A Few Issues in MHD Turbulence

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* *Introduction*

• Some **examples** of MHD turbulence in astro, geophysics (& engineering)

• Equations, **invariants**, exact laws, phenomenologie**s**

* *Decay versus forced case*

• Is there any difference with fluid turbulence in the decay (unforced) case?

• What are the **features** of such a flow, both spatially and spectrally?

• The generation of magnetic fields (the **dynamo** problem)

* *Discussion*

The need to access higher Reynolds numbers to lead to better scaling *laws*

• Can **modeling** of MHD flows help understand their properties?

• Can **adaptive mesh refinement** help unravel their characteristic features?

* *Conclusion*

References

Classical books:

Paul Roberts ; Moffatt ; Zeldovich ; more recently: Davidson

Also: Parker (astrophysics) ; Priest (solar), …

Reviews (paper copy to be in the library):

Boozer, 2004
Brandenburg & Subramanian, 2004
Pouquet (Les Houches '93; San Miniato '96; Heraklion '96)

Research papers *(pdfs available, some paper copies)*

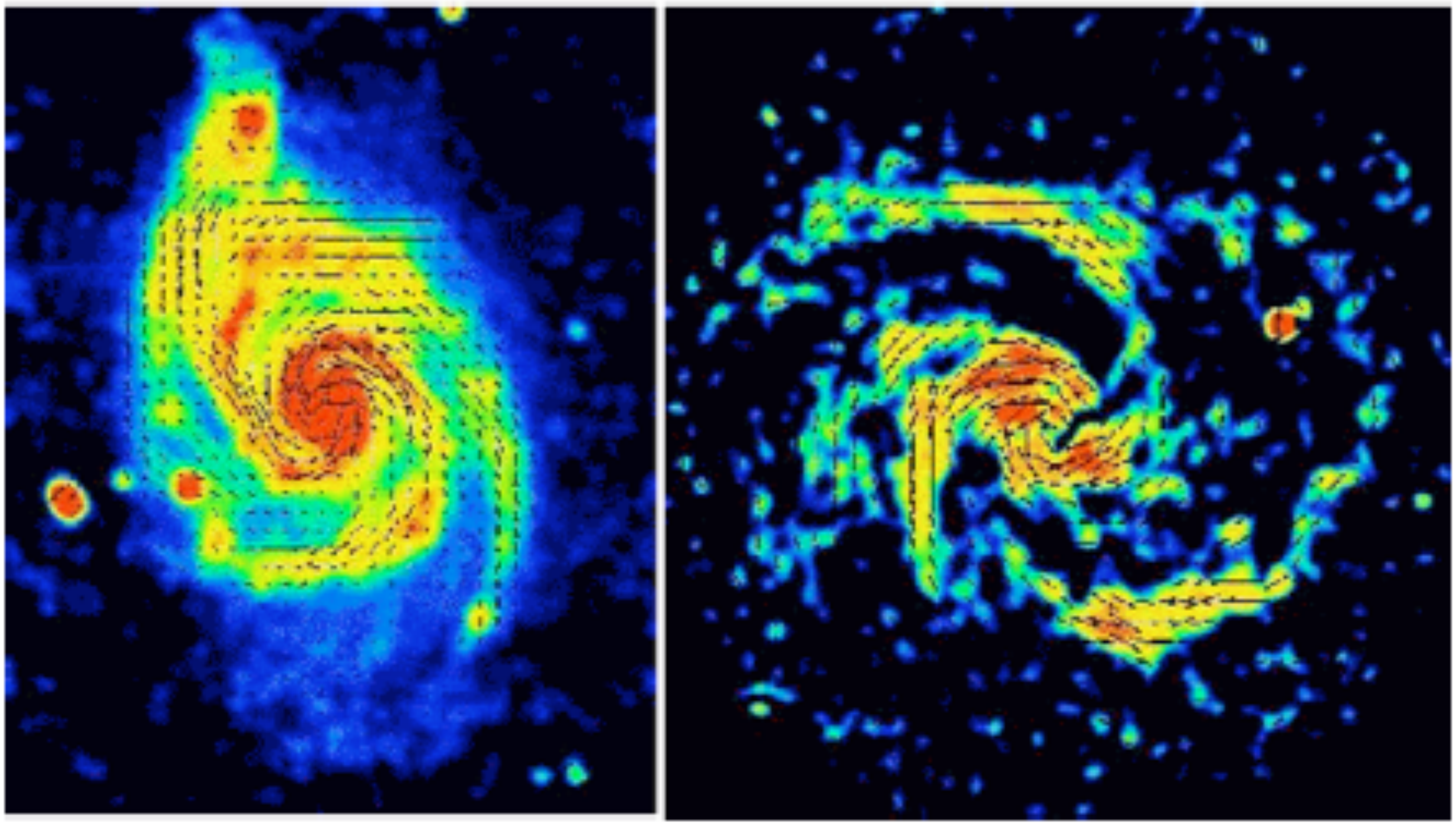
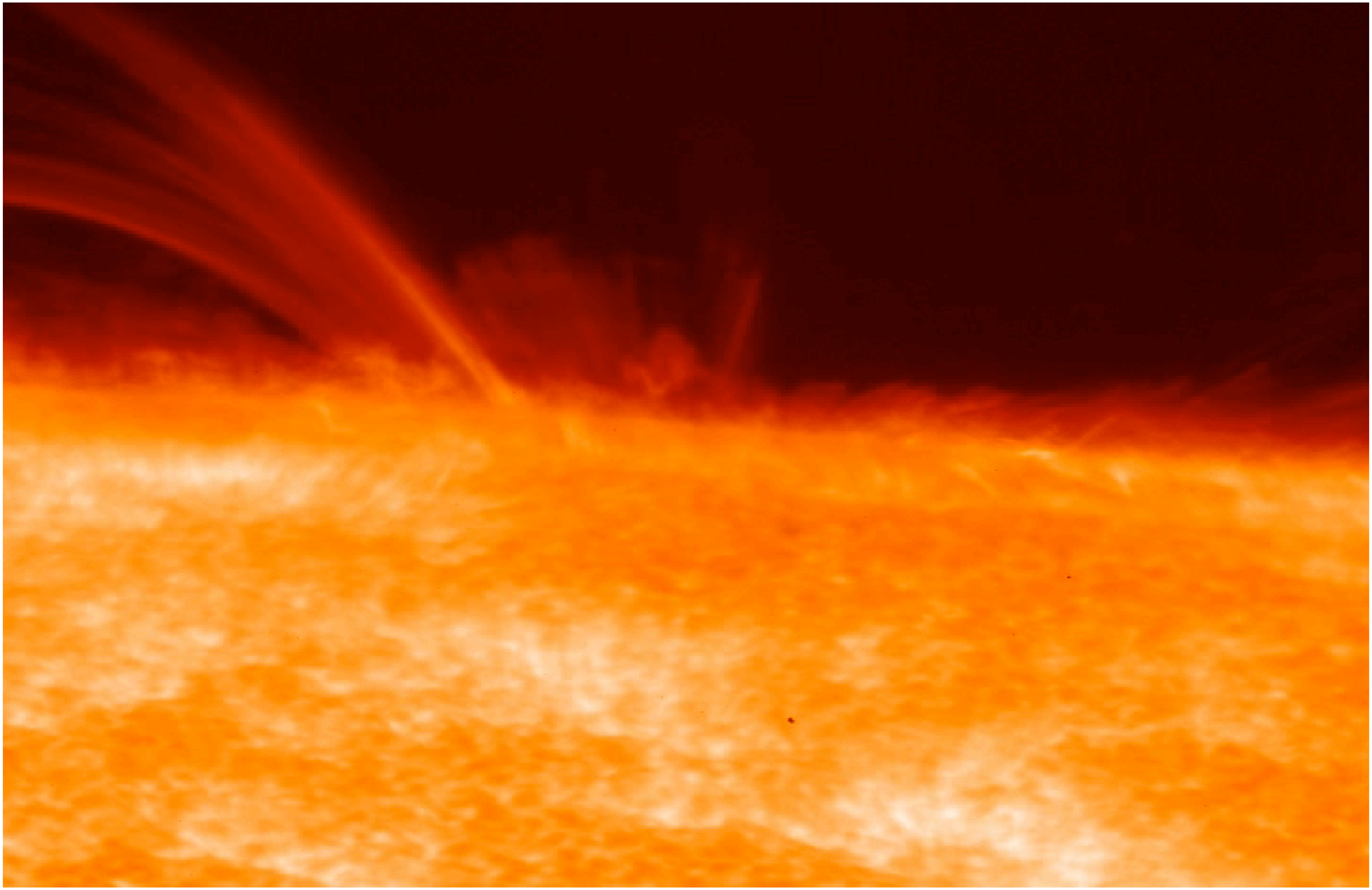


Fig. 2.8. Left: M51 in 6 cm, total intensity with magnetic field vectors. Right: NGC 6946 in 6 cm, polarized intensity with magnetic field vectors. The physical extent of the images is approximately $28 \times 34\,\mathrm{kpc}^2$ for M51 (distance 9.6 Mpc) and $22 \times 22\,\mathrm{kpc}^2$ for NGC 6946 (distance 7 Mpc). (VLA and Effelsberg. Courtesy R. Beck.)

Observations of galactic magnetic fields (after Brandenburg & Subramanian, 2005)

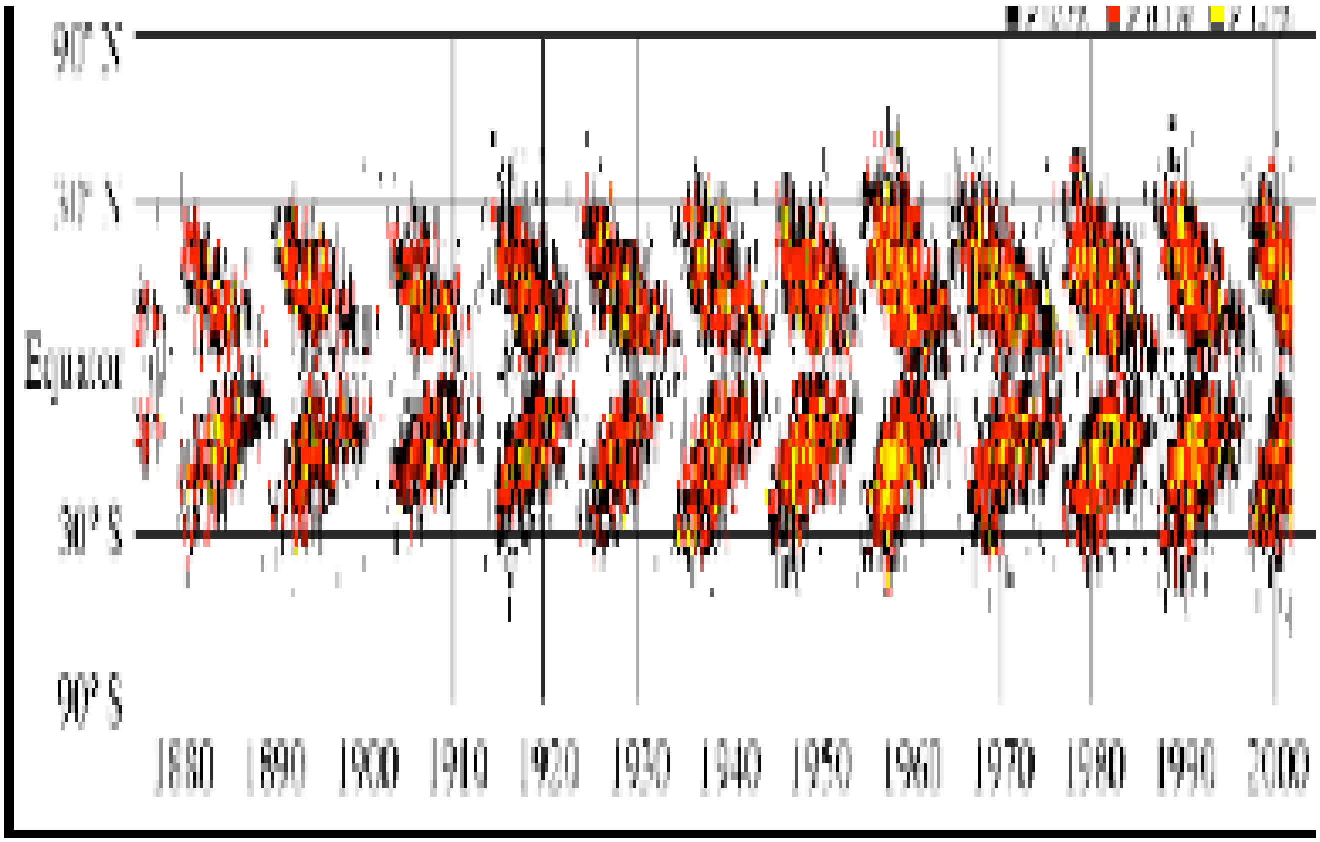
HINODE SOLAR-B telescope

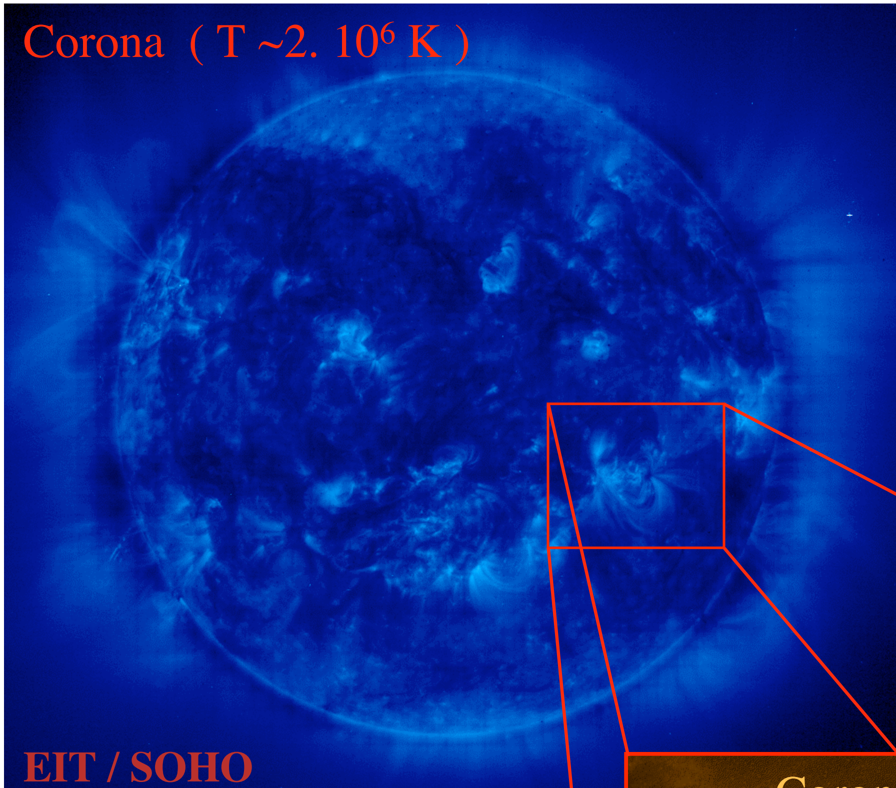
(November 2006)

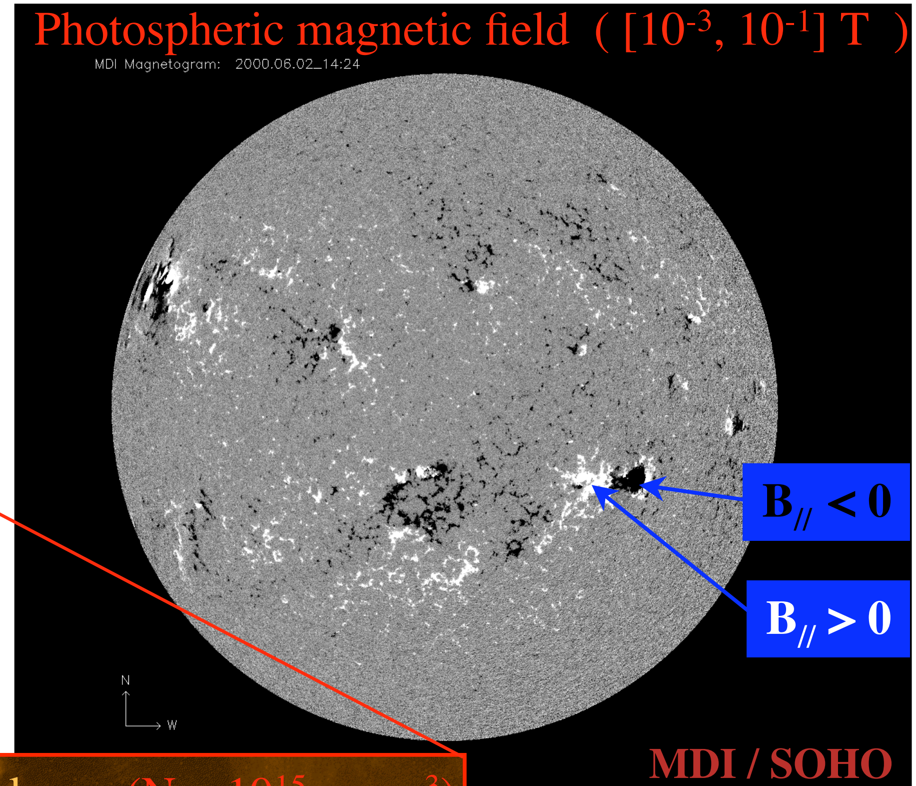


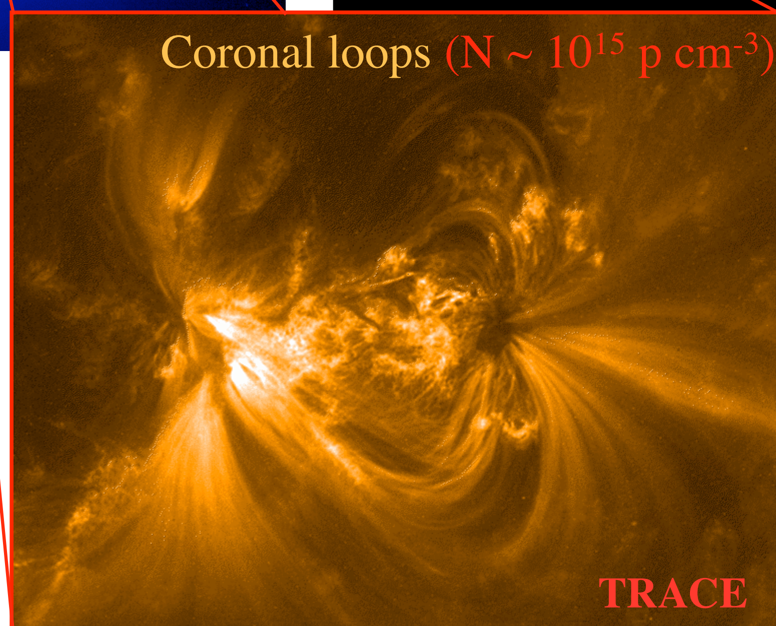
Cyclical reversal of the solar magnetic field over the last 130 years

- Cycle ~ 11 years
- Maunder minimum
- Prediction of next cycle because of long-term memory in the system (Dikpati, 2007)
- Other sun-like stars have a cyclical dynamo as well







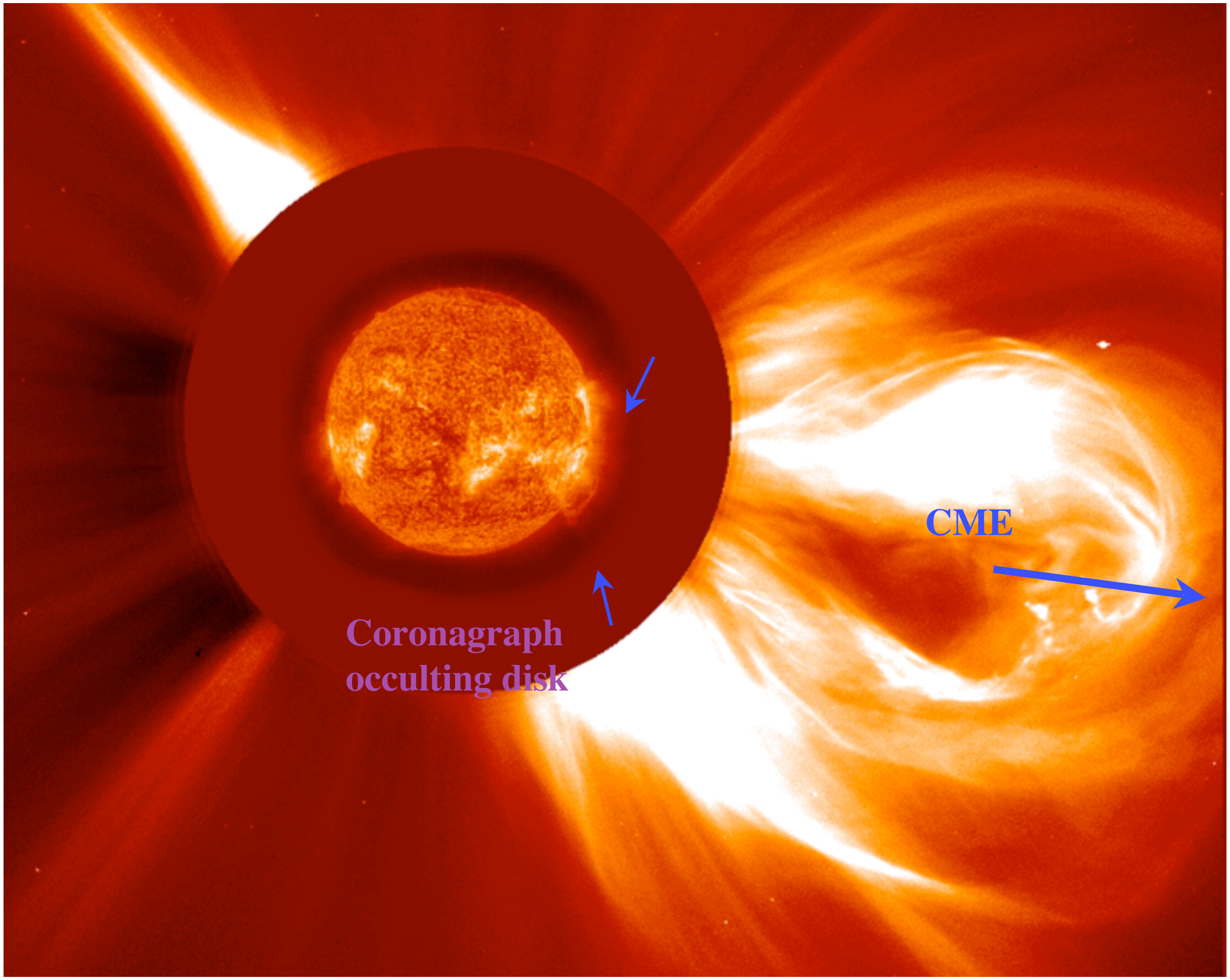


The sun

Slide from P. Démoulin

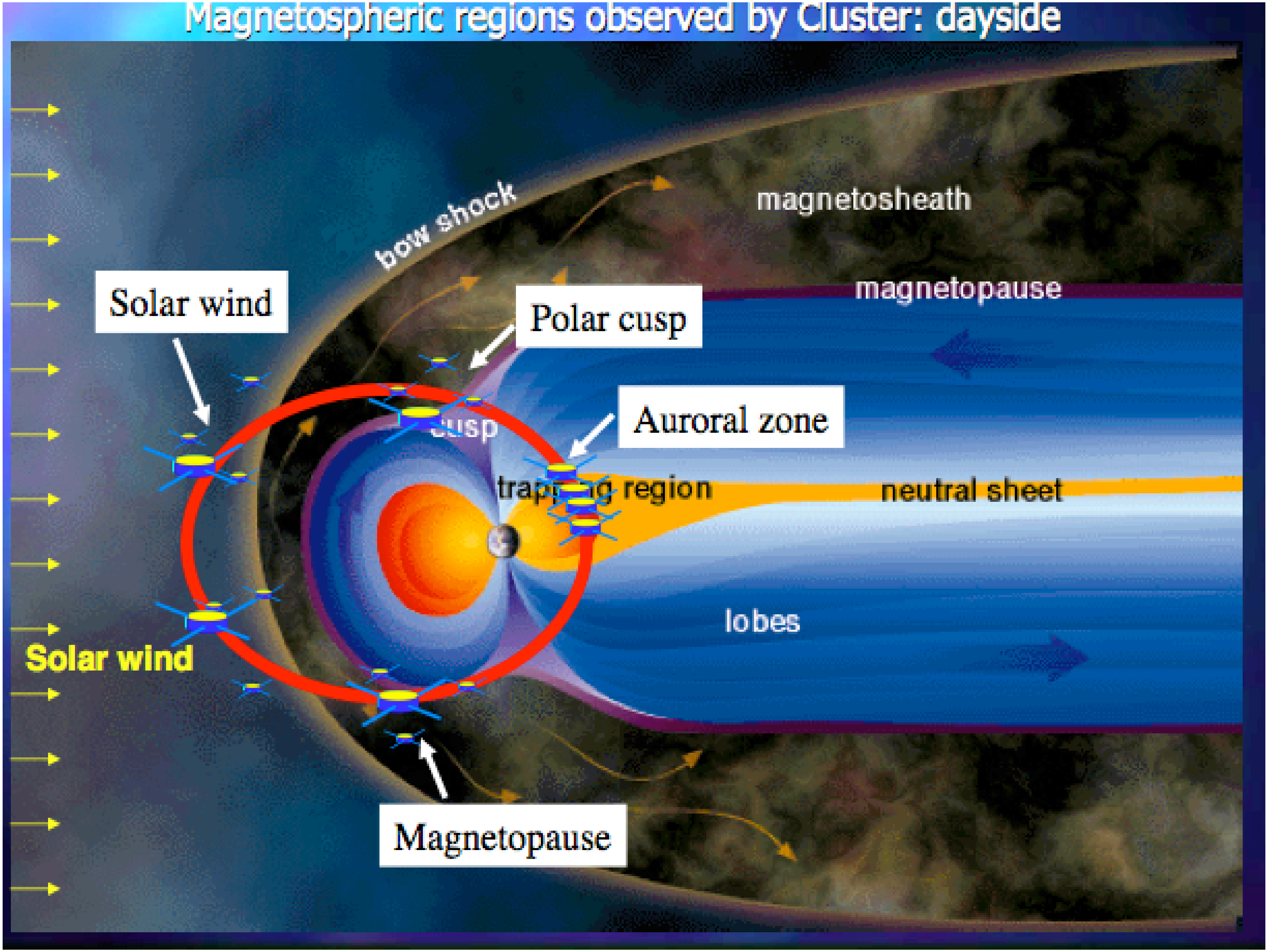
Coronal Mass Ejection (CME)

Destabilization & launch of a coronal magnetic structure in the interplanetary space



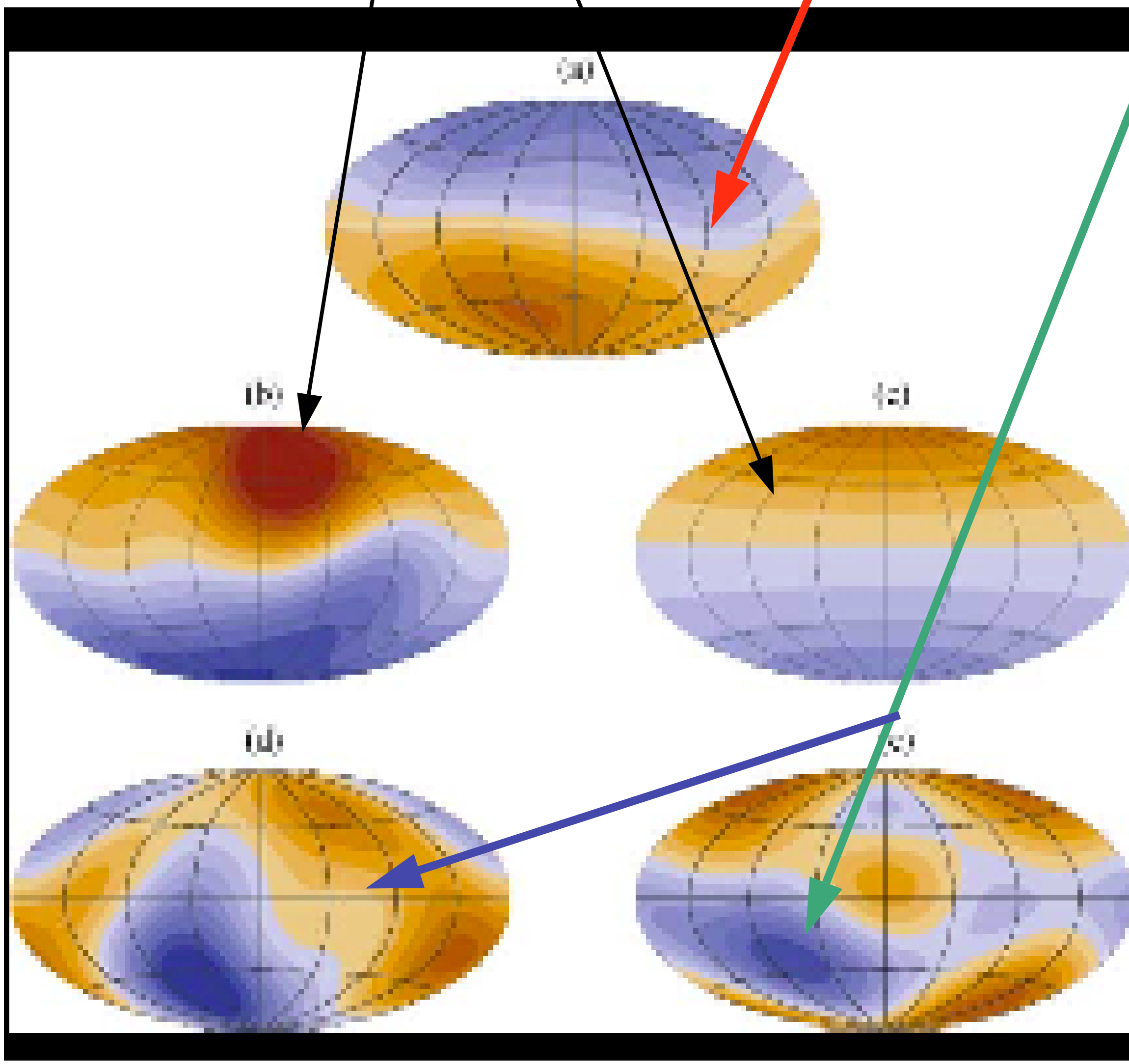
**EIT,
LASCO/
SOHO
5 dec. 2003**

Slide from P. Démoulin


Magnetospheric regions observed by Cluster: dayside
bow shock
magnetosheath
magnetopause
Solar wind
Polar cusp
Auroral zone
cusp
trapping region
neutral sheet
lobes
Solar wind
Magnetopause

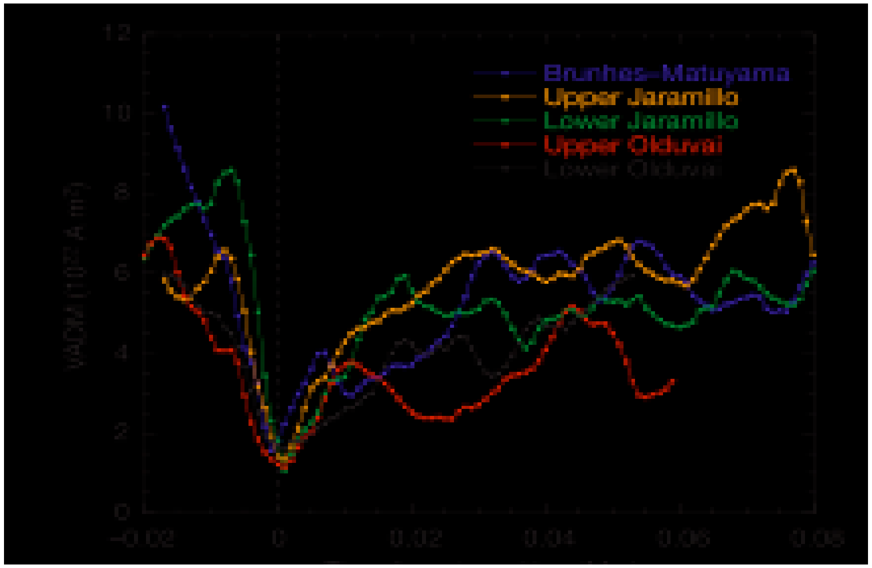
Surface (1 bar) radial magnetic fields for
Jupiter, **Saturne** & **Earth**, **Uranus** & **Neptune**

(*16-degree truncation, Sabine Stanley, 2006)*



Axially dipolar

Quadrupole ~ dipole

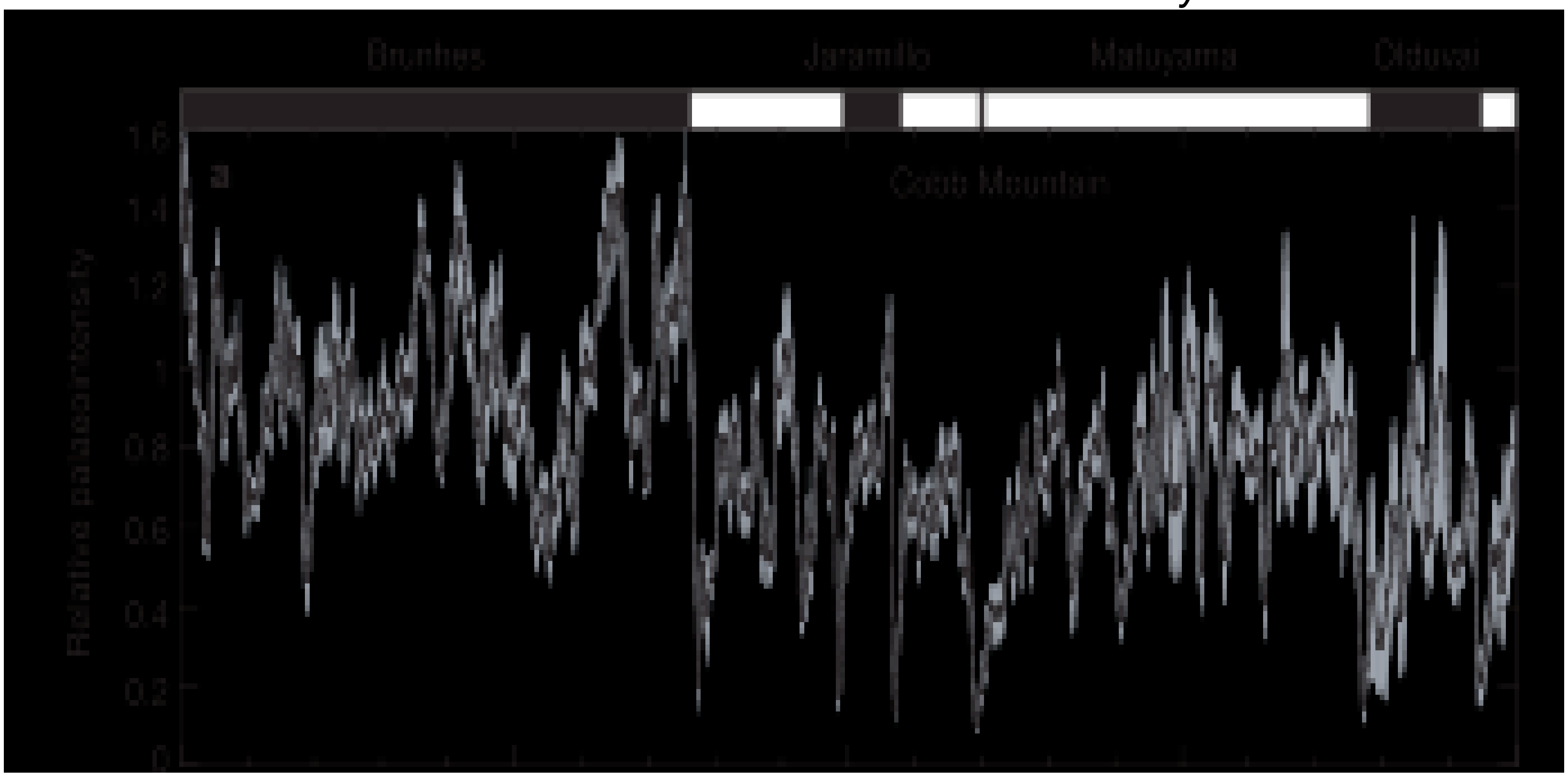


Reversal of the Earth’s magnetic field over the last 2Myrs

(Valet, Nature, 2005)

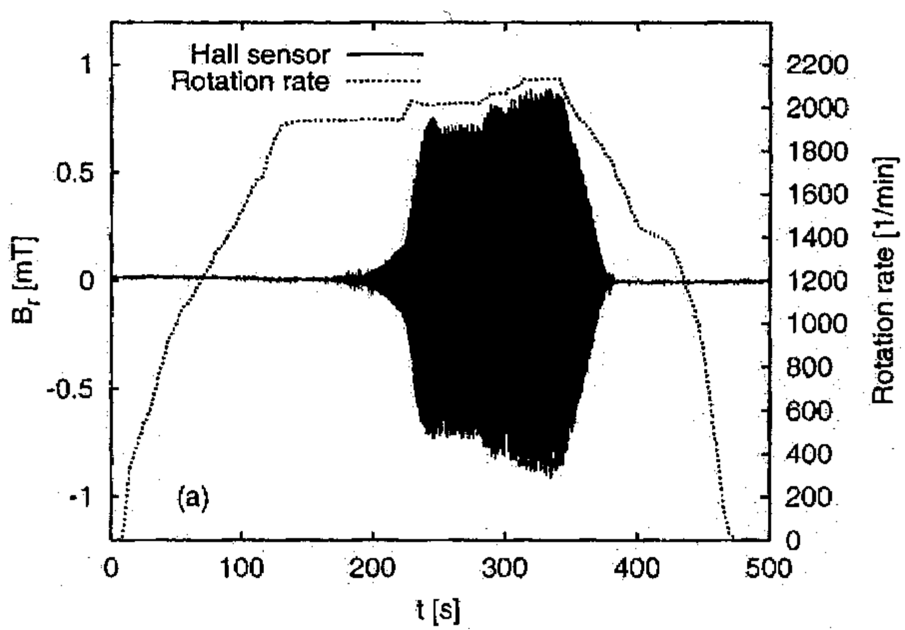
Temporal assymmetry of reversal process

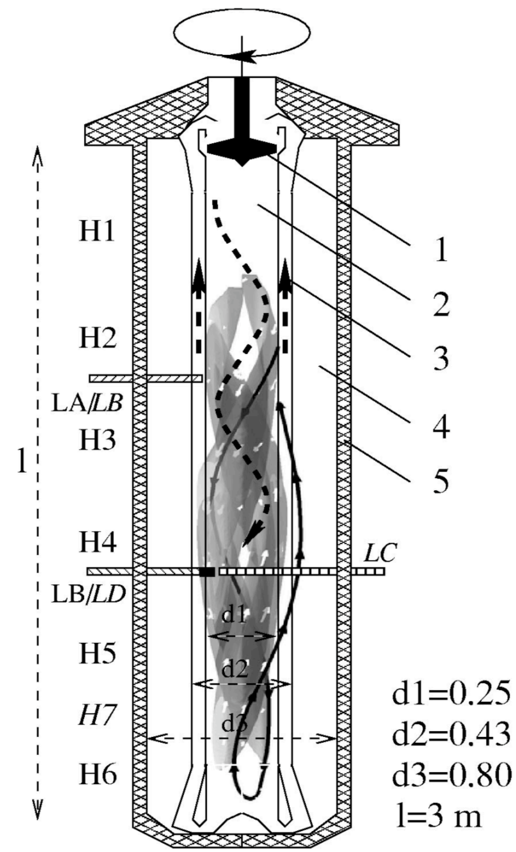
Brunhes *Jamarillo* *Matuyama* *Olduvai*



Experimental dynamo with a constrained flow: Riga

Gailitis et al., *PRL* **84** (2000)

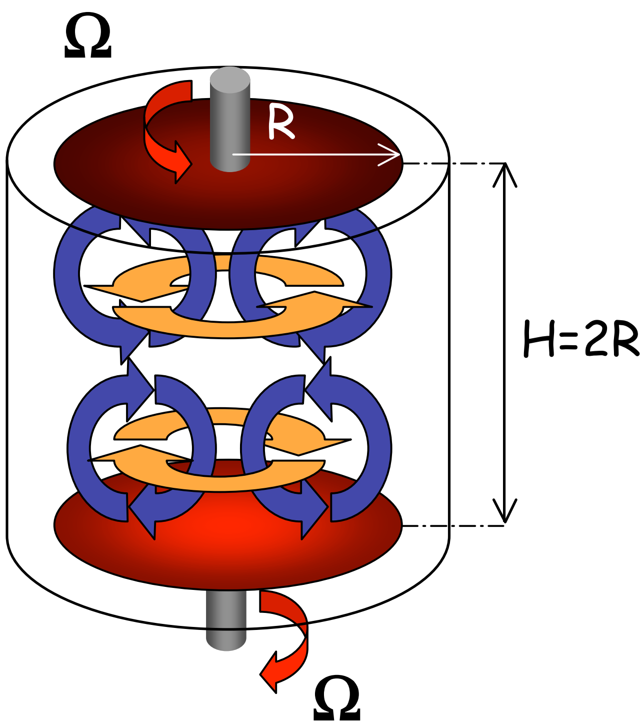


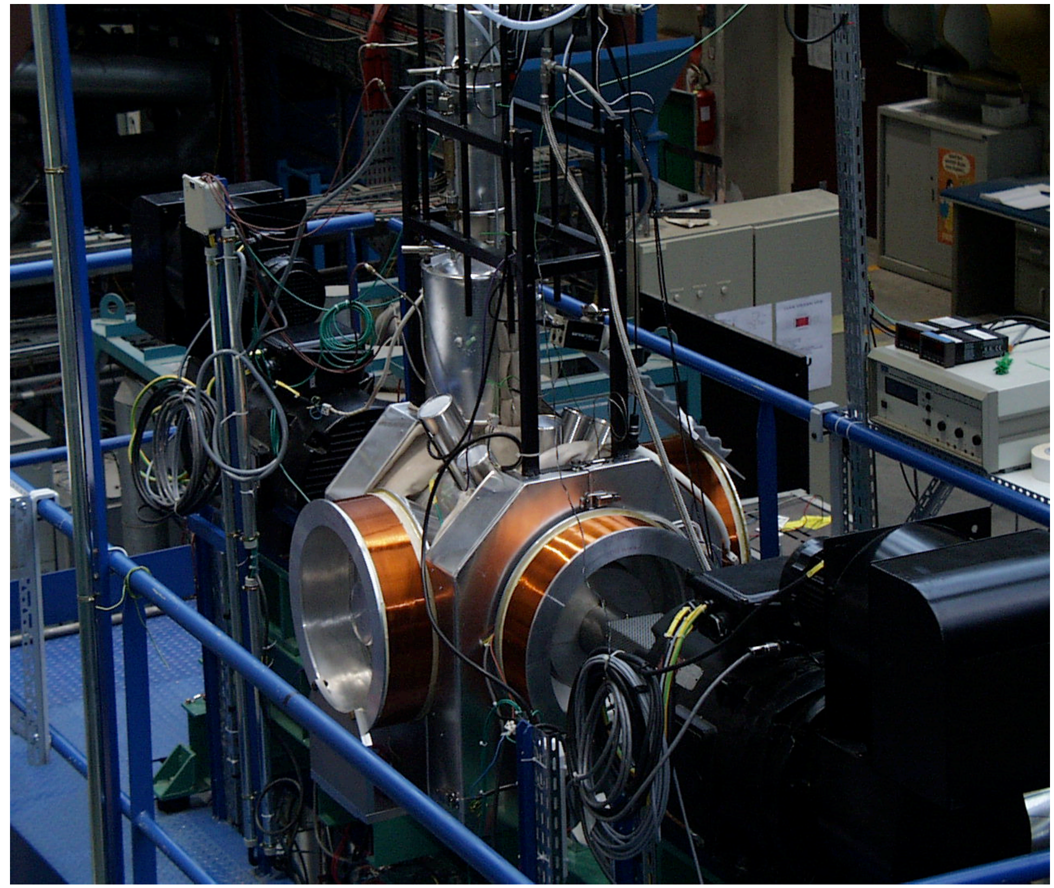


Also: Karlsruhe, with a Roberts flow (see special issue Magnetohydrodynamics **38**, 2002)

Experimental dynamo at Cadarache with the Taylor-Green (TG) **turbulent** flow

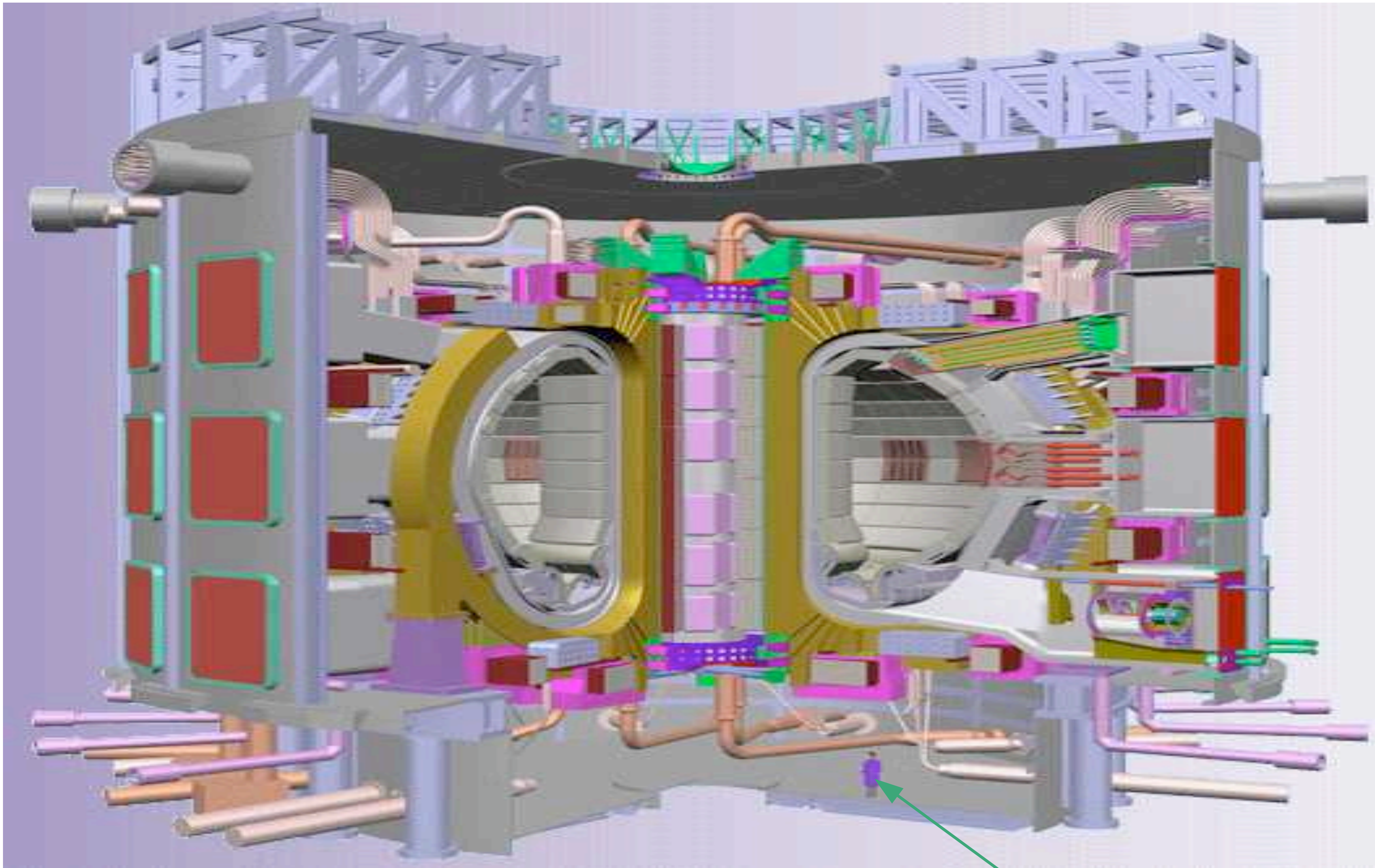
*Bourgoin et al PoF **14** ('02), **16** ('04)…*





Numerical computation at a magnetic Prandtl number $P_M=1$ *(Nore et al., PoP, **4**, 1997)* leads to a dynamo, but $P_M \sim 10^{-6}$ in liquid sodium: does it matter?

Experimental dynamo in 2007!



ITER *(Cadarache)*

The MHD equations [1]

- P is the pressure, **j** = ∇ × **B** is the current, **F** is an external force, ν is the viscosity, η the resistivity, **v** the velocity and **B** the induction (in Alfvén velocity units); incompressibility is assumed, and ∇.B = 0.
  Finally, **B** (like **ω**) is an axial vector.

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \mathcal{P} + \underline{\mathbf{j} \times \mathbf{B}} + \nu \nabla^2 \mathbf{v} + \mathbf{F}$$

Lorentz force

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B} \ ,$$

*Maxwell's equations with **v** << c (no displacement current)*

The MHD equations [2]

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \mathcal{P} + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{v} + \mathbf{F}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B} \ ,$$

Batchelor analogy B → $\omega = \nabla \times v$:

Stretching of magnetic field lines by velocity gradients, and growth of B^2 (generation of magnetic fields or dynamo problem) in the kinematic (linear) regime (velocity given, neglecting the Lorentz force for the case of weak B fields)

The MHD equations [3]

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \mathcal{P} + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{v} + \mathbf{F}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B} \ ,$$

Elsässer variables: $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{B}$, $2\nu^{\pm} = \nu \pm \eta$

→ $\partial_t \mathbf{z}^{\pm} + \mathbf{z}^{-/+} . \nabla \mathbf{z}^{\pm} = - \nabla P + \nu^{\pm} \Delta \mathbf{z}^{\pm} + \mathbf{F}$

Obvious exact solutions: $\mathbf{z}^{\pm} = 0$

The MHD equations [again and last]

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \mathcal{P} + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{v} + \mathbf{F}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B} \ ,$$

At smaller scales, other terms have to be added in a generalized Ohm's law: the ambipolar drift at low ionisation (as in the interstellar medium), the Hall current in highly ionized plasmas (as in the Solar Wind below the ion skin depth), an anisotropic pressure gradient, …

Is MHD a good limit for the large scale properties of all such flows?

There are analytically derived or numerically found differences, e.g. in the (alpha) dynamo (Zweibel, 1998; Mininni et al., 2005), or for the reconnection problem of magnetic field lines, but …

Governing Parameters in MHD

- $R_V = U_{rms} L_0 / \nu >> 1$

  --> extended inertial range for the velocity field

- Magnetic Reynolds number

$R_M = U_{rms} L_0 / \eta$

* Magnetic Prandtl number: $P_M = R_M / R_V = \nu / \eta$

P_M is high in the interstellar medium;

P_M is low in the liquid core of the Earth, in liquid metals, in laboratory experiments, in the solar convection zone.

* Other: E_M/E_V ? B_0 ?

The MHD invariants ($\nu = \eta = 0$)

* Energy: **E^T**$=1/2< v^2 + B^2 >$

or $E^{\pm} = 1/2< (z^{\pm})^2 >$

* Cross helicity: **H^C**$= < v.B >$

And:

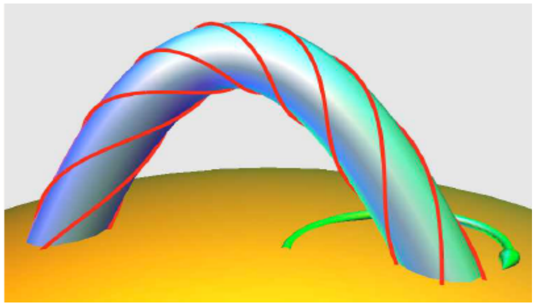
* 3D: Magnetic helicity: **H^M**$=< A.B >$ with $B= \nabla \times A$ *(Woltjer, mid '50s)*

* 2D: $E^A= < A^2 >$ *(+)* *[A: magnetic potential]*

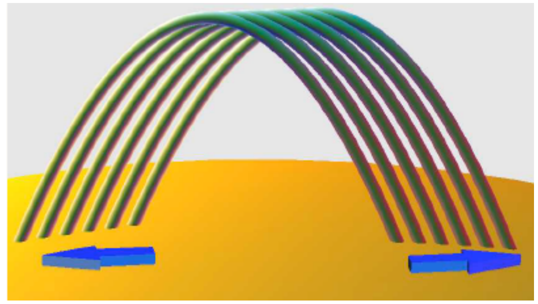
Also:

Alfvén theorem for magnetic flux conservation
(analogous to the theorem of Kelvin)

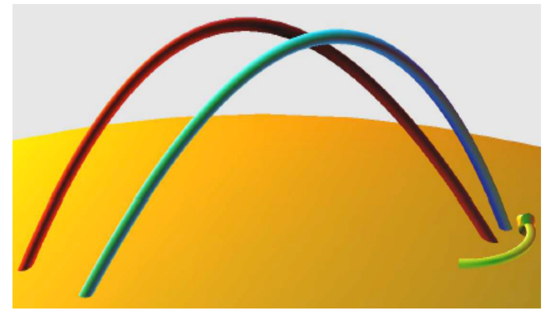
What does magnetic helicity represent?



Twisted flux tube



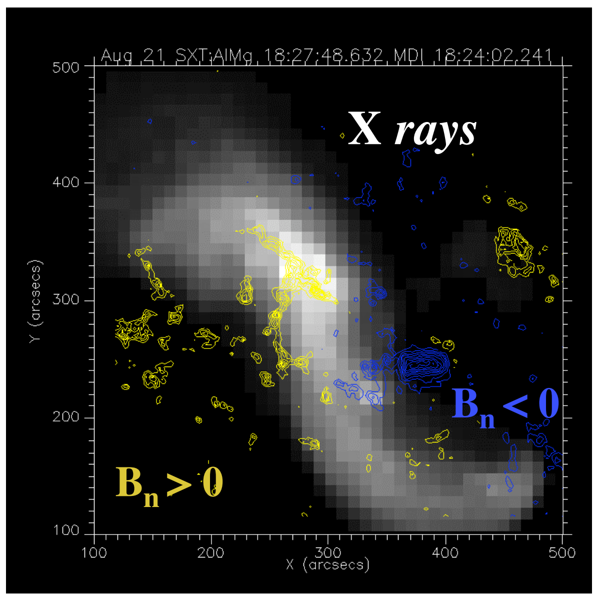
Sheared arcade

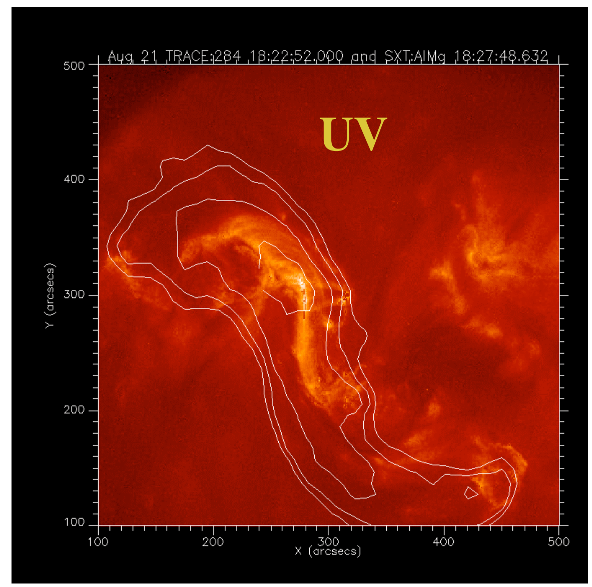


Braided flux tubes

In the corona:
e.g. sigmoids

H^M is present in
any non potential
magnetic configuration





X *rays* & UV emissions : trace of field lines

Slide: P. Démoulin

The MHD invariants [2]

1) **Direct** cascade to small scales, in two and in three space dimensions:

* Energy $E^T = 1/2 < v^2 + B^2 >$, cross helicity $H^C = < v.B >$

or $E^{\pm} = 1/2 < (z^{\pm})^2 >$

2) ***Inverse*** *cascade to large scales:*

** 3D: Magnetic helicity:* $H^M = < A.B >$ *with* $B = \nabla \times A$

** 2D:* $E^A = < A^2 >$ *[A: magnetic potential]*

Evidence for both direct and inverse cascades:

^ Statistical mechanics (truncated ensemble equilibria)

^ Closure models (e.g., the Eddy Damped Quasi-Normal Markovian, or EDQNM)

^ Numerical simulations

Exact laws in MHD [1]

à la Yaglom (1949), and Antonia et al. (1997)

$\delta F(r) = F(x+r) - F(x)$: structure function for field F ;
longitudinal component $\delta F_L(r)$

$$< \delta z^{-/+}_L \Sigma_i \delta z_i^{\pm\ 2} > = -[4/d]\ \varepsilon^{\pm}\ r$$

in dimension d, *with* $\varepsilon^{\pm} = - d_t E^{\pm} = \varepsilon^T \pm \varepsilon^C$, *and omitting dissipation and forcing*
(Politano and Pouquet, 1998, PRE 57 & GRL 25)

Note: $z^+ z^+ z^- \sim (v+B)^2 (v-B) \sim \mathcal{E}^+$ *(linked to an observed lack of equipartition between kinetic and magnetic energy)*

Other exact laws for the H^M and E^A MHD invariants *Gomez et al., 2003, PRE 68*
P. Caillol, DEA (Nice and Paris)

Exact laws in MHD [2]

In terms of V and B, we have:

$$< \delta v_L \delta v_i^2 > + < \delta v_L \delta b_i^2 > -2 < \delta b_L \delta v_i \delta b_i > \quad = - (4/d)\, \varepsilon^T r$$

$$- < \delta b_L \delta b_i^2 > - < \delta b_L \delta v_i^2 > +2 < \delta v_L \delta v_i \delta b_i > \quad = - (4/d)\, \varepsilon^c r$$

with $\varepsilon^T = - d_t E^T$ and $\varepsilon^c = - d_t H^c$

* v-dominated regime, vs. B-dominated regime vs. Alfvénic (v~B) regime?

(cf. Ting et al 1986)

* ***Dynamical role of the correlation*** *between the velocity and the magnetic field in the mixed regime (GRL 25, 1998; also Boldyrev, 2006).*

Exact law for kinetic helicity

Batchelor analogy B $\longrightarrow$ ω

$H^v = < \mathbf{v} . \boldsymbol{\omega} >$

$$< \delta v_L \, \delta v_i \, \delta\omega_i > - (1/2) < \delta\omega_L \, \delta v_i^2 > = - (4/3) \, \varepsilon^h \, r$$

with $\varepsilon^h = - d_t H^v$

Hence, a dynamical role for the correlations between the velocity and the vorticity

Von Karman equation for kinetic helicity: Chkhetiani (JETP 63, 1996)

Exact law: Gomez et al., PRE 61, 2000

Theoretical approaches in MHD

- Linearisation around a strong uniform magnetic field $\mathbf{B}_0$: Alfvén waves in the incompressible case (~ 1950).
- Weak MHD turbulence [WT] (*Galtier et al., 2000*) : 3-wave interactions, leading to exact $k_\perp^{-2}$ spectrum.
- Statistical equilibria of truncated non-dissipative systems: prediction of an inverse cascade for magnetic helicity in 3D
- Fully developed turbulence: closure models for MHD turbulence (DIA, TFM, EDQNM) *(Kraichnan, '50s and beyond):*

→ Computation of transport coefficients *(as well as with WT)*

e.g. saturation of the nonlinear dynamo, through a combination of Alfvén waves equilibration and the inverse cascade of magnetic helicity, and use as LES (Baerenzung et al, 2007)

Other approaches (~ 1980): shell models → intermittency.

Phenomenology of MHD turbulence [1]

- Is MHD like fluids? → Kolmogorov spectrum: $\mathbf{E_{K41}(k) \sim k^{-5/3}}$

Or

- Slowing-down of energy transfer to small scales because of Alfvén waves propagation along a (quasi)-uniform field B_0: → $\mathbf{E_{IK}(k) \sim k^{-3/2}}$

  (Iroshnikov - Kraichnan (IK), mid '60s)

  $\tau_{transfer} \sim \tau_{NL} * [\tau_{NL}/\tau_A]$, *or 3-wave interactions but still with isotropy.*
  Eddy turn-over time $\tau_{NL} \sim l/u_l$ *and wave (Alfvén) time* $\tau_A \sim l/B_0$

- Weak turbulence theory *(Galtier et al PoP 2000)*: anisotropy develops and the exact spectrum is: $\mathbf{E_{WT}(k) \sim k_\perp^{-2} f(k_{//})}$

IK -compatible when isotropy is assumed: $\tau_{NL} \sim l_\perp/u_l$ *and* $\tau_A \sim l_{//}/B_0$, $f(\mathbf{k}_{//}) = k_{//}^{1/2}$ & $k_{//} \sim k_\perp$

Or $\mathbf{k_\perp^{-5/3}}$ *(Goldreich Sridhar, APJ 1995)*? Or $\mathbf{k_\perp^{-3/2}}$ *(Nakayama, 2001; Boldyrev, PRL 2006)*?

Phenomenology of MHD turbulence [2]

- $E_{K41}(k) \sim k^{-5/3}$ as observed in the Solar Wind (SW) and in DNS

- $E_{IK}(k) \sim k^{-3/2}$ as observed in SW, in DNS, and in several closure models
  e.g. Podesta et al APJ (2007), Mason et al arXiv (2007), Yoshida (2007)

- $E_{WT}(k) \sim k_{\perp}^{-2}$ as may have been observed in the Jovian magnetosphere, and in a recent DNS on a grid of 1536^3 points *(more on that later)*

- **Is it a lack of universality of MHD turbulence? If so, what are the parameters that govern the (plausible) classes of universality?**
  The presence of a strong guiding uniform magnetic field may be one.

*** Or is it a lack of resolving power?**

*** Or is an energy spectrum the right way to analyze / understand MHD?**

Recent results using direct numerical simulations and models of MHD

Recent results using direct numerical simulations and models of MHD

I. High-res decay run (no forcing)

- Temporal evolution of maximum of current and vorticity
- Roll-up of current sheets
- Alignment of fields in small-scale structures
- Energy dissipation and scaling laws
- Energy spectra and anisotropy
- Intermittency
- Energy transfer and non-local interactions in Fourier space

II. The dynamo at low magnetic Prandtl number

- The validity of the Lagrangian-averaged (alpha) model
- Combining three approaches
- Is Adaptive Mesh Refinement useful?

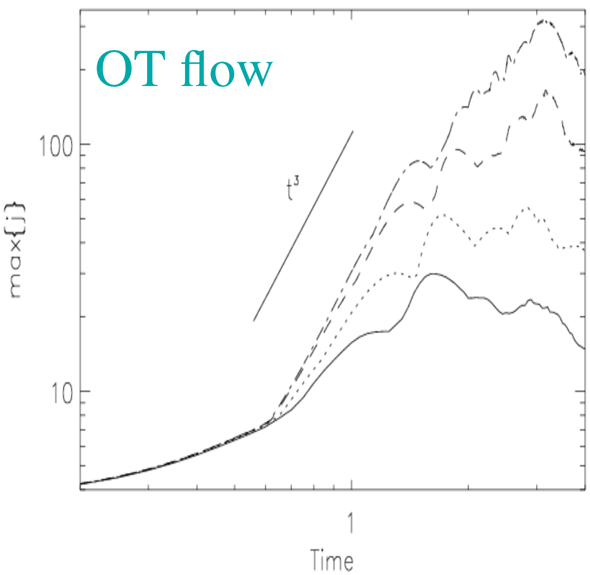
Numerical set-up

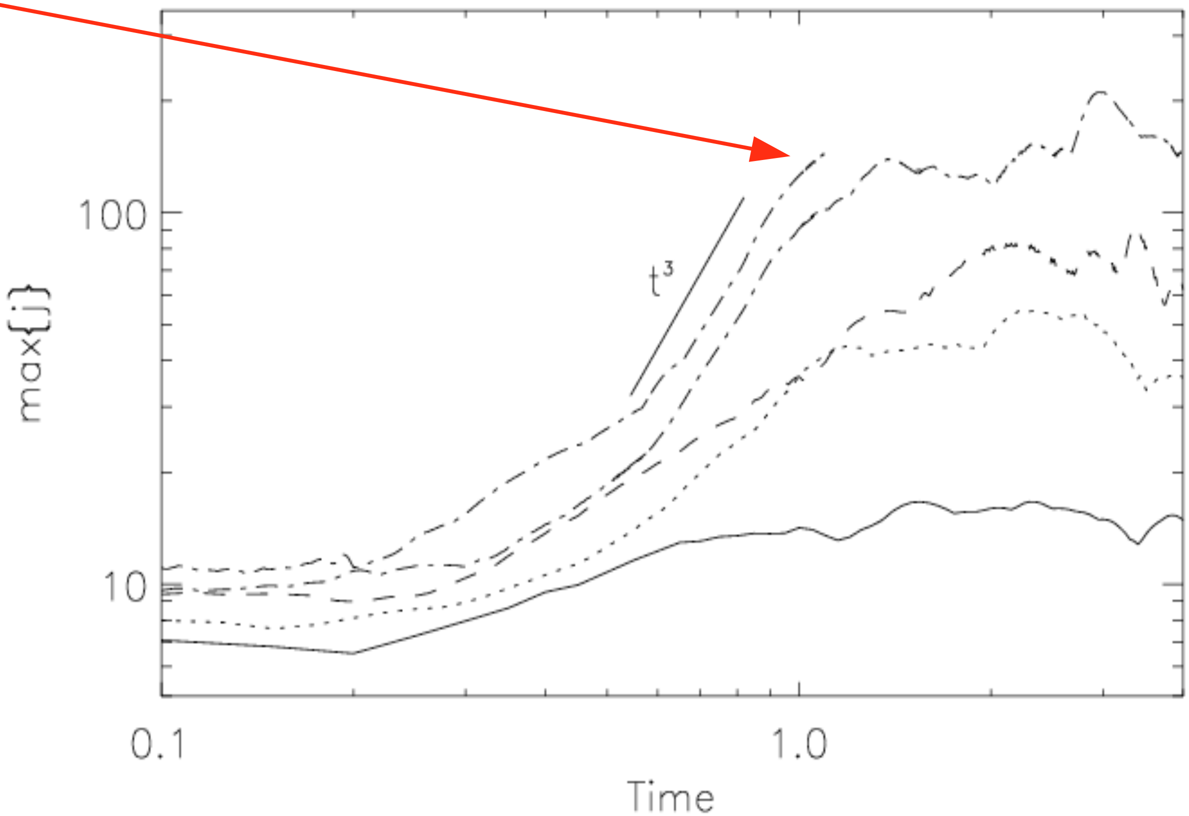
- Periodic boundary conditions, pseudo-spectral code; from 64^3 to 1536^3 points, de-aliased with the 2/3 rule

- No uniform magnetic field imposed
- Decay run (F=0) , or forcing at k_F~3 with small initial magnetic field (dynamo problem)

^ Orsag-Tang configuration for reconnection
^ ABC flow: Beltrami (helical) + random noise at small scale
^ Taylor-Green configuration: no global helicity

J_{max} for a **random flow**, resolutions up to 1536^3 grid points (R_V from 690 to 10100)





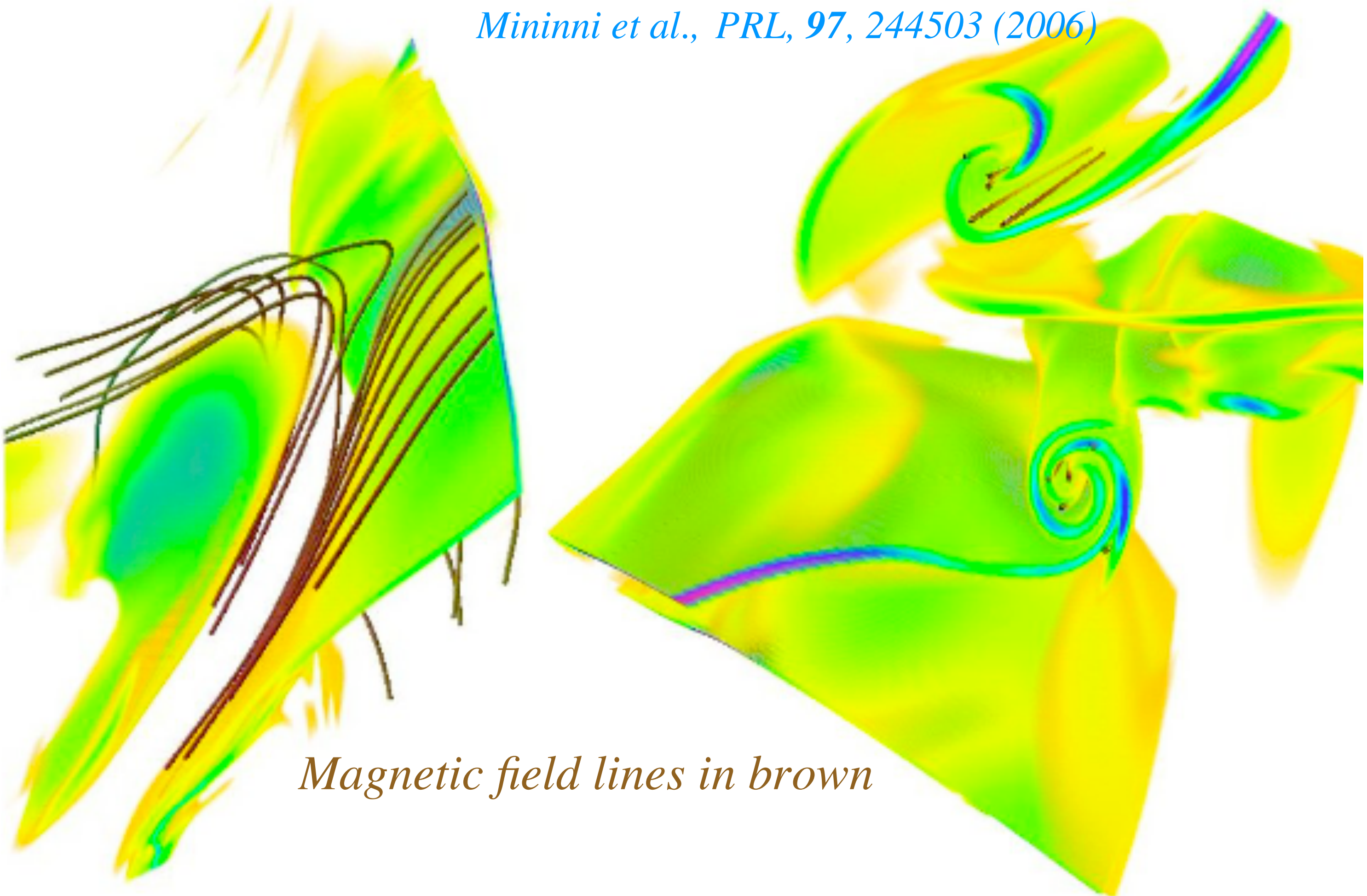
- Linear phase followed by t^3 growth of the current maximum

MHD decay simulation @ NCAR on **1536³** points

Visualization freeware: VAPOR http://www.cisl.ucar.edu/hss/dasg/software/vapor

Zoom on individual current structures: folding and roll-up

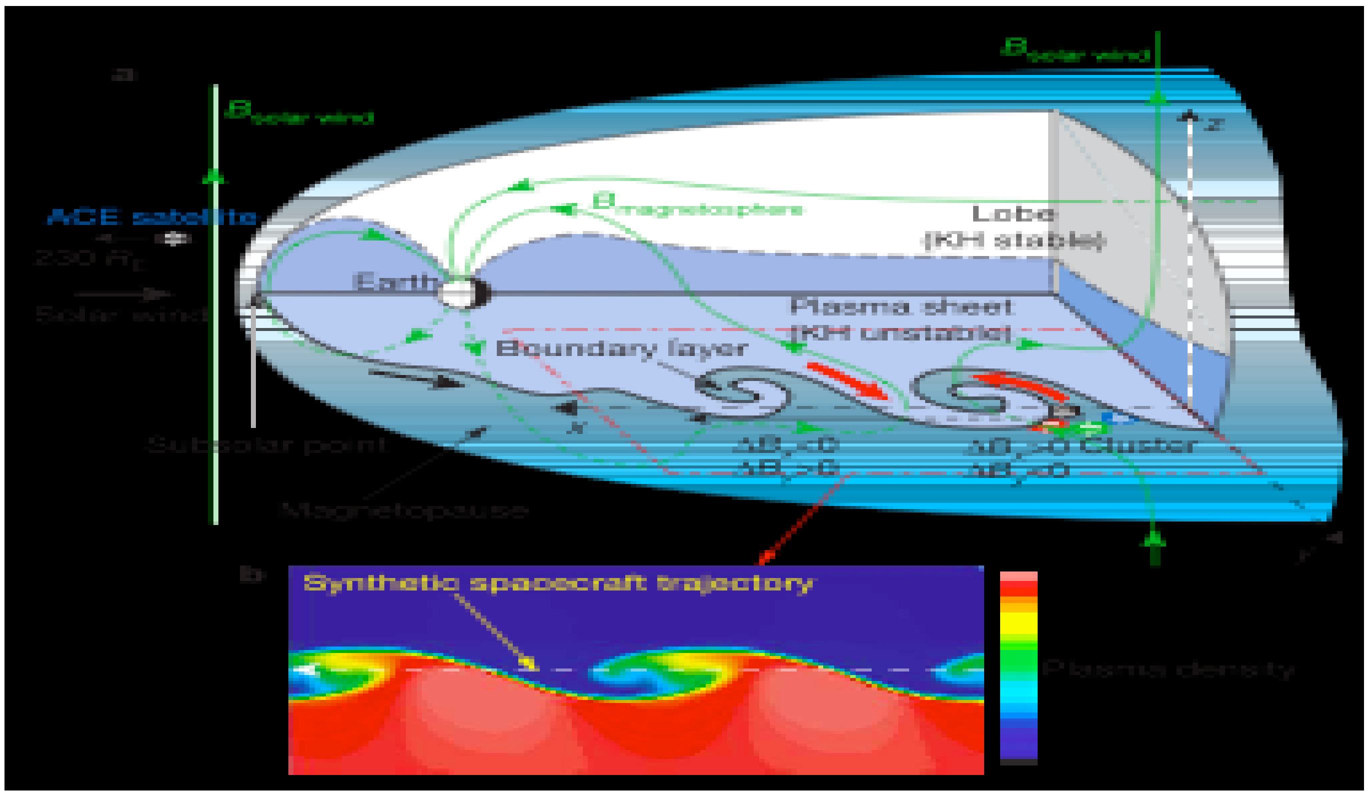
*Mininni et al., PRL, **97**, 244503 (2006)*



Magnetic field lines in brown

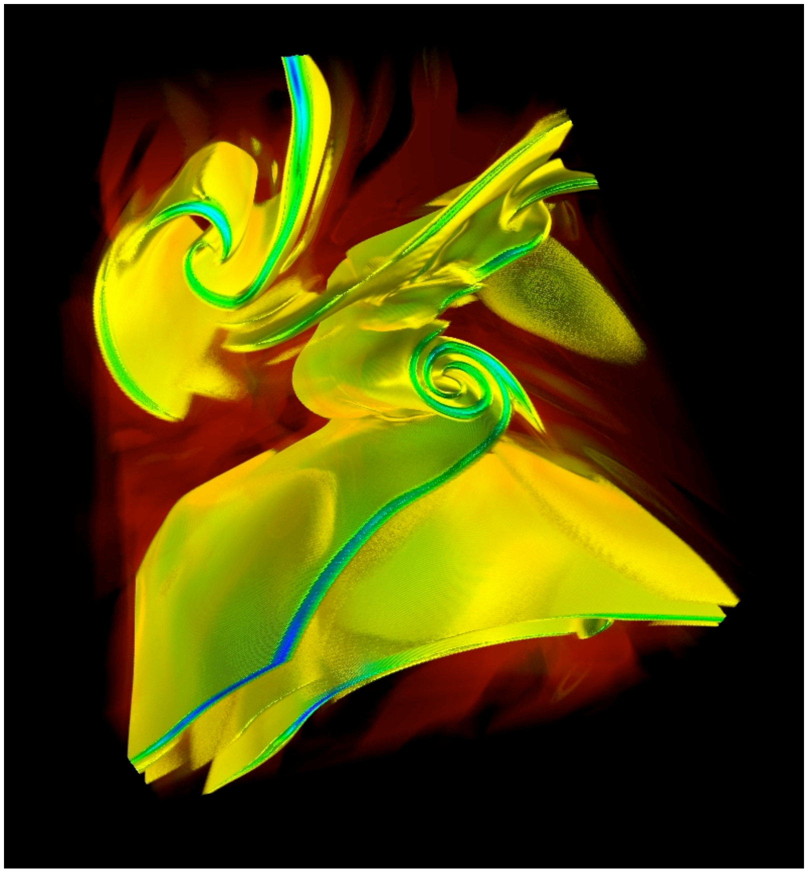
Kelvin-Helmoltz observed in the magnetosphere with Cluster.

Recent observations (and computations as well) of Kelvin-Helmoltz roll-up of current sheets

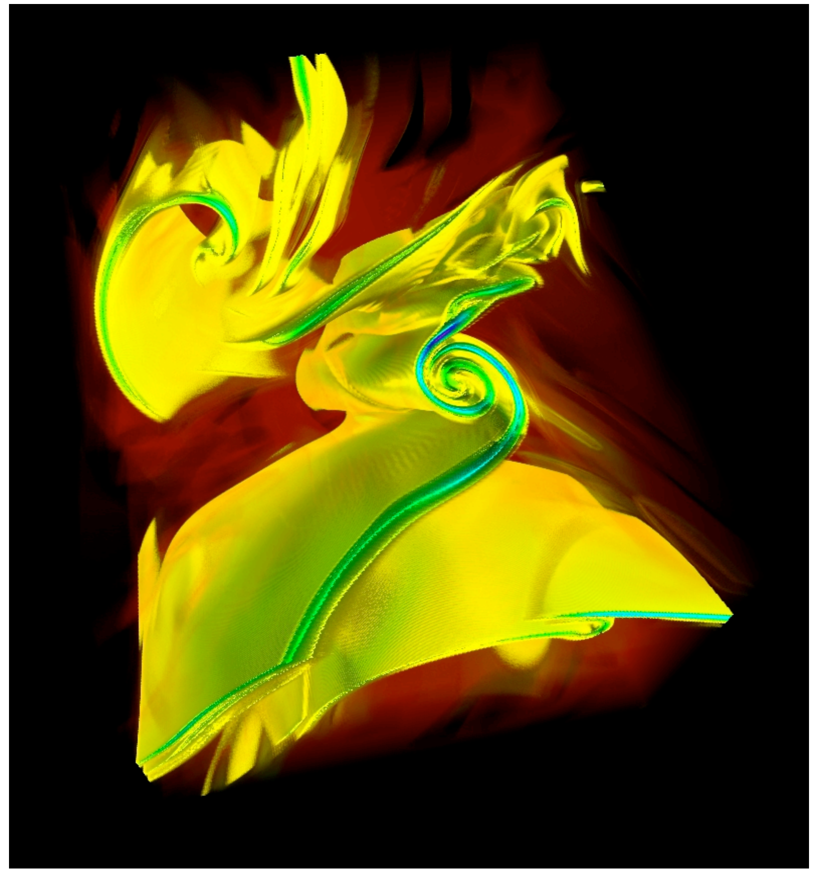


Hasegawa et al., Nature (2004); Phan et al., Nature (2006), …

Current and vorticity are correlated in the rolled-up sheet



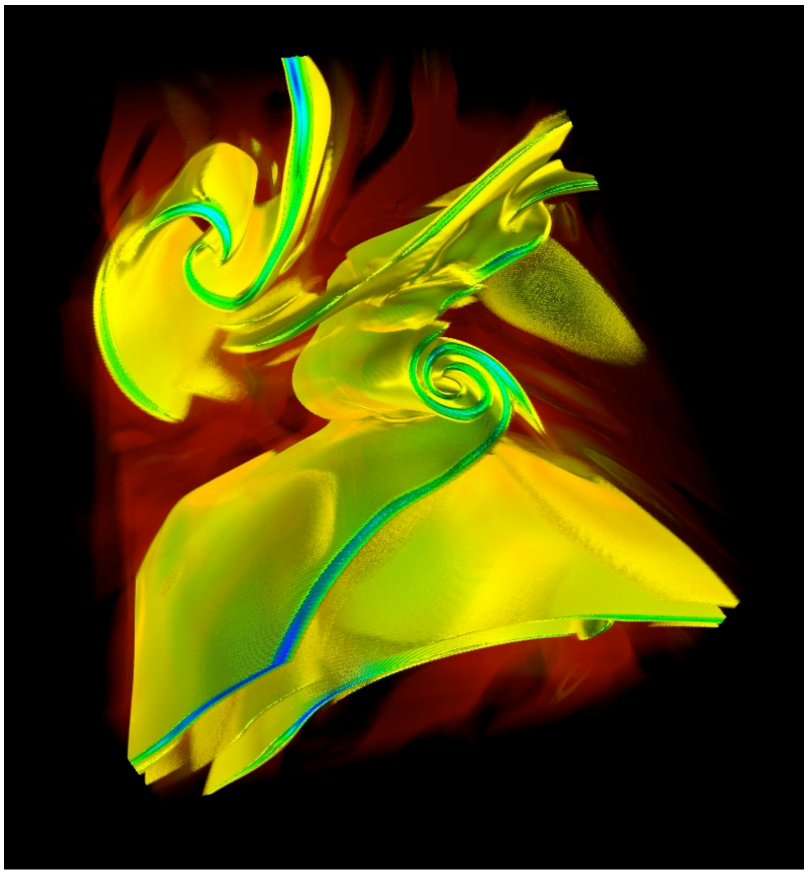
Current J^2



Vorticity ω^2

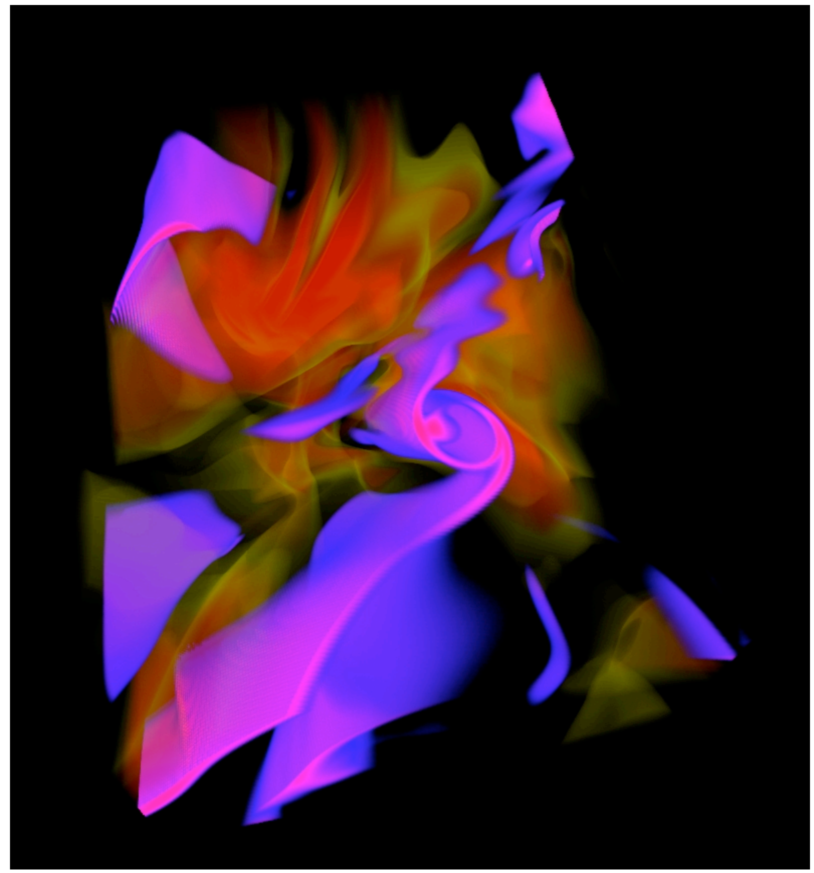
1536^3 run, early time

V and B are aligned in the rolled-up sheet, *but not equal (B^2 ~$2V^2$)*



Current J^2

1536^3 run, early time

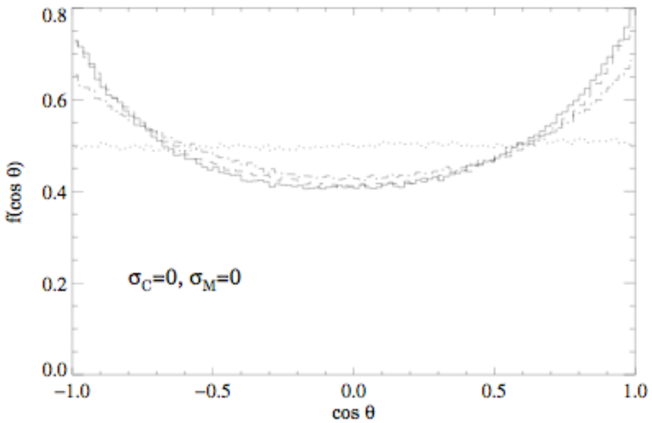


cos(**V**, **B**)

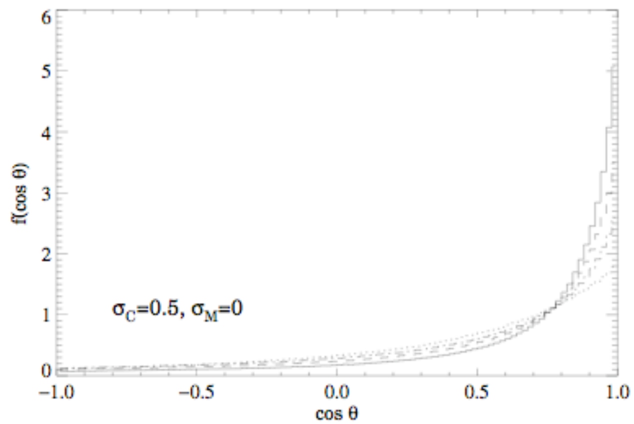
H_c : Velocity - magnetic field correlation

PdFs of cos(v,B):

- Flow with weak normalized total cross helicity H_c



- Flow with strong H_c



Matthaeus et al. arXiv.org/abs/0708.0801

Velocity - magnetic field correlation [3]

- Local map in 2D of **v** & **B** alignment:

  |*cos* (**v**,**B**)| > 0.7 (black/white)

  (otherwise, grey regions).

  Note that the global normalized correlation coefficient is ~ 10^{-4}

Weakening of nonlinear terms in MHD,

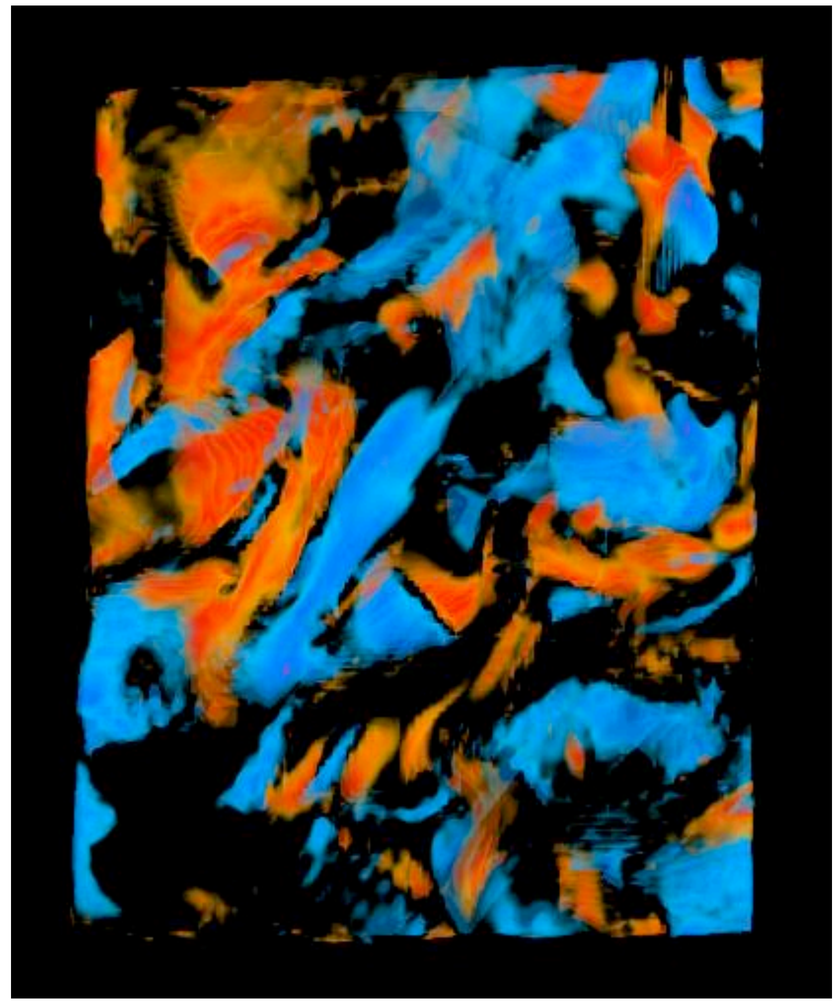
*similar to the Beltramisation (**v** // ω) of fluids*

Vorticity $\omega=\nabla \mathrm{x} \mathbf{u}$ & Relative helicity intensity $h=cos(\mathbf{u},\omega)$

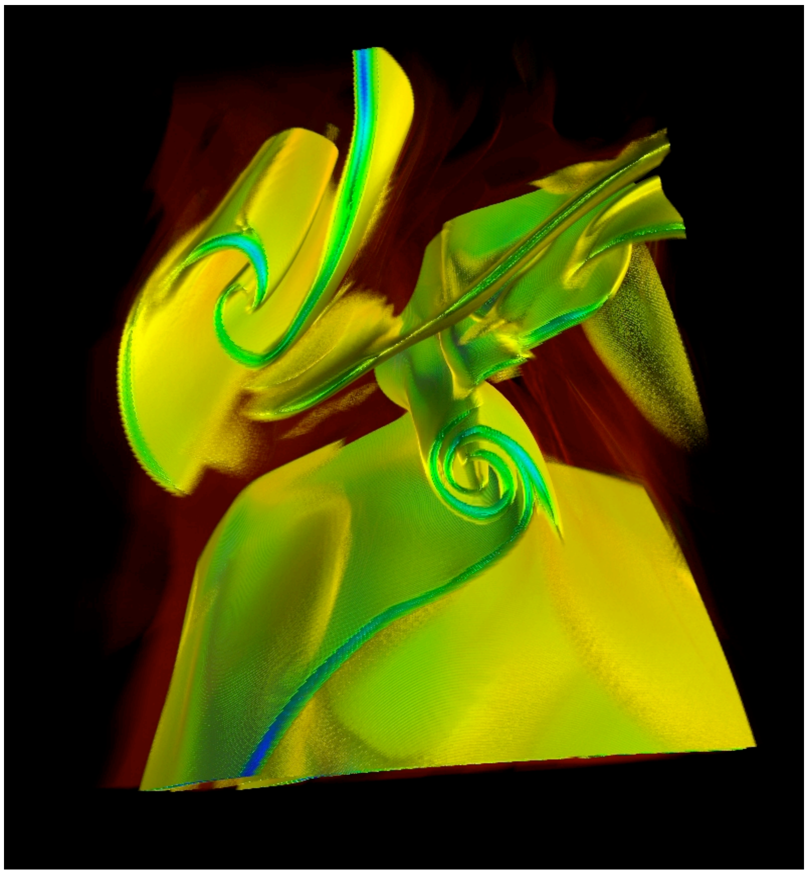
- Local u-ω alignment (Beltramization). Tsinober & Levich, Phys. Lett. (1983); Moffatt, J. Fluid Mech. (1985); Farge, Pellegrino, & Schneider, PRL (2001), Holm & Kerr PRL (2002).

--> no mirror symmetry, together with weak nonlinearities in the small scales

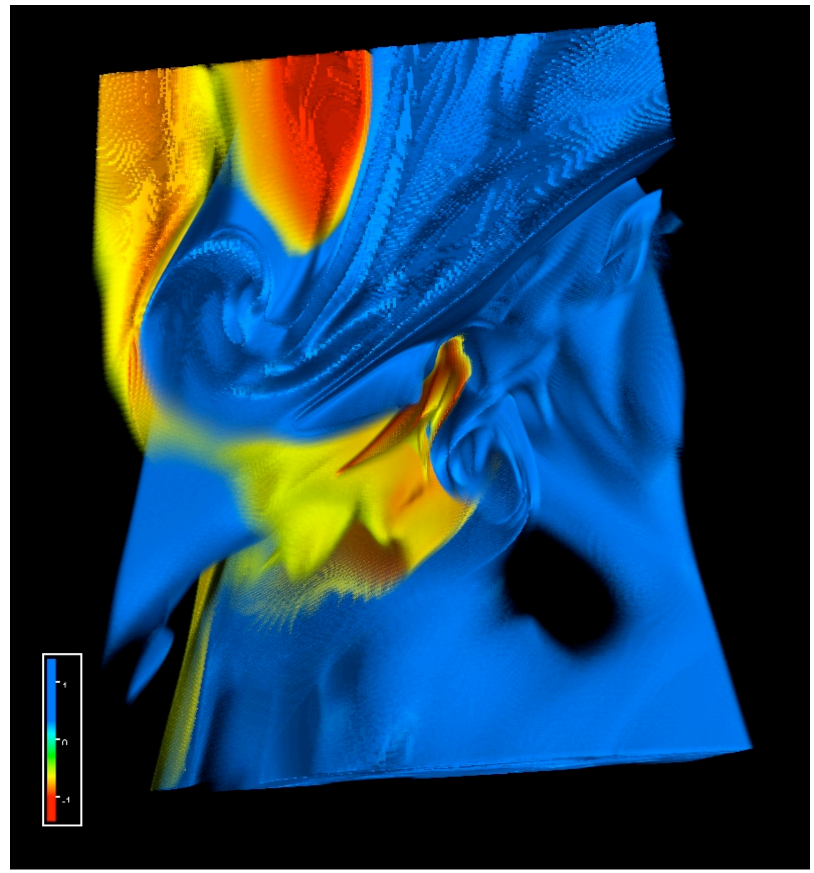
Relative Helicity: Blue h> 0.95
Red h<-0.95



Strong *relative* magnetic helicity (~ ± 1):
change of topology across sheet



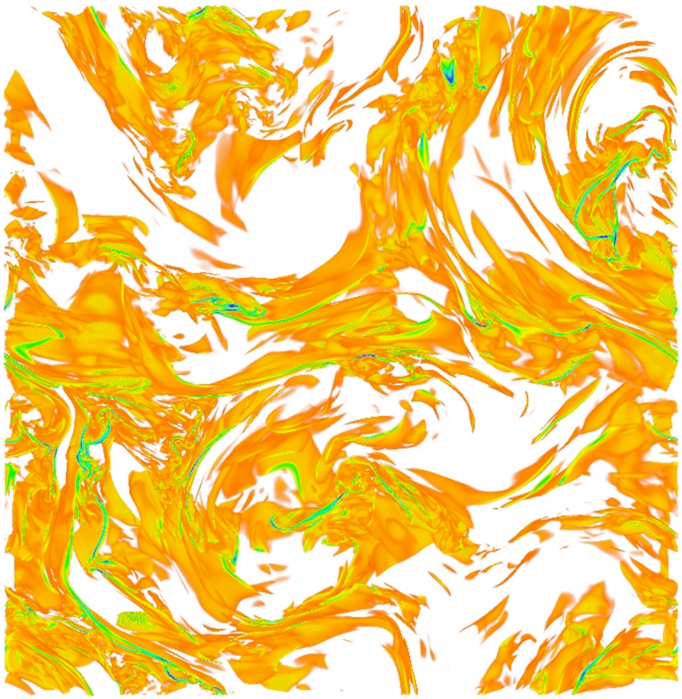
Current J^2



cos(**A, B**) , with **B=∇xA**

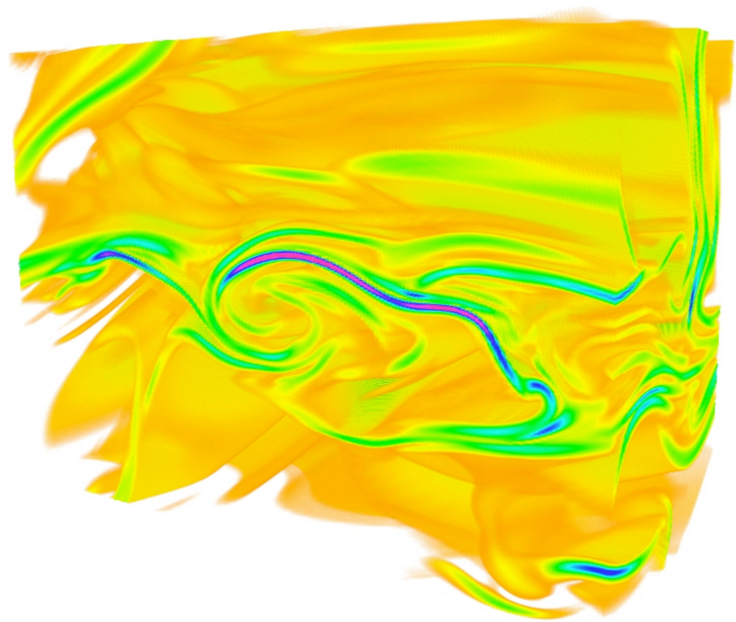
1536^3 run, early time

Current at peak of dissipation



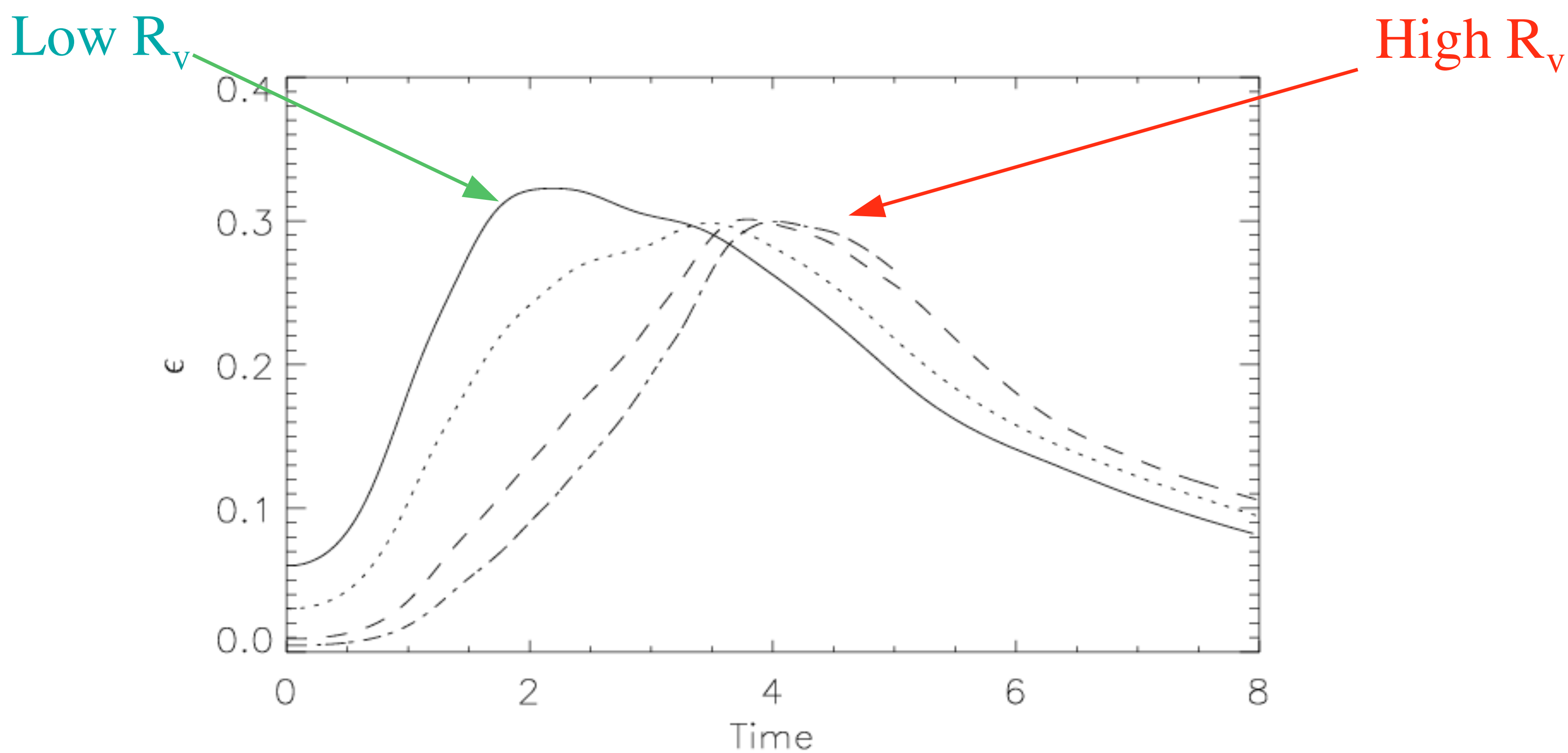
Global view

Zoom



Energy dissipation rate in MHD for several R_V

OT- vortex



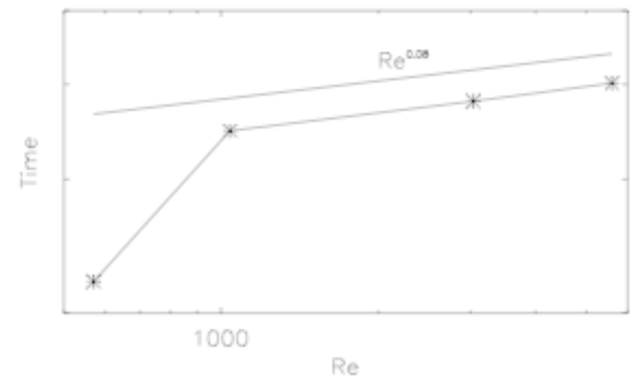
Orszag-Tang simulations at different Reynolds numbers (factor of 10)

- Is the energy dissipation rate, ε, constant in MHD turbulence at large Reynolds, as presumably it is in 2D-MHD in the reconnection phase?

There is evidence of constant ε in the hydro case (Kaneda et al., 2003)

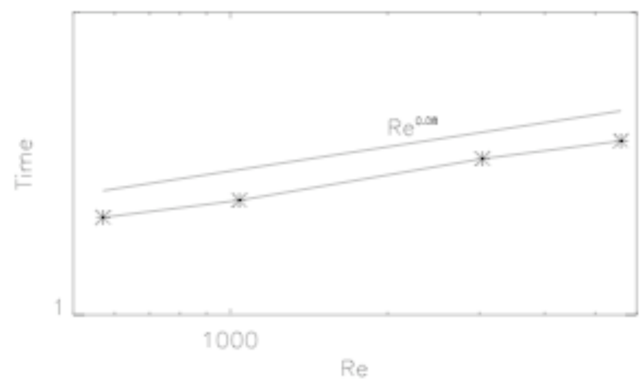
Scaling with Reynolds number of max. current & dissipation

Time $T_{max}^{(1)}$ at which
global **maximum of dissipation**
is reached in (ABC+ random) flow



and

Time $T_{max}^{(2)}$ at which the current reaches its first maximum



Both scale as $\mathbf{R_v^{0.08}}$

MHD decay run at peak of dissipation [1]

Energy spectra compensated by $k^{3/2}$

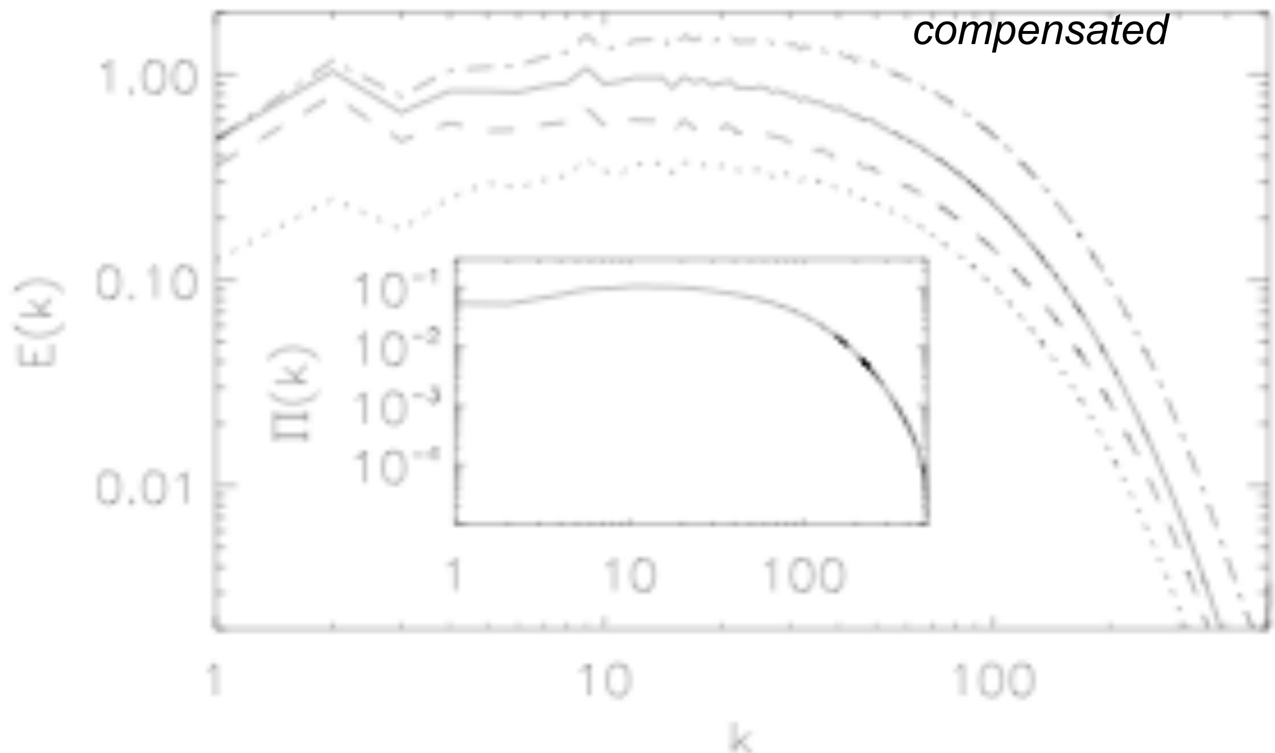
Solid: E^T

Dash: E^M

Dot: E^V

Insert: energy flux

Dash-dot: $k^{5/3}$*-compensated*

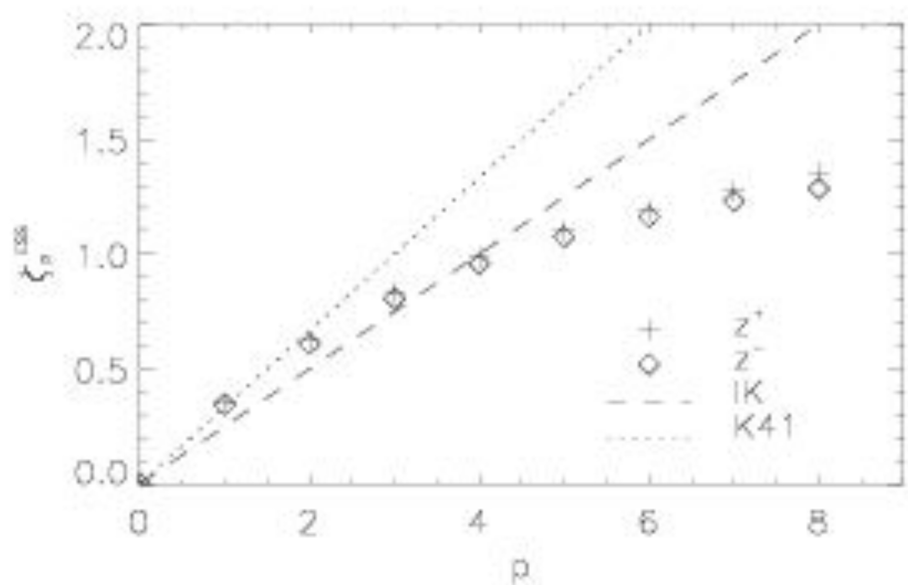


MHD decay run at peak of dissipation [2]

- Anomalous isotropic exponents for Elsässer variables, *and for V and B fields*

Note $\zeta_4 \sim 1$*,*

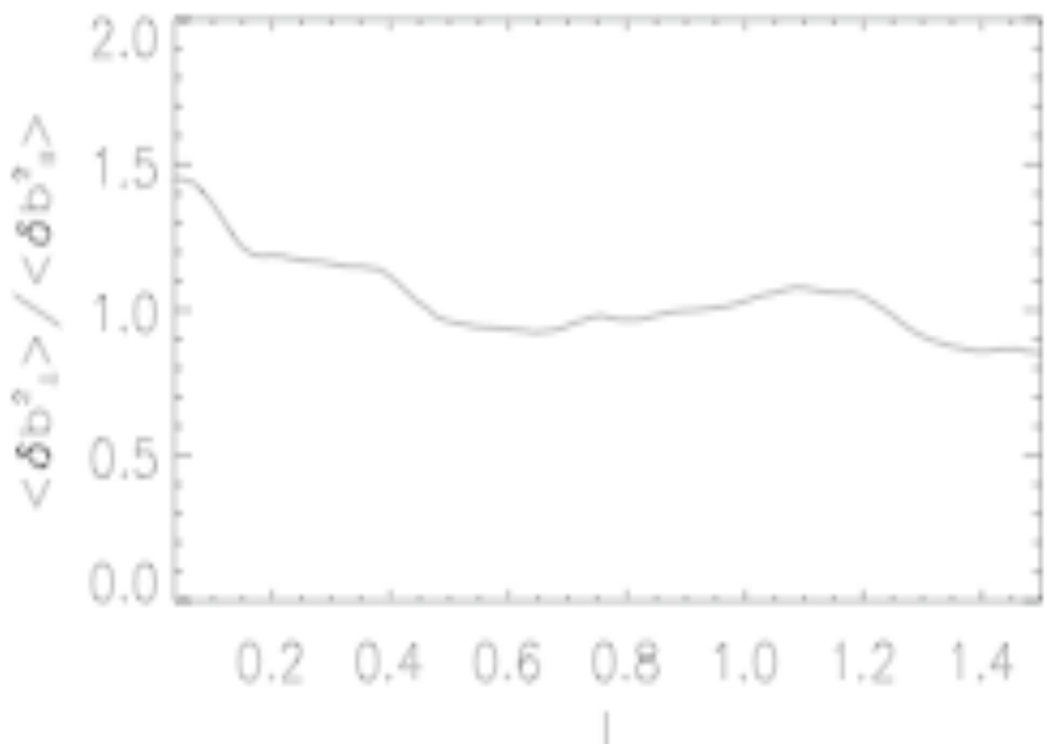
i.e. far from fluids and with more intermittency



MHD decay run at peak of dissipation [3]

Isotropy ratio

$R = S^{2(b)}_{\perp} / S^{2(b)}_{//}$



Isotropy obtains in the first inertial domain, and anisotropy develops at smaller scales

R is proportional to the so-called Shebalin angles

MHD decay run at peak of dissipation [4]

$L^{1/2}$ compensation of S_2 structure functions.

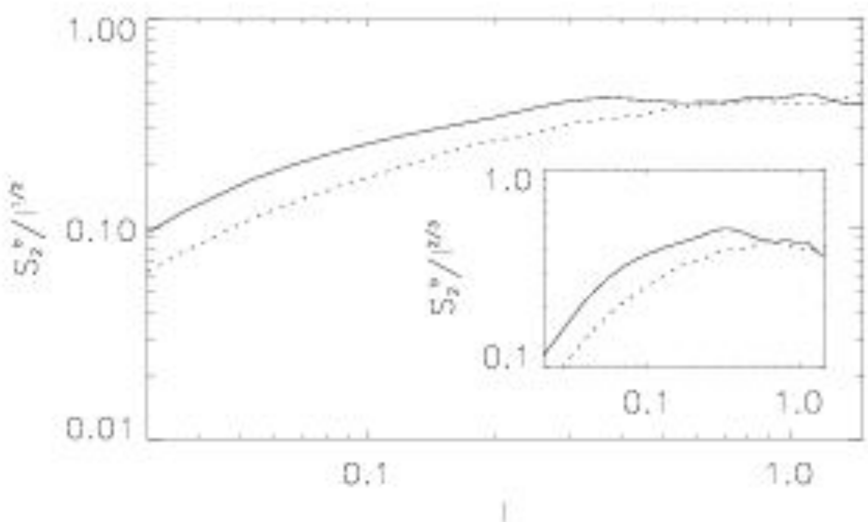
Flat at large scales with equipartition of the perp. and // components, hence

$E(k) \sim k^{-3/2}$

Solid: perpendicular

Dash: parallel

Insert: $l^{2/3}$-compensated



MHD decay run at peak of dissipation [5]

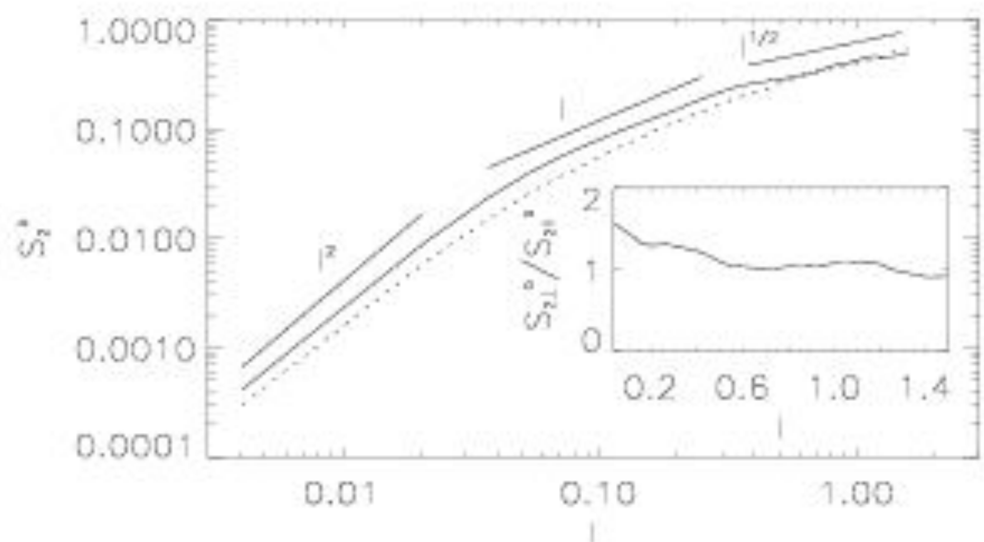
Structure function S_2 ,
with 3 ranges:
L^2 (regular) at small scale

L at intermediate scale, as for weak turbulence: $E_k \sim k_\perp^{-2}$, i.e. weak wave turbulence?

$L^{1/2}$ at largest scales ($E_k \sim k^{-3/2}$)

Solid: perpendicular
Dash: parallel

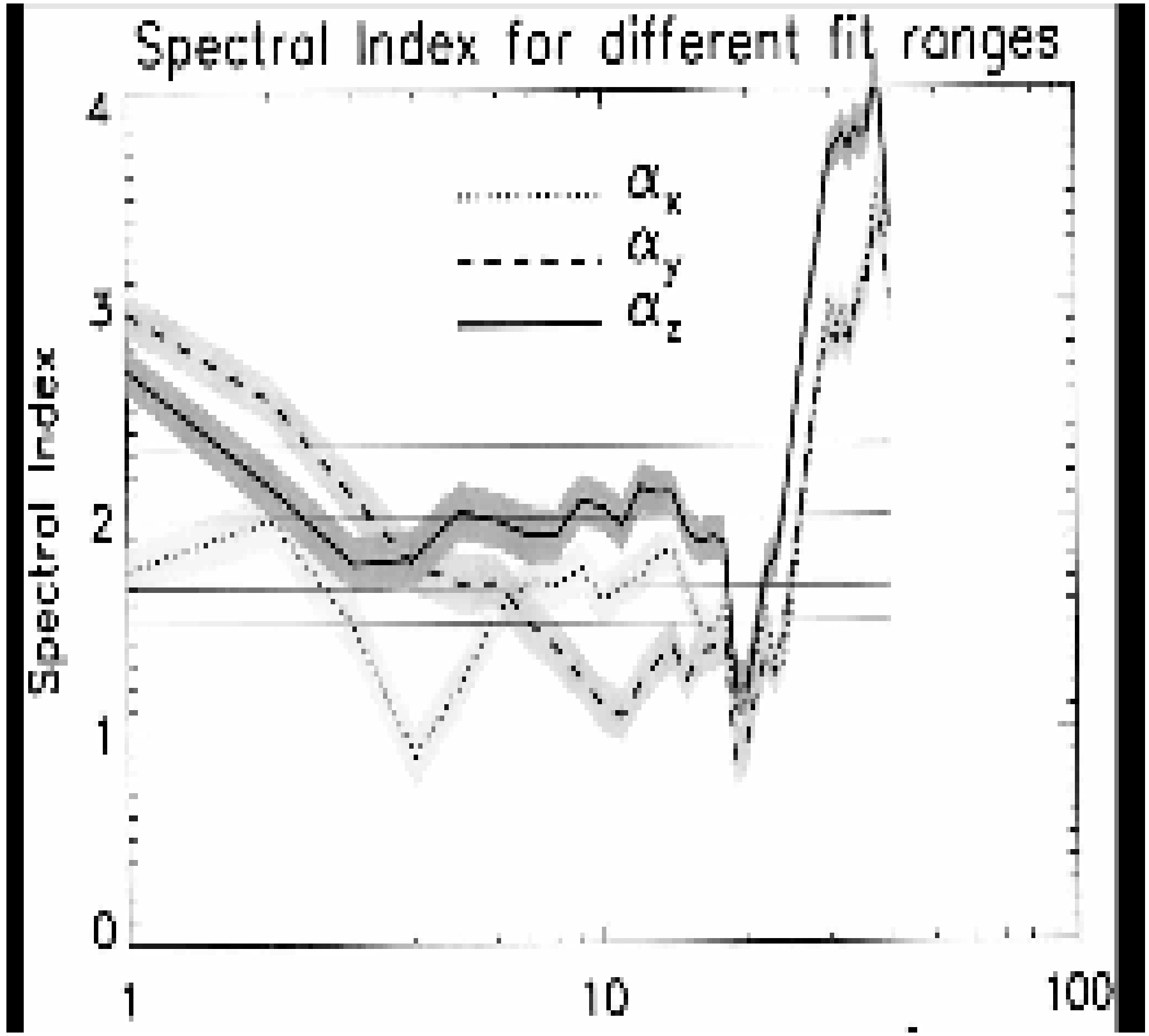
Insert: anisotropy ratio



- Evidence of weak MHD turbulence in the Jovian magnetosphere

- with a $k_{\perp}^{-2}$ spectrum

(Saur et al., A&A 386, 2002)



Kolmogorov-compensated Energy Spectra: $k^{5/3}$ E(k)

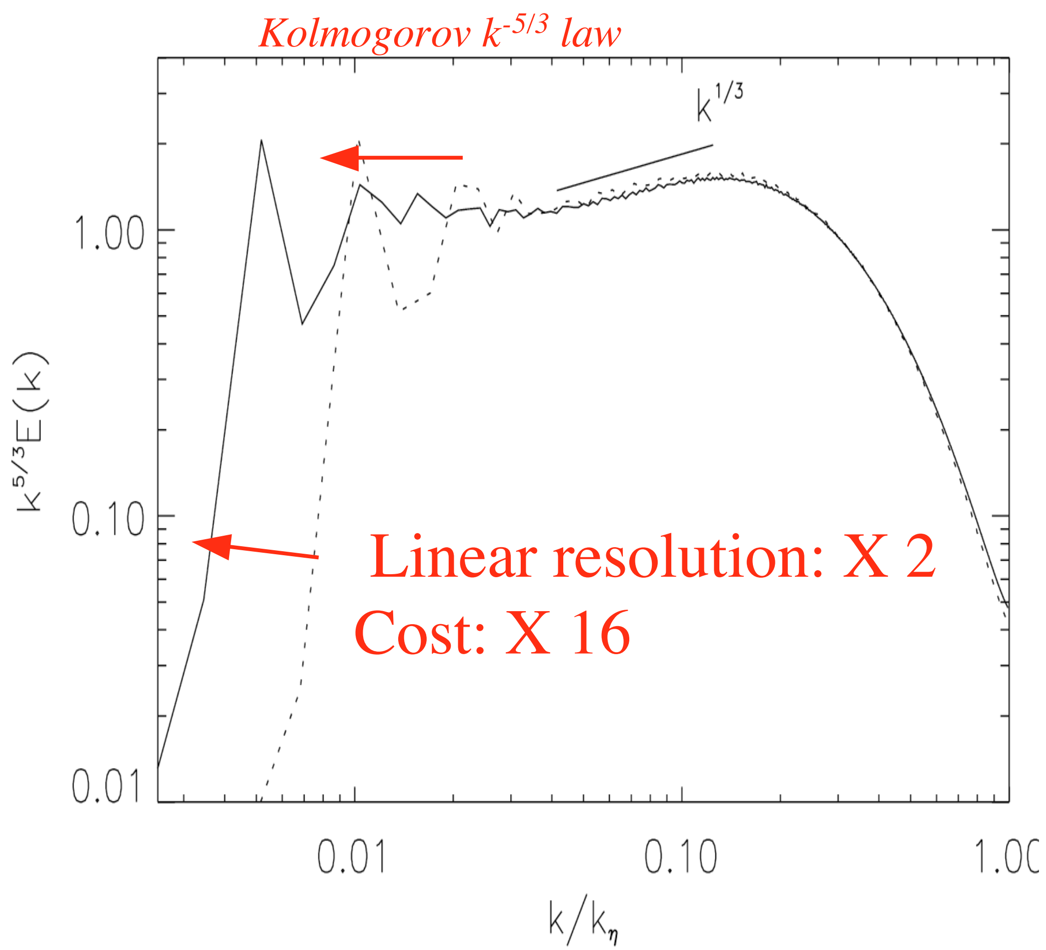
Navier-Stokes, ABC flow

Small Kolmogorov $k^{-5/3}$ law
(flat part of the spectrum here)

It increases in length as the Reynolds number increases

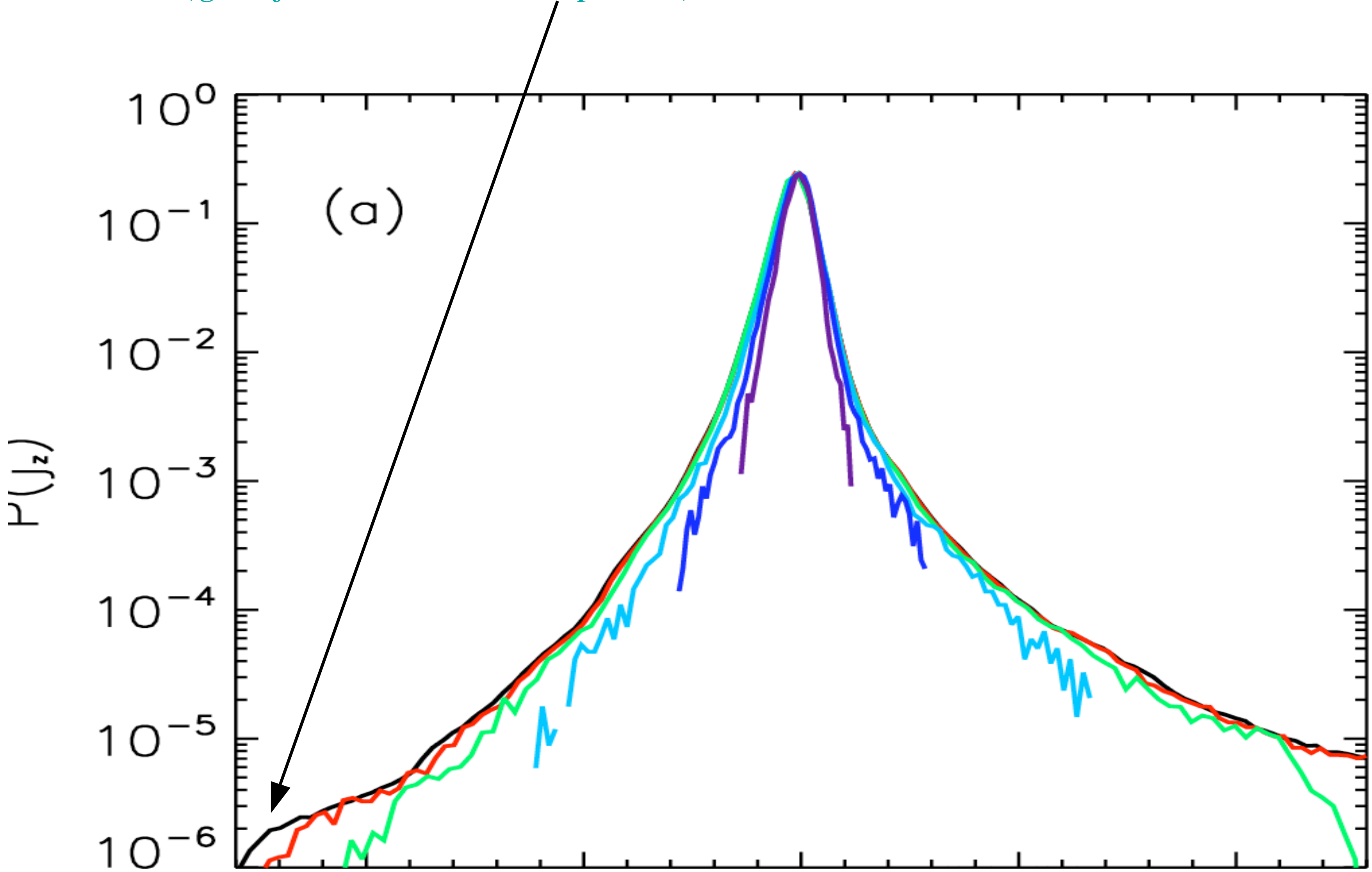
- Bottleneck at dissipation scale

Solid: 2048^3, $R_v = 10^4$, $R_\lambda \sim 1200$
Dash: 1024^3, R_v=4000

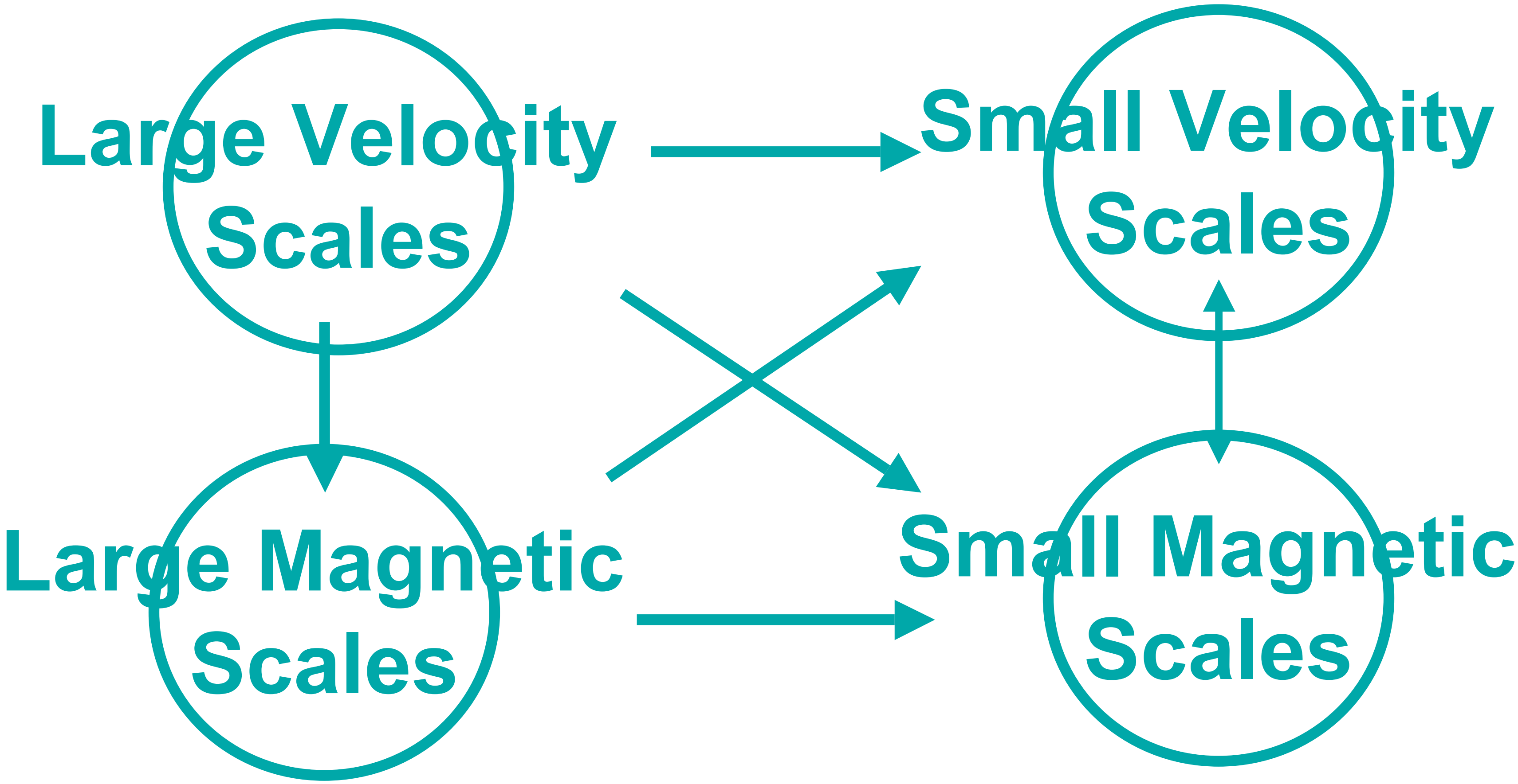


Extreme events at high R_v unraveled by high-resolution runs

(grid from 48^3 to 1536^3 points)

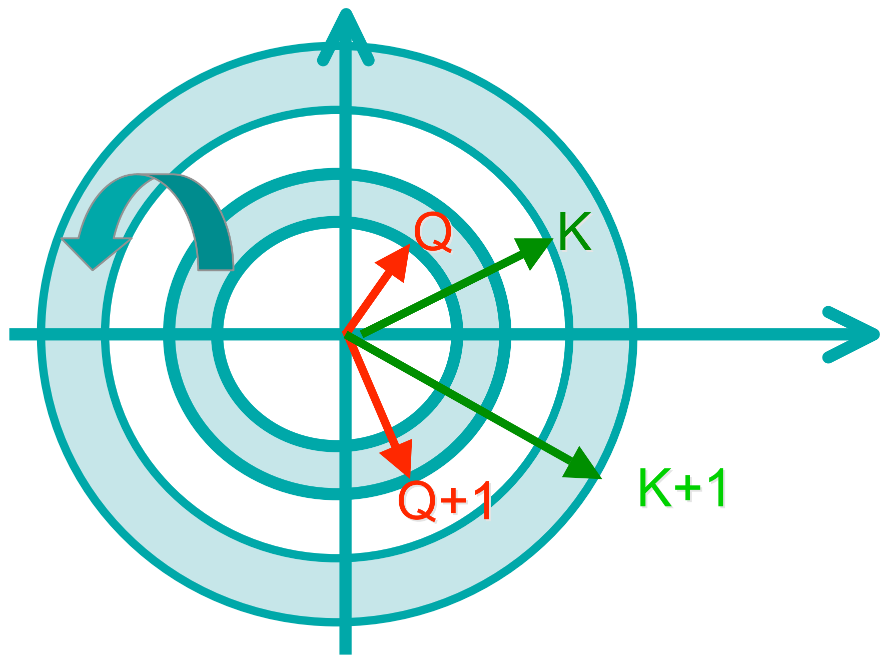


MHD cascade of energy



Energy Transfer

Let $\mathbf{u}_K(x)$ be the velocity field with wave numbers in the range $K < |k| < K+1$



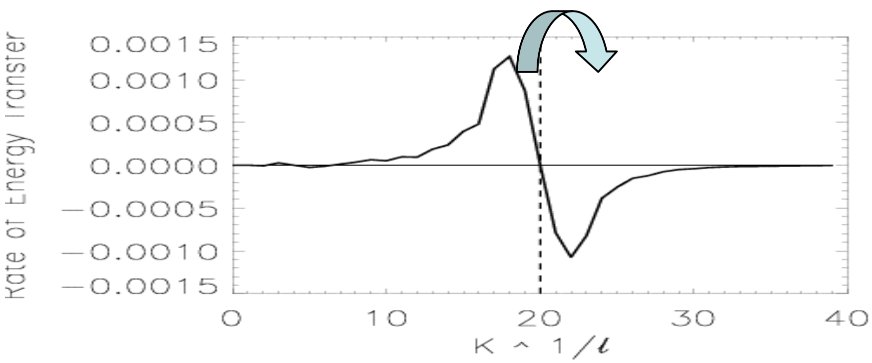
^ Sharp filters
^ Isotropy

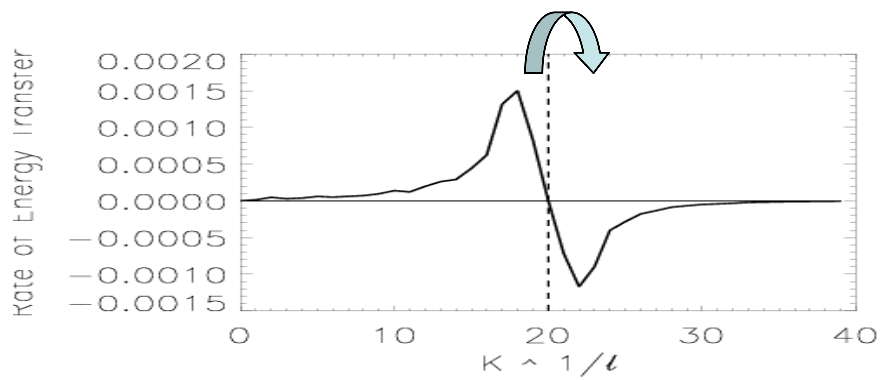
Fourier space

Rate of energy transfer in **MHD**

1024^3 runs, either T-G or ABC forcing

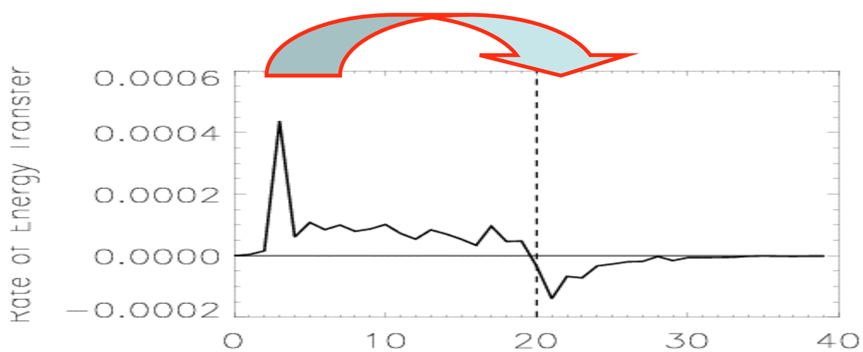
Advection terms





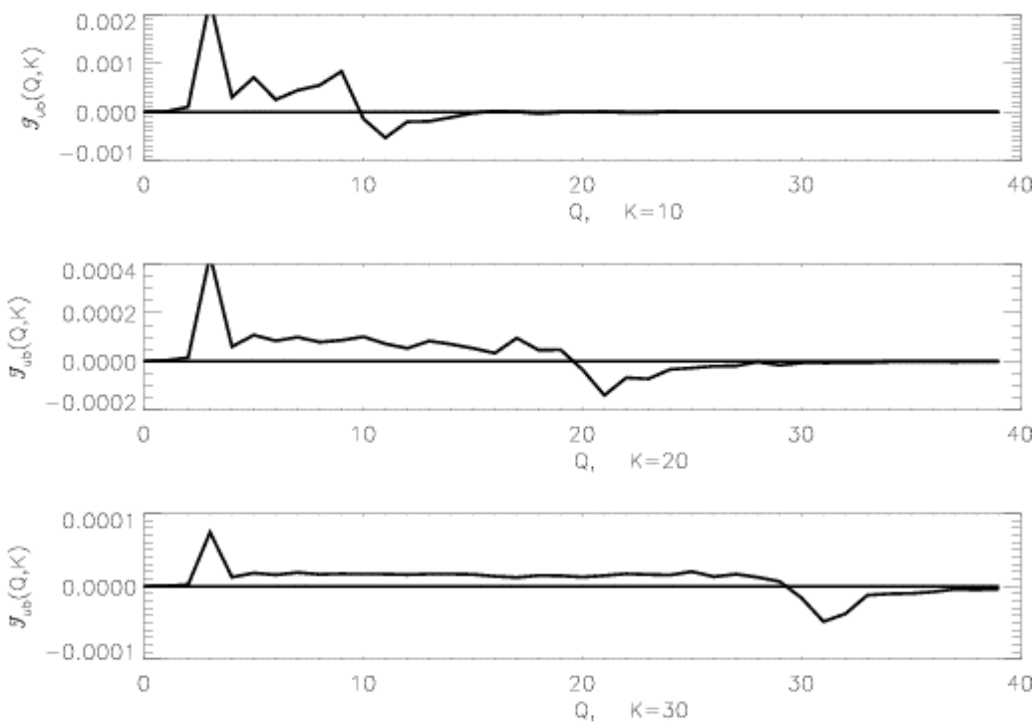
$R_\lambda \sim 800$

New: all scales contribute to energy transfer through the Lorentz force



The non-local energy transfer seems to be absent in decay runs (Debliquy et al., PoP 05)

Rate of energy transfer $T_{ub}(Q,K)$ from u to b for different K shells



K= 10

K= 20

K= 30

The magnetic field at a given scale receives energy in equal amounts from the velocity field from all larger scales *(but more from forcing scale)*

The dynamo problem of generation of magnetic field at small magnetic Prandtl number

- Is a turbulent dynamo possible at all?
- Is the magnetic field present at small scales?

Small Prandtl Number: No Problem!

P_M is ratio of two (linear) diffusion coefficients

- Take into account turbulent diffusivities:
- Dimensionally, $[\nu] = U \,.\, L$

$$\longrightarrow \quad \nu^{turb} \sim U_{rms} L_0 \sim \eta^{turb}$$

$$\longrightarrow \quad P_M^{turb} \sim 1$$

Note: renormalization group (Forster et al., 1977) & stochastic models

Thus, dynamos for all P_M should behave similarly.

Is this correct?

Note: $R^{turb} = U_{rms} L_0 / \nu^{turb} \sim 1$ *as well*

Small magnetic Prandtl number: Big problem numerically

- $P_M << 1$: it is 10^{-6} in liquid metals

→ Resolve two dissipative ranges, the inertial range and the energy containing range

And

Run at a magnetic Reynolds number R_M larger than some critical value (R_M governs the importance of stretching of magnetic field lines over Joule dissipation)

→ Resort to modeling of small scales

Lagrangian-averaged (or alpha) Model for Navier-Stokes and MHD (LAMHD):

the velocity & induction are smoothed on lengths α_V & α_M, but not their sources (vorticity & current)

$$\mathbf{v} = \mathbf{u_s} + \delta\mathbf{v}, \ \mathbf{B} = \mathbf{B_s} + \delta\mathbf{B},$$

$$G_\alpha(\mathbf{r}, t) = \exp[-r/\alpha]/4\pi\alpha^2 r.$$

$$\mathbf{u_s} = G_{\alpha_V} \otimes \mathbf{v}, \ \mathbf{B_s} = G_{\alpha_M} \otimes \mathbf{B},$$

-->

$$\mathbf{v} = (1 - \alpha_V^2 \nabla^2)\ \mathbf{u_s} \ \text{and} \ \mathbf{B} = (1 - \alpha_M^2 \nabla^2)\ \mathbf{B_s}$$

Equations preserve invariants (in modified - filtered L_2 --> H_1 form)

McIntyre (mid '70s), Holm (2002), Marsden, Titi, … Montgomery & AP (2002)

Lagrangian-averaged NS & MHD **Non-dissipative** Model Equations

- $\partial v / \partial t + u_s \cdot \nabla v = - v_j \nabla u^j{}_s - \nabla P_* + j \times B_s,$
- $\partial B_s / \partial t + u_s \cdot \nabla B_s = B_s \cdot \nabla u_s$
- The above equations have invariants that differ in their formulation from those of the primitive equations: the filtering prevents the small scales from developing.
- For example, kinetic energy invariant $E_V = \langle v^2 \rangle /2$

  --> $E_{V,\, \alpha \text{ model}} = \langle v^2 + \alpha^2 \omega^2 \rangle /2$

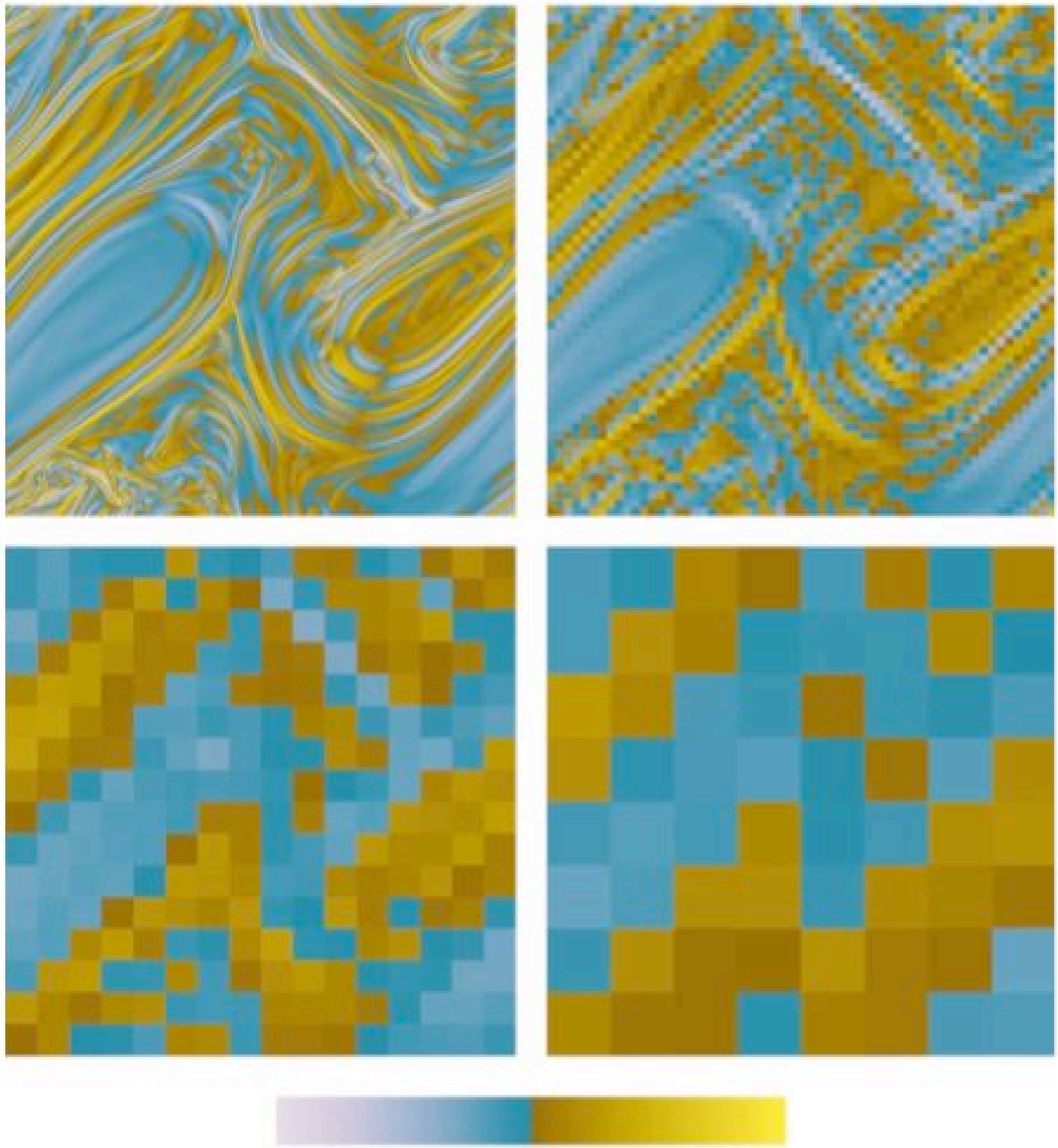
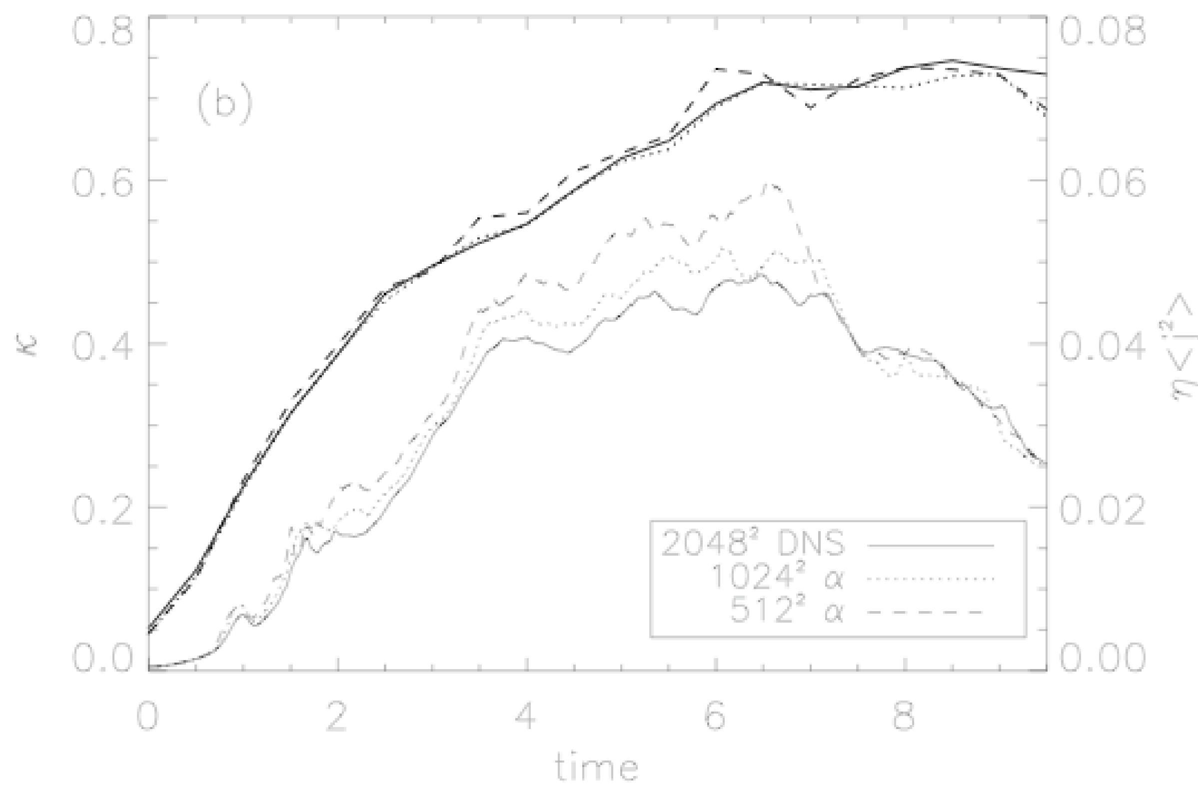


FIG. 3. (Color) The coarse-grained signed measure of the current J at time $t=7.3$ for four different box sizes, namely $l/L=0.001$, $l/L=0.016$, $l/L=0.059$, $l/L=0.12$, from top to bottom. Colors range from cyan for negative J values to yellow for positive ones, going through blue and brown. Cancellations at large scales are responsible for the decrease in magnitude of the measure.

Cancellation exponent κ and magnetic dissipation: comparison with LAMHD



κ=[d-d_F]/2 *(Noullez, 2002)*

Dynamo Test in Three-dimensional MHD at P_M=1

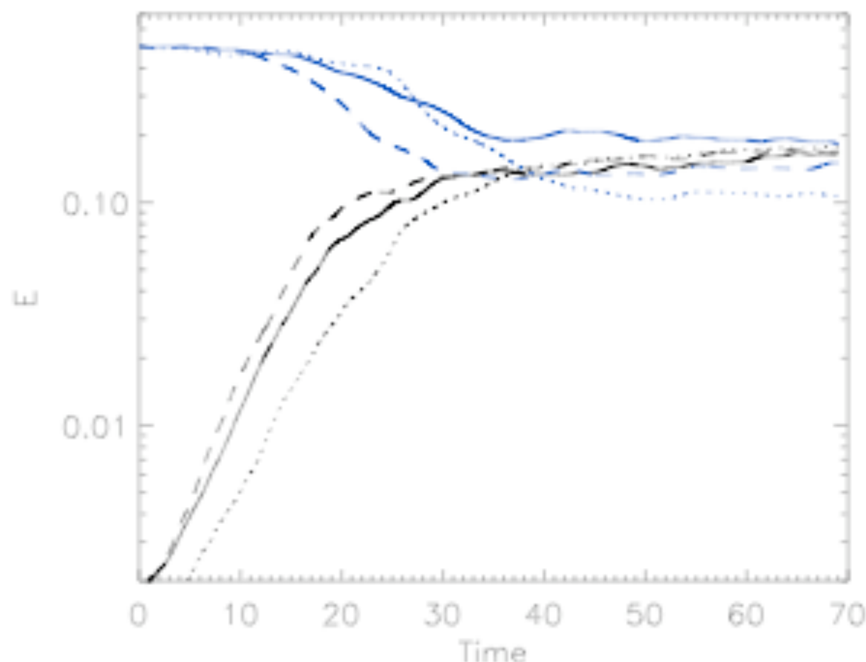
- Comparison of DNS at 256^3 grid resolution (solid line) and α runs at 128^3 or 64^3 resolutions (dash or dot)

- Dynamo in a Beltrami (fully helical) ABC flow at k_0=3

Phys. Fluids 17; Phys Rev E 71 (2005)

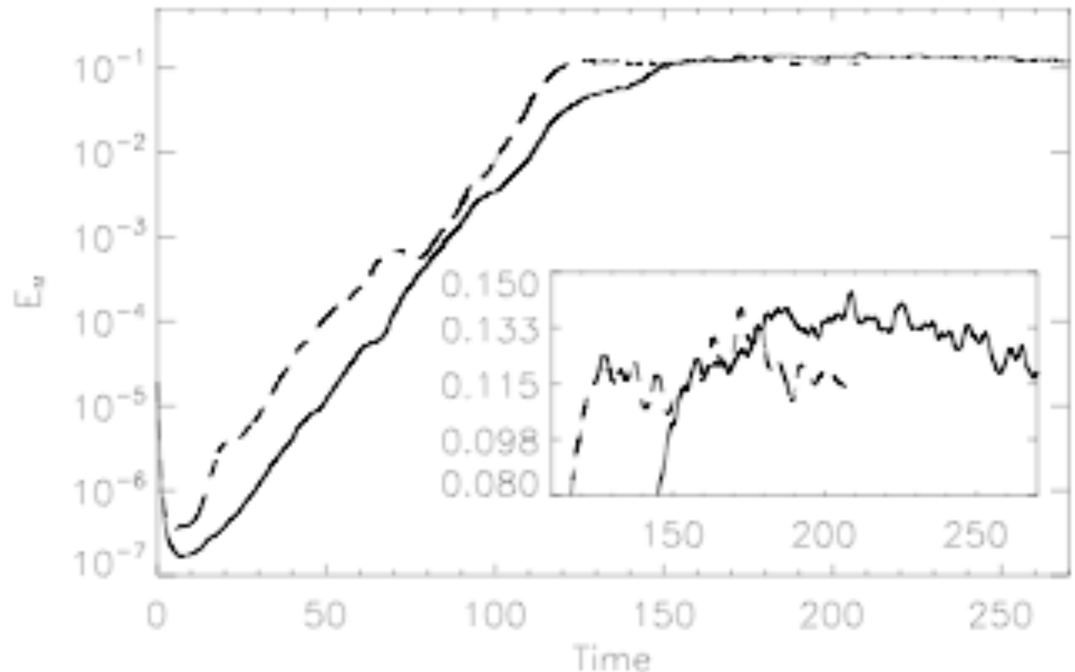
Dynamo regime: the growth of magnetic energy at the expense of kinetic energy:

all three runs display similar temporal evolutions and energy spectra



Comparison of DNS and Lagrangian model

- $R_M = 41$, R_v=820,
  $P_M = 0.05$ dynamo



- *Solid line: DNS*
- *- - - : LAMHD*
- Linear scale in inset

Comparable growth rate an
saturation level of Direct
Numerical Simulation and model

Large-Eddy Simulation (LES)

- Add to the momentum equation a turbulent viscosity $\nu_t(k,t)$ (*à la Chollet-Lesieur*) (no modification to the induction equation

(study of similar LES for MHD in progress, Baerenzung et al.)

$$\nu(k,t) = 0.27[1 + 3.58(k/K_c)^8]\sqrt{E_V(K_c,t)/K_c}$$

with K_c a cut-off wave-number

The first numerical dynamo within a turbulent flow at a magnetic Prandtl number below $P_M \sim 0.25$, down to 0.02 *(Ponty et al., PRL* **94***, 164502, 2005).*

Turbulent dynamo at $P_M \sim 0.002$ on the Roberts flow (Mininni, 2006).

Turbulent dynamo at $P_M \sim 10^{-6}$, using second-order EDQNM closure (Léorat et al., 1980)

Critical magnetic Reynolds number for dynamo action -->

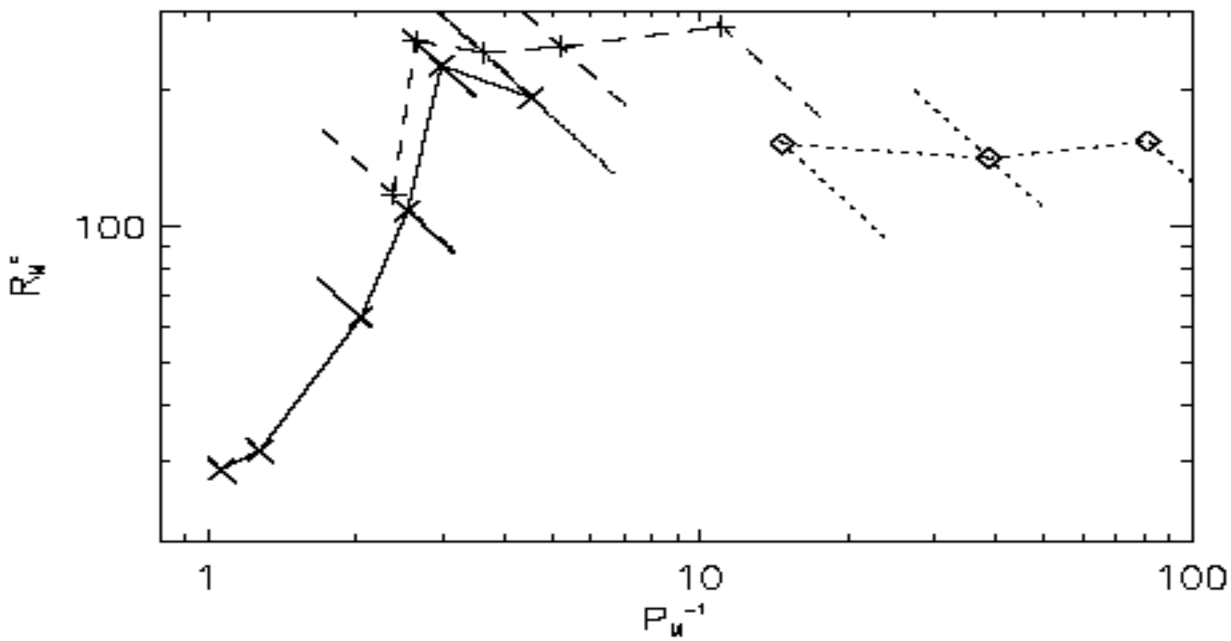
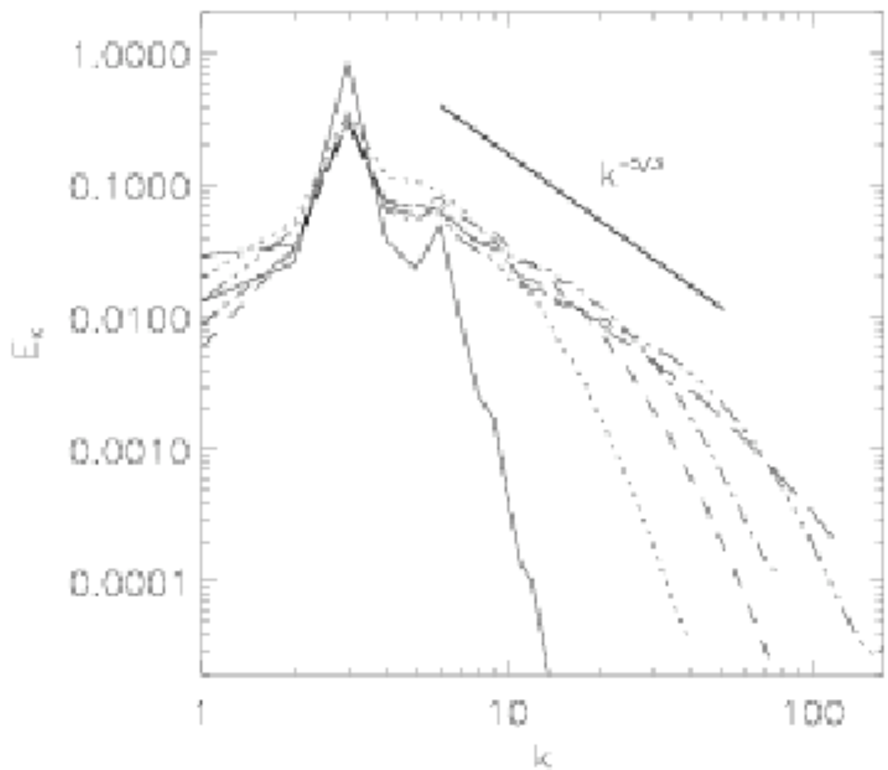


FIG. 1: Evolution of R_M^c for dynamo action with the inverse of P_M. Symbols are: × (DNS), + (LAMHD), and ◇ (LES). Transverse thin lines indicate error bars in the determination of R_M^c, as the distance between growing and decaying runs at a constant R_V.

Kinetic energy spectra as a function of magnetic Prandtl number



Magnetic Energy Spectra

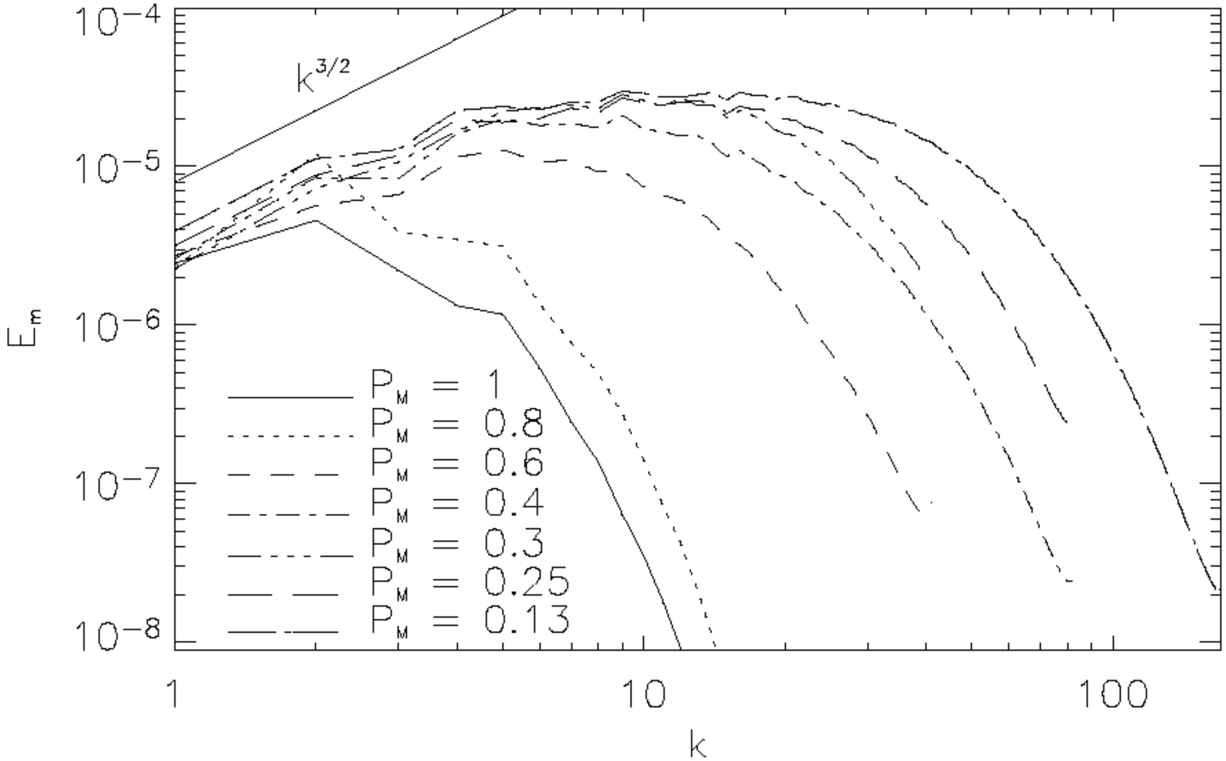


FIG. 2: Magnetic spectra for $P_M = 1$ to $P_M = 0.13$, at a time within the linear growth of magnetic energy. The curves are shifted vertically for better comparison.

Kazantsev (Kraichnan) model (1968) with δ-correlated velocity fluctuations

Another way to go to higher Reynolds numbers …

Can we go beyond Moore's law?

Doubling of speed of processors every 18 months

--> doubling of resolution for DNS in 3D every 6 years …

◊ Develop models of turbulent flows (Large Eddy Simulations, closures, Lagrangian-averaged, …)

◊ Improve numerical techniques

◊ Be patient

- Is Adaptive Mesh Refinement (AMR) a solution?

- *If so, how do we adapt? How much accuracy do we need?*

The need for Adaptive Mesh Refinement

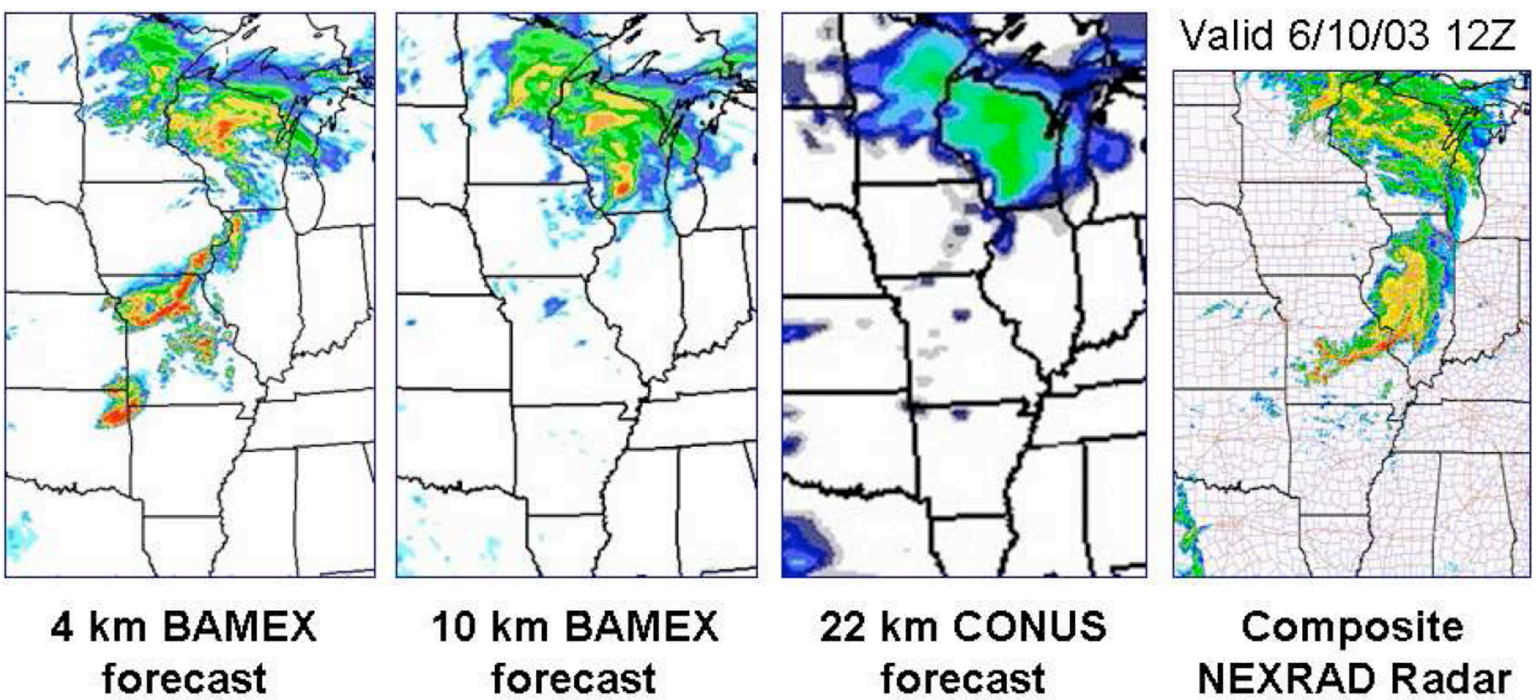
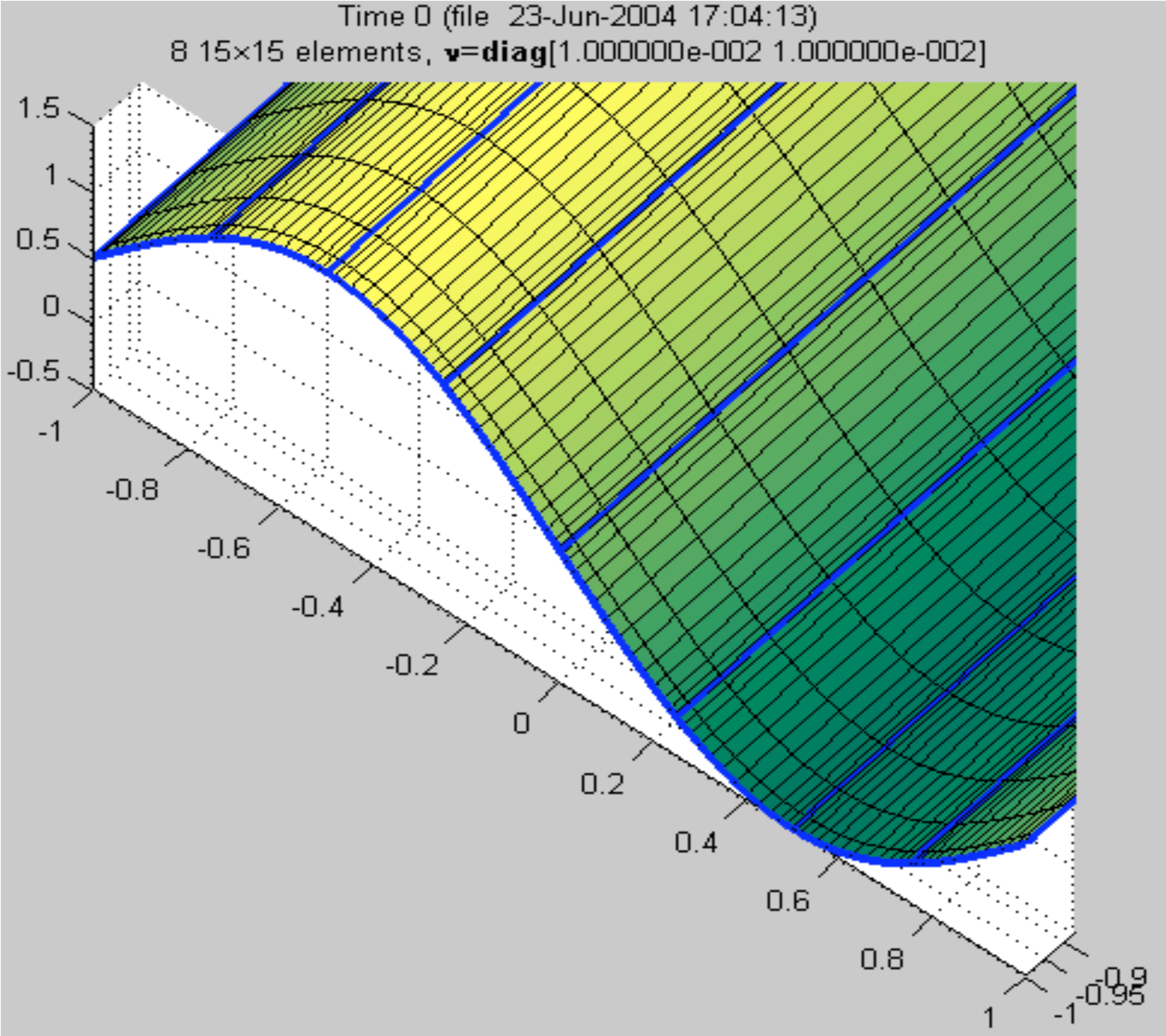


Figure 1: From Bill Skamarock, showing the lack of convergence with model resolution.

Burgers translating front ($\nu = 10^{-2}$)



Adaptive mesh refinement of the Burgers advection-diffusion equation, using spectral elements (GASpAR, NCAR)

Rosenberg et al., J. Comp. Phys., 2006

FD: since the 1980s

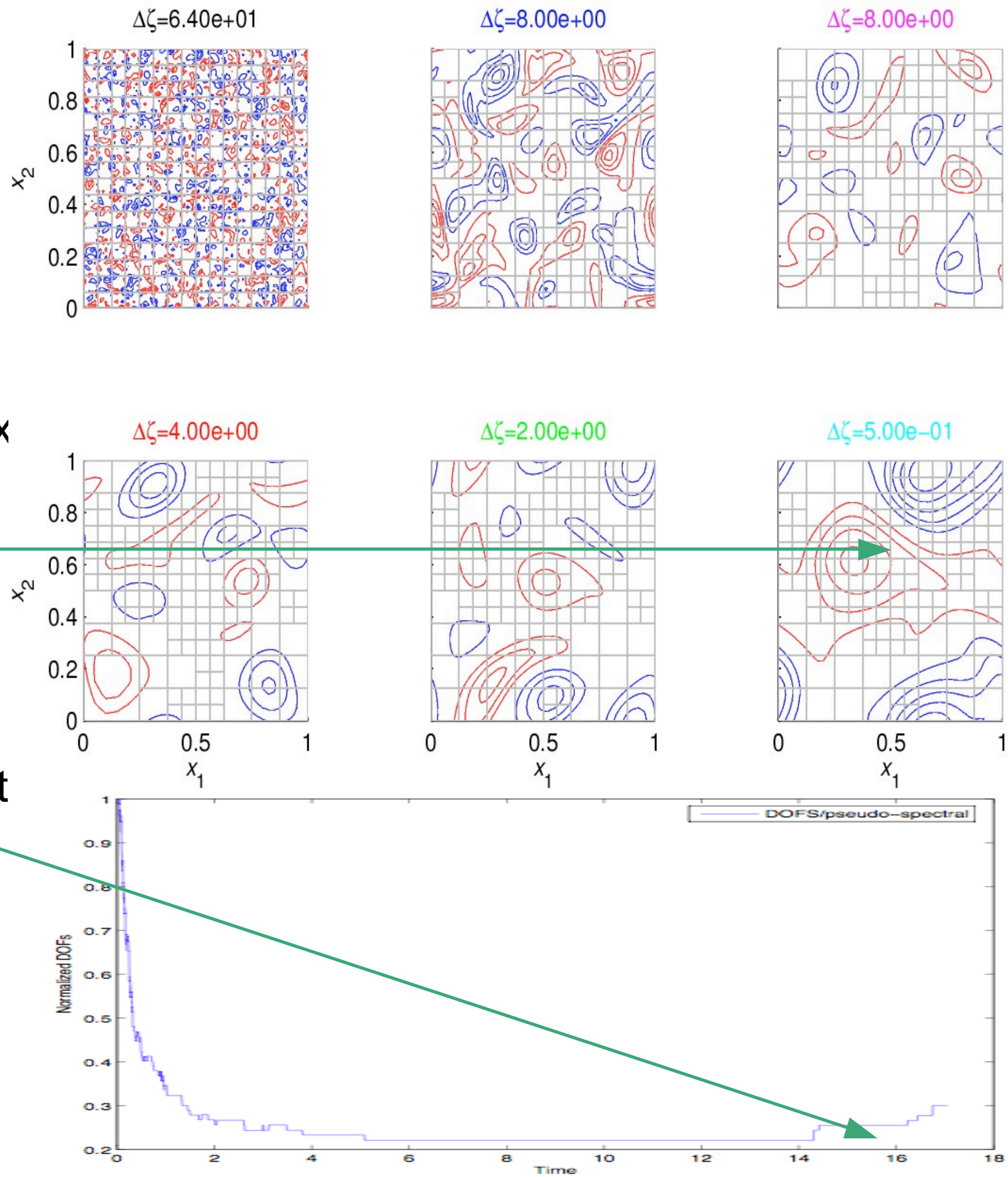
AMR on 2D Navier-Stokes

Aimé Fournier et al., 2007

- Decay for long times (incompressible)
- Formation of dipolar vortex structures

- Gain in the number of degrees of freedom (~ 4) with adaptive mesh refinement (AMR), compared to an equivalent pseudo-spectral code (periodic boundary conditions)

(but ….)

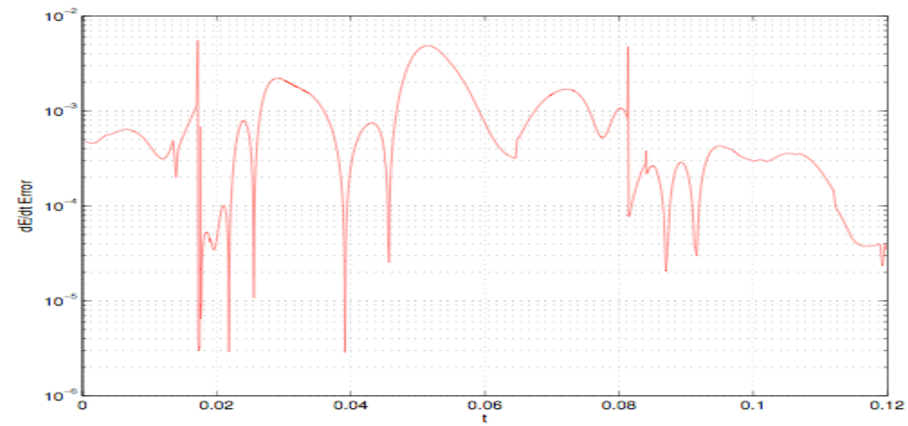


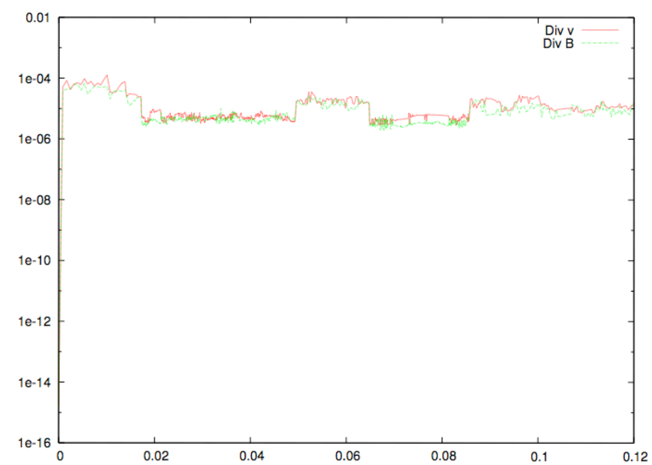
2D -MHD OT vortex with AMR

- Error in temporal derivative of total energy (compared to dissipation)

is ~ 10^{-3}

(computed every 10 time steps)





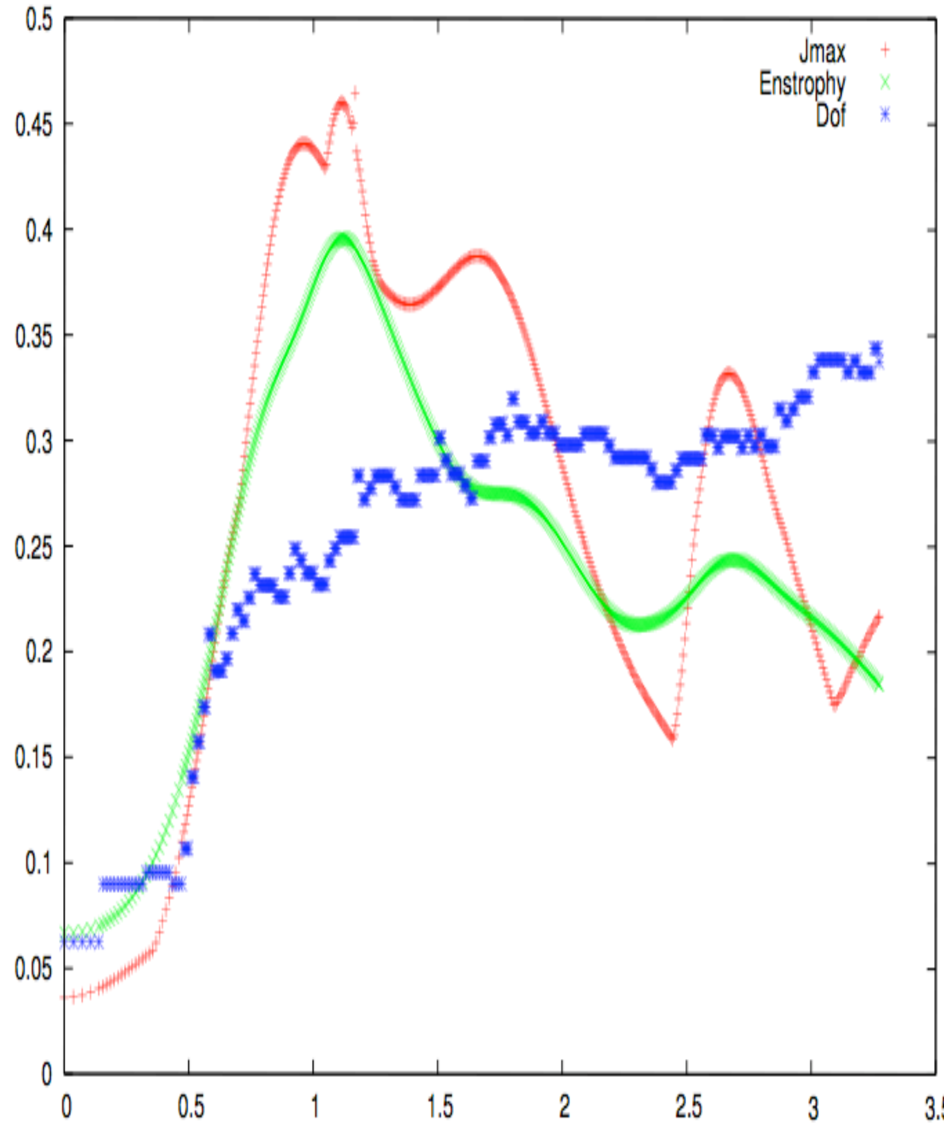
- Error in ∇.v is ~ 10^{-5} *(controlled by code parameter)*

AMR in 2D - MHD turbulence

- Magnetic X-point configuration in 2D
- Temporal variation of:
- Dissipation
- J_{max}
- Degrees of freedom

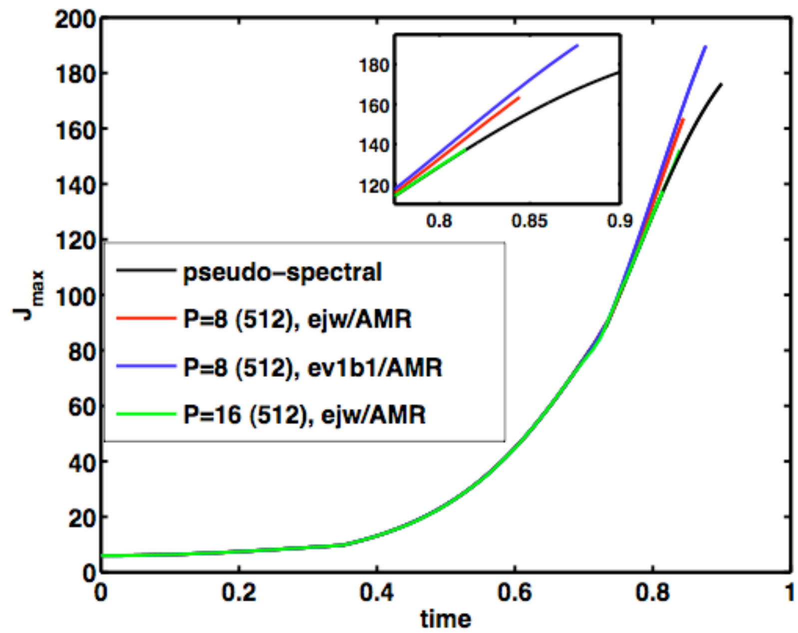
normalized by the number of modes in a pseudo-spectral code at the same R_v,
~33%

Refinement and coarsening criteria …



AMR in 2D - MHD turbulence

- Accuracy matters when looking at Max norms, here the current



Discussion

• What is the effect of the non-locality of nonlinear energy transfer observed in MHD on the flow dynamics, *e.g*. on the dynamo problem (generation of magnetic fields)?

• What is the physical origin of the t^3 evolution of the current and vorticity maxima in MHD *(time-dependent velocity shear?)*?

• Can we derive a dynamic model for the Kelvin-Helmoltz rolling-up of current and vorticity sheets? Are Alfvén vortices, as observed for example in the magnetosphere, present in MHD at high Reynolds number?

• *Quantification of anisotropy in MHD, including in the absence of a large-scale magnetic field*

• *Large-scale coherent forcing versus random forcing, i.e. universality?*

Thank you for your attention!