August 21st, 2007, Summer School at Cargèse,
Small-Scale Turbulence :
Theory, Phenomenology and Applications

DNS Study of the Universality at Small Scales of Turbulence at High Reynolds Number

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Frontiers of Computational Science

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Computation on: VPP500, VPP5000 at Nagoya Univ. C.C.
& Earth Simulator

21st Century COE Program
`Frontiers of Computational Scien

Frontiers of Computational Science

Outline of Talk

I) Background Idea & Overview of our DNS

II) Some Results of DNS Data-analysis

III) Coarse Grained Statistics

IV) a Comment on Universality

On 23rd (Thursday):

Small Scale Anisotropy in High Re-Turbulence

Universality of the 2nd Kind, in Turbulent Shear Flow, MHD & Stratified Turbulence

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Objective

= To Understand/Explore
the Physics/Universality of Turbulence
by Direct Numerical Simulation (DNS)
of Canonical Turbulence

universality
= statistical property insensitive to the detail of external large scale conditions, such as the boundary conditions, forcing, initial conditions, etc.

Kolmogorov’s Idea

- Universality

  at sufficiently large Re

  at sufficiently small scale

Re & scale can be only finite in any real turbulence

What is the meaning of ``at **sufficiently large** Re” ?

``at **sufficiently small scale**” ?

Box Turbulence

simple but keep the essence of turbulence dynamics

- Essence of turbulence dynamics
  - Convection (non-linear) +
  - Incompressible (pressure, non-local)+
  - Dissipation

NS equation

- Simple conditions
  - for the understanding of inherent dynamics

•Periodic BC
•Forcing
→Controllable

Basic Equation

Equation (Navier-Stokes)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

B.C. 2π–periodic in x,y,z

$$\mathbf{u}(\mathbf{x}) = \sum_{|\mathbf{k}|<k_{\max}} \hat{\mathbf{u}}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Forcing

$$\mathbf{f}(\mathbf{x}) = \sum_{\mathbf{k}} \hat{\mathbf{f}}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \hat{\mathbf{f}}(\mathbf{k}) = c(k)\hat{\mathbf{u}}(\mathbf{k})$$

$$c(k) = \begin{cases} c & (k < 2.5) \\ 0 & \text{otherwise} \end{cases}$$

cf.Jimenez et al.

Use of Spectral method
with phase shift method
to remove alias error

See Yokokawa et al. (2002)

Spectral Method

- Space derivative, pressure solver

= algebraic (Fast & Accurate)

Mass conservation, Most cpu time in FD

- High Re → U >> u: small eddies,

high accuracy required: more cpu time in FD

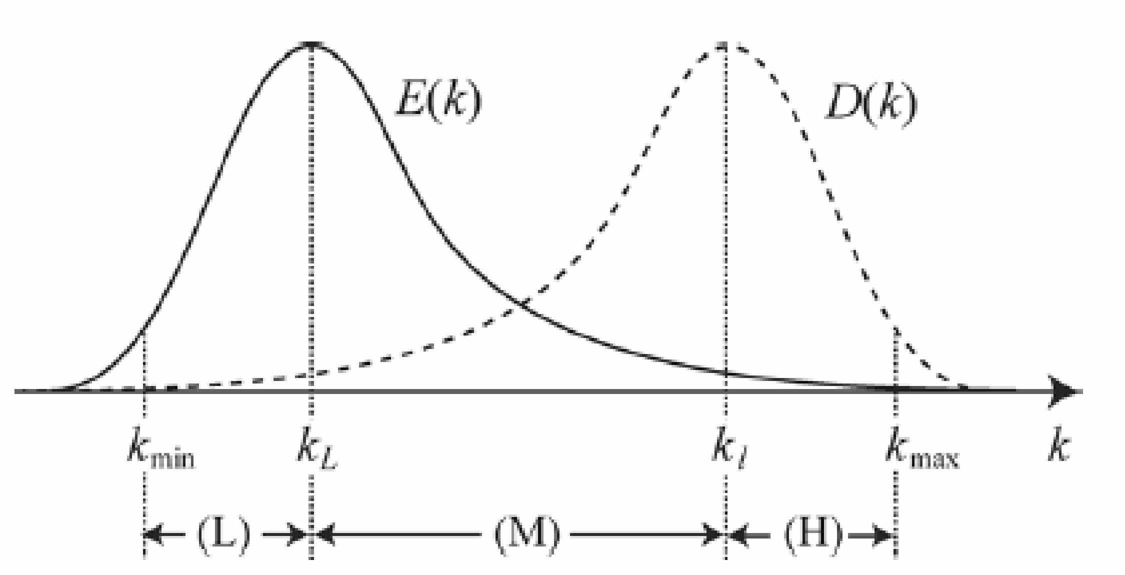
- Most cpu time is consumed by 3D FT

3DFFT

A Constraint on DNS

Wave-number range

$[k_{min},k_{max}]$



From Kaneda & Ishihara (2006)

# of grid points in one direction

$$\frac{N}{2} \sim \frac{k_{max}}{k_{min}} = \frac{1/L}{k_{min}} \times \frac{1/\eta}{1/L} \times \frac{k_{max}}{1/\eta}$$

cf. Yamazakai et al.(2002)

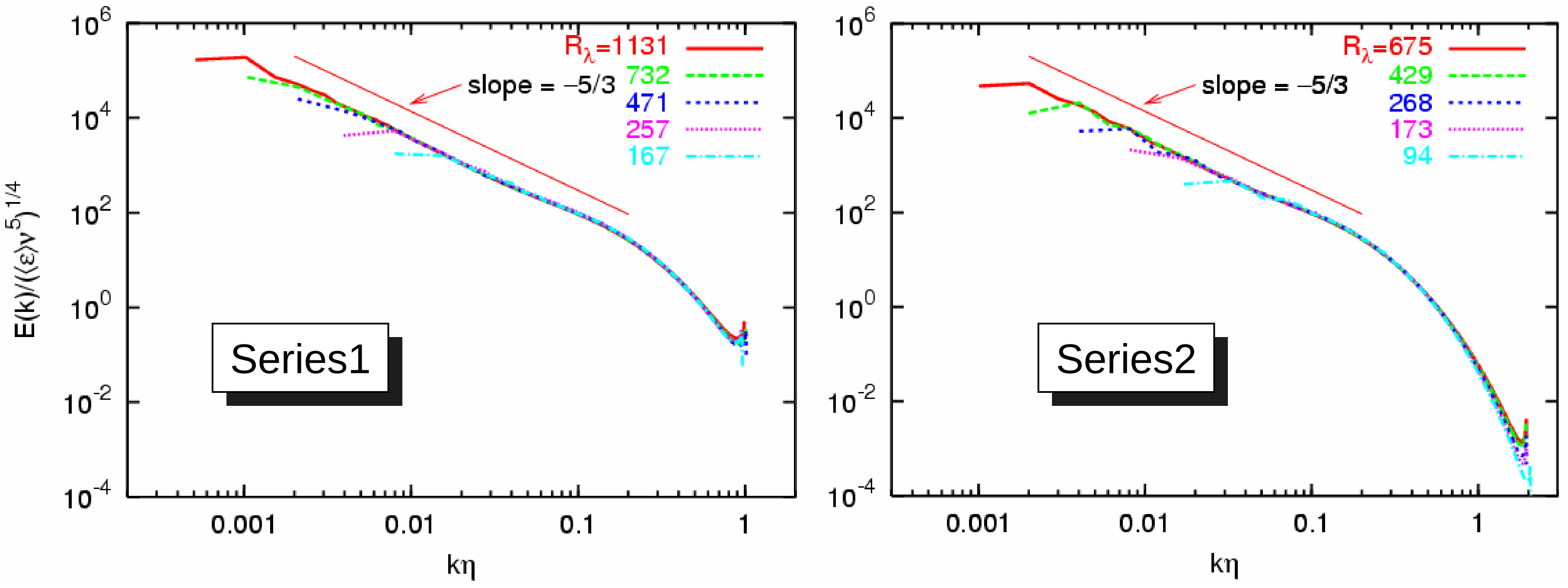
Two Series of DNS

Possible on
the Earth Simulator

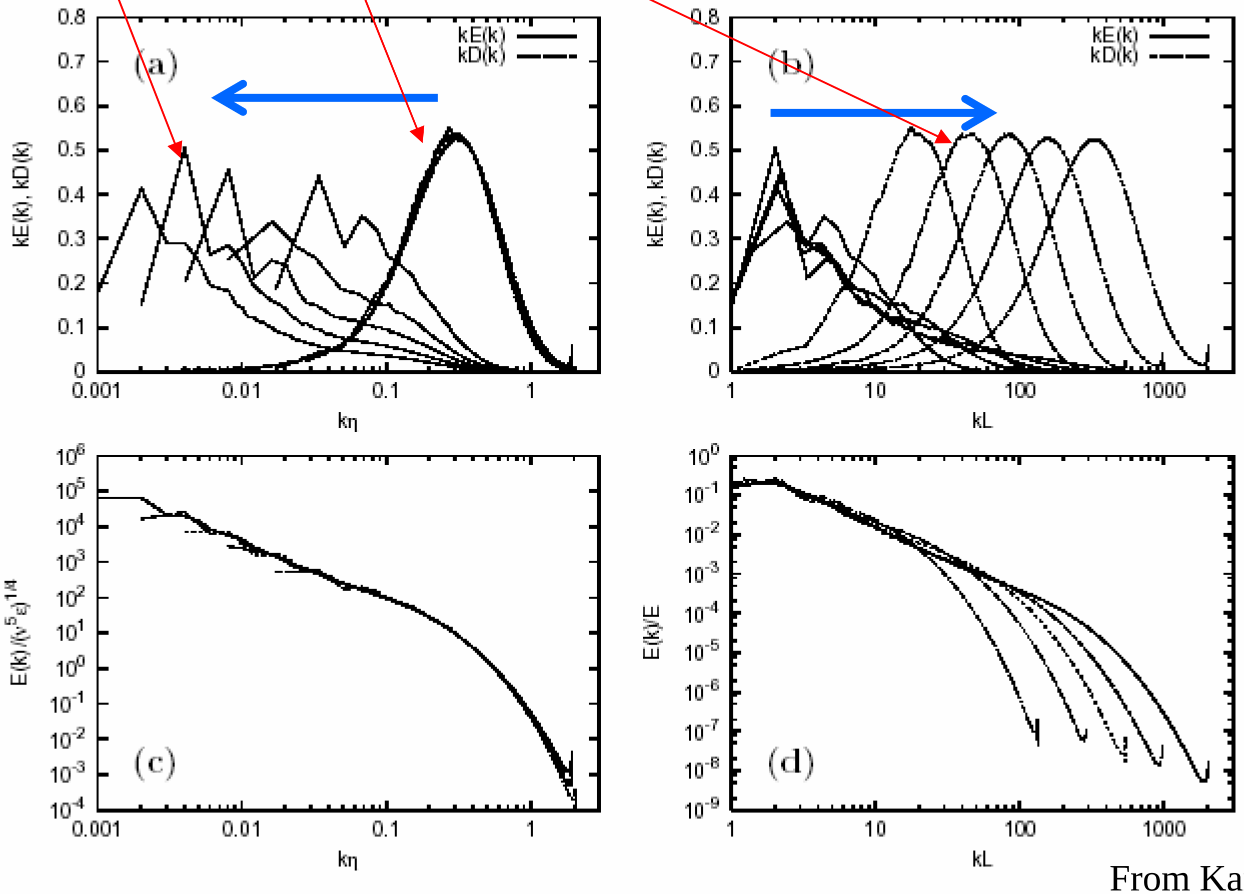
VPP

	256^3	512^3	1024^3	2048^3	4096^3	N
➢Series 1($k_{\max}\eta=1$)	167	257	471	732	1131	R_λ
➢Series 2($k_{\max}\eta=2$)	94	173	268	429	675	

From Kaneda & Ishihara (2006)



E(k), D(k)=k^2E(k) vs. kL or kη



From Kaneda & Ishihara (2006)

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Degree of freedom: $M=4\times 4096^3 \sim 2.7\times 10^{11}$
Number of nonlinear couplings: $M^2 \sim 7.6\times 10^{22}$

II-1:

Energy Dissipation as Re → ∞

In the limit of the viscosity $\mu \rightarrow 0$ (Re → ∞), ?

Navier-Stokes Eq.

$$\rho[\frac{\partial \mathbf{u}}{\partial t}+\boxed{(\mathbf{u}\cdot\nabla)\mathbf{u})}]=-\nabla p+\boxed{\mu\Delta\mathbf{u}}+\mathbf{f}$$

$$\nabla\cdot\mathbf{u}=0$$

Reynolds number → ∞

μ → 0,

Ideal Fluid→ d'Alembert's Paradox

Prandtl's Boundary Layer Theory

Singular PDE （μ →0, =0?)

Energy Dissipation as Re → ∞

1. In the limit of the viscosity ν → 0 (Re → ∞),

   a) dissipation ε→ 0 ? i.e., $\boxed{``\nu \to 0"}$ ~ $\boxed{``\nu=0"}$ (as in d'Alembert's paradox)

   or

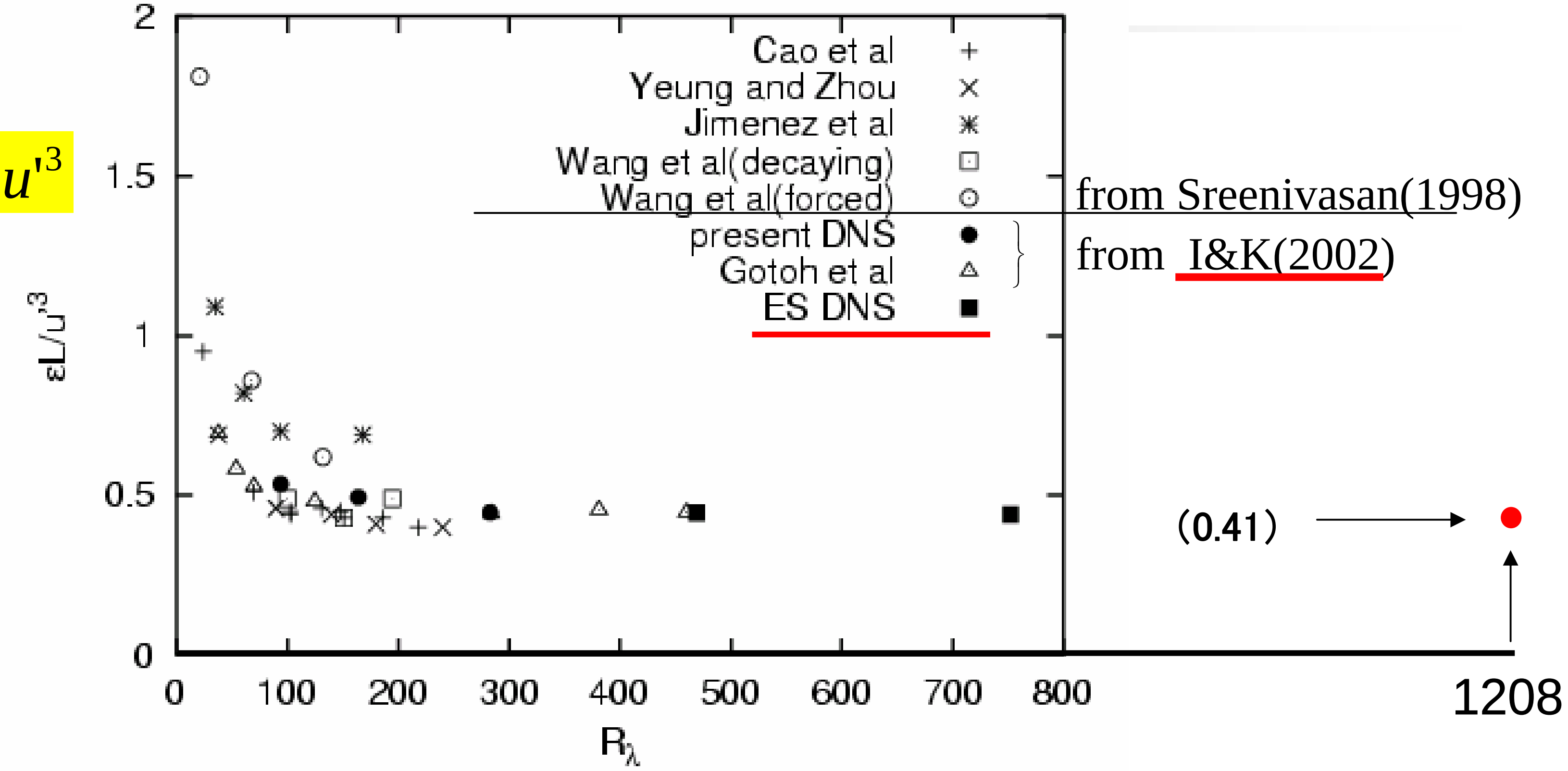
   b) ε → ≠ 0, i.e.,

$$\boxed{\underset{\text{finite}}{\varepsilon} \propto \underset{0}{\nu} \, \underset{\infty}{\underline{\langle(\text{grad } u)^2\rangle}}}$$

2. b) is a basic premise of theories of turbulence including Kolmogorov's.

Normalized energy dissipation vs. R_λ

$\alpha = \varepsilon L / u'^3$

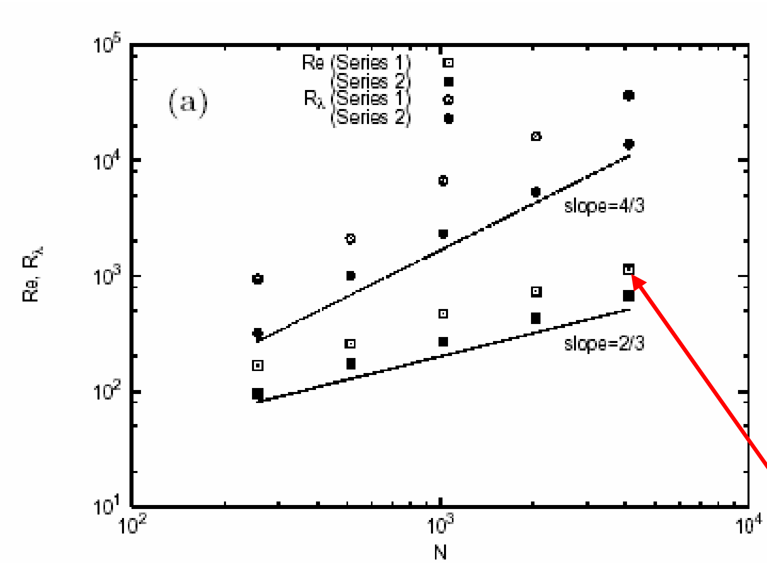


suggests α approaches to a constant as $R_\lambda \rightarrow \infty$

From Kaneda et al. (2003)

Scaling in N & Re

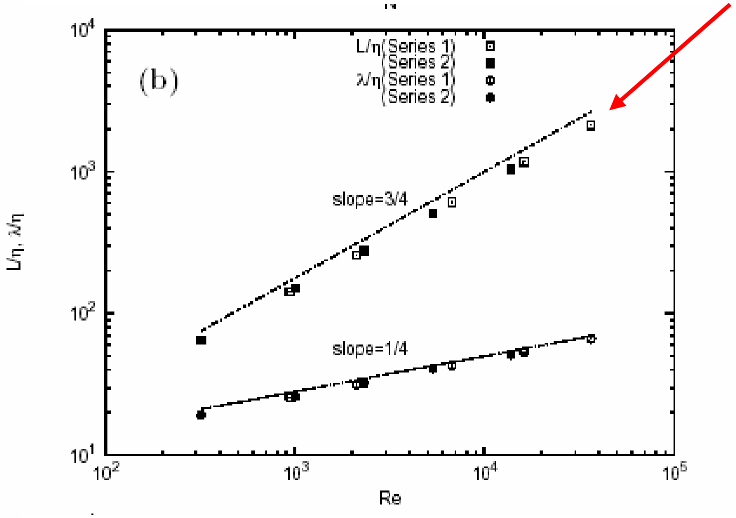
Re, R_λ vs.N



R_λ ~1200

L/η, λ/η vs. Re

L/η~2000



From Kaneda & Ishihara (2006)

$N \propto Re^{3/4}$

$R_\lambda \propto Re^{1/2}$

$L/\eta \propto Re^{3/4}$

$\lambda/\eta \propto Re^{1/4}$

II-2: Energy Spectrum

Does the energy spectrum follow Kolmogorov’s law ?

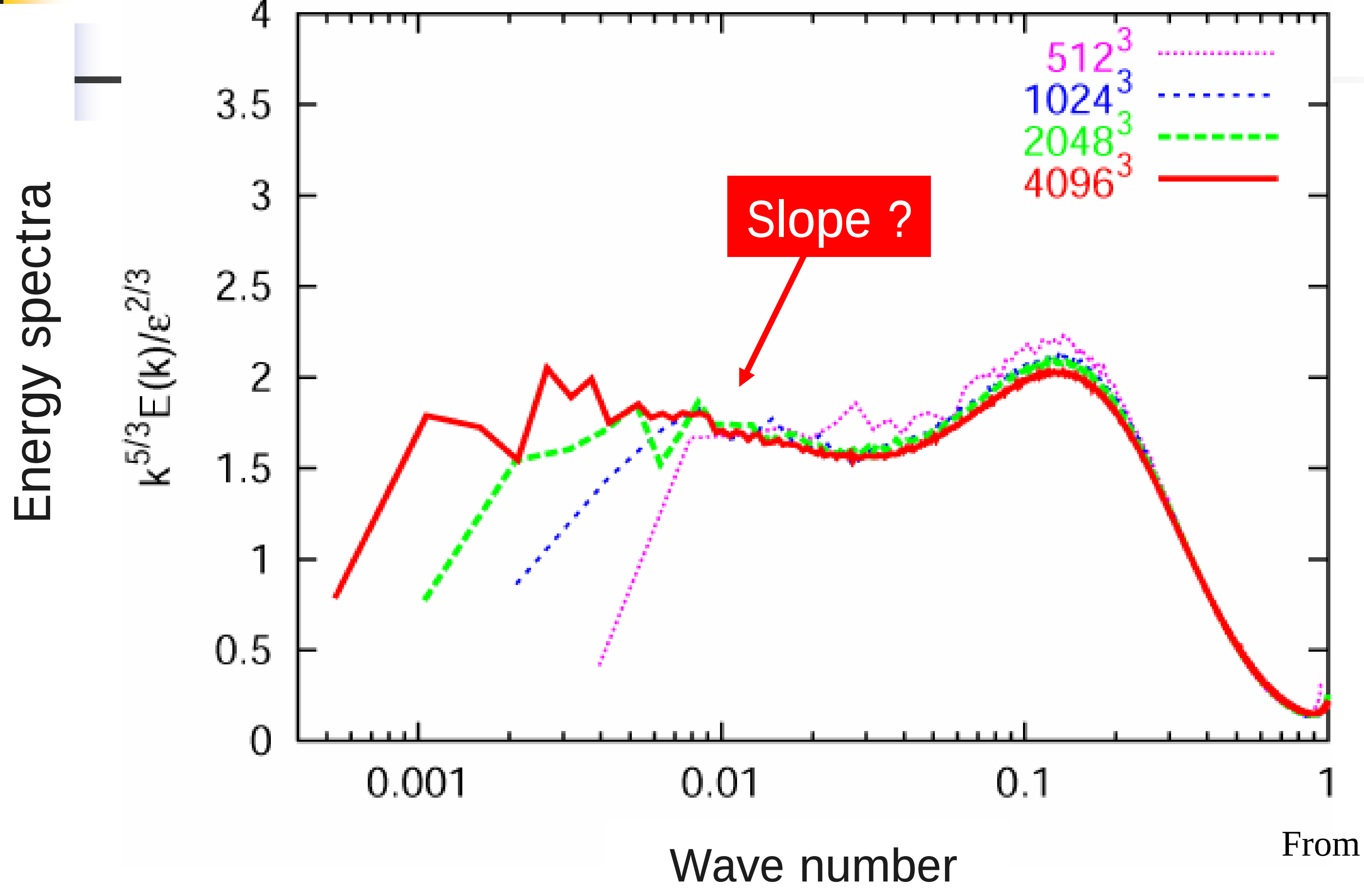
$$E\,(k) = (\varepsilon\,\nu^{5})^{1/4}\ f\,(k\eta)$$

i) Inertial Subrange
ii) Near Dissipation Range

Kaneda et al. (2003)
Ishihara et al. (2005)

II-2-i:

Does the energy spectrum in the inertial range follow Kolmogorov’s law ?

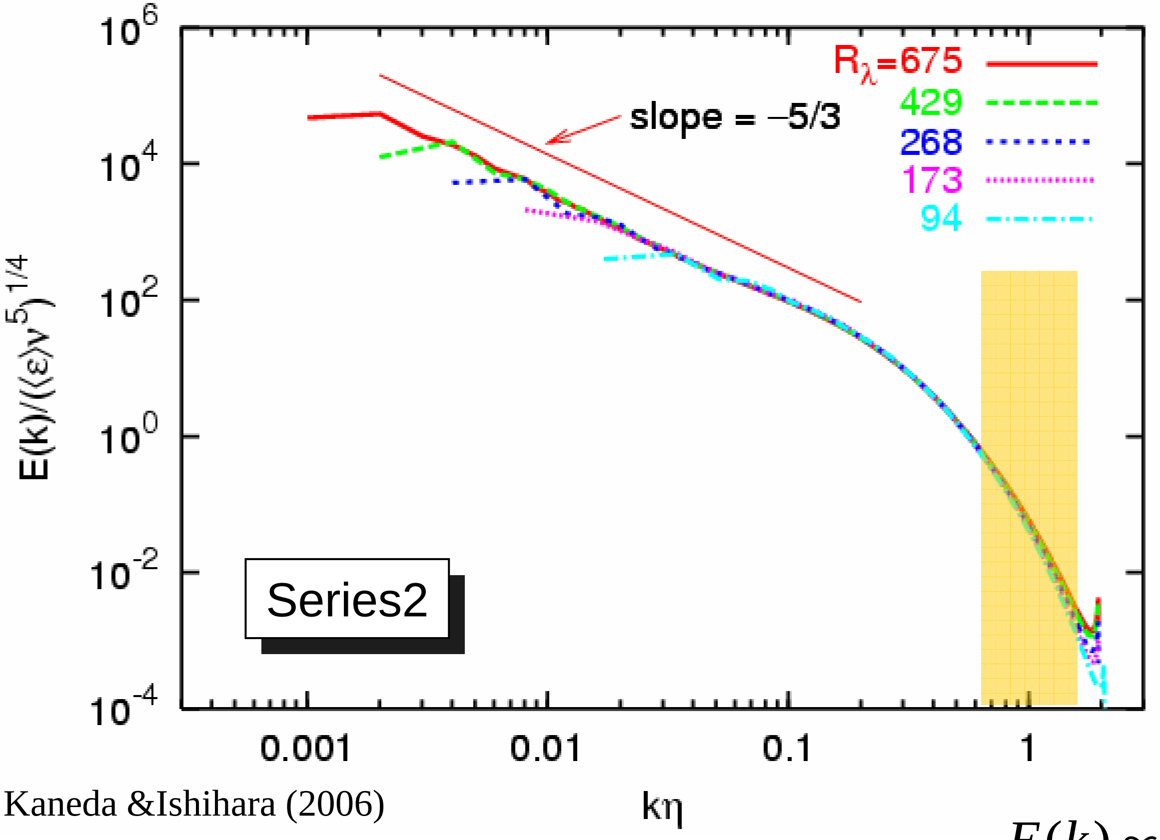


Slope ?

From Kaneda et al. (2003)

- A little difference from Kolmogorov’s law is observed.

II-2 – i :
Energy spectrum in the near dissipation range



Kaneda &Ishihara (2006)

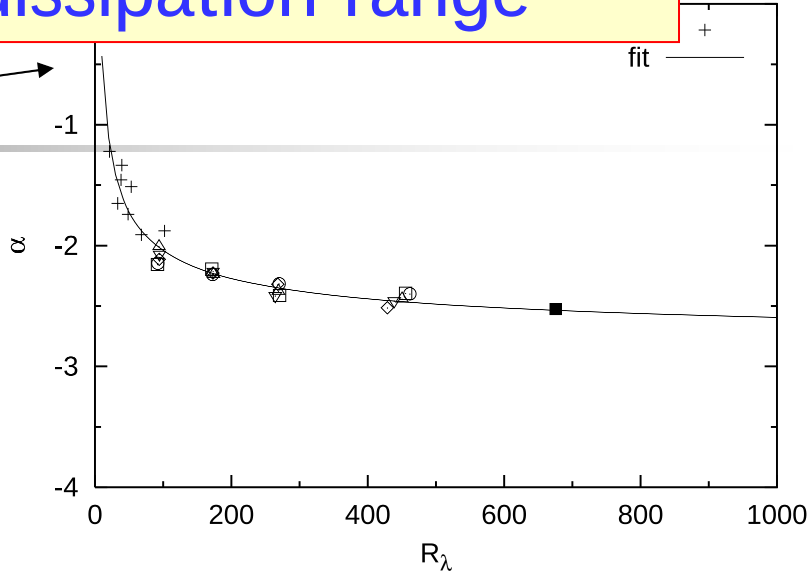
$$E(k) \propto C(k\eta)^{\alpha} \exp[-\beta(k\eta)]$$

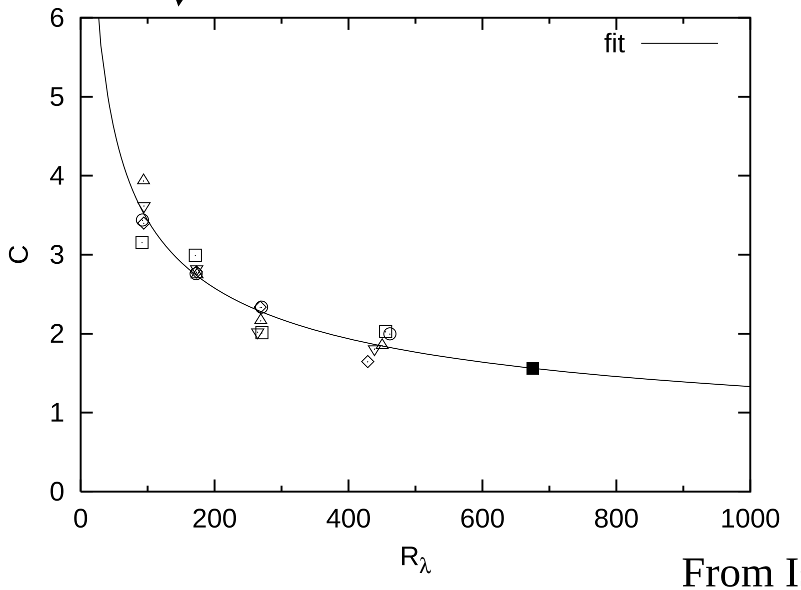
Martinez, et al.(1997)

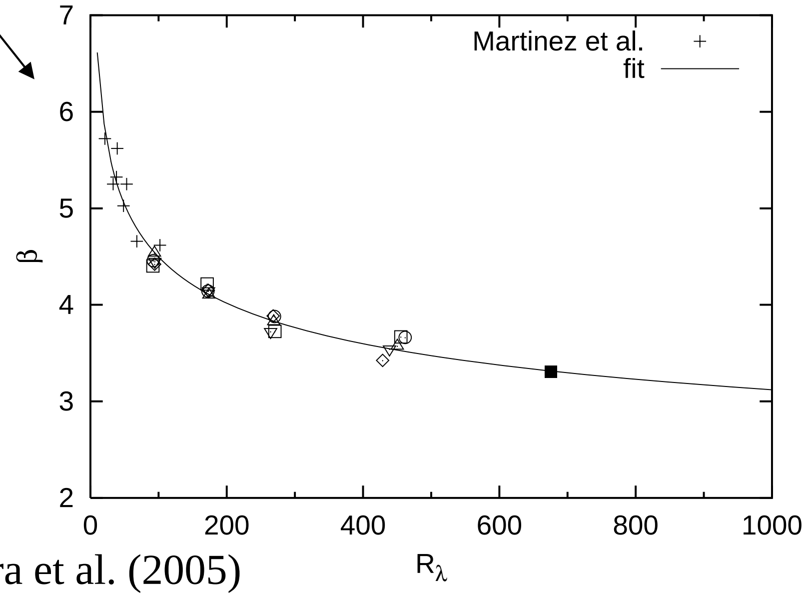
II-2-ii:
Energy spectrum at dissipation range

$$E(k) = C(k\eta)^{\alpha}\exp[-\beta(k\eta)^{n}]$$

(n=1)







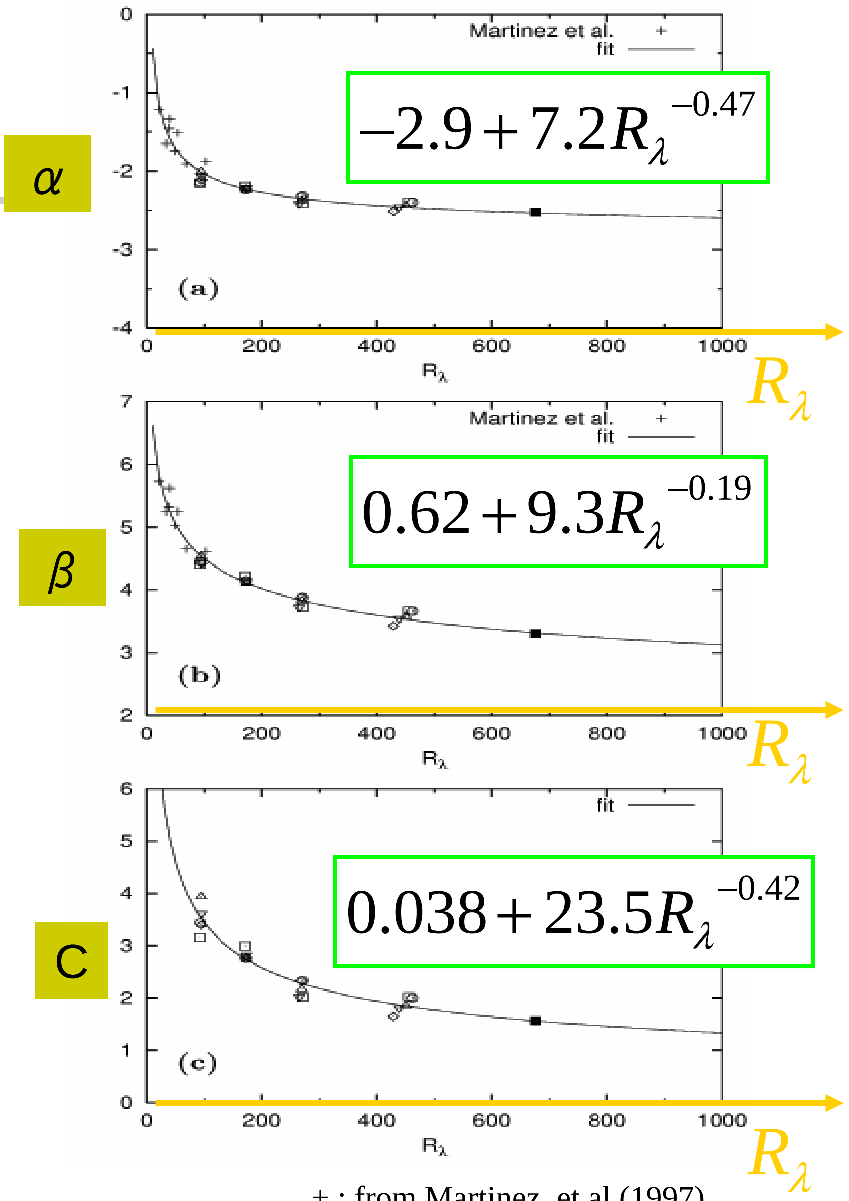
From Ishihara et al. (2005)

Dependence on R_λ

α, β and C approach to constants as $R_\lambda \rightarrow \infty$, but **the approach is slow**

e.g., $\beta(R_\lambda)-\beta_\infty / \beta_\infty \sim 2.61$, even at R_λ =10,000

From Ishihara et al. (2005)



$$-2.9+7.2R_\lambda^{-0.47}$$

$$0.62+9.3R_\lambda^{-0.19}$$

$$0.038+23.5R_\lambda^{-0.42}$$

+ : from Martinez, et al.(1997)
the others: present DNS

II-3:
Statistics of Velocity Gradients

i. PDF
ii. Skewness
iii. Flatness
iv. Acceleration

Ishihara et al. (2007)

II: Summary

1. DNS has reached at a stage where
   inertial subrange with $L/\eta \sim 1000$,

2. Some light on Asymptotic Small –Scale Statistics
   1. Normalized Energy Dissipation D (→ const)

   2. Dissipation range spectrum (→ converge to some form, but very slow)

   3. Statistics of velocity derivatives.
      Skewness, Flatness, 4th order moments
      depend on Re (→ some kind of power laws, different at high R)

   4. PDF of Eulerian and Lagrangian accelerations
      Wider for Lagrangian than Eulerian
      Wider for higher time derivatives

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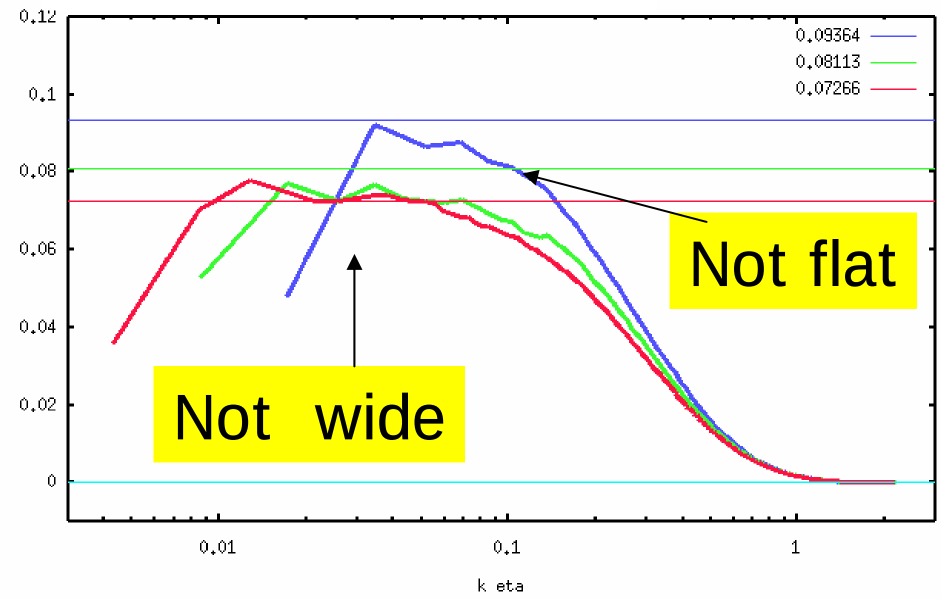
Some difference
from DNS by lower resolution

energy transfer through wave number k

$$\Pi(k) = \int_k^\infty T(k)dk$$



Not flat

Not wide

$k\eta$

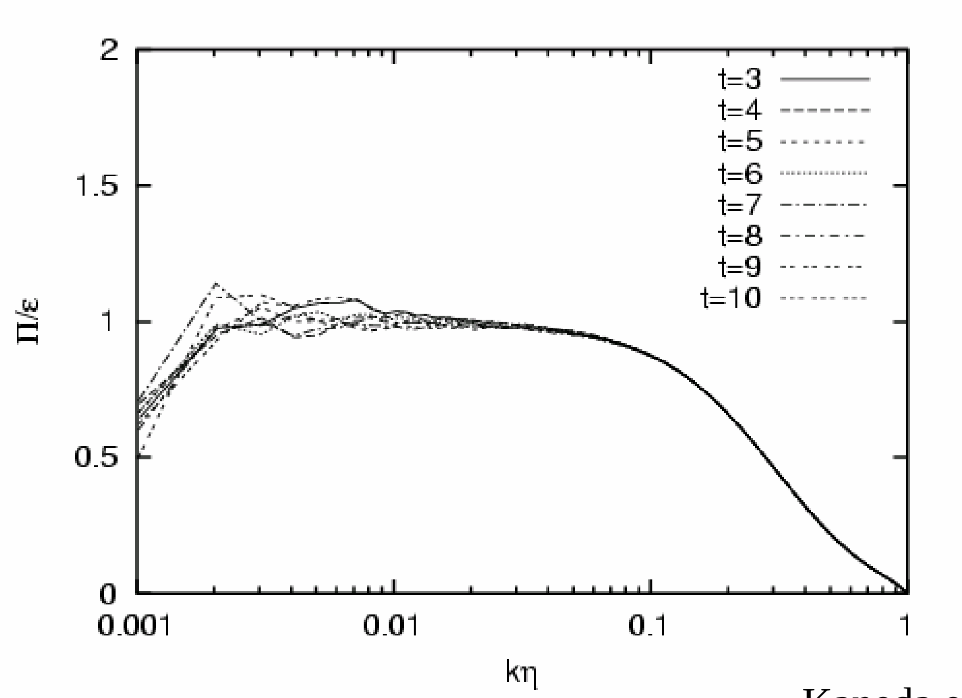
Ishihara & Kaneda (2002)

Some difference

from DNS by lower resolution

$$\Pi \overset{?}{=} \varepsilon$$ (width, flat, stationarity)

$$\Pi(k) = \int_k^\infty T(k)dk$$



Kaneda et al. (2003)

N=2048, $k_{\max}\eta \sim 1$ R_λ~732

III-1

Statistics of Energy Transfer

Aoyama et al. (2005)

Statistics of Energy Transfer

Energy transfer from Grid to Sub-Grid scales:

Recall D.Pullin's lecture

$$\boxed{T = -\tau_{ij}\overline{S}_{ij},}$$

$$\boxed{\tau_{ij} = (\overline{u_i u_j} - \overline{u_i}\ \overline{u_j}) - \frac{2}{3}\delta_{ij}q, \quad q = \frac{1}{2}\left(\overline{u_k u_k} - \overline{u_k}\ \overline{u_k}\right),}$$

Pimeori, et al. PF(1990),
Pimeori, et al. PF(1991)
Domaradki, et al. PF(1993)
Cerutti & Meneeveau, PF(1998)
Chen et al.PRL(2003)

Spectral cut-off filter at k_c

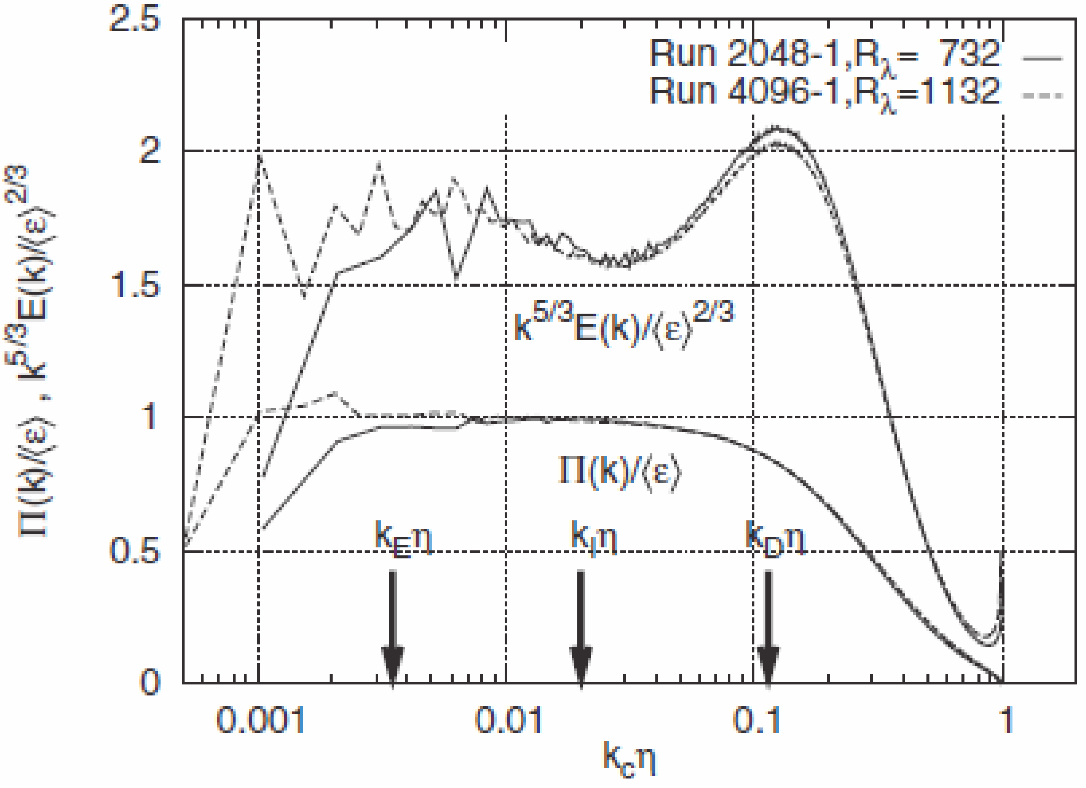
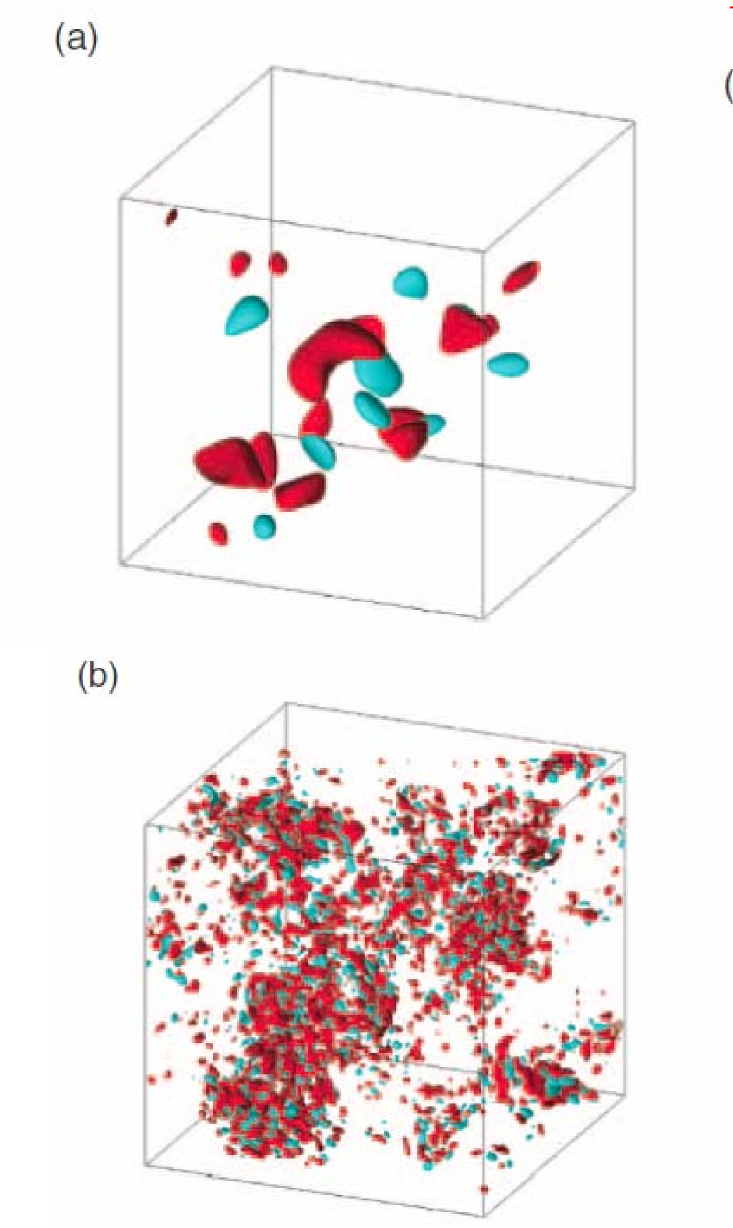
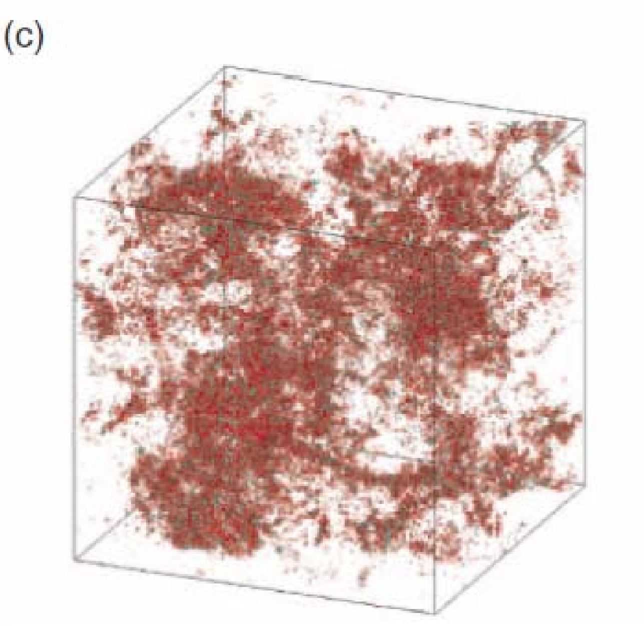


Fig. 4. $\Pi(k)$ and the positions of the cut-off wave numbers, in Run 2048-1 and Run 4096-1.

Aoyama et al. (2005)

Strong Backward & Forward Transfer Regions





Iso-T surface

● $T = \langle T \rangle + 3\sigma_T$

forward

● $T = \langle T \rangle - 3\sigma_T$

backward

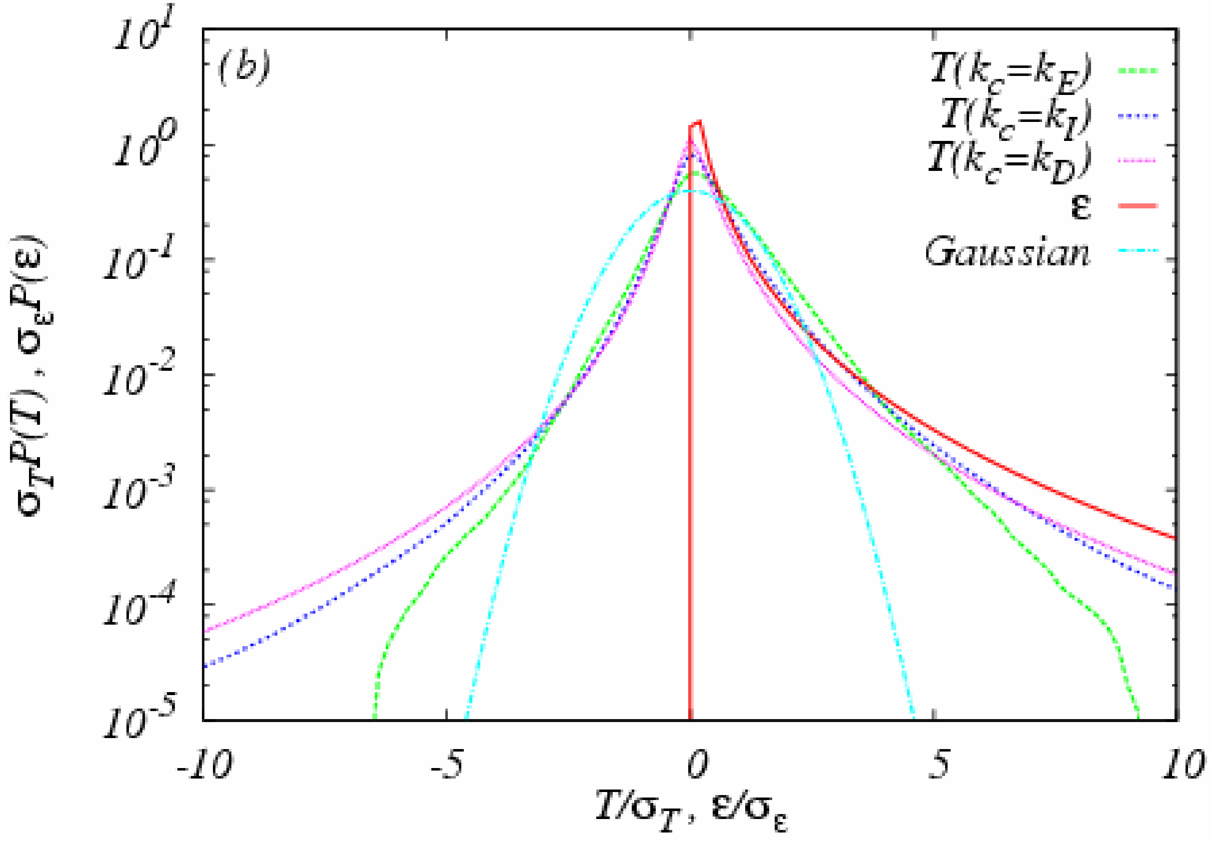
Fig. 7. Snapshots of intense transfer regions in Run 2048-1. Equi-contour surfaces of $T = \langle T \rangle + 3\sigma$ (red) and $T = \langle T \rangle - 3\sigma$ (blue), where σ is the root mean square of $\Delta T = T - \langle T \rangle$, and $k_c = k_{\mathrm{I}}$. (a) $k_c = k_{\mathrm{E}}$, (b) $k_c = k_{\mathrm{I}}$, (c) $k_c = k_{\mathrm{D}}$.

Aoyama et al. (2005)

PDF of T (& ε)

$$T = -\tau_{ij}\overline{S}_{ij},$$

$$\tau_{ij} = (\overline{u_i u_j} - \overline{u_i}\;\overline{u_j}) - \frac{2}{3}\delta_{ij} q, \quad q = \frac{1}{2}(\overline{u_k u_k} - \overline{u_k}\;\overline{u_k}),$$



Aoyama et al. (2005)

Volume ratio of Backscatter Region (T＜0)

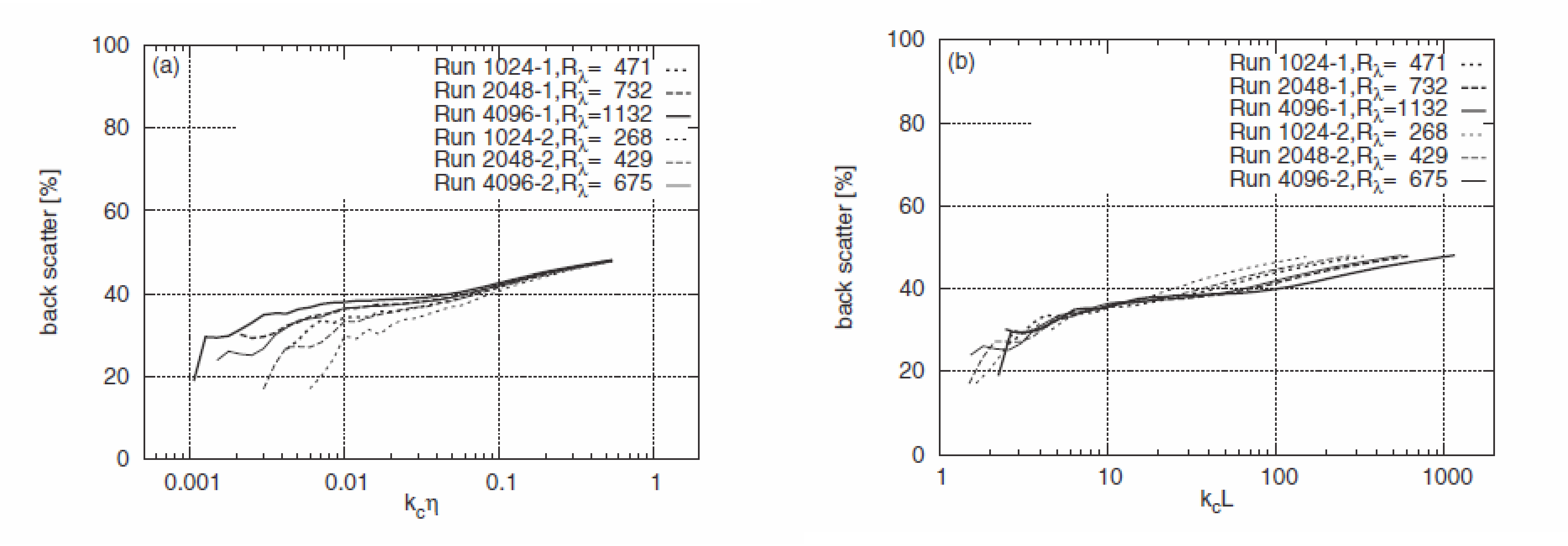
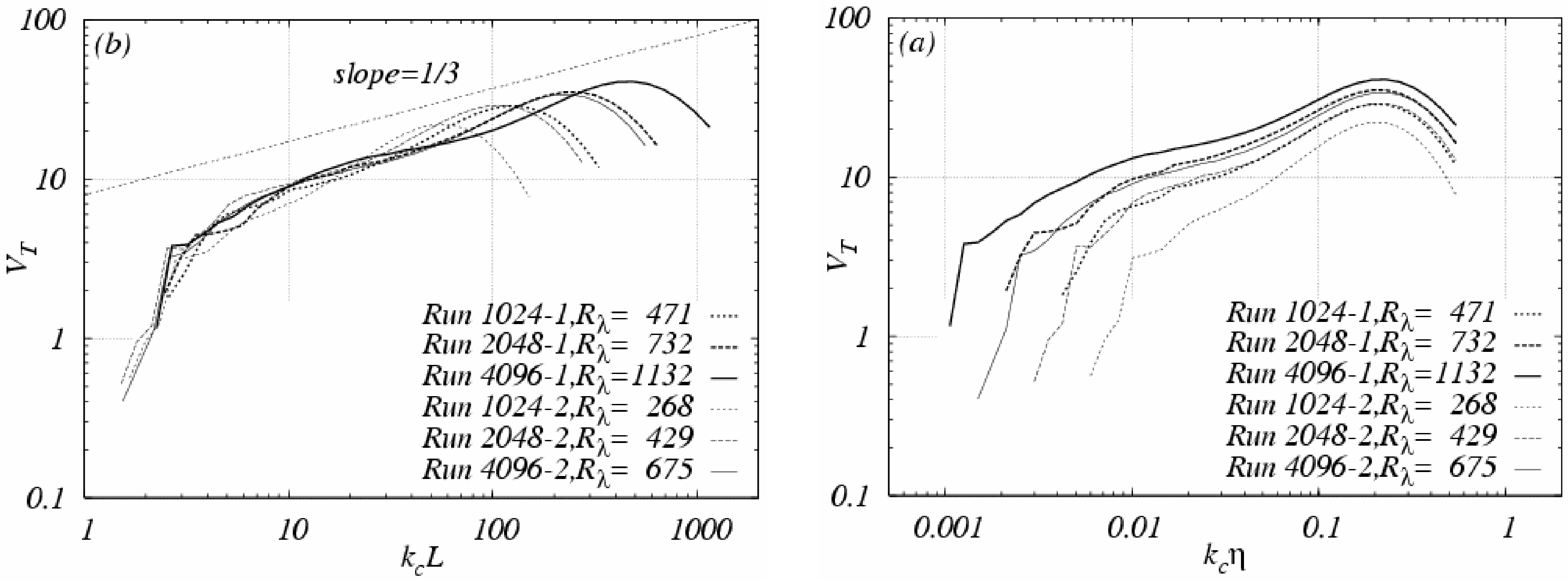


Fig. 6. Volume ratio of the region of backward transfer vs $k_c\eta$ in (a) and vs k_cL in (b).

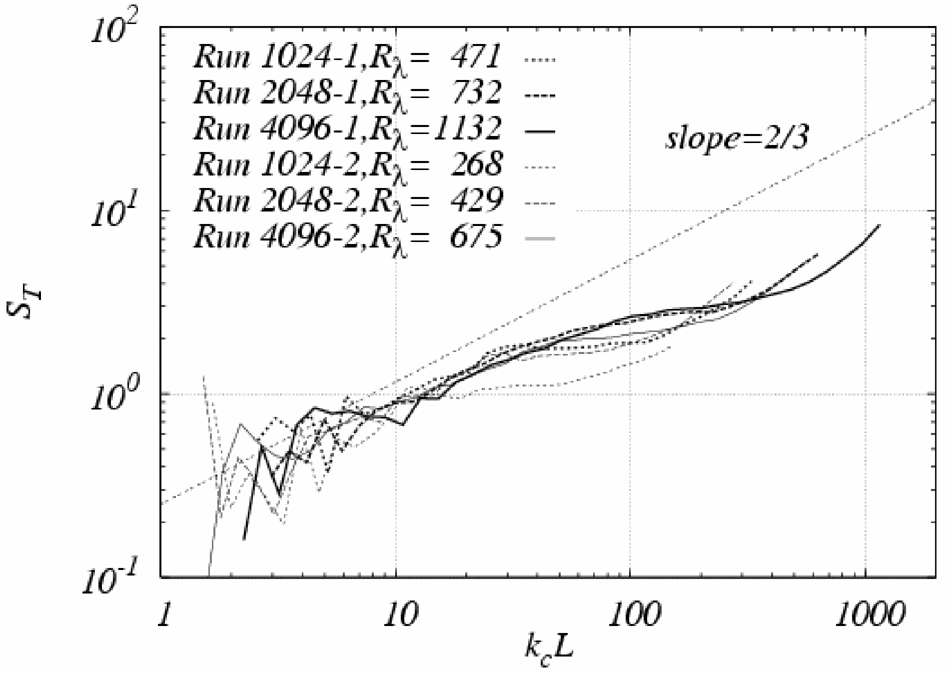
Aoyama et al. (2005)

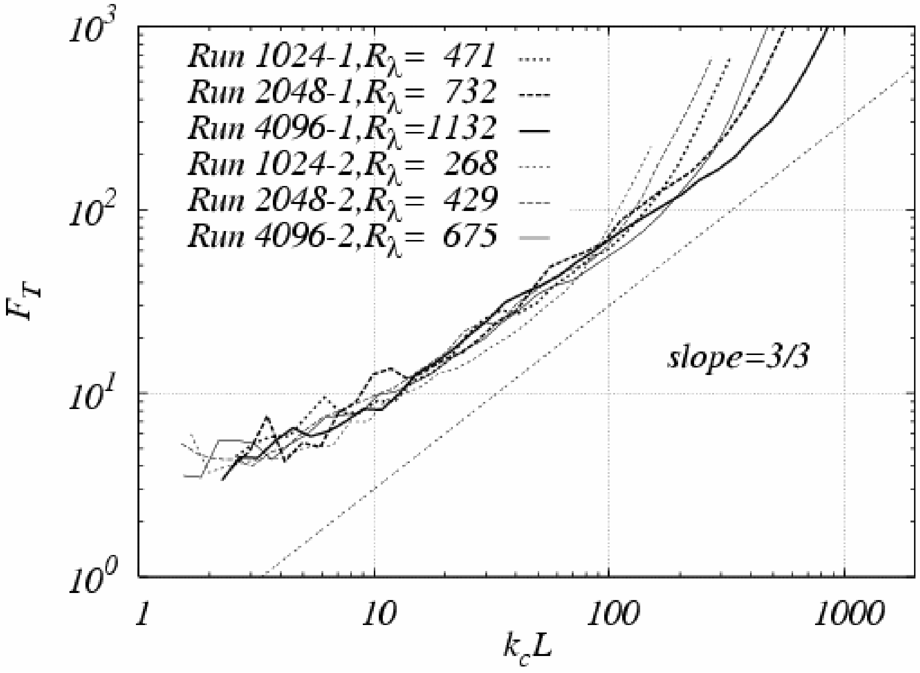
Variance



Aoyama et al. (2005)

Skewness & Flatness





Aoyama et al. (2005)

III-1: Summary (statistics of T)

- Intermittency of T is lager for larger kc
- Backward T region ~ 40 %
- Variance, skewness, flatness of T、

  scales in the ISR as (kc-dependent; not independent)

$$T_{variance} \propto (kcL)^{\alpha} \Rightarrow \alpha \approx \frac{1}{3}$$

$$T_{skewness} \propto (kcL)^{\beta} \Rightarrow \beta \approx \frac{2}{3}$$

$$T_{flatness} \propto (kcL)^{\gamma} \Rightarrow \gamma \approx \frac{3}{3}$$

III-2: Energy Dissipation

Energy Cascade as a Multiplicative Random Process

Kaneda & Morishita (2007)

Energy Cascade as a Multiplicative Process

energy dissipation rate ε_r averaged over a sphere of radius r

$$\varepsilon_n(\boldsymbol{x}) = \frac{3}{4\pi r_n^3} \int_{|\boldsymbol{x}'|<r_n} \varepsilon(\boldsymbol{x}+\boldsymbol{x}') d^3\boldsymbol{x}' \qquad \varepsilon(\boldsymbol{x}) = \frac{\nu}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2$$

$$\frac{\varepsilon_n}{\varepsilon_0} = \frac{\varepsilon_1}{\varepsilon_0}\frac{\varepsilon_2}{\varepsilon_1} \cdot\cdot \frac{\varepsilon_n}{\varepsilon_{n-1}} = \alpha_1 \alpha_2 \cdot\cdot \alpha_n$$

$$\frac{r_n}{L} = a^n, \quad 0 < a < 1$$

a: scale ratio

log

$$\log\frac{\varepsilon_n}{\varepsilon_0} = \log\alpha_1 + \log\alpha_2 + \,..\, + \log\alpha_n$$

$$\varepsilon_{n+1} = \alpha_n \varepsilon_n$$

$$\alpha_n(\mathbf{x}) \equiv \frac{\varepsilon_n(\mathbf{x})}{\varepsilon_{n-1}(\mathbf{x})}$$

cascade ratio

Energy Cascade as a Random Walk

$$x_n = \log \frac{\varepsilon_n}{\varepsilon_0}$$

$$x_n - x_{n-1} = u_n \Delta t$$

$$x_n = \sum_{i=1}^{n} u_i \Delta t$$

$$t_n = \log(L / r_n)$$
$$= \log(1 / a^n)$$
$$= -n \log a = n\Delta t$$

$$\Delta t \equiv -\log a$$

$$u_n = \frac{x_n - x_{n-1}}{t_n - t_{n-1}} = \frac{(\Delta x)_n}{(\Delta t)_n} = \frac{\log \alpha_n}{\Delta t}$$

$$\alpha_n(\mathbf{x}) \equiv \frac{\varepsilon_n(\mathbf{x})}{\varepsilon_{n-1}(\mathbf{x})}$$

cf. Naert, A. *etal*. PRE(1997),

In the limit, a →1

$$\frac{d\hat{\epsilon}(\tau)}{d\tau} = u(\tau)\hat{\epsilon}(\tau),$$

$$u(\tau) = d \log \hat{\epsilon}(\tau)/d\tau, \ \tau = \log(r_0/r) \text{ and } \hat{\epsilon}(\tau) = \epsilon(r).$$

PDF of $u_n(\boldsymbol{x}) = \dfrac{\log \alpha_n(\boldsymbol{x})}{\Delta t}$

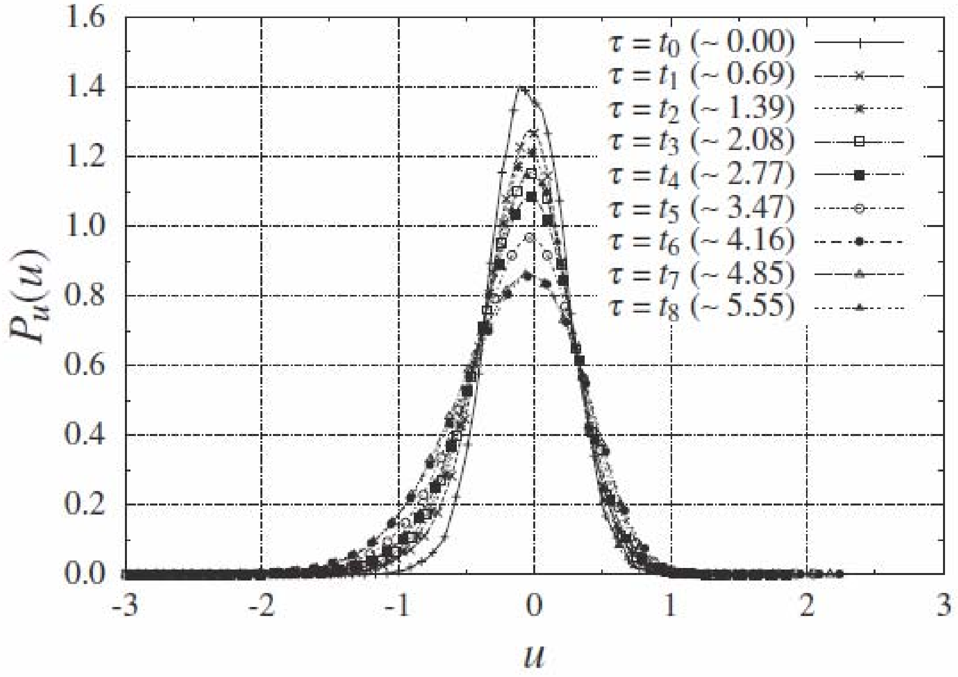


Fig. 1. The pdf P_u of the virtual velocity defined by eq. (2), for $\tau = t_n \equiv n \log 2$ $(n = 0, 1, 2, \ldots, 8)$ in Run2048. $a = 2$.

Kaneda & Morishita (2007)

Scale (in-)Dependence of u

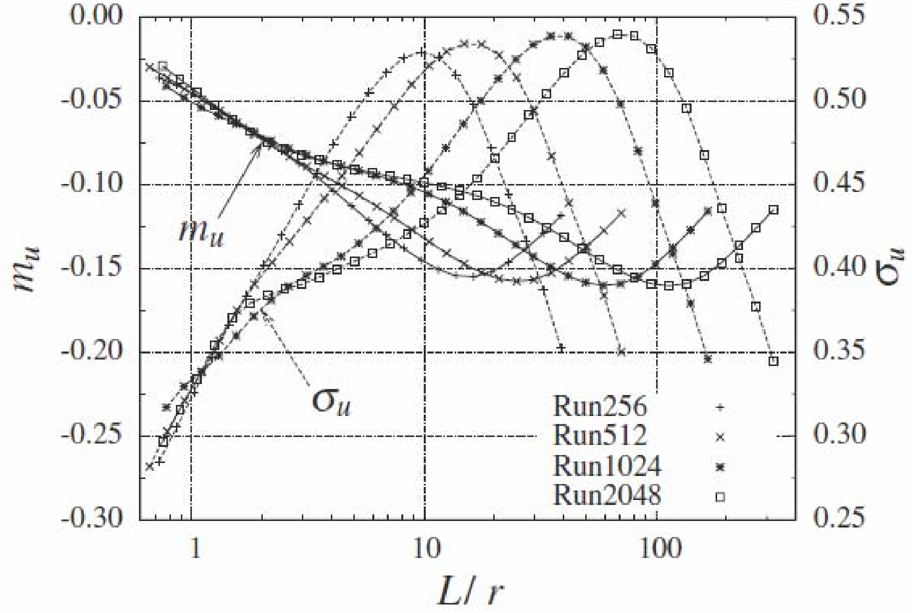


Fig. 2. Mean m_u (solid lines, left scale) and standard deviation σ_u (dotted lines, right scale) vs L/r in Run256, Run512, Run1024, and Run2048. $a = 2^{1/4}$.

Kaneda & Morishita (2007)

Correlation between u’s, Similarity ?

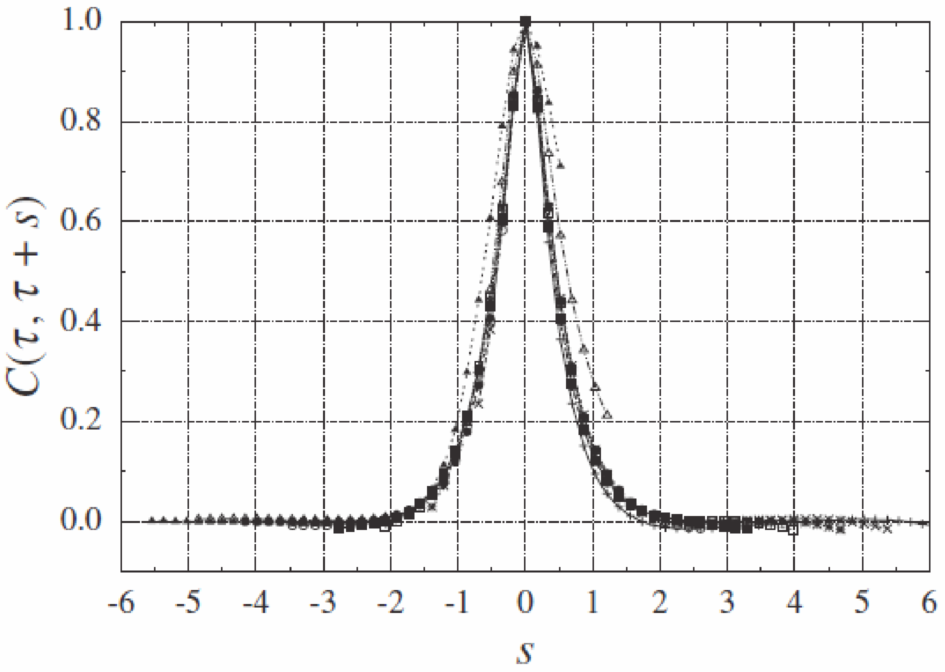


Fig. 3. Correlation C between $\tilde{u}(\tau)$ and $\tilde{u}(\tau + s)$ for $\tau = t_n \equiv n \log 2$ $(n = 0, 1, \ldots, 8)$ in Run2048. $a = 2^{1/4}$. The meanings of the lines are the same as in Fig. 1.

Kaneda & Morishita (2007)

Effect on Non-Markovian Statistical Dependence

$$D(p) \equiv \frac{\langle (\epsilon_n/\epsilon_0)^p \rangle}{\langle (\alpha_{n-1})^p \rangle \langle (\alpha_{n-2})^p \rangle \cdots \langle (\alpha_0)^p \rangle}. \qquad (4)$$

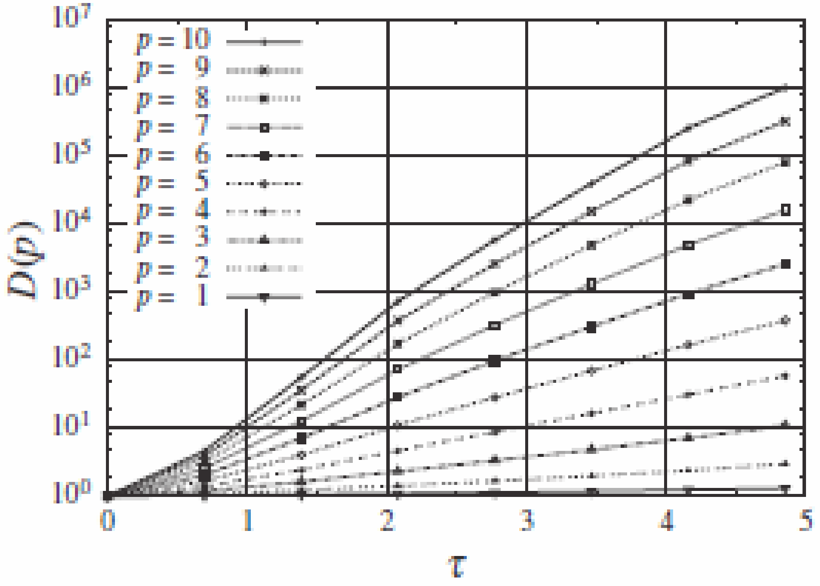


Fig. 4. DNS values of ratio $D(p)$ defined by eq. (4) vs τ in Run2048. $a = 2$.

Kaneda & Morishita (2007)

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