TURBULENCE MODELS FOR PREDICTING HEAT TRANSFER IN BOUNDARY LAYER FLOWS

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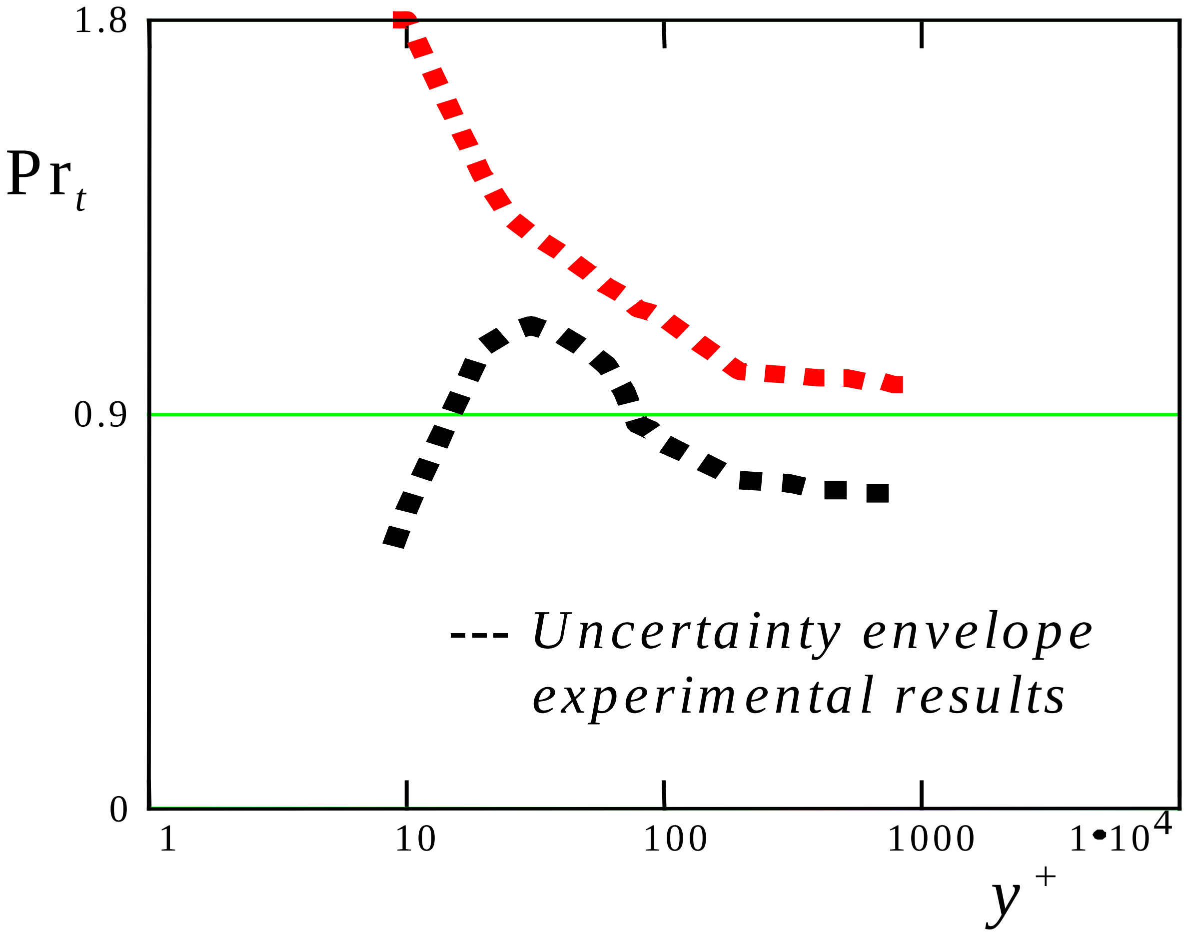
- In order to improve the thermal field calculations in near wall region, a two equation model for thermal field was developed
- This model are applicable to flows where the real value of turbulent Pranndtl number is unknown, having much higher generality than the traditional constant Prandtl number models.
- The eddy diffusivity for turbulent heat is expressed using the temperature point correlation   and the dissipation rate of temperature fluctuations.
- The model was tested for different available experimental data with zero pressure gradient, favorable pressure gradient and adverse pressure gradient flows.

- 1. Motivation
- 2. Turbulence modeling
- 3. Results
- 4. Conclusions

Motivation

- In heat transfer calculations, the turbulent Prandtl number is usually taken constant and the turbulent heat fluxes are directly determined from turbulent momentum fluxes.
- The eddy diffusivity for heat is estimated via the eddy viscosity considering a constant value of turbulent Prandtl number
- Measurements and direct simulation *) data showed that the hypothesis of constant turbulent Prandtl number could not adequately reflect the physical phenomenon, even for simple wall shear flows.

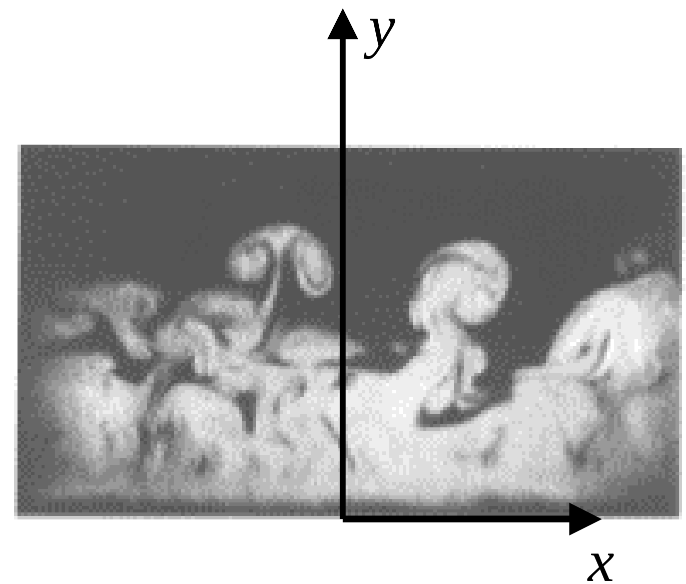
Krishnamoorthy, L.V. and Antonia, R.A., *Temperature dissipation measurements in a turbulent boundary layer*, J. Fluid Mech. Vol. 176, pp.265-281,1987.
Antonia R.A., *Behavior of the turbulent Prandtl number near the wall*, Int. J. Heat Mass Transfer, 23, pp. 906-908, 1980.
Spalart, P.R., *Direct simulation of a turbulent boundary layer up to* , J. Fluid Mech. Vol. 187, pp. 61-98, 1988.



- In the fully turbulent region the turbulent Prandtl number reach 0.7 - 0.9, values normally assumed for wall shear flow calculations and increases towards a wall, where is about 1.10.

Mean flow equations

Applying the mass (Favre) average and introducing the boundary layer and the gradient (Boussinesq) hypothesis



$$-\rho\overline{u'v'} = \rho\nu_t \frac{\partial U}{\partial y} = \mu_t \frac{\partial U}{\partial y}$$

$$-\rho\overline{v't'} = \rho\alpha_t \frac{\partial T}{\partial y} = \frac{\mu_t}{\mathrm{Pr}_t} \frac{\partial T}{\partial y}$$

$$\mathrm{Pr}_t = \nu_t / \alpha_t$$

Mean flow equations

$$\frac{\partial}{\partial x}(\rho U) + \frac{\partial}{\partial y}(\rho V) = 0$$

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial U}{\partial y}\right] - \frac{dP}{dx}$$

$$\rho U \frac{\partial H}{\partial x} + \rho V \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left\{\rho(\alpha + \alpha_t)\frac{\partial H}{\partial y} + [(\mu + \mu_t) - \rho(\alpha + \alpha_t)]U \frac{\partial U}{\partial y}\right\} +$$

$$+ \frac{\partial}{\partial y}\left[\left\{(\mu - \rho\alpha)\frac{\partial H}{\partial y} + \left(\frac{\mu_t}{\sigma_k} - \rho\alpha_t\right)\right\}\frac{\partial k}{\partial y}\right]$$

$$y = 0 \quad U = V = 0 \quad H = H_w (T = T_W)$$

$$y = \delta \quad U = U_e \quad H = H_e$$

Turbulence models

$$\mu_t = \rho C_\mu f_\mu \frac{k^2}{\varepsilon} \qquad \mu_t \propto k.\tilde{t} \qquad \tilde{t}_e = k/\varepsilon \quad , \quad \hat{V}_e = \sqrt{k} \quad , \quad L_e = k^{\frac{3}{2}}/\varepsilon$$

$$\rho\left(u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y}\right) = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + \mu_t\left(\frac{\partial u}{\partial y}\right)^2 - \rho\varepsilon$$

$$\rho\left(u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y}\right) = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + \left[C_{\varepsilon 1} f_1 \mu_t \left(\frac{\partial u}{\partial y}\right)^2 - C_{\varepsilon 2} f_2 \rho\varepsilon\right]\frac{\varepsilon}{k}$$

,

The various models differ trough the use of different damping functions , different terms *D* and *E* and different closure constants , depending on some dimensionless parameters*)

$$R_y = \frac{\sqrt{k}\,y}{\nu}, \quad Re_T = \frac{k^2}{\nu\varepsilon}, \quad y^+ = \frac{u_\tau y}{\nu} \qquad u_\tau = \sqrt{\frac{\tau_w}{\rho}}, \quad \tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

Lam C.K.G., Bremhorst K.A., *Modified form of the k- model for predicting wall turbulence*, Journal of Fluid Engineering, Vol. 103, pp. 456-460, 1981.

Chien K.Y., *Predictions of channel and boundary -layer flows with a low Reynolds number turbulence model*, AIAA Journal, Vol. 20, pp. 33-38, 1981

Turbulence model for thermal field

The modeled transport equations for the two dimensional turbulent boundary layer*)

$$\rho U \frac{\partial \overline{t^2}}{\partial x} + \rho V \frac{\partial \overline{t^2}}{\partial y} = \frac{\partial}{\partial y}\left[\left(\rho\alpha + \frac{\rho\alpha_t}{\sigma_h}\right)\frac{\partial \overline{t^2}}{\partial y}\right] + 2\rho\alpha_t\left(\frac{\partial T}{\partial y}\right)^2 - 2\rho\varepsilon_t$$

$$\rho U \frac{\partial \varepsilon_t}{\partial x} + \rho V \frac{\partial \varepsilon_t}{\partial y} = \frac{\partial}{\partial y}\left[\left(\rho\alpha + \frac{\rho\alpha_t}{\sigma_\phi}\right)\frac{\partial \varepsilon_t}{\partial y}\right] + C_{p1} f_{p1} \frac{\varepsilon_t}{\overline{t^2}}\rho\alpha_t\left(\frac{\partial T}{\partial y}\right)^2 +$$

$$+ C_{p2} f_{p2} \frac{\varepsilon_t}{k}\mu_t\left(\frac{\partial U}{\partial y}\right)^2 - C_{d1} f_{d1}\rho\frac{\varepsilon_t^2}{\overline{t^2}} - C_{d2} f_{d2}\rho\frac{\varepsilon}{k}\varepsilon_t$$

$$\varepsilon_t = \alpha\overline{\left(\partial t/\partial x_j\right)^2}$$

Sommer, T.P., So, R.M.C., and Lai,Y.G., *A near wall two equation model for turbulent heat fluxes,* Int. J. Heat Mass Transfer 35, pp. 3375-3387, 1992

- the eddy diffusivity for heat

$$\alpha_t \propto k\tau_m$$

$$\tau_m$$

$$\tau_m \propto \left(\tau_u^l \tau_t^m\right) = \tau_u R_\tau^m,$$

$$l + m = 1$$

$$R_\tau = \tau_t / \tau_u$$

the time scale ratio

- the composite (hybrid) time scale which depends on
  - the velocity field time scale;
  - the temperature field time scale

In the external region, the characteristic scales are

$$\tilde{t}_e = k/\varepsilon \quad , \qquad \hat{V}_e = \sqrt{k} \quad , \qquad L_e = k^{\frac{3}{2}}/\varepsilon$$

In the internal region, the Kolmogorov time scale

$$\tilde{t}_i = (\nu/\varepsilon)^{\frac{1}{2}} \quad , \qquad \hat{V}_i = (\nu\varepsilon)^{\frac{1}{4}} \quad , \qquad L_i = \left(\nu^3/\varepsilon\right)^{\frac{1}{4}}$$

The turbulent time scale for the whole boundary layer can be written as

$$\tilde{t} = \tilde{t}_e + f_t.\tilde{t}_i = \tilde{t}_e.\left(1 + \frac{f_t}{\sqrt{\mathrm{Re}_T}}\right) \qquad \mathrm{Re}_T = k^2/(\nu\varepsilon)$$

where the damping function should satisfy the asymptotic conditions:

$$f_t \rightarrow 1 \text{ for } Re_T \rightarrow 0 \text{ (near the wall)}$$

$$f_t \rightarrow 0 \text{ for } Re_T \rightarrow \infty \text{ (in the inertial turbulent region)}$$

$$f_t = 1 - e^{-\frac{1}{Re_T}}$$

$$f_{p1} = 1$$

$$\rho U \frac{\partial \varepsilon_t}{\partial x} + \rho V \frac{\partial \varepsilon_t}{\partial y} = \frac{\partial}{\partial y}\left[\left(\rho\alpha + \frac{\rho\alpha_t}{\sigma_\phi}\right)\frac{\partial \varepsilon_t}{\partial y}\right] + C_{p1} f_{p1} \frac{\varepsilon_t}{t^2} \rho\alpha_t \left(\frac{\partial T}{\partial y}\right)^2 +$$

$$+ C_{p2} f_{p2} \frac{\varepsilon_t}{k} \mu_t \left(\frac{\partial U}{\partial y}\right)^2 - C_{d1} f_{d1} \rho \frac{\varepsilon_t^2}{t^2} - C_{d2} f_{d2} \rho \frac{\varepsilon}{k} \varepsilon_t$$

$$f_{p2} = \frac{\sqrt{\mathrm{Re}_T}}{\sqrt{\mathrm{Re}_T} + 1 - \exp\left(-1/\mathrm{Re}_T\right)}$$

$$f_{d1} = \left\{1 - \exp\left(-y^+/D_1^+\right)\right\}^2$$

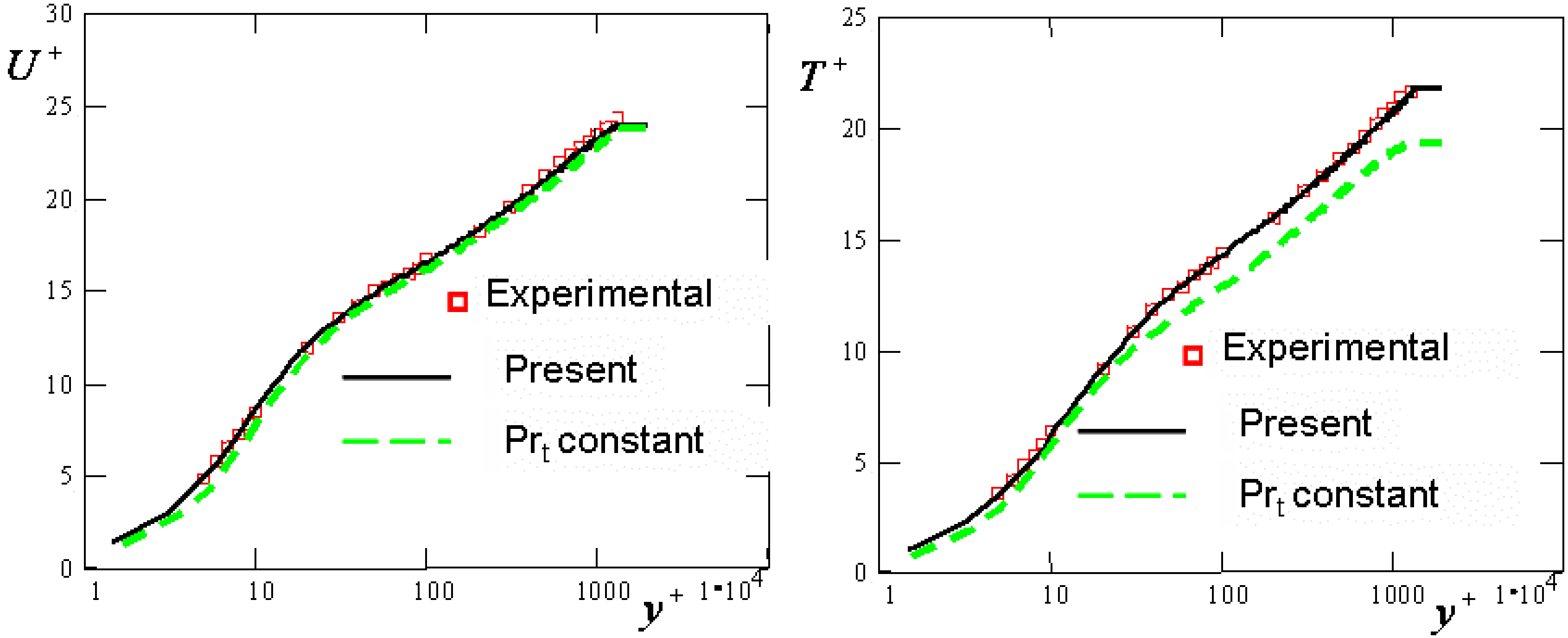
$$f_{d2} = \frac{C_{2\varepsilon} f_2 - 1}{C_{d2}} \cdot \frac{\sqrt{\mathrm{Re}_T}}{\sqrt{\mathrm{Re}_T} + 1 - \exp\left(-1/\mathrm{Re}_T\right)} \cdot \left[1 - \exp\left(-y^+/D_2^+\right)\right]^2$$

$$\alpha_t = C_\lambda f_\lambda k \left\{ \frac{k}{\varepsilon} \frac{\overline{t^2}}{\varepsilon_t} \right\}^{\frac{1}{2}}$$

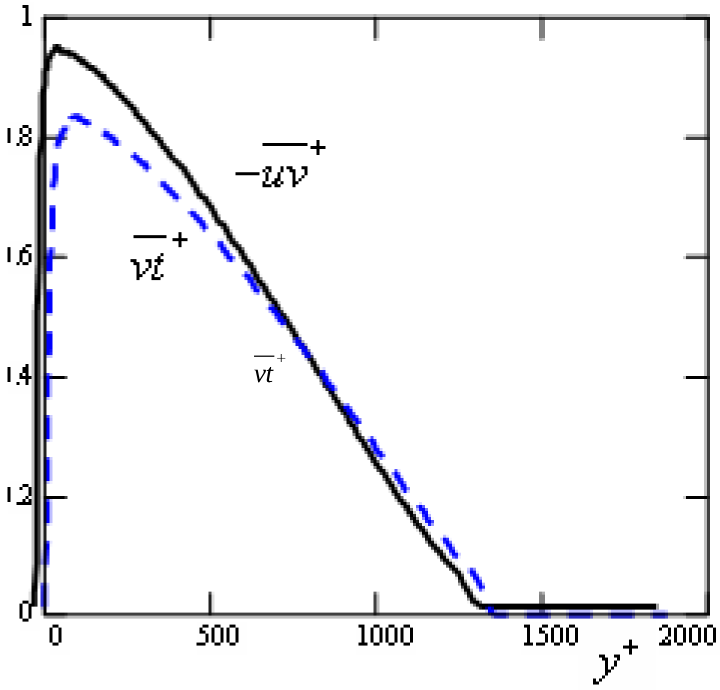
$$f_\lambda = \left[\frac{C_{\lambda 1}\left(1 - f_{\lambda 1}\right)}{\sqrt[4]{\mathrm{Re}_T}} + f_{\lambda 1} \right] \left[1 + \frac{1 - \exp\left(-1/\mathrm{Re}_T\right)}{\sqrt{\mathrm{Re}_T}} \right]$$

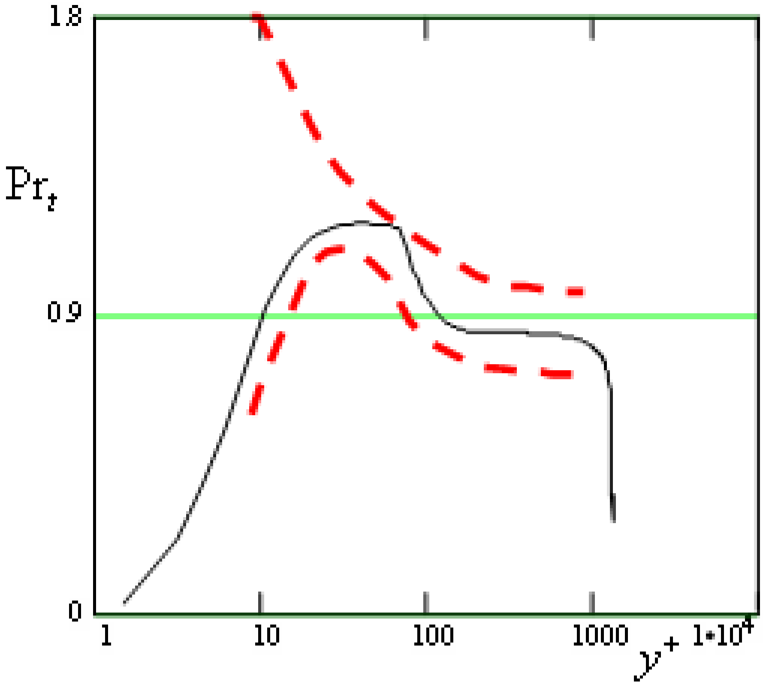
$$f_{\lambda 1} = \left[1 - \exp\left(-y^+ / A_\lambda^+\right) \right]^2$$

Zero pressure gradient flow
(experimental Simpson, Whitten, and Moffat)

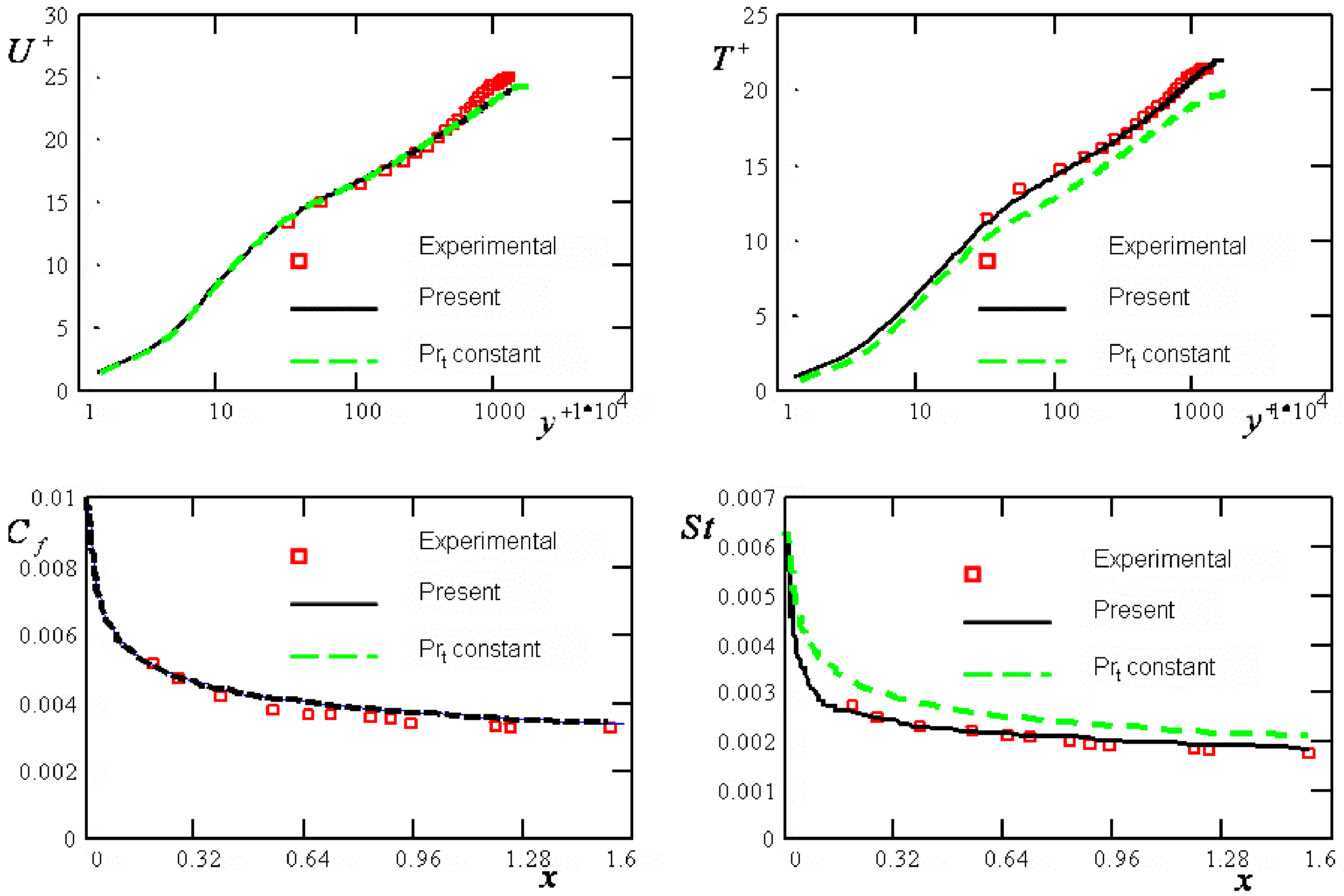


Turbulent boundary layer on heated plate at constant pressure (Bell 3000)

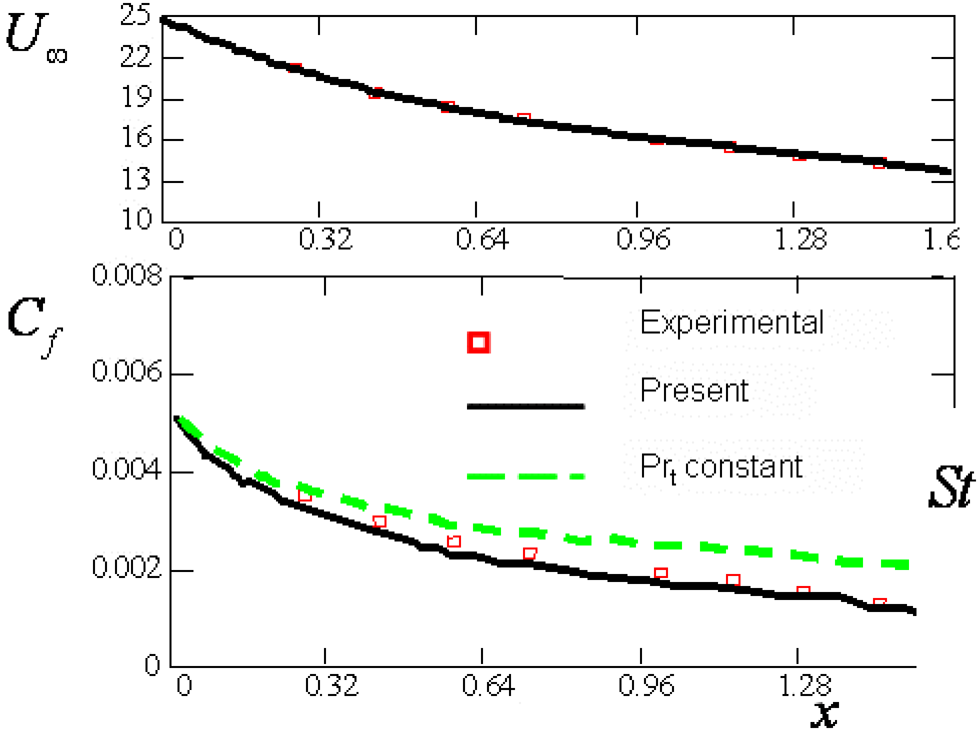


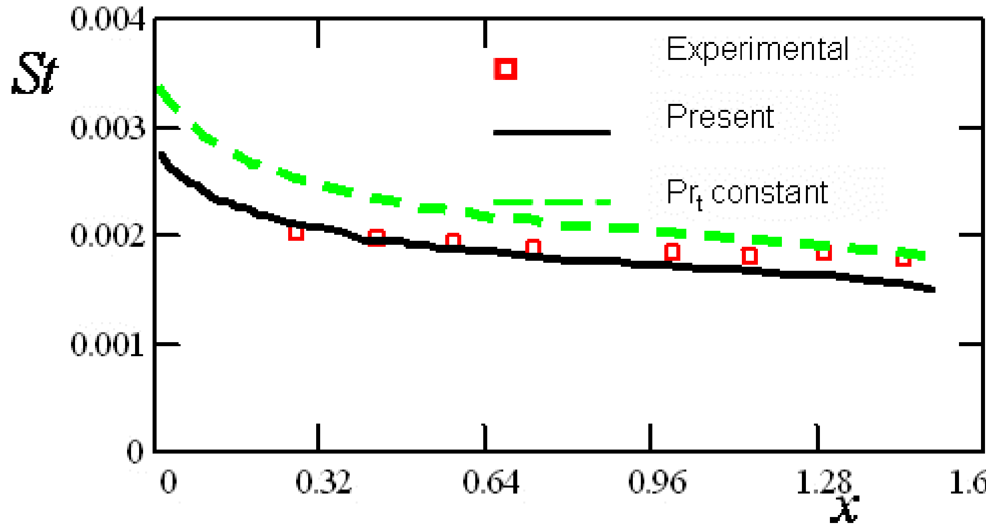


Boundary layer on heated plate (Bell 3200)



Adverse pressure gradient heat transfer flows (Bell 3200)





Conclusions

- (*i*) The presented near wall model uses the same set of model constants as that used in the standard model, and away from the wall the used model will reduce to the standard model. Thus it can be used in both near wall turbulence and high Reynolds number turbulence. The Kolmogorov time scale is used as its lower bound. By using this time scale to reformulate the dissipation equation, the singularity in the standard dissipation equation is removed as the wall is approached.
- (*ii*) A near wall variable Prandtl number turbulence model has been derived by Sommer et. al. is used in the present study with the change of the damping function in the calculation of the turbulent thermal diffusivity , and the new near wall model is used for the velocity field calculations is found to give the best results.
- (*iii*) The present prediction technique is tested for both the velocity field and the thermal field with different available experimental data for the zero pressure gradient, favorable pressure gradient, and adverse pressure gradient heat transfer flows, all the results show a good improvement in the present prediction method against the constant turbulent Prandtl number calculations