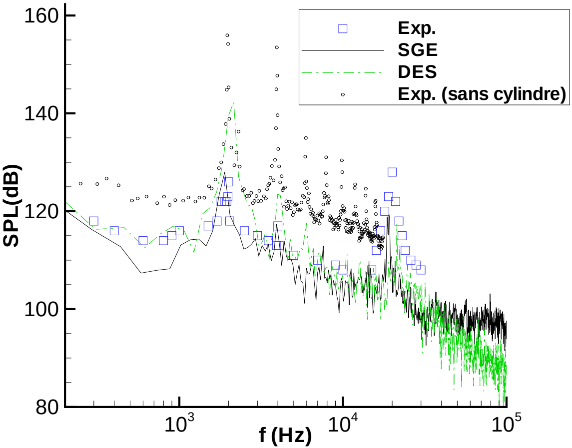
left : DES right : LES

Fluctuation profiles in DES and in LES

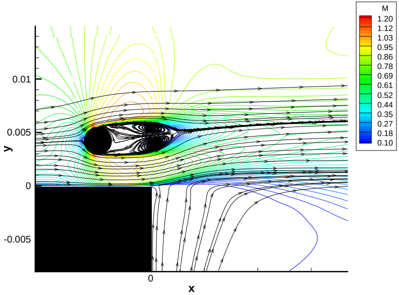


from left to right : mean velocity (top) and Reynolds stress $\overline{u'v'}$ (bottom) at $x = 0$, $x = L/5$ and $x = 4L/5$.

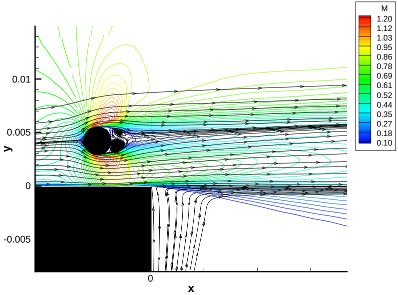
pressure spectra near the rear wall of the cavity



Mean iso-Mach lines

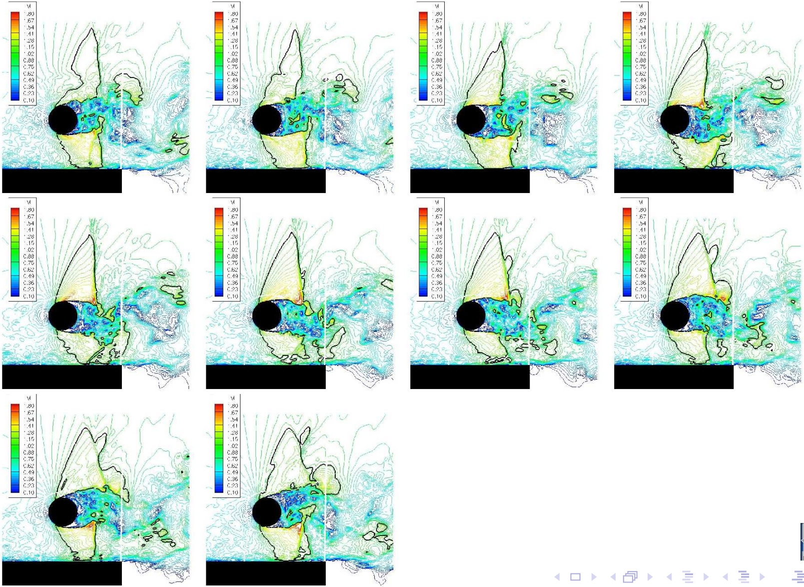


DES

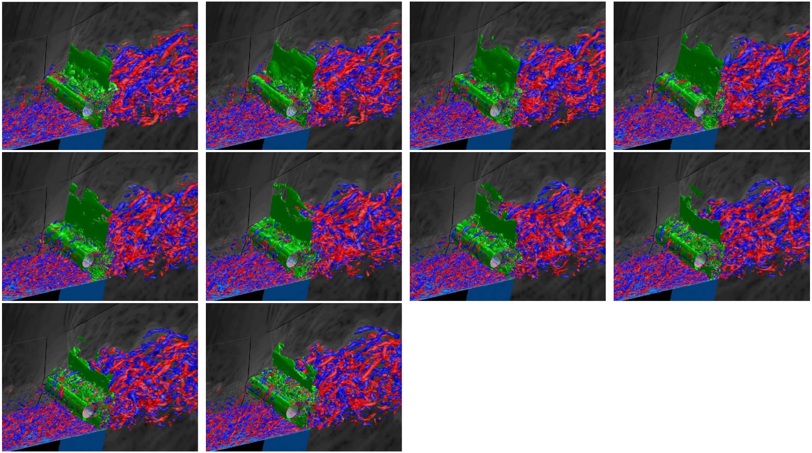


LES

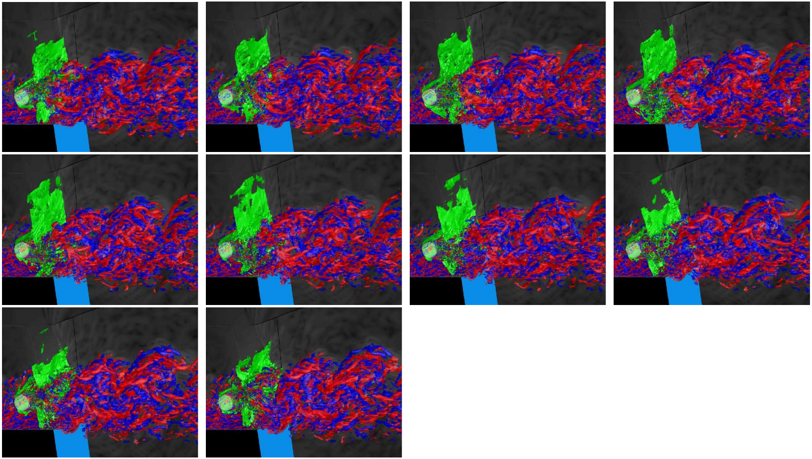
Instantaneous iso Mach lines



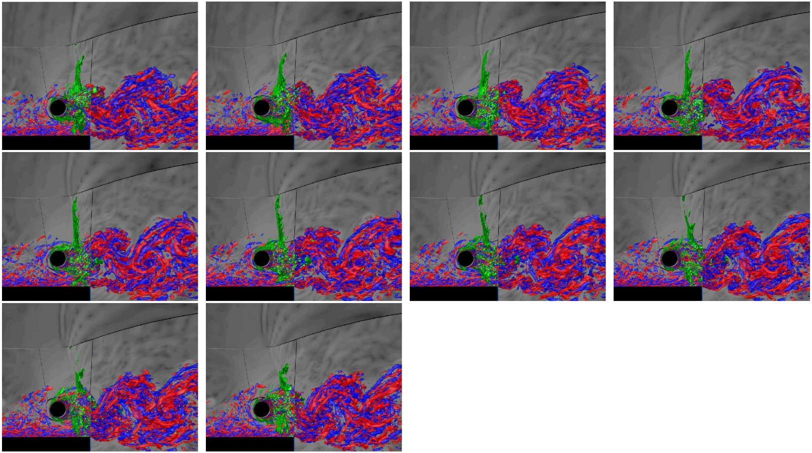
Schlieren, $Q = 2\,(U_\infty/d)^2$ and $|\partial_x \rho||\mathrm{div}\vec{u}| = 1.3\,d^2/(\rho_\infty U_\infty)$



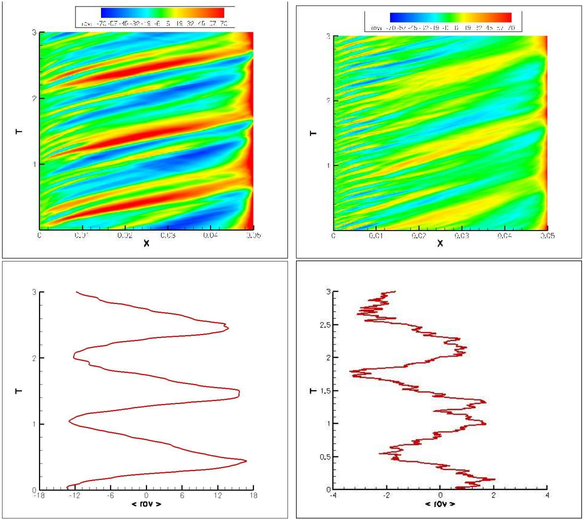
Schlieren, $Q = 2\,(U_\infty/d)^2$ and $|\partial_x \rho||\mathrm{div}\vec{u}| = 1.3\, d^2/(\rho_\infty U_\infty)$

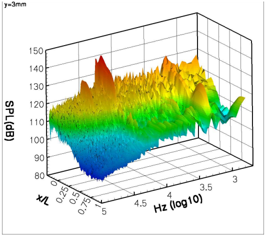


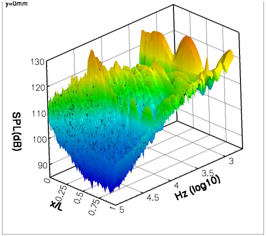
Schlieren, $Q = 2\,(U_\infty/d)^2$ and $|\partial_x \rho||\mathrm{div}\vec{u}| = 1.3\,d^2/(\rho_\infty U_\infty)$

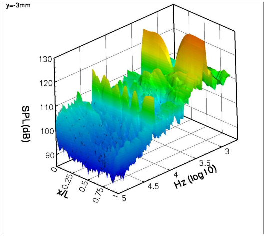


Mass flow rate through grazing plane : baseline (left) and LES with cylinder (right)









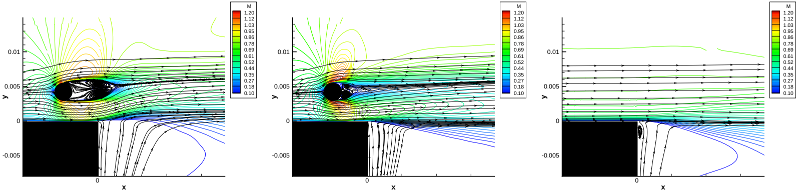
$y = 3mm$, $y = 0mm$ $y = -3mm$.

1st Rossiter : log(2000)Hz= 3.3 cylinder wake : log(20000)Hz= 4.3

- node at $x = 0.5L$ for 1st Rossiter for $y \leq 0$ as in Larchevêque *et al.*(2004)
- 6 antinodes at 20kHz : acoustic modes, since

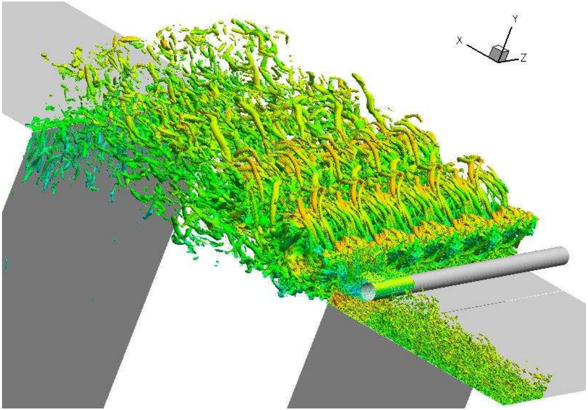
$$f_{n_x,n_y,n_z} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_z}{W}\right)^2 + \left(\frac{n_y + \frac{1}{2}}{D}\right)^2}, \text{ hence, } n_x = \frac{2f_{n_x,0,0}L}{c} \sim 6$$

Mean-flow deflection or mischievous fore-edge mean vortex ?



DES LES Baseline

Movies ? Will it work ? Cross fingers !



$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = \frac{1}{2\rho}\nabla^2 P > 0 \text{ and } \underline{\text{grad}}\ \rho :$$

slice zoom perspective zoom front zoom back zoom side

- Numerical recovery of Rossiter mode mitigation at $L/D \sim .41$
- Precursor boundary-layer fluctuation generator needed
- together with $y^+ \sim 1$ mesh refinement around the cylinder
- high local “generalized" CFL's, well tolerated if convergence sufficiently pushed
- rule of thumb “CLF / N $\sim$ constant" confirmed
- Regarding the physics …

- At $L/d \sim 0.41$, no obvious mean flow deflection observed $\longrightarrow$ against H_3
- H1 rebutted by Illy *et al.* (control effective at lower forcing frequencies : z/d more important than d alone)
- Mean flow in shear layer remains inflectional : against H_2.
- The fore-edge recirculation bubble noticed by Larchevêque disappeared when control applied : much welcome outsider ! ! !
- anyway, total lack of universality wrt various aspect ratios helps making us cautious.

Summary I

- Some soft spots
  - Spectral cusp near cutoff wavenumber
  - Compressible LES formalism
  - Shock-capturing schemes
- Some highs
  - "Selective" function in use in applied configurations
  - Filtered Structure-Function model can retrieve cusp behaviour and enable vorticity backscatter (daSilva et al.) $\longrightarrow$ High-pass filtered models (Stolz etal)
  - Existence of SGS models based on the 3rd-order structure function (Shao et al)
  - High generlized CFLs possible in certain cases (eg transonic cavity flow)

Summary II

- Recovery of phase-averaged coherent structures, mean-flow bifurcations, mode switching . . .
- Importance of "realistic" upstream fluctuations : LES vs DES
- Sufficiently high fidelity to tackle active/reactive control

Thanks I

- E. Briand
- C. Brun
- F. Daude
- E. David
- F. Delcayre
- F. Ducros
- Y. Dubief
- E. Garnier
- L. Larchevêque
- I. Mary
- J.H. Silvestrini

Thanks II

- LEGI Grenoble, M. Lesieur, O. Métais
- ONERA Chatillon

Thank you
for your
attention