The inertial range of solar wind MHD turbulence

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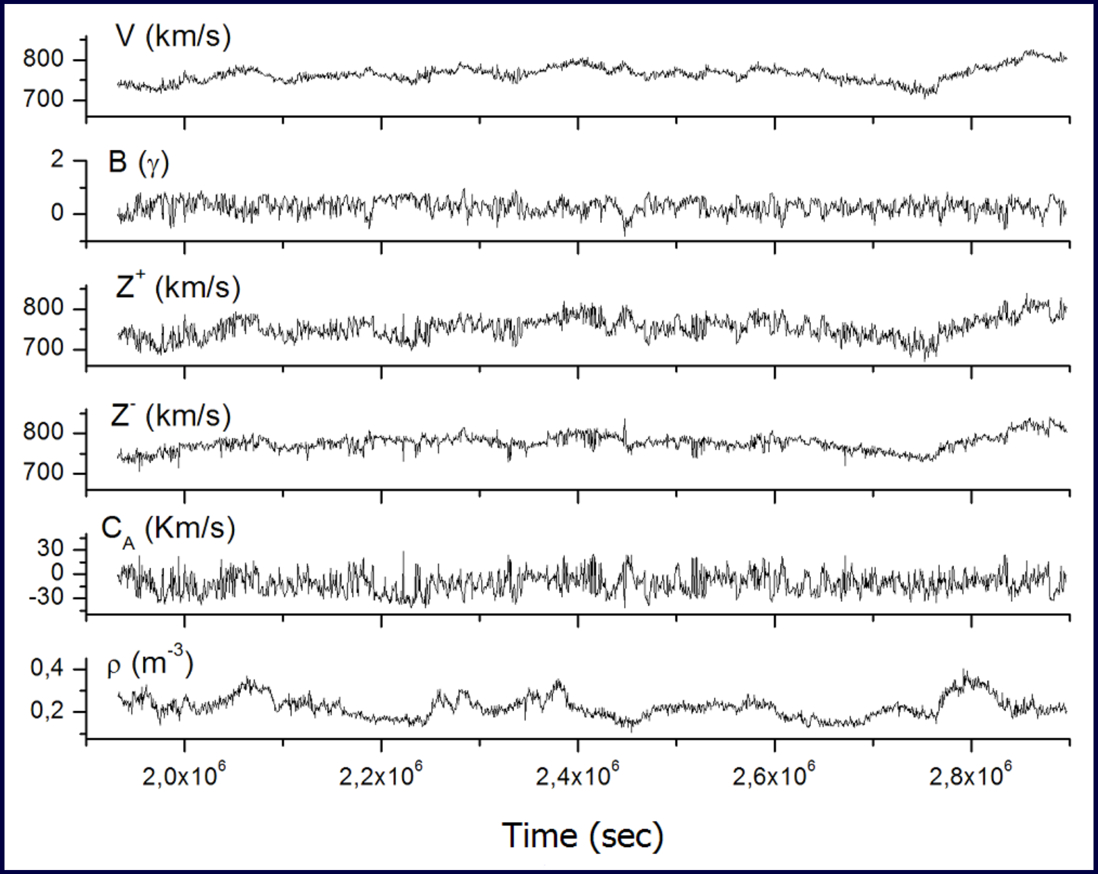
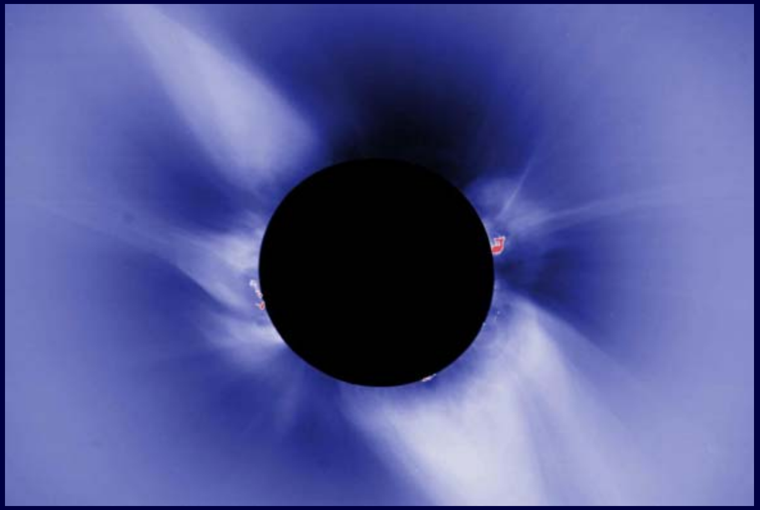
Cargese, August 2007

The solar wind as a wind tunnel



In situ measurements of high amplitude fluctuations for all fields (velocity, magnetic, temperature...) A unique possibility to measure low-frequency turbulence in plasmas over a wide range of scales.



$R = 10^4$

Laboratory

$R = 10^6$

Atmosphere

$R \approx 10^{10}$

Astrophysical flows

For a recent review: http://www.livingreview.org
R. Bruno & V. Carbone, Living Review in Solar Physics, (2005)

Turbulence is the result of nonlinear dynamics of Navier-Stokes (fluid flow) or MHD (plasma flow) equations.

$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \, \partial_\alpha^2 u_i$$

$$\partial_i u_i = 0$$

$$R = \frac{\text{Nonlinear}}{\text{Dissipative}} \approx \frac{UL}{\upsilon}$$

Nonlinear

Dissipative

Incompressible Navier-Stokes equation

u $\rightarrow$ velocity field

P $\rightarrow$ pressure

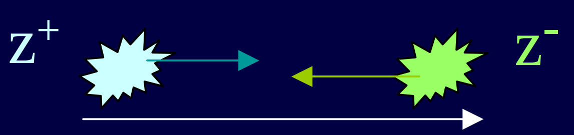
ν $\rightarrow$ kinematic viscosity

MHD

$$\partial_t z_i^{\pm} + z_\alpha^{\mp} \partial_\alpha z_i^{\pm} = -\partial_i P + \nu \, \partial_\alpha^2 z_i^{\pm}$$

$$z_i^{\pm} = u_i \pm b_i = u_i \pm B_i / \sqrt{4\pi\rho}$$

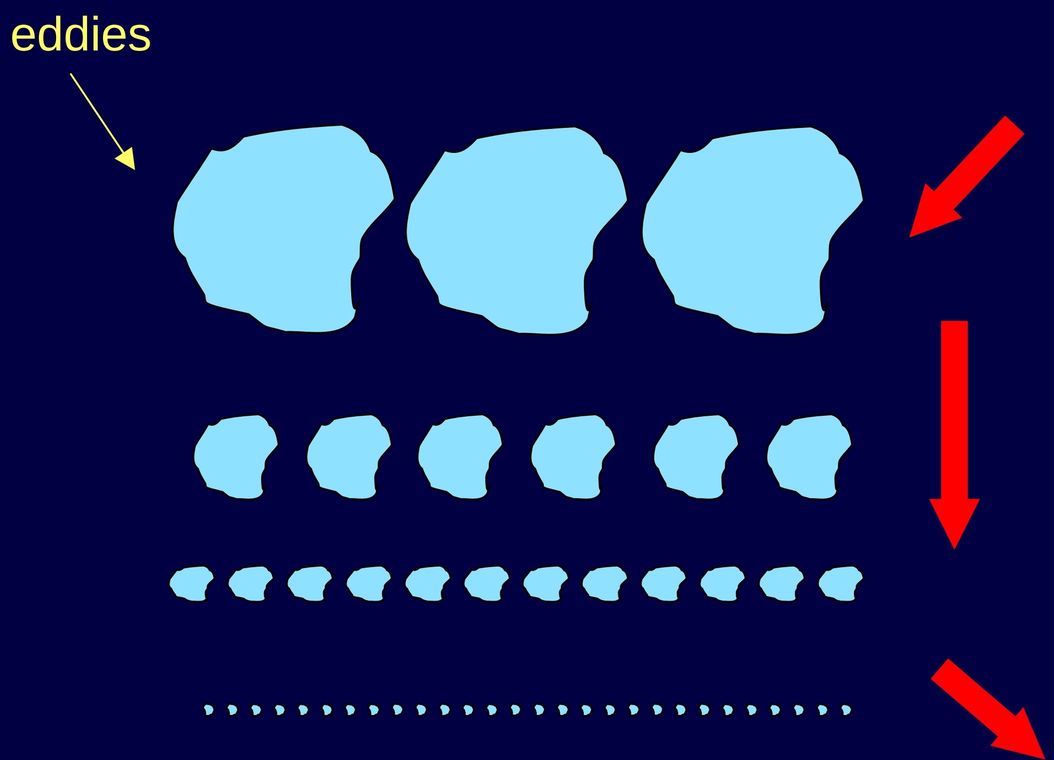
Hydromagnetic flows: the same "structure" of NS equations



Elsasser variables

Nonlinear interactions in MHD happens only between fluctuations propagating in opposite direction with respect to the magnetic field
$\rightarrow$ slow down of interactions

The energy cascade



Energy injection (e)

Integral scale *L*

Non-linear energy transfer (e)

Inertial range

Energy dissipation (e)

Dissipative scale l_d

In the inertial range:

$$\Delta u_\ell \approx \varepsilon^{1/3}\ell^{1/3} \;\rightarrow\; \frac{1}{2}\left\langle \Delta u_\ell^2 \right\rangle = S_2(\ell) \approx \ell^{2/3} \;\rightarrow\; E(k) \approx k^{-5/3}$$

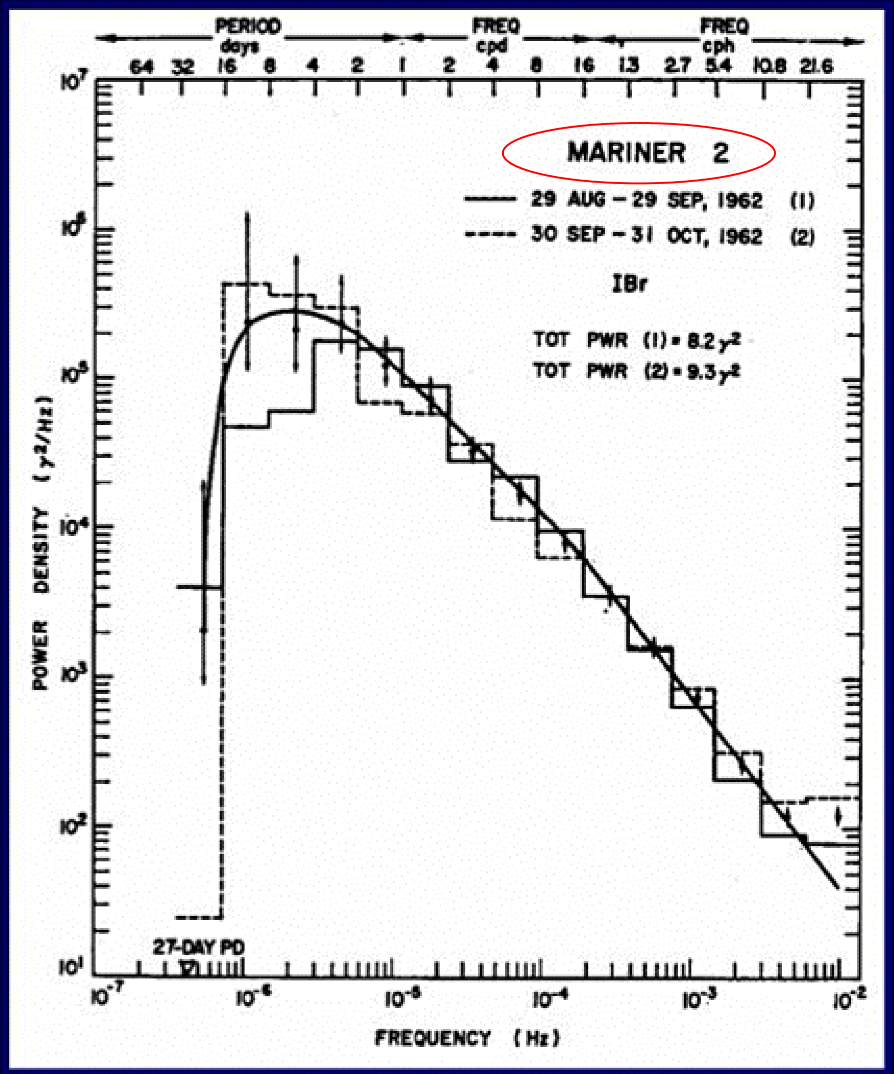
The Kolmogorov spectrum

Also, for the higher order moments (structure functions):

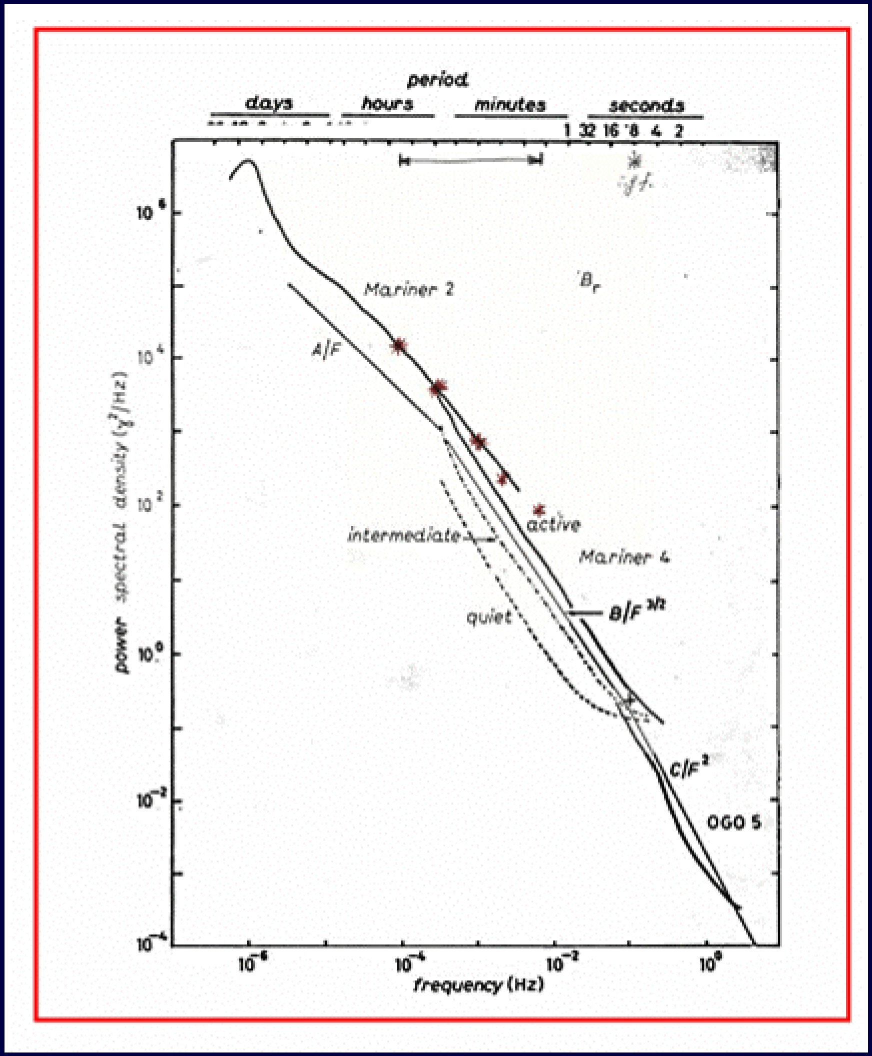
$$\left\langle \Delta u_\ell^p \right\rangle = C_p \varepsilon^{p/3} \ell^{p/3}$$

The Kolmogorov spectrum is observed

Magnetic field fluctuation



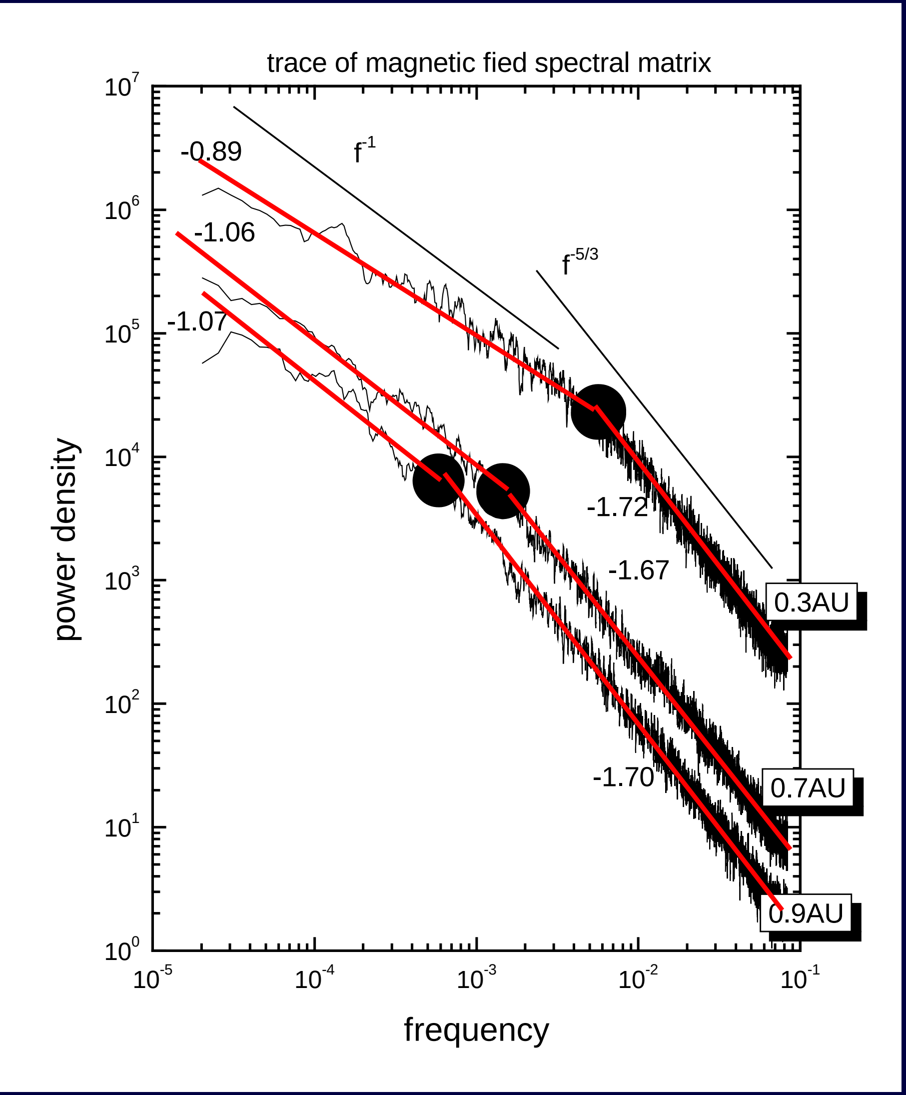
Coleman, 1968



Russel, 1972

The Kolmogorov spectrum is observed

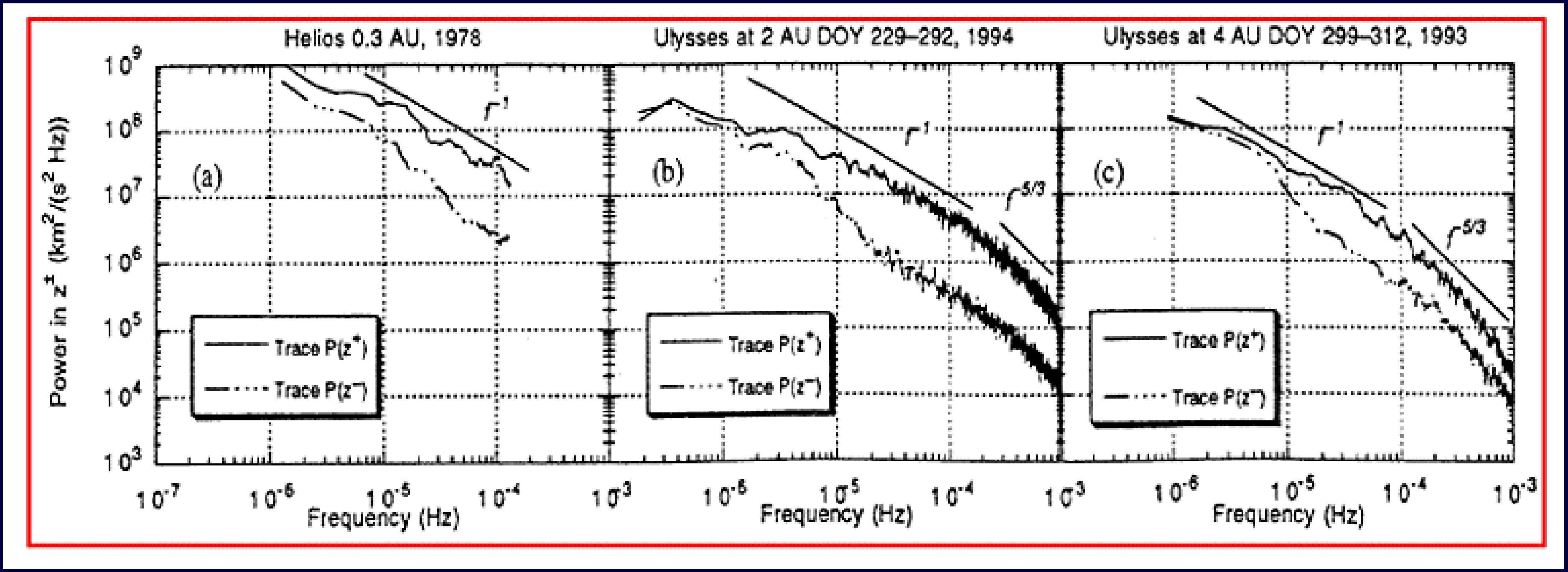
Power spectral density of the magnetic field, Helios 2 data



Bavassano et al., 1982

The Kolmogorov spectrum is observed

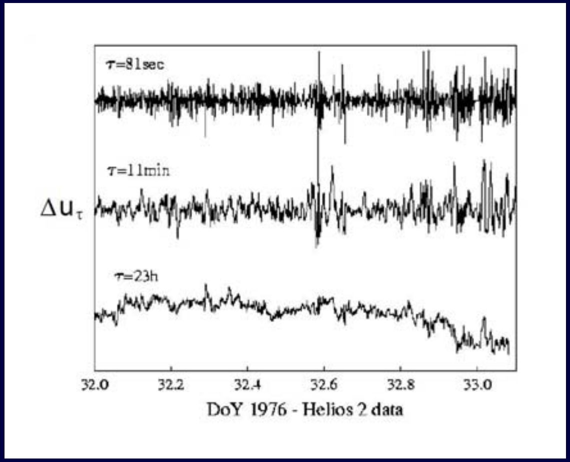
Power spectra of the Elsasser variables, Helios 2 and Ulysses data



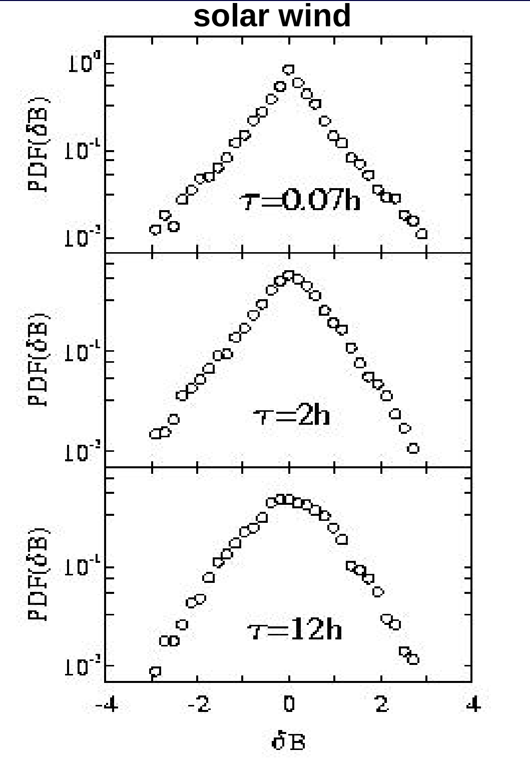
Goldstein et al., 1995

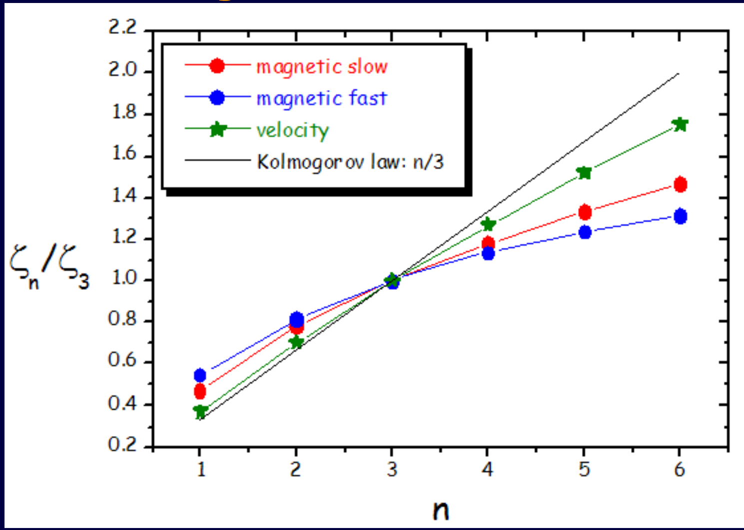
From many evidences, the inertial range was estimated as extending from the scale of the second to the one of the hour, whereas larger scales are characterized by f^{-1} spectrum (attributed to large scale wind structure?)

Behaviour of the structure function scaling exponents: intermittency



solar wind radial velocity fluctuations at different scales





$$S_p(r) = \left\langle \left[u(t+\tau) - u(t) \right]^p \right\rangle \approx \tau^{\varsigma_p}$$

Since turbulence is non-gaussian, the 2-th order moment (thus the spectrum) CANNOT play any privileged role.

So we need a more rigorous definition of the INERTIAL RANGE !

Fluids: an exact law from Navier-Stokes

$$\Delta u_i = [u_i(x+\ell) - u_i(x)]$$

Two-points differences along the LONGITUDINAL direction

Under the conditions of homogeneity and isotropy, in the stationary state an exact relation can be derived from Navier-Stokes equation:

$$\left\langle \Delta u_\ell \Delta u_i^2 \right\rangle = 2\nu \frac{\partial}{\partial \ell} \left\langle \Delta u_i^2 \right\rangle - \frac{4}{3} \langle \varepsilon \rangle \ell$$

$< \varepsilon >$ = averaged energy dissipation rate

In the **inertial range**
$\nu \rightarrow 0$
(a FORMAL definition of inertial range!)

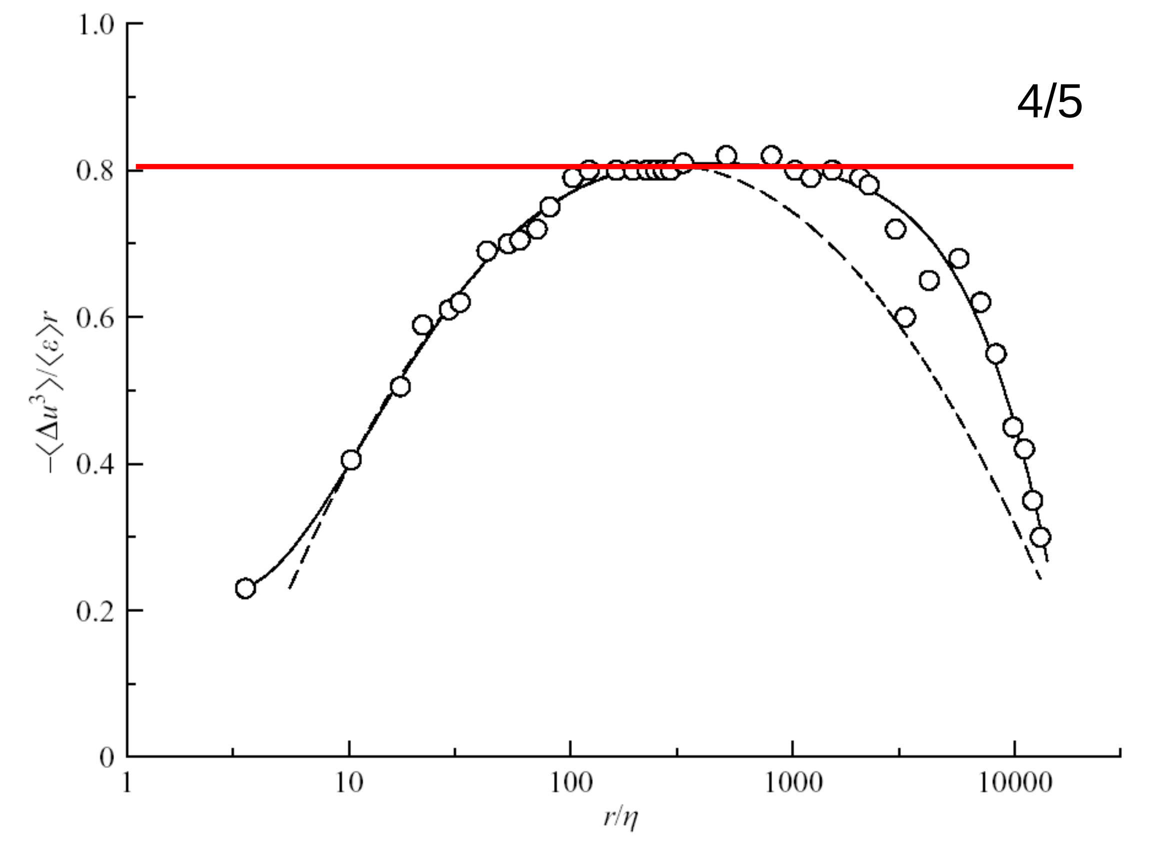
$$\Rightarrow \quad \boxed{\left\langle \Delta u_\ell^3 \right\rangle = -\frac{4}{5} \langle \varepsilon \rangle \ell}$$

Kolmogorov 4/5 law

a) The negative sign IS CRUCIAL!!! (NO absolute values!!) → energy cascade
b) The third-order moment of fluctuations is related to the energy dissipation rate and is different from zero

The 4/5-law in action!

Sreenivasan & Dhruva (1998), atmospheric turbulence



The 4/5-law represents a cornerstone for modeling of turbulence.
Any attempt to describe turbulence MUST satisfies this law.

How can we evidence the *presence of a nonlinear energy cascade in MHD*? **An exact relation for MHD turbulence**

$$Z_i^{\pm} = V_i \pm \frac{B_i}{\sqrt{4\pi\rho}}$$

Elsässer variables

$$\Delta Z_i^{\pm} = Z_i^{\pm}(x_i') - Z_i^{\pm}(x_i) \qquad x_i' = x_i + \ell_i$$

Two-points differences

$$\varepsilon_{ij}^{\pm} = \upsilon \left\langle \left(\partial_i Z_j^{\pm}\right)\left(\partial_j Z_i^{\pm}\right) \right\rangle$$

Pseudo-energy dissipation rate tensor

Third-order mixed moment

inhomogeneities

$$\frac{\partial}{\partial \ell_k} \left\langle \Delta Z_k^{\mp} \left(\Delta Z_i^{\pm} \Delta Z_j^{\pm} \right) \right\rangle = -\left\langle Z_k^{\mp} (\partial_k' + \partial_k) \left(\Delta Z_i^{\pm} \Delta Z_j^{\pm} \right) \right\rangle - \Pi_{ij} + 2\upsilon \frac{\partial^2}{\partial \ell_k^2} \left\langle \Delta Z_i^{\pm} \Delta Z_j^{\pm} \right\rangle - \frac{4}{3} \frac{\partial}{\partial \ell_k} \left(\varepsilon_{ij}^{\pm} \ell_k \right)$$

Pressure term

Dissipative term

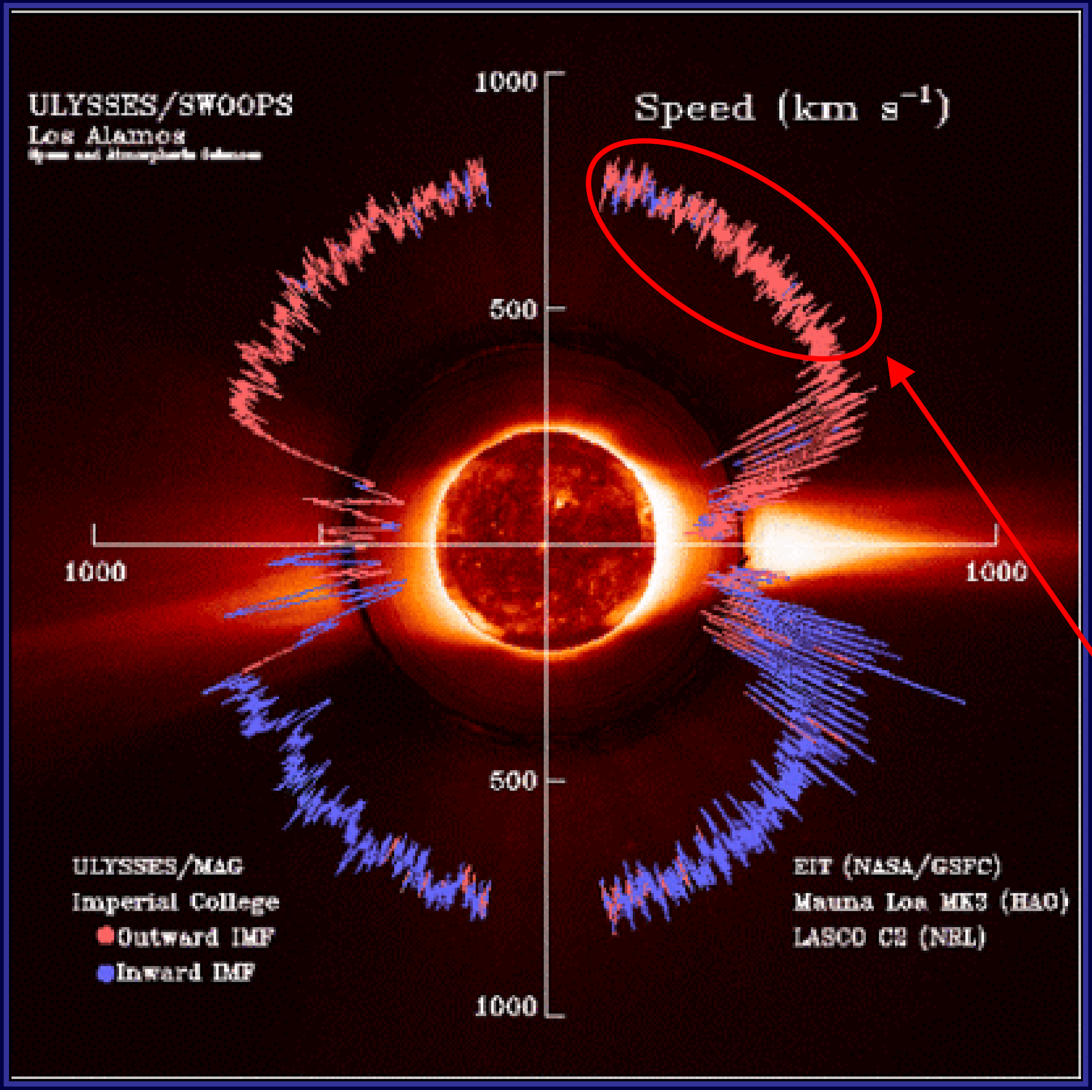
homogeneous
isotropic
In the inertial range

Yaglom-like equation for the MHD

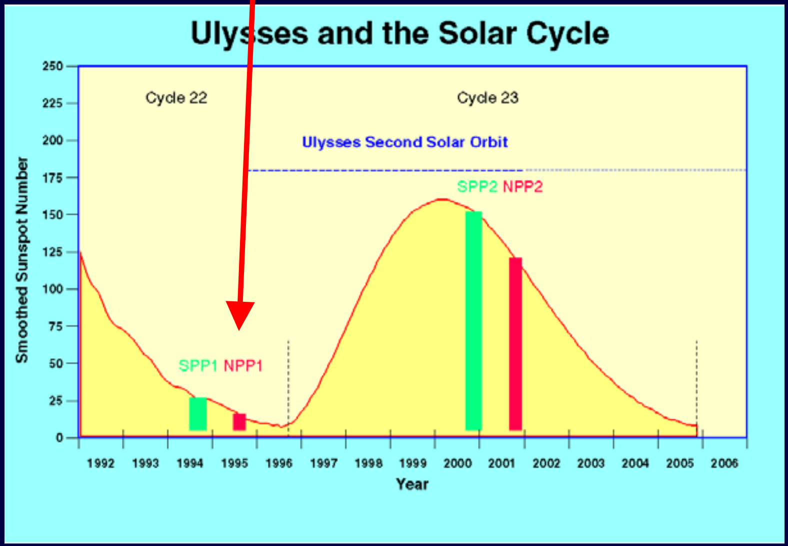
$$\left\langle \Delta Z_\ell^{\mp} \left(\Delta Z_i^{\pm} \right)^2 \right\rangle = -\frac{4}{3} \varepsilon^{\pm} \ell$$

Politano & Pouquet (1995)

Ulysses data: 1994-1996



Low solar activity (1994-1996)

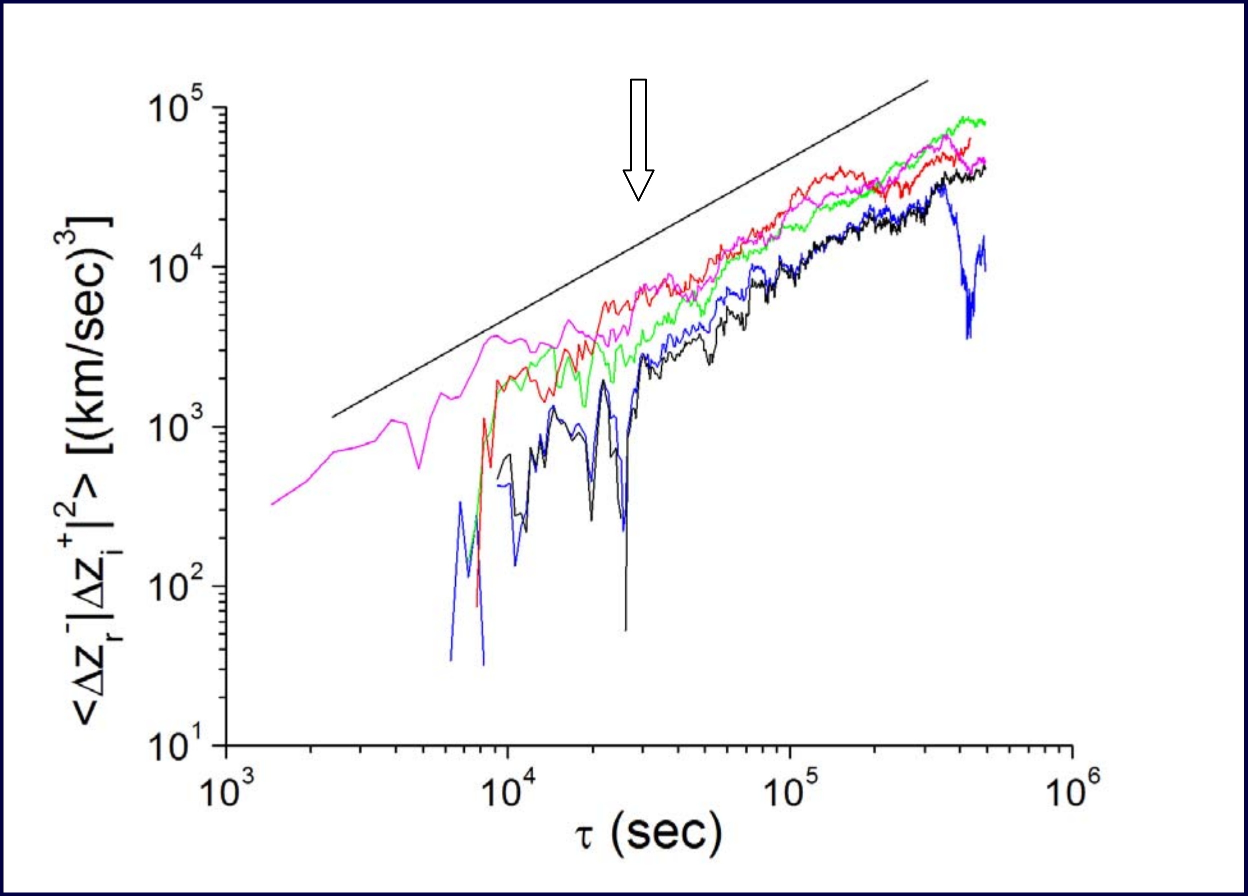


High latitude ($\theta > 35°$)

8 minutes averages of velocity, magnetic field and density are used to build the Elsasser fields $Z^{\pm}$

Running windows of 10 days (2000 data points each) have been used to avoid radial distance and latitudinal variations, as well as non-stationariety effects.

The Yaglom relation is present in most periods of datasets of Ulysses spacecraft



Although the solar wind is known to be characterized by the inhomogeneity and anisotropy, the collapse onto the Yaglom law appears very robust in many periods of about 10 days.

(Low frequency) solar wind can be described in the framework of MHD turbulence

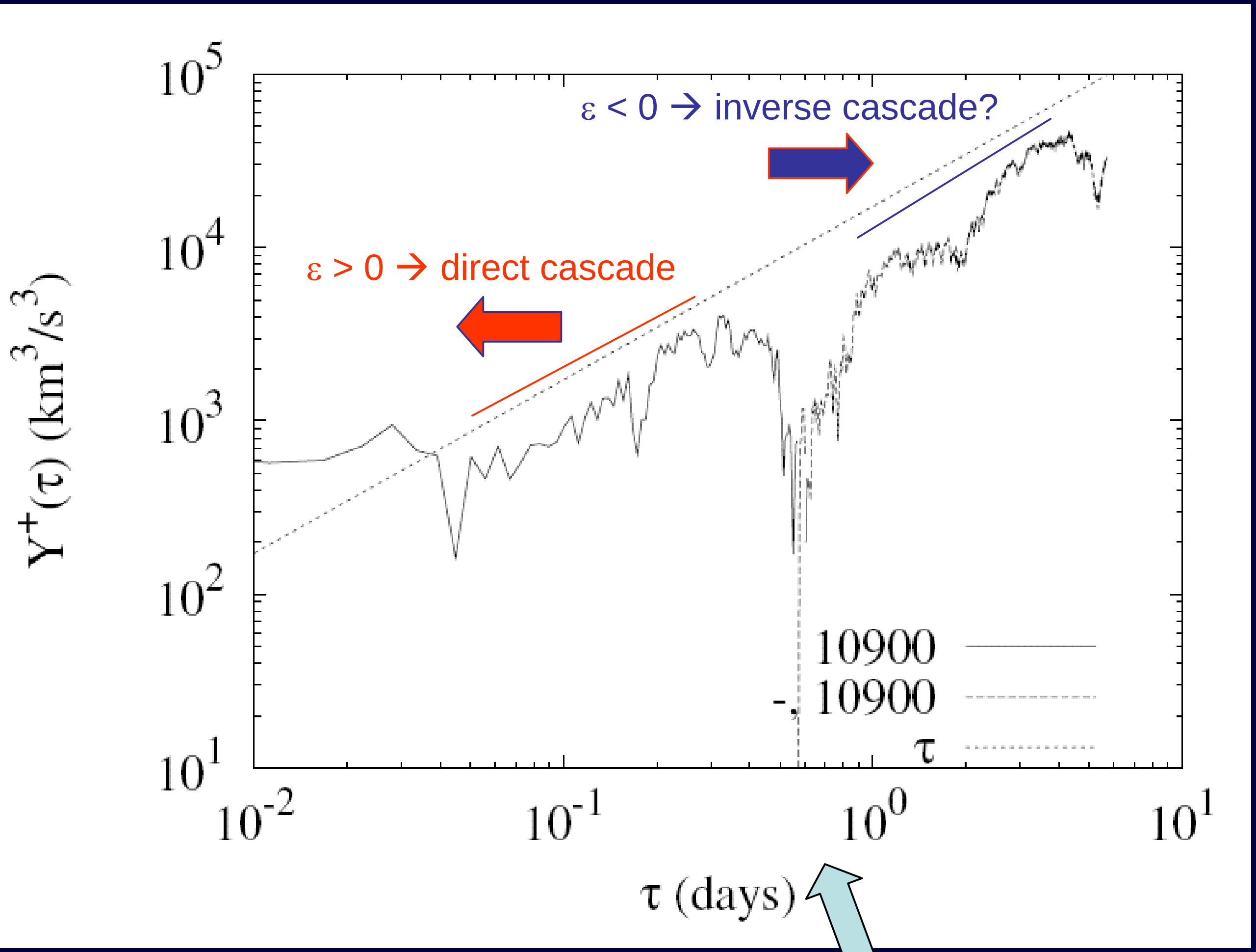
Taylor's hypothesis to transform length scales in time scales

$$\left\langle \Delta Z_r^{\mp} \left| \Delta Z_i^{\pm} \right|^2 \right\rangle = \frac{4}{3} \langle V \rangle \varepsilon^{\pm} \tau$$

Inertial range: up to 1-5 days!!!

L. Sorriso-Valvo, R. Marino et al., (PRL,2007)

More features observed from Ulysses 1996 data



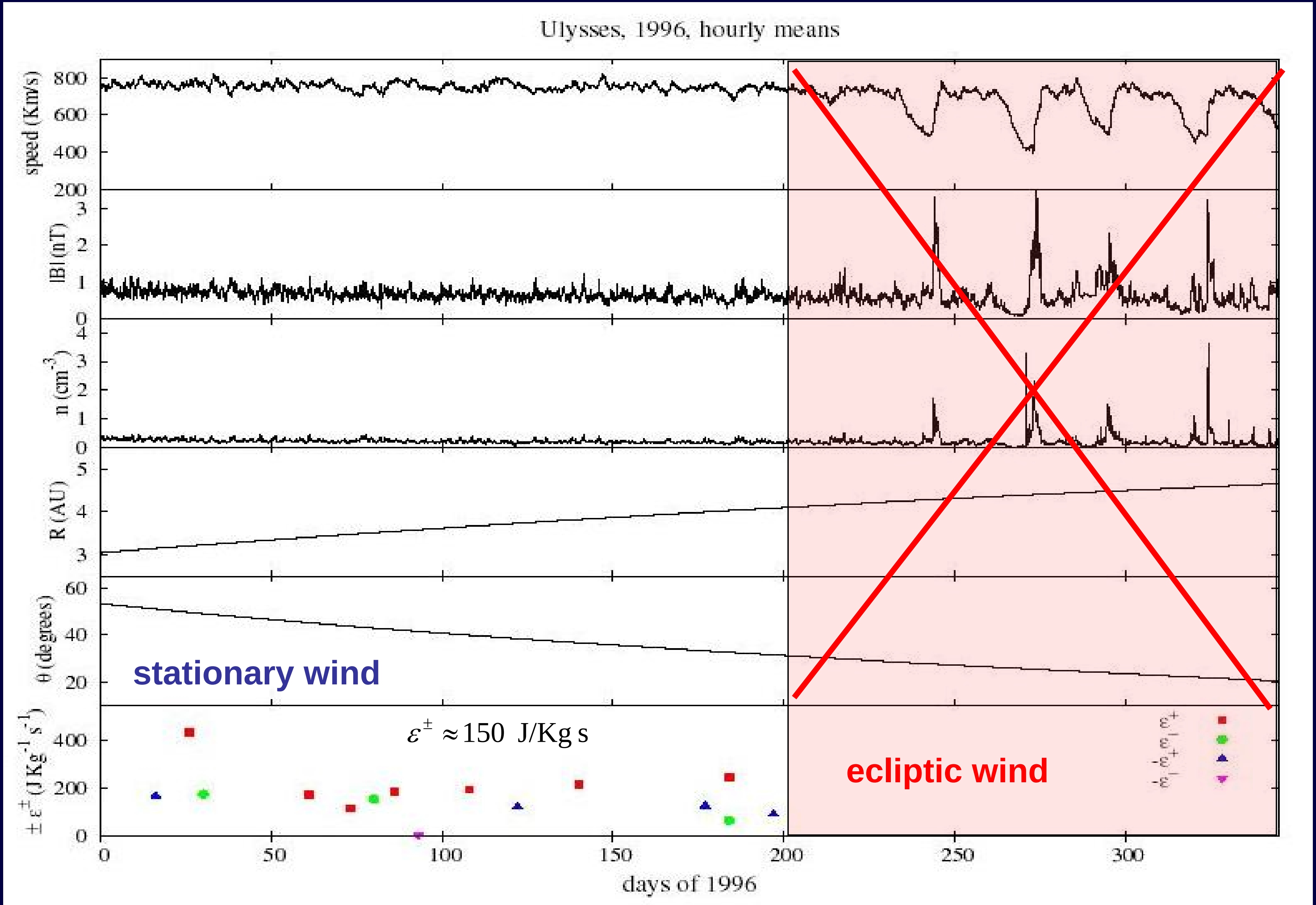
~ 10 hours

The observed *energy flux can also be negative, maybe implying an INVERSE cascade of pseudo-energy, or display both direct and inverse cascade.*

Not understood yet. Role of large scale magnetic and wind structure? 2D-like behaviour?

L. Sorriso-Valvo, R. Marino et al., (PRL,2007)

The energy transfer rate from Ulysses 1996 data



Conclusions

• An exact law for the scaling of the mixed third order moment in MHD turbulence has been obtained

• Ulysses 1996 high latitude, low solar activity, locally stationary data have been used to observe such exact law.

• A new estimate of the inertial range and a first estimate of the pseudo-energy transfer (and dissipation) rate in solar wind turbulence have been made.