August 23rd, 2007, Summer School at Cargèse,
Small-Scale Turbulence :
Theory, Phenomenology and Applications

Small-Scale Anisotropy in High Reynolds Number Turbulence

-- Universality of the 2nd Kind --

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Frontiers of Computational Science

Turbulence
= a system of huge degree of freedom

A paradigm of
Studies of systems of huge degree of freedom

→

Thermodynamics and Statistical Mechanics
for thermal equilibrium state

Analogy with
Statistical Mechanics for Near Equilibrium System

I. **Two kinds of universality**

characterizing the macroscopic state of the equilibrium system

not only

a) Equilibrium state itself, like Boyle-Charles' law

but also

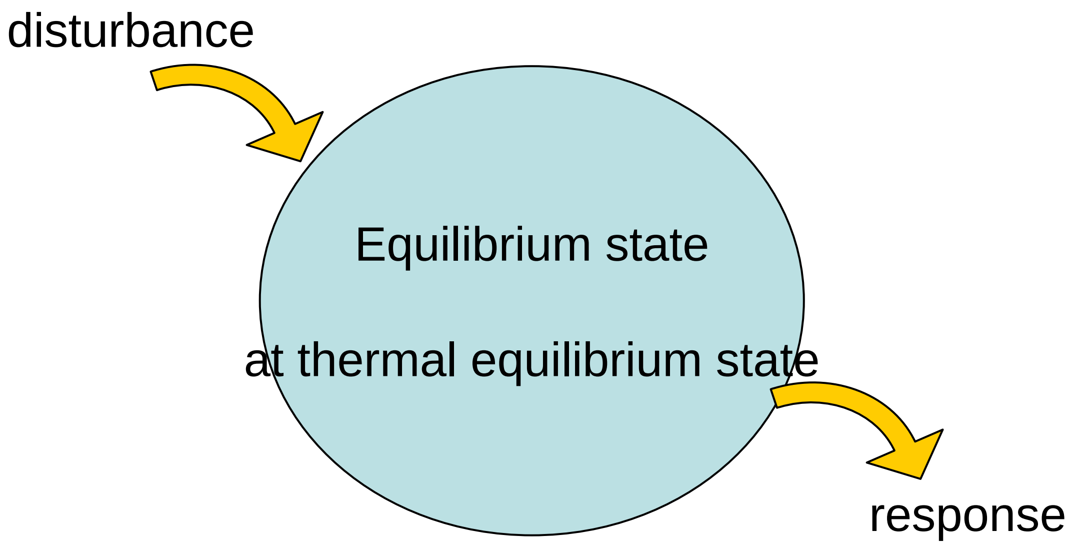
b) **Response** to disturbance ⟷ **Universality of the 2nd Kind**

II . Thermal equilibrium state ←→ Universal equilibrium state at small scale

influence of external force, mean flow, etc. at small scale

may be regarded as disturbance.

How dose the equilibrium state respond to the disturbance ?


disturbance
Equilibrium state
at thermal equilibrium state
response

Universality in Response to disturbances, near equilibrium state

1905, Einstein, $D=\mu kT$,

the first example of FD-elation→ Perrin's experiment.

1928, Nyquist's theorem on thermal noise:

$$P(f)=4kT\ Re(Z(f))$$

1931, Onsager's reciprocal theorem:

$$J = C X, \qquad C={}^{T}C$$

generalized flux, generalized force

$$J \quad = \quad CX$$

Generalized Flux vs. Generalized Force

e.g.

Density Flux vs. Density gradient ; $J = C \text{ grad } \rho$

Heat Flux vs. Temperature gradient ; $J = C \text{ grad } T$

Electric Current vs. External electric field ; $J = \sigma E = \sigma \text{ grad } \phi$,
(I=E/R, Ohm's law)

Momentum Flux vs. Strain rate

$$\tau_{ij} = C_{ijmn} S_{mn},$$

(Newton's law)

coupling only between tensors of the same order.

Thermal Equilibrium system

Disturbance

X
grad ϕ
grad T
grad c

Equilibrium state

Linear response

F=CX	(Hooke's law)
J = C' grad ϕ = σ E	(Ohm's law)
J = C'' grad T	(Fourier's law)
J = C''' grad c	(Fick's law)

History: Universality in Response to disturbances, near equilibrium state

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the first example of FD-elation → Perrin's experiment.

1928, Nyquist's theorem on thermal noise:

$$P(f)=4kT\ Re(Z(f))$$

1931, Onsager's reciprocal theorem:

$$J = C\,X, \qquad C={}^{T}C$$

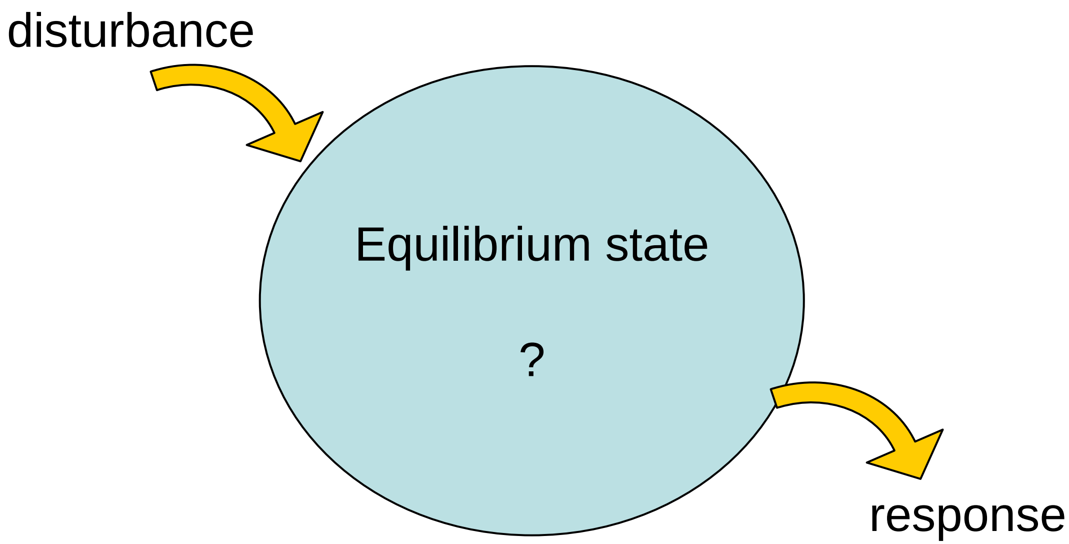
generalized flux, generalized force

1950-60, Nakano, Kubo

Linear Response Theory

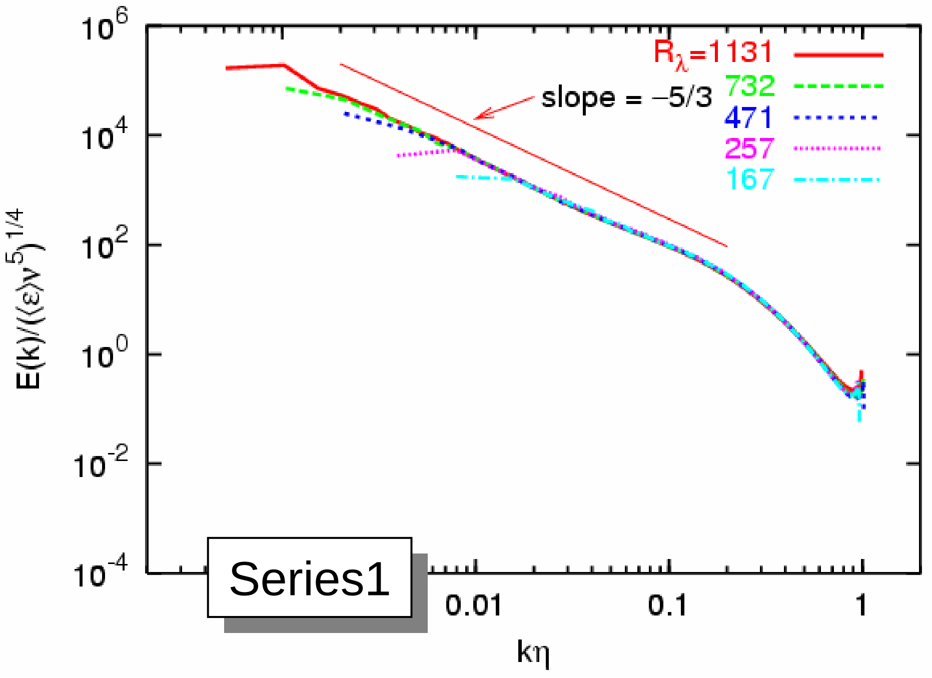
Equilibrium State of Turbulence

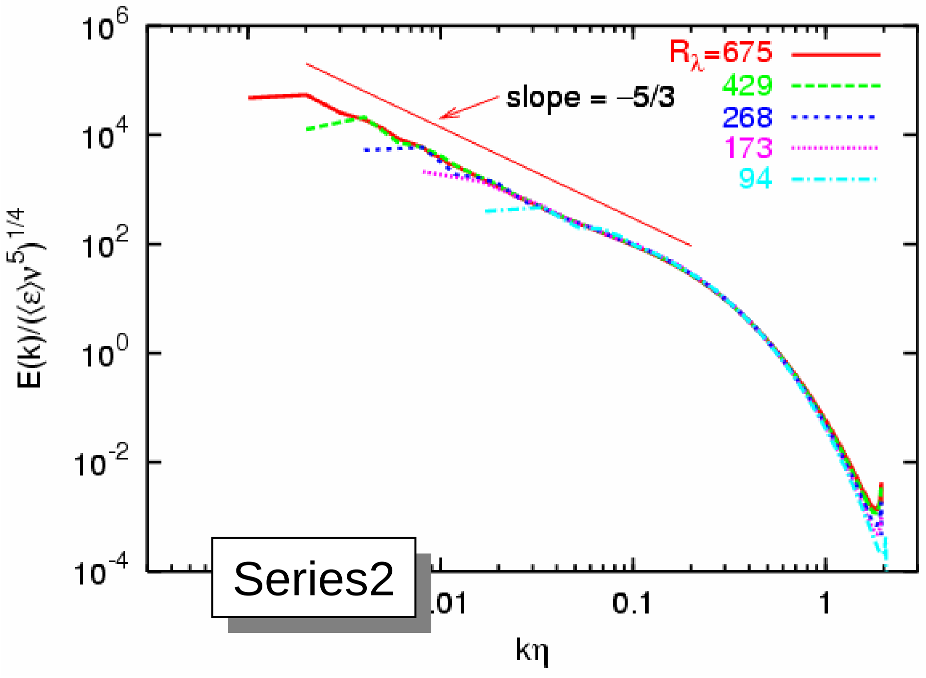
Turbulence ?



DNS

Energy Spectrum a la Kolmogorov (K41)





From Kaneda &Ishihara (2006)


disturbance
Equilibrium state
a la Kolmogorov
K41
response

Disturbance

1. Mean Shear
2. Buoyancy by Stratification
3. Magneto-Hydrodynamic Force

I. Mean Shear

See Ishihara et al. (2002)

Turbulent Shear Flow

NS-equation   Local co-ordinate $\boldsymbol{v} = <\boldsymbol{v}> + \tilde{\boldsymbol{v}}$

$$\frac{\partial}{\partial t}\tilde{\boldsymbol{v}}(\boldsymbol{r},t) = -(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}} - \nabla \boldsymbol{q} + \nu\nabla^2\tilde{\boldsymbol{v}} + \boxed{\boldsymbol{M},}$$

$$M_i = \underline{S_{mn}}r_n\frac{\partial \tilde{v}_j}{\partial r_m} + S_{ij}\tilde{v}_j$$

Effect of mean shear   **Local strain rate of mean flow**

for $r \ll L$, $<\boldsymbol{v}> \sim S\boldsymbol{r}$

$$(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}} \sim v_\ell^2/\ell, \qquad \nu\nabla^2\tilde{\boldsymbol{v}} \sim \nu v_\ell/\ell^2, \qquad \boldsymbol{M} \sim S v_\ell,$$

$$\tau_N \sim \ell/v_\ell, \qquad \tau_\nu \sim \ell^2/\nu, \qquad \tau_E \sim 1/S,$$

$$\frac{\boldsymbol{M}}{(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}}} \sim \frac{S v_\ell}{v_\ell^2/\ell} = \frac{S\ell}{v_\ell} \propto S\ell^{2/3}/\epsilon^{1/3} \ll 1$$

for $\ell \ll \ell_E = (\epsilon^{1/3}/S)^{3/2}$.

Effect of Mean Flow

Homogenous Mean Shear Flow: $U_i = S_{ij} x_j$

- in the inertial subrange of homogeneous turbulent shear flow;

$$Q_{ij}(\mathbf{k}) = \left\langle u_i(\mathbf{k}) u_j(-\mathbf{k}) \right\rangle \quad \mathbf{= ?}$$

Let $\delta(k) = \tau_k / T$, (where $\tau_k = 1/(\varepsilon^{1/3} k^{2/3})$, $T = 1/S$)

$$\delta(k) \equiv S/[kv(k)] \sim S/(k^{2/3} \epsilon^{1/3}) \qquad k = 1/l$$

Assume (1) $\delta \ll 1$, for large enough k

(2) expand in powers of δ

Two kind of measures

characterizing the universal equilibrium state

$<u_i(\mathbf{k})u_j(-\mathbf{k})> = \; Q_{ij}(\mathbf{k}) = \underline{Q_{ij}{}^{0}(\mathbf{k})} + \underline{C_{ij\alpha\beta}(\mathbf{k})}S_{\alpha\beta}$

Not only

1) Equilibrium state itself

$$Q_{ij}^{(0)}(\mathbf{k}) = \frac{C_K}{4\pi}\varepsilon^{2/3}k^{-11/3}P_{ij}(\mathbf{k})$$

$$P_{ij}(\mathbf{k}) = \delta_{ij} - \hat{k}_i\hat{k}_j, \; \hat{k}_i = k_i / k$$

but also

2) Response to disturbance

Similar to stress vs. rate of strain relation:

$$\tau_{ij} = C_{ij\alpha\beta}\, S_{\alpha\beta}$$

(justified by the Linear response theory for non-equilibrium system)

Anisotropic part

at small scale of turbulent shear flow

Kolmogorov's isotropic equilibrium spectrum

$$\langle u_i(\mathbf{k})u_j(-\mathbf{k})\rangle = Q_{ij}(\mathbf{k}) = Q_{ij}{}^{0}(\mathbf{k}) + \underline{C_{ij\alpha\beta}(\mathbf{k})S_{\alpha\beta}}$$

response

$$C_{ij\alpha\beta}(\mathbf{k}) = \underline{a(k)}\left[P_{i\alpha}(\mathbf{k})P_{j\beta}(\mathbf{k}) + P_{i\beta}(\mathbf{k})P_{j\alpha}(\mathbf{k})\right] + \underline{b(k)}P_{ij}(\mathbf{k})\hat{k}_\alpha \hat{k}_\beta$$

$$\underline{a(k)} = A\varepsilon^{1/3}k^{-13/3}, \quad \underline{b(k)} = B\varepsilon^{1/3}k^{-13/3}$$

cf. Lumley(1967)
Cambon & Rubinstein (2006)

Only 2 (universal) parameters, A and B

Is this correct?
What are the values of A and B?

Method of numerical experiment

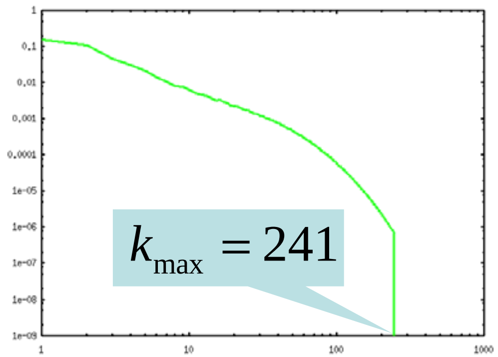
DNS of homogeneous turbulence with the simple shear flow

$$\mathbf{U} = \begin{pmatrix} Sx_2 \\ 0 \\ 0 \end{pmatrix}$$

$$S = 0.5, 1.0$$

Initial condition:
isotropic turbulence

Resolution: 512^3 $k_{\max}\eta = 1$



Observe $E_{ij}(k) = \sum_{p=k} Q_{ij}(\mathbf{p}),\quad E_{ij}^{ab}(k) = \sum_{p=k} \hat{p}_a \hat{p}_b Q_{ij}(\mathbf{p})$

especially,

$$E_{12}(k), E_{ii}^{12}(k), E_{11}^{12}(k), E_{22}^{12}(k), E_{33}^{12}(k), E_{12}^{11}(k), E_{12}^{22}(k), E_{12}^{33}(k)$$

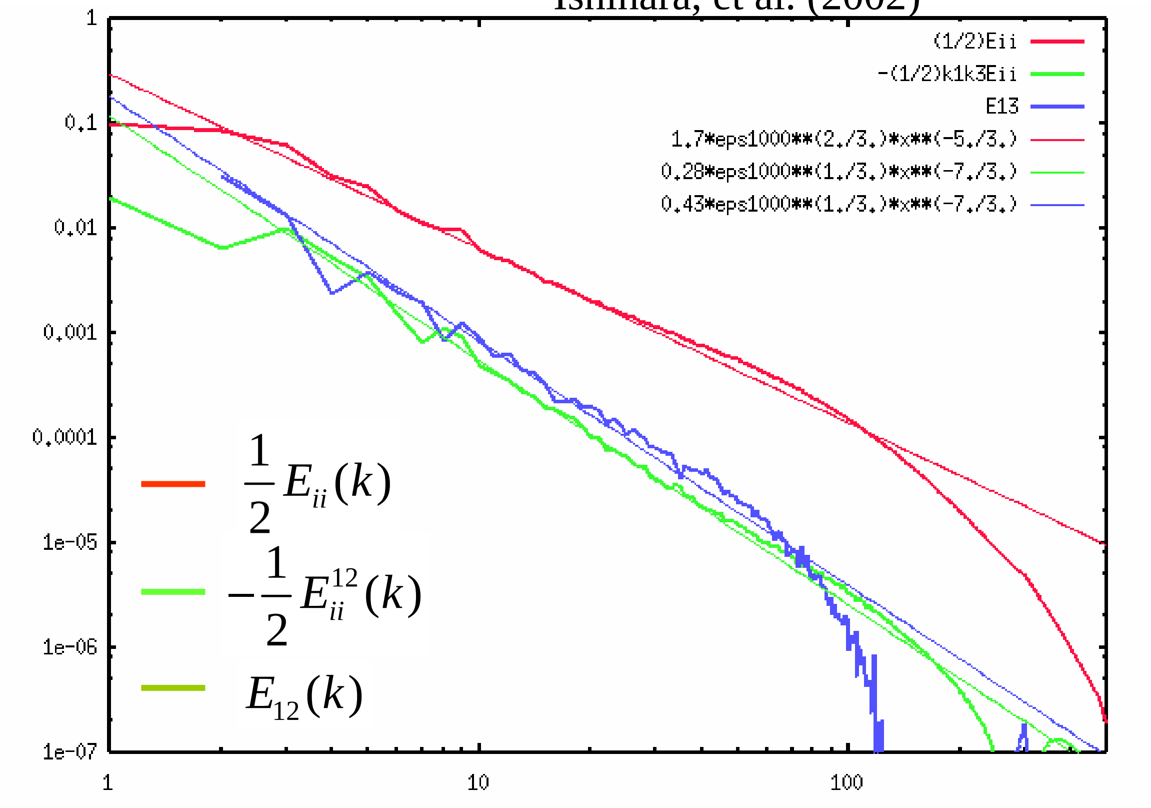
$$E_{12}(k) = \frac{4\pi}{15}(7A - B)\xi, E_{12}(k),\ \frac{1}{2}E_{ii}^{12}(k) = \frac{4\pi}{15}(-A + B)\xi, \ldots \qquad \xi = \varepsilon^{1/3}k^{-7/3}S$$

⇨ **Estimate A and B, and check consistency.**

Anisotropic Energy spectrum of homogeneous turbulent shear flow

Mean Flow $\mathbf{U} = \begin{pmatrix} Sx_2 \\ 0 \\ 0 \end{pmatrix}$

Ishihara, et al. (2002)



$$E_{ij}(k) = \sum_{p=k} \langle u_i(\mathbf{p}) u_j(-\mathbf{p}) \rangle$$

$$E_{ij}^{ab}(k) = \sum_{p=k} \hat{p}_a \hat{p}_b \langle u_i(\mathbf{p}) u_j(-\mathbf{p}) \rangle$$

Theoretical predictions:

$$\frac{1}{2}E_{ii}^{12}(k) = \frac{4\pi}{15}(-A+B)\xi$$

$$E_{12}(k) = \frac{4\pi}{15}(7A-B)\xi$$

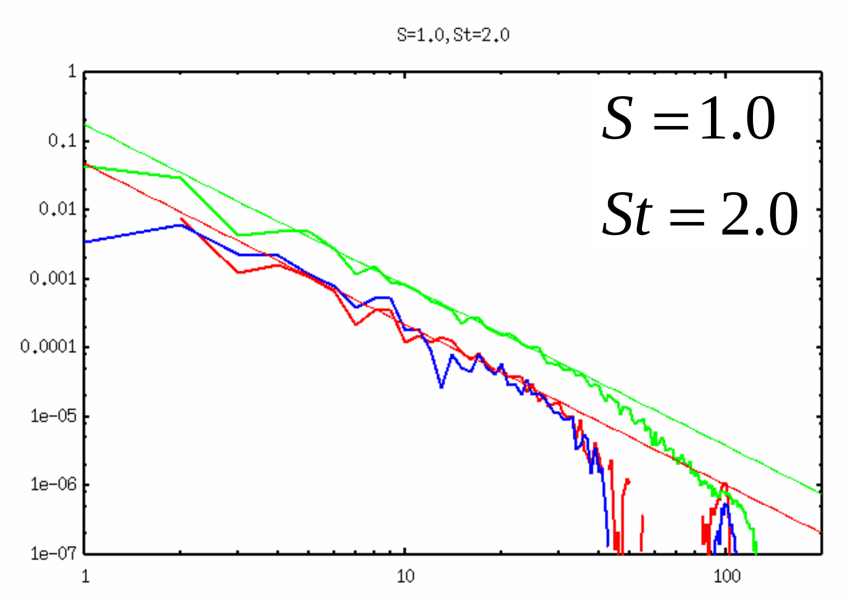
$$E_{12}^{11}(k) = E_{12}^{22}(k) = \frac{4\pi}{105}(13A-3B)\xi$$

$$E_{12}^{33}(k) = \frac{4\pi}{105}(23A-B)\xi, \ldots$$

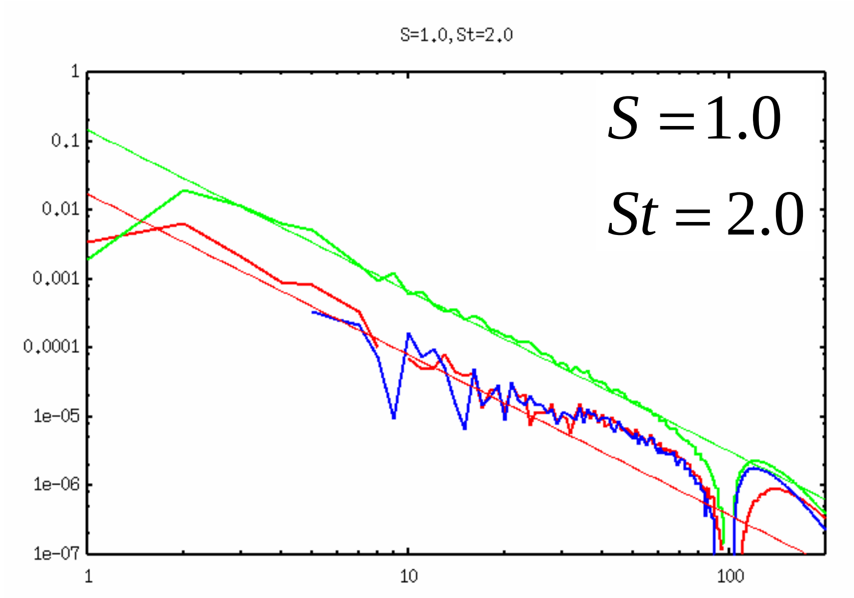
where $\xi = S\varepsilon^{1/3}k^{-7/3}$

Consistency

$-E_{12}^{11}(k), -E_{12}^{22}(k), -E_{12}^{33}(k)$



$-E_{11}^{12}(k), -E_{22}^{12}(k), -E_{33}^{12}(k)$



Ishihara et al.(2002)

Theoretical predictions:

$$E_{12}^{11}(k) = E_{12}^{22}(k) = \frac{4\pi}{105}(13A - 3B)\xi \qquad E_{11}^{12}(k) = E_{22}^{12}(k) = \frac{16\pi}{105}(-2A + B)\xi$$

$$E_{12}^{33}(k) = \frac{4\pi}{105}(23A - B)\xi \qquad E_{33}^{12}(k) = \frac{8\pi}{105}(A + 3B)\xi$$

See also Yoshida et al. (2003)

II. Stratification

See Kaneda & Yoshida (2004)

Boussinesq approximation

$$\begin{aligned}
\frac{\partial}{\partial t}\mathbf{u} &= -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \nu\nabla^2\mathbf{u} \boxed{-N\rho\mathbf{e}_3}, \\
\nabla\cdot\mathbf{u} &= 0, \\
\frac{\partial}{\partial t}\rho &= -(\mathbf{u}\cdot\nabla)\rho + \kappa\nabla^2\rho \boxed{+Nu_3},
\end{aligned}$$

Buoyancy by Stratification

$$\tau_N \sim \ell/v_\ell, \qquad \tau_\nu \sim \ell^2/\nu, \qquad \tau_E \sim 1/N,$$

$$\tau_N/\tau_E \sim \delta(k) \equiv N/[kv(k)] \quad \ll 1,$$

$$B_i(\mathbf{k},t) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{r}\langle u_i(\mathbf{x}+\mathbf{r},t)\rho(\mathbf{x},t)\rangle e^{-i\mathbf{k}\cdot\mathbf{r}}.$$

$$P(\mathbf{k},t) = \frac{1}{(2\pi)^3}\int d^3\mathbf{r}\langle \rho(\mathbf{x}+\mathbf{r},t)\rho(\mathbf{x},t)\rangle e^{-i\mathbf{k}\cdot\mathbf{r}},$$

$$
\begin{aligned}
Q_{ij}(\mathbf{k}) &= Q_{ij}^{(0)}(\mathbf{k}) + \Delta Q_{ij}(\mathbf{k}), & \Delta Q_{ij}(\mathbf{k}) &= Q_{ijm}^{(1)}(\mathbf{k})N_m + Q_{ijmn}^{(2)}(\mathbf{k})N_m N_n + \ldots, \\
P(\mathbf{k}) &= P^{(0)}(\mathbf{k}) + \Delta P(\mathbf{k}), & \Delta P(\mathbf{k}) &= P_m^{(1)}(\mathbf{k})N_m + P_{mn}^{(2)}(\mathbf{k})N_m N_n + \ldots, \\
B_i(\mathbf{k}) &= B_i^{(0)}(\mathbf{k}) + \Delta B_i(\mathbf{k}), & \Delta B_i(\mathbf{k}) &= B_{im}^{(1)}(\mathbf{k})N_m + B_{imn}^{(2)}(\mathbf{k})N_m N_n + \ldots,
\end{aligned}
$$

$$\boxed{\mathbf{N} = N\mathbf{e}_3}$$

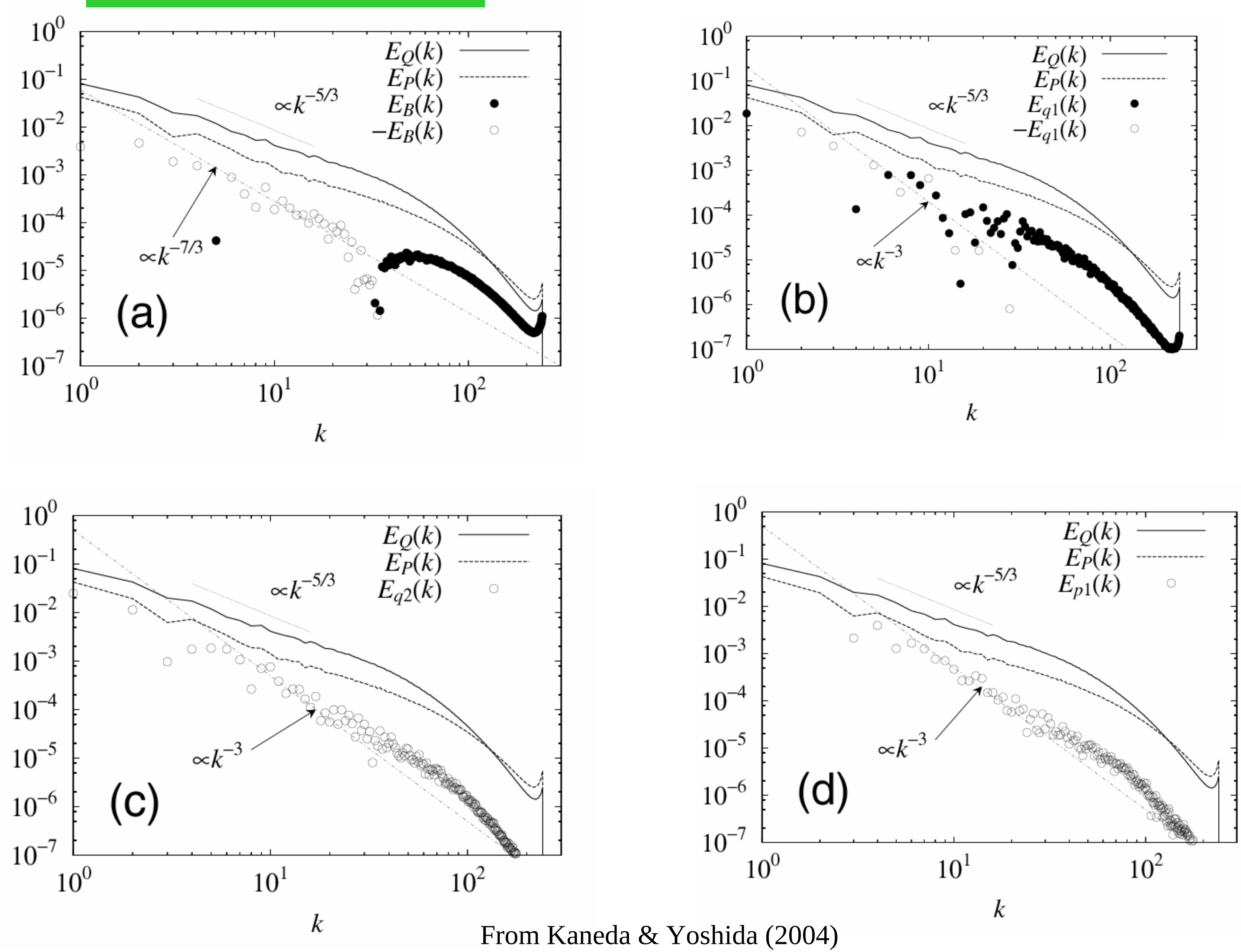
$$
\begin{aligned}
\Delta Q_{ij}(\mathbf{k}) &= \left[\boxed{q_1(k)}P_{i3}(\mathbf{k})P_{j3}(\mathbf{k}) + \boxed{q_2(k)}P_{ij}(\mathbf{k})\frac{k_3^2}{k^2} + \boxed{q_3(k)}P_{ij}(\mathbf{k})\right]N^2, \\
\Delta P(\mathbf{k}) &= \left[\boxed{p_1(k)}\frac{k_3^2}{k^2} + \boxed{p_2(k)}\right]N^2, \\
\Delta B_i(\mathbf{k}) &= \boxed{b(k)}P_{i3}(\mathbf{k})N,
\end{aligned}
$$

Kolmogorov scaling:

$$q_i(k) = \alpha_i k^{-5}, \qquad p_j(k) = \beta_j k^{-5}, \qquad b(k) = \gamma\epsilon^{1/3}k^{-13/3},$$

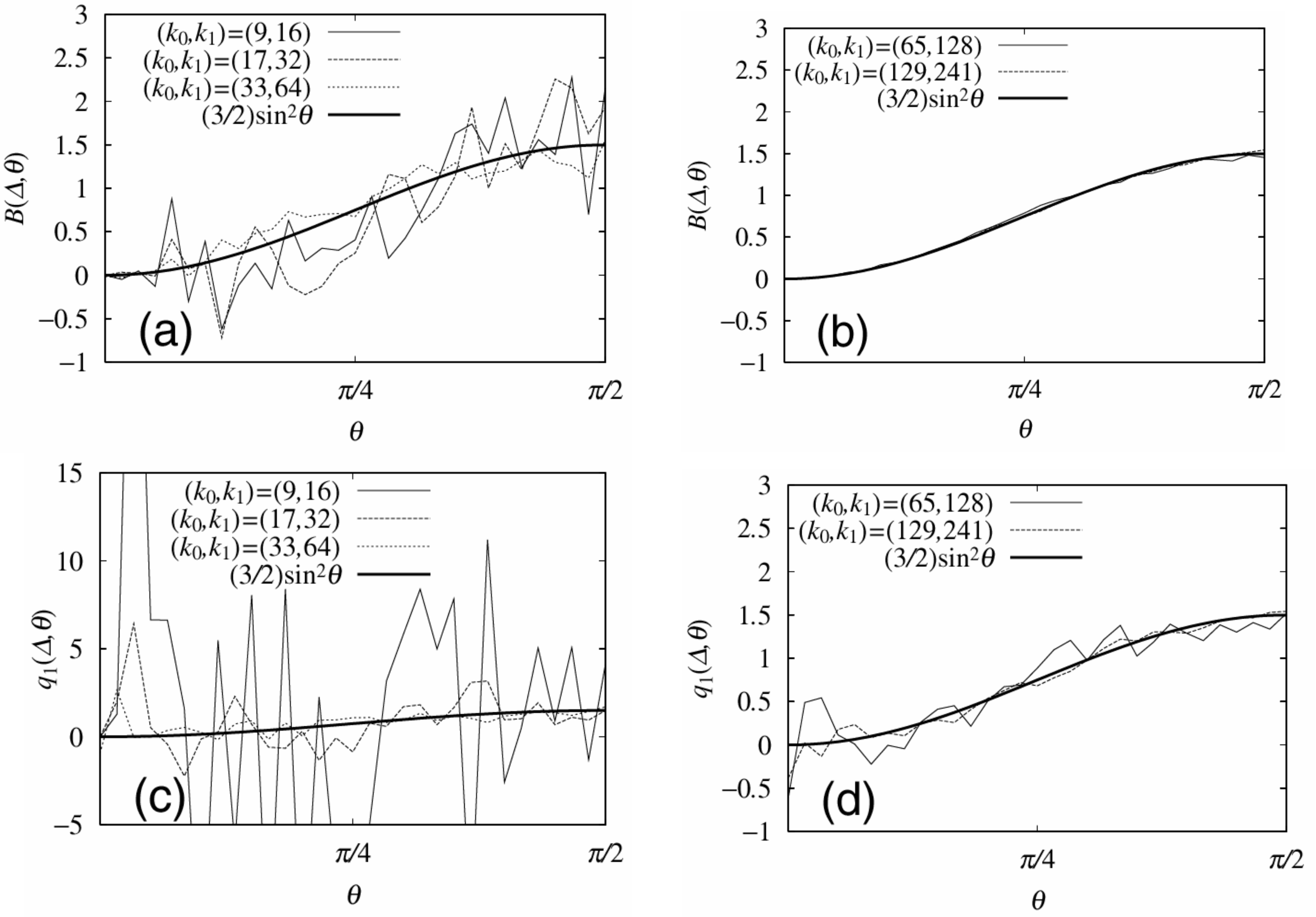
Boussinesq Turbulence

Scaling

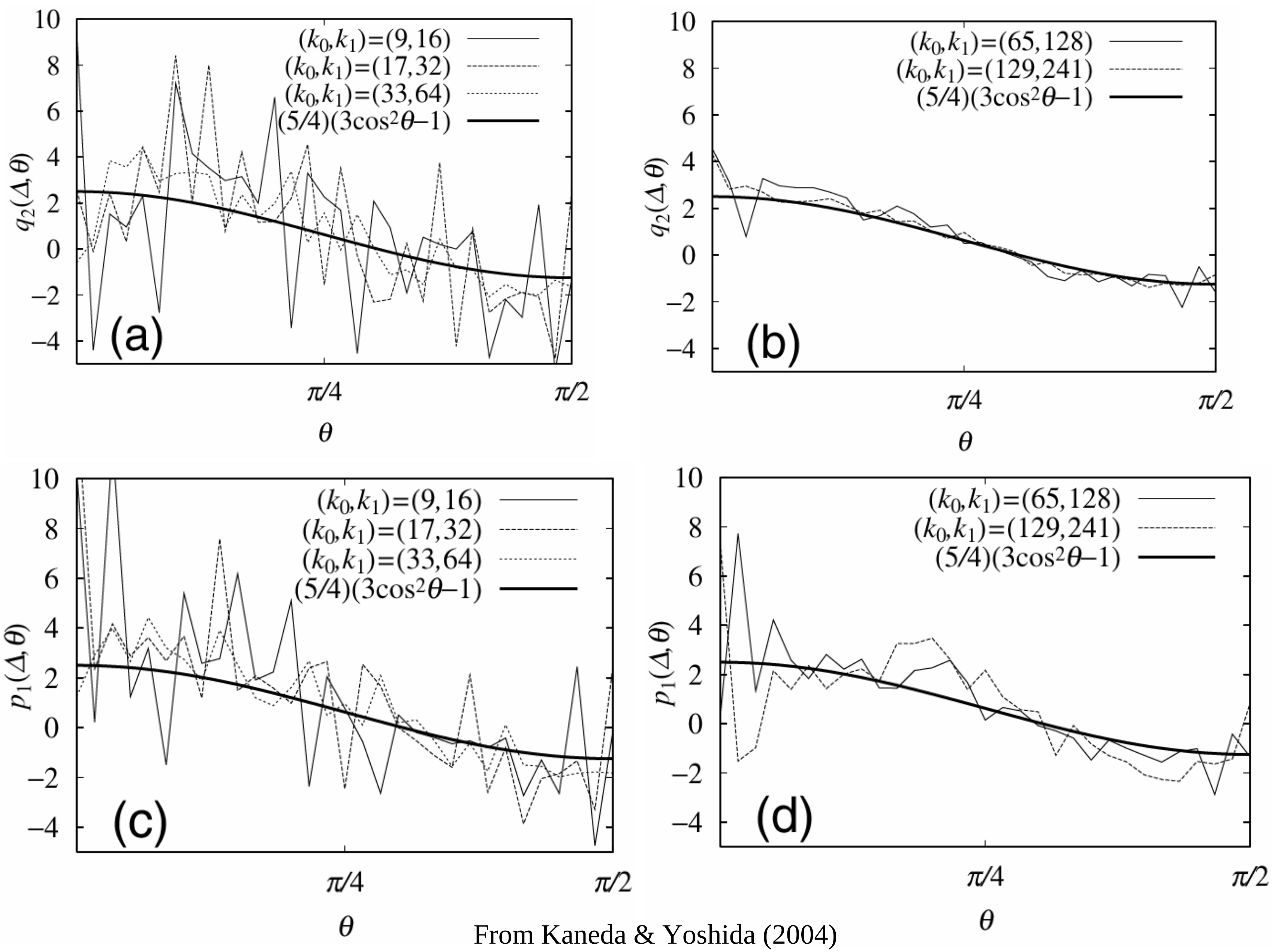


From Kaneda & Yoshida (2004)

Angular dependence



From Kaneda & Yoshida (2004)



From Kaneda & Yoshida (2004)

III. MHD

See Ishida & Kaneda (2007)

MHD approximation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\mathrm{grad})\,\boldsymbol{u} = -\frac{1}{\rho}\mathrm{grad}p + \nu\triangle\boldsymbol{u} + \boxed{\frac{1}{\rho}\boldsymbol{F}},$$

$$\mathrm{div}\boldsymbol{u} = 0,$$

Magnetic force

$$\frac{\partial \boldsymbol{B}}{\partial t} = (\boldsymbol{B}\cdot\mathrm{grad})\,\boldsymbol{u} - (\boldsymbol{u}\cdot\mathrm{grad})\,\boldsymbol{B} + \eta_{\mathrm{e}}\triangle\boldsymbol{B}$$

$$\mathrm{div}\boldsymbol{B} = 0$$

$$\boldsymbol{F} = \frac{1}{\mu_{\mathrm{e}}}(\boldsymbol{B}\cdot\mathrm{grad})\,\boldsymbol{B}$$

quasi-static approximation

$$\begin{aligned}\boldsymbol{F}' &= \frac{1}{\mu_{\mathrm{e}}}(\boldsymbol{B}\cdot\mathrm{grad})\,\boldsymbol{B} \\ &\simeq \frac{1}{\mu_{\mathrm{e}}}(\boldsymbol{B}_0\cdot\mathrm{grad})\,\boldsymbol{b} \quad + O(b^2) \\ &= -\frac{\sigma_{\mathrm{e}}}{\rho_{\mathrm{e}}}\triangle^{-1}(\boldsymbol{B}_0\cdot\mathrm{grad})^2\,\boldsymbol{u}\end{aligned}$$

$$(\boldsymbol{v}\cdot\mathrm{grad})\boldsymbol{v}\sim\frac{v_\ell^2}{\ell},\quad \nu\Delta\boldsymbol{v}\sim\frac{\nu v_\ell}{\ell^2},\quad \boldsymbol{F}\sim M v_\ell,\qquad \left(M\equiv\frac{\sigma_e B_0^2}{\rho}\right).$$

$$\frac{|\boldsymbol{F}|}{|(\boldsymbol{v}\cdot\mathrm{grad})\boldsymbol{v}|}\sim\delta_f\equiv\boxed{M\ell/v_\ell,}\longrightarrow\quad |\boldsymbol{F}|\ll|(\boldsymbol{v}\cdot\mathrm{grad})\boldsymbol{v}|\quad\text{for }\delta_f\ll 1.$$

Turbulent Shear Flow

NS-equation Local co-ordinate $\boldsymbol{v} = < \boldsymbol{v} > + \tilde{\boldsymbol{v}}$

$$\frac{\partial}{\partial t}\tilde{\boldsymbol{v}}(\boldsymbol{r},t) = -(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}} - \nabla\boldsymbol{q} + \nu\nabla^2\tilde{\boldsymbol{v}} + \boxed{\boldsymbol{M},}$$

$$M_i = \underline{S_{mn}} r_n \frac{\partial\tilde{v}_j}{\partial r_m} + S_{ij}\tilde{v}_j$$

<u>Effect of mean shear</u> **Local strain rate of mean flow**

for $r \ll L$, $< \boldsymbol{v} > \sim S\boldsymbol{r}$

$$(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}} \sim v_\ell^2/\ell, \qquad \nu\nabla^2\tilde{\boldsymbol{v}} \sim \nu v_\ell/\ell^2, \qquad \boldsymbol{M} \sim S v_\ell,$$

$$\tau_N \sim \ell/v_\ell, \qquad \tau_\nu \sim \ell^2/\nu, \qquad \tau_E \sim 1/S,$$

$$\frac{\boldsymbol{M}}{(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}}} \sim \frac{S v_\ell}{v_\ell^2/\ell} = \boxed{\frac{S\ell}{v_\ell}} \propto S\ell^{2/3}/\epsilon^{1/3} \ll 1$$

for $\ell \ll \ell_E = (\epsilon^{1/3}/S)^{3/2}$.

Disturbance : X

External force
due to the magnetic field

Isotropic turbulence
(Equilibrium state)

Response; Anisotropic

C X

Approximately linear in “X”

$$Q_{ij}(\boldsymbol{k}) = Q_{ij}^{(0)}(\boldsymbol{k}) + \underline{\Delta Q_{ij}(\boldsymbol{k})},$$

$$\Delta Q_{ij}(\boldsymbol{k}) = C_{ijmn}(\boldsymbol{k})\alpha_{mn}$$

$$\boxed{\alpha_{mn} = \frac{\sigma_{\mathrm{e}} B_{0m} B_{0n}}{\rho}}$$

response of isotropic turbulence

$$\begin{aligned}
\Delta Q_{ij}(\boldsymbol{k}) &= C_{ijmn}(\boldsymbol{k})\alpha_{mn} \\
&= [\frac{1}{2}\boxed{q_1(k)}\,(P_{im}(\boldsymbol{k})P_{jn}(\boldsymbol{k}) + P_{jm}(\boldsymbol{k})P_{in}(\boldsymbol{k})) \\
&+ \boxed{q_2(k)}P_{ij}(\boldsymbol{k})\frac{k_m k_n}{k^2} + \boxed{q_3(k)}P_{ij}(\boldsymbol{k})\delta_{mn}]\alpha_{mn}
\end{aligned}$$

3 (universal) parameters in the inertial subrange

anisotropic parts

- To detect $q_1(k)$ and $q_2(k)$, define

$$a(\boldsymbol{k}) \equiv \frac{k^2+k_3^2}{k^2-k_3^2}Q_{33}(\boldsymbol{k}) - Q_{11}(\boldsymbol{k}) - Q_{22}(\boldsymbol{k})$$

$$= \alpha_{33} q_1(k) \sin^2\theta$$

$$b(\boldsymbol{k}) \equiv \tilde{b}(\boldsymbol{k}) - \frac{1}{4\pi k^2}\langle \tilde{b}(\boldsymbol{k})\rangle_k,$$

$$\tilde{b}(\boldsymbol{k}) \equiv \frac{k_3^2}{k^2-k_3^2}Q_{33}(\boldsymbol{k}) + Q_{11}(\boldsymbol{k}) + Q_{22}(\boldsymbol{k})$$

$$= \alpha_{33} q_2(k)(\cos^2\theta - \frac{1}{3})$$

$$\cos\theta = \frac{k_3}{k}$$

$$\boxed{< f(\boldsymbol{k}) >_k = \int_0^{\pi} \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \, [k^2 \sin\theta \, f(\boldsymbol{k})]}$$

Theoretical prediction (1)

- Angular dependence

$$E_a(k,\theta) \equiv \frac{[a(\boldsymbol{k})]_{k,\theta}}{[a(\boldsymbol{k})]_k} = \frac{3}{2}\sin^2\theta,$$

$$E_b(k,\theta) \equiv \frac{[b(\boldsymbol{k})]_{k,\theta}}{[b(\boldsymbol{k})(3\cos^2\theta - 1)]_k} = \frac{5}{4}(3\cos^2\theta - 1)$$

$$[f(\boldsymbol{k})]_{k,\theta} \equiv \frac{\int_0^{2\pi} \mathrm{d}\phi\, [k\sin\theta f(\boldsymbol{k})]}{2\pi k\sin\theta}.$$

$$[f(\boldsymbol{k})]_k \equiv \langle f(\boldsymbol{k})\rangle_k/(4\pi k^2)$$

Theoretical prediction (2)

- Scale dependence
  - Assumptions
    - $q_i(k)$ are functions of only ϵ and k in the inertial subrange

$$q_i(k) = C_i \epsilon^{2/3} k^{-11/3},$$

$$\begin{aligned} E_a(k) &\equiv \langle a(\boldsymbol{k}) \rangle_k := \frac{16}{15} \pi \alpha_{33} C_2 \epsilon^{1/3} k^{-7/3} \\ E_b(k) &\equiv \langle (3\cos^2\theta - 1) b(\boldsymbol{k}) \rangle_k = \frac{8}{3} \pi \alpha_{33} C_1 \epsilon^{1/3} k^{-7/3} \end{aligned}$$

Comparison between theory and DNS

- Angular dependence
  - Theory $$E_a(k,\theta) = \frac{3}{2}\sin^2\theta$$
  - DNS

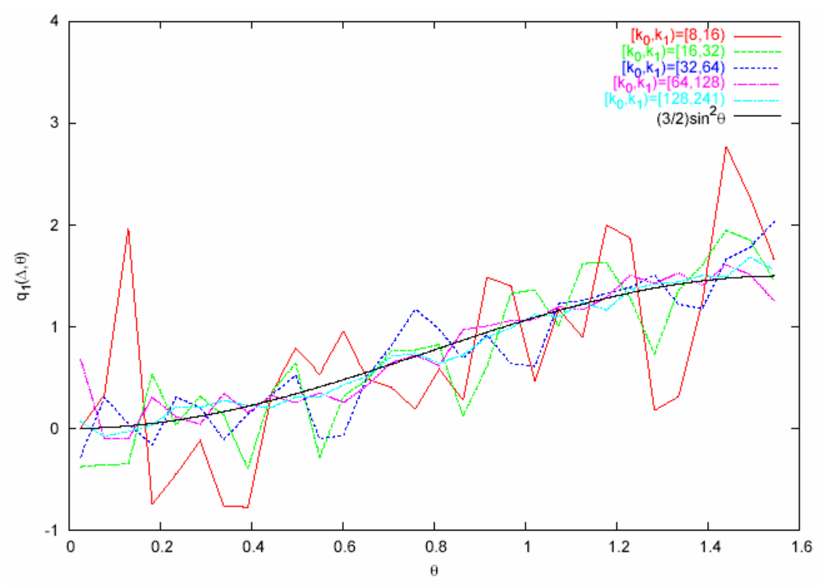
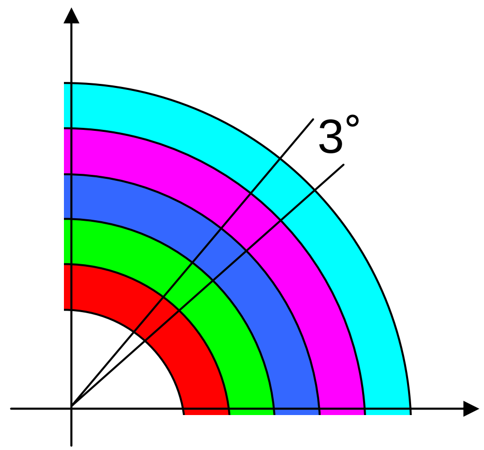


FIG. 8: $q_1(\Delta,\theta)$

From Ishida & Kaneda (2007)

- for the DNS
  - Every 3°
  - Shells
    - $[k_0,k_1)=[8,16)$
    - $[k_0,k_1)=[16,32)$
    - $[k_0,k_1)=[32,64)$
    - $[k_0,k_1)=[64,128)$
    - $[k_0,k_1)=[128,241)$



Comparison between theory and the DNS

- Angular dependence
  - Theory

$$E_b(k,\theta) = \frac{5}{4}(3\cos^2\theta - 1)$$

  - DNS

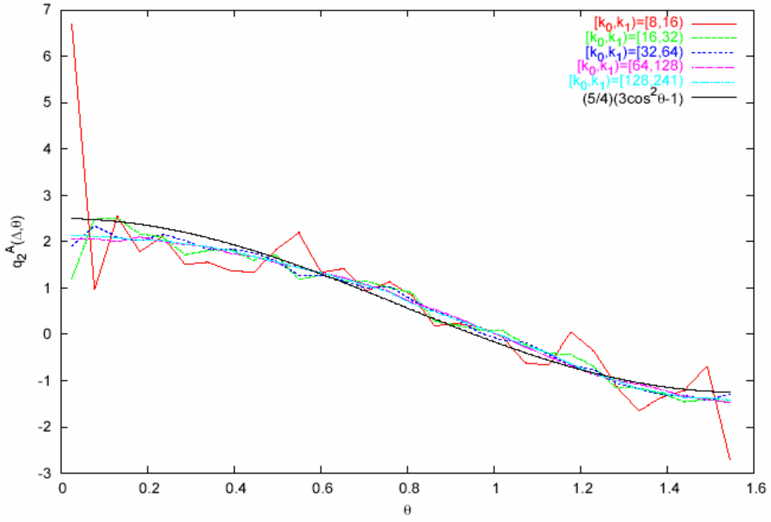


FIG. 9: $q_2^A(\Delta,\theta)$

From Ishida & Kaneda (2007)

Comparison between theory and DNS

- Scale dependence

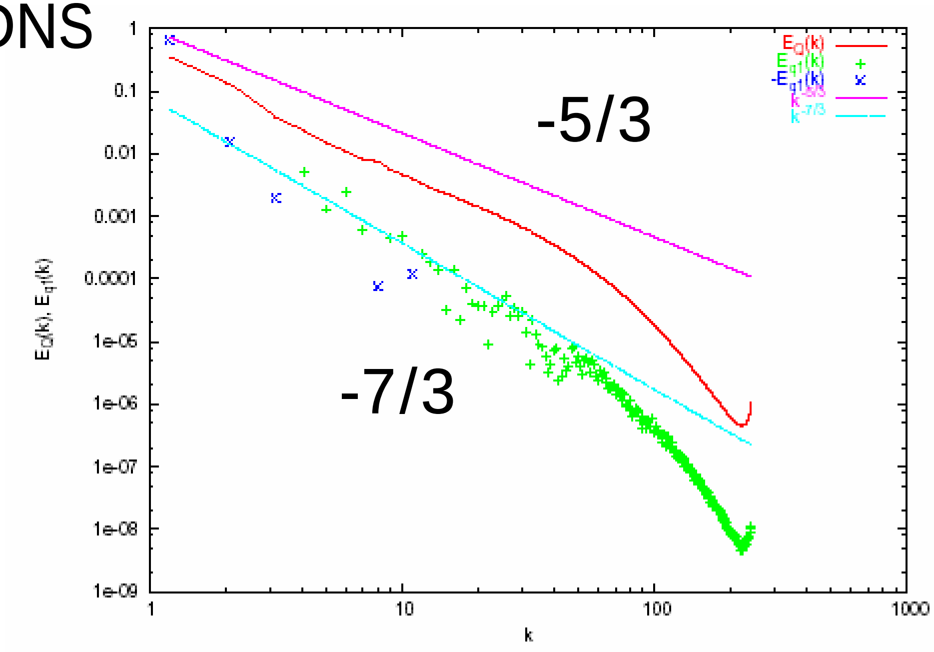
$k^{-7/3}/k^{-5/3}=k^{-2/3}$

Theory

$$E_a(k) = \frac{8}{3}\pi\alpha_{33}C_1\epsilon^{1/3}k^{-7/3}$$

$$E_b(k) = \frac{16}{15}\pi\alpha_{33}C_2\epsilon^{1/3}k^{-7/3}$$

DNS



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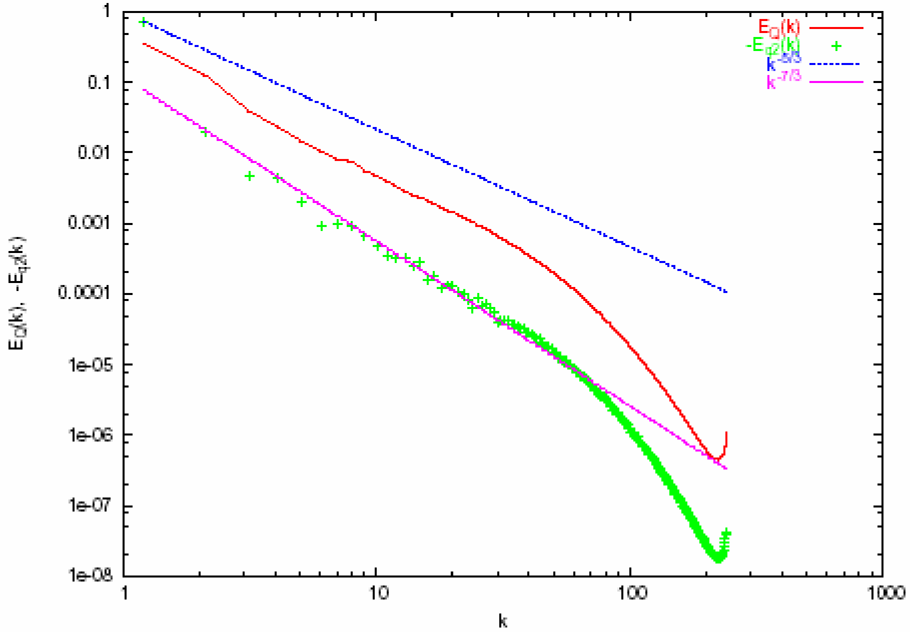
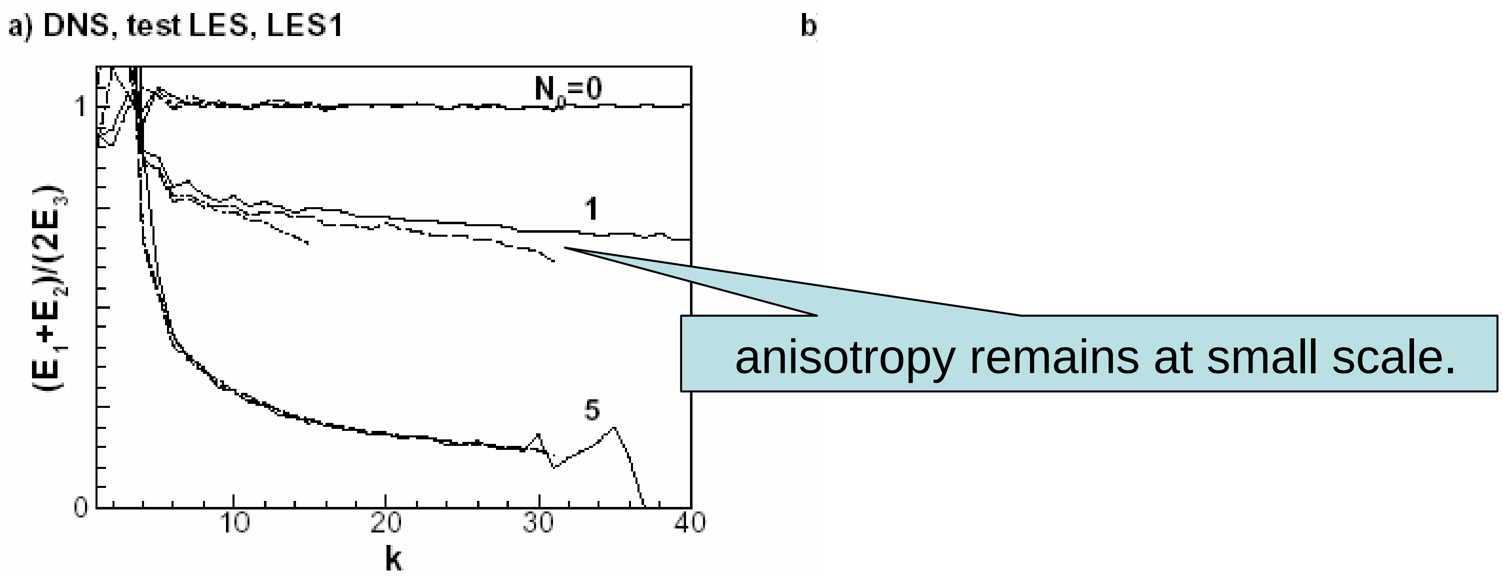


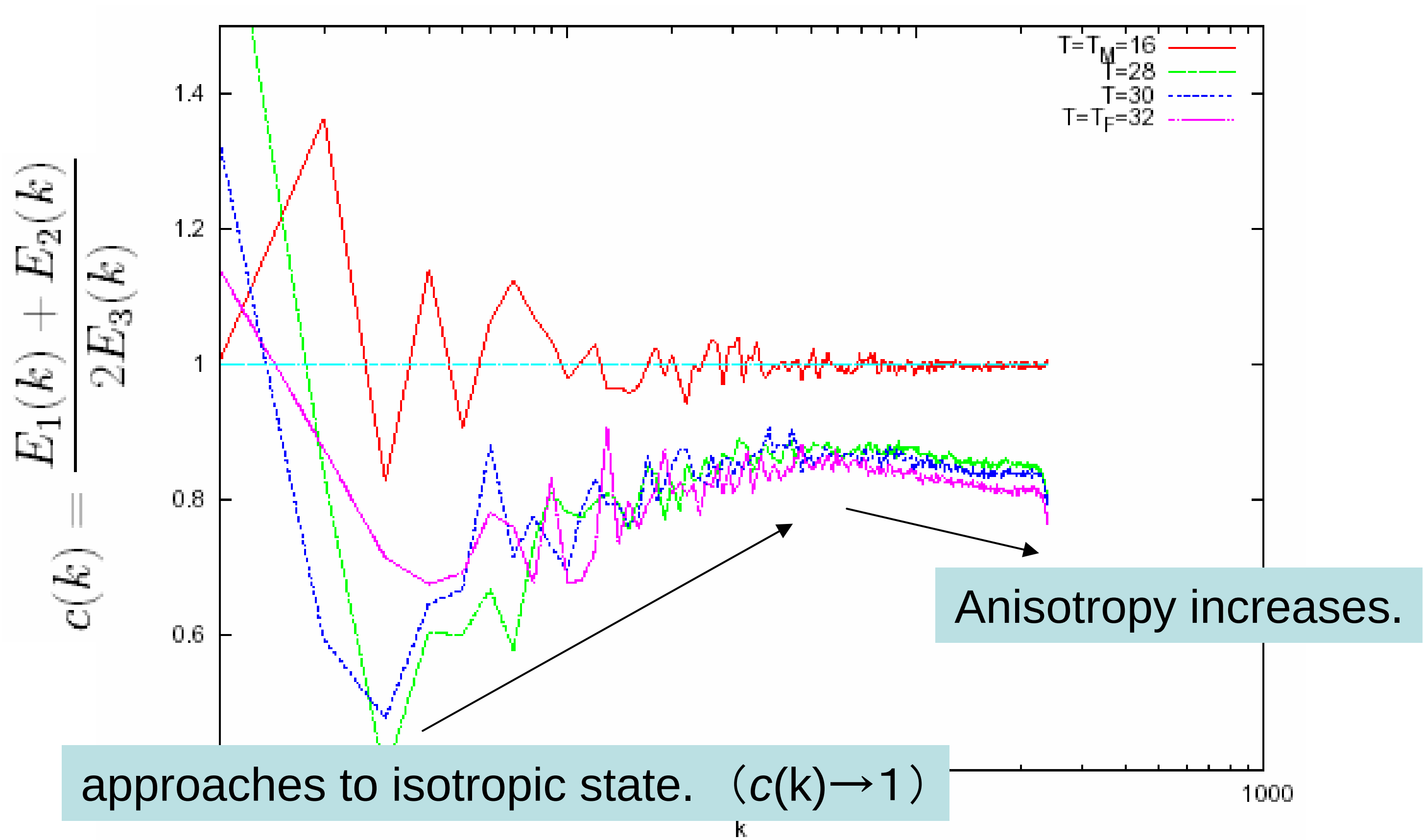
$$c(k) = \frac{E_1(k) + E_2(k)}{2E_3(k)}$$

$$E_i(k) = \frac{1}{2} \sum_{k-1/2<|\boldsymbol{k}|<k+1/2} (v_i(\boldsymbol{k}) \cdot v_i^*(\boldsymbol{k})), \qquad i = 1, 2, 3$$



From Vorobev et al.(Phys. Fluids **17**, 125105 (2005))

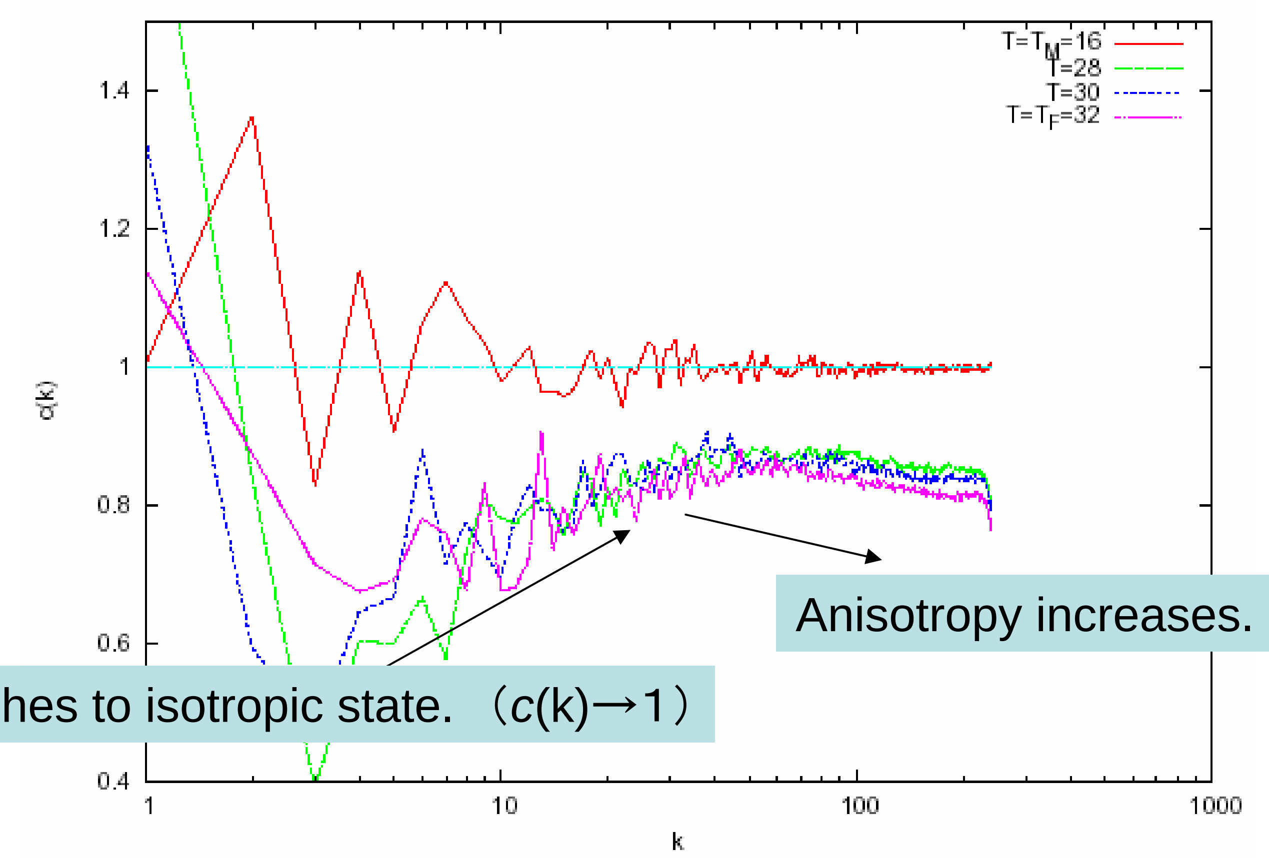
Results in our DNS



From Ishida & Kaneda (2007)

Results in our DNS

$$c(k) = \frac{E_1(k) + E_2(k)}{2E_3(k)}$$

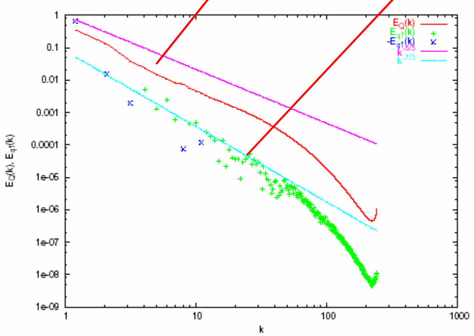


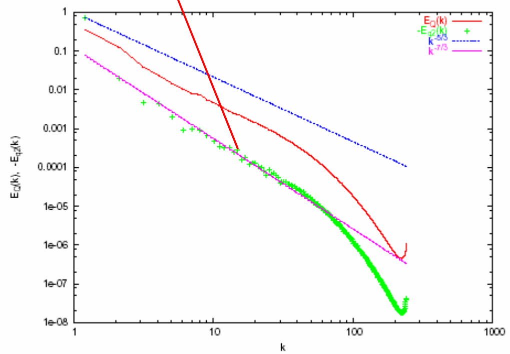
From Ishida & Kaneda (2007)

- Anisotropy : *c(k)*
  - DNS $$c(k) = \frac{E_1(k) + E_2(k)}{2E_3(k)}$$

$$c_{\text{theory}}(k) \equiv \frac{(8/3)[Q^{(0)}(k) + \alpha_{33}q_3(k)] + \alpha_{33}[(4/15)q_1(k) + (16/15)q_2(k)]}{(8/3)[Q^{(0)}(k) + \alpha_{33}q_3(k)] + 2\alpha_{33}[(16/15)q_1(k) + (4/15)q_2(k)]}$$

$$= 1 - \frac{21E_a(k) - 15E_b(k)}{20E(k) + 14E_a(k) - 10E_b(k)}$$



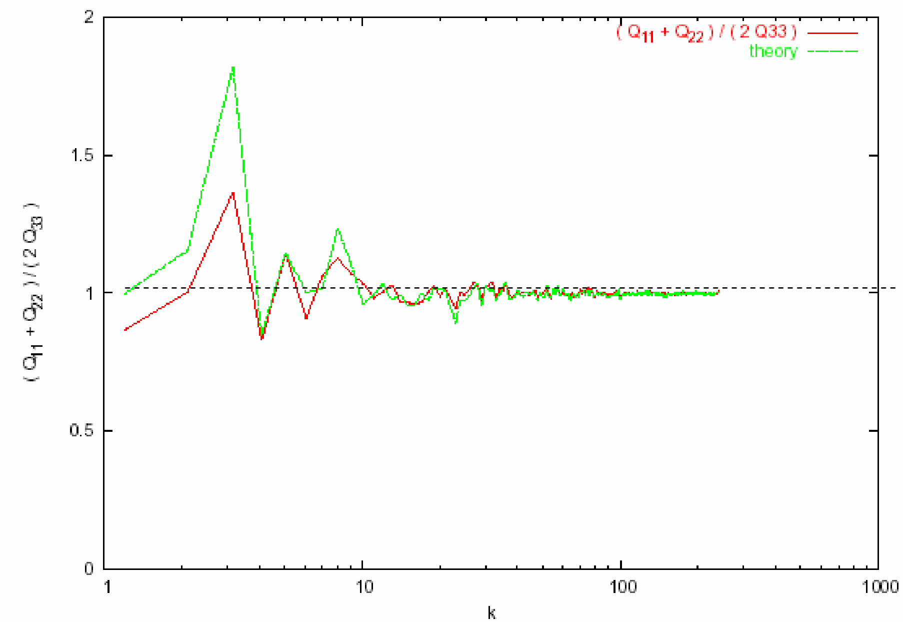


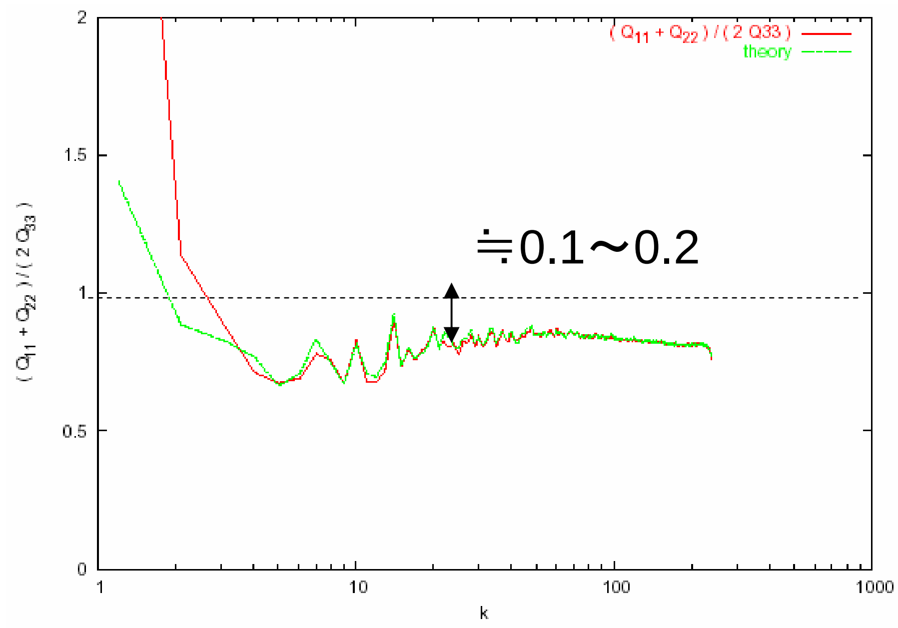
Comparison between theory and the DNS

Comparison with the DNS and the theory

$$c(k) = \frac{E_1(k) + E_2(k)}{2E_3(k)}$$

$$k^{-7/3}/k^{-5/3} = k^{-2/3}$$





$T=T_M=16$ magnetic field OFF

$T=T_F=32$ magnetic field ON

From Ishida & Kaneda (2007)

Summary -1

For turbulence,
We don't know the pdf nor Hamiltonian in contrast to thermal equilibrium state

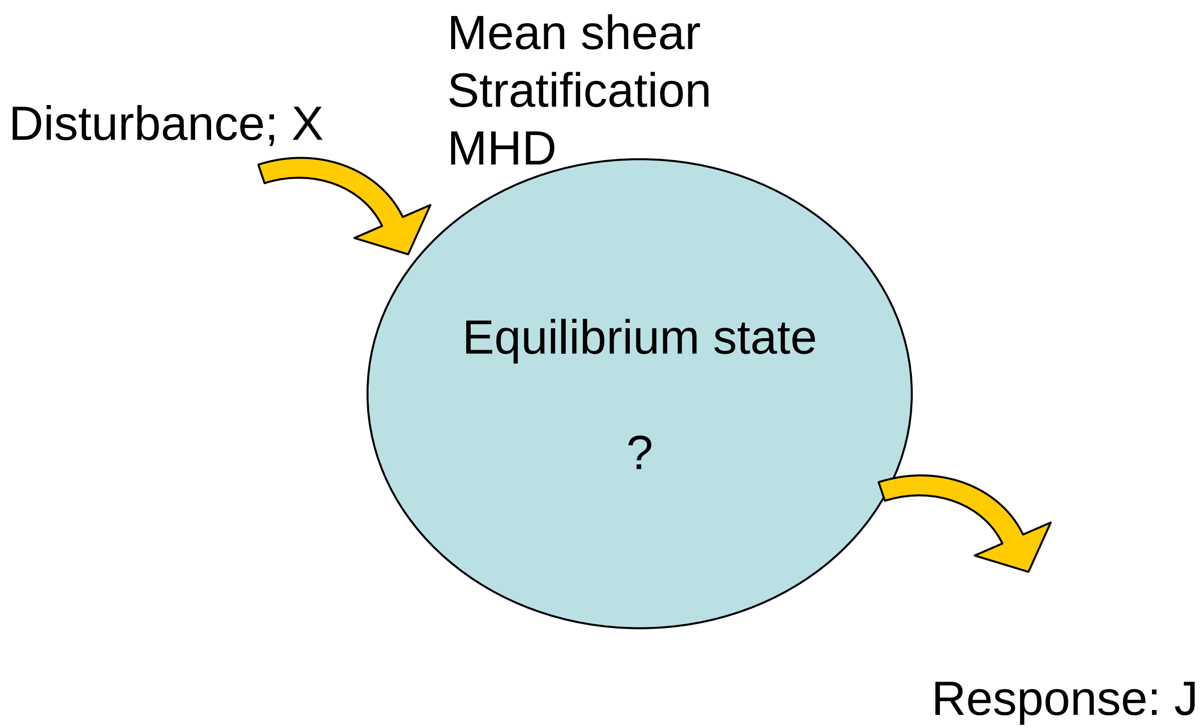
But we may assume
1) the existence of equilibrium state at small scale
(Universal) Local equilibrium state *a la* K41,
disturbance can be treated as perturbation to the inherent equilibrium state determined by the NS-dynamics

and see

2) the response ← small disturbance
disturbances tested:
Mean Shear, Stratification, MHD
(→ others, e.g., scalar field under mean scalar gradient).

But
3) The **anisotropy may remain large** even at small scales,
if Re is not high enough, so that the inertial subrange is not wide.

Summary -2



$$J_{ij} = \Delta Q_{ij} = C_{ijkm} X_{km}$$

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The end