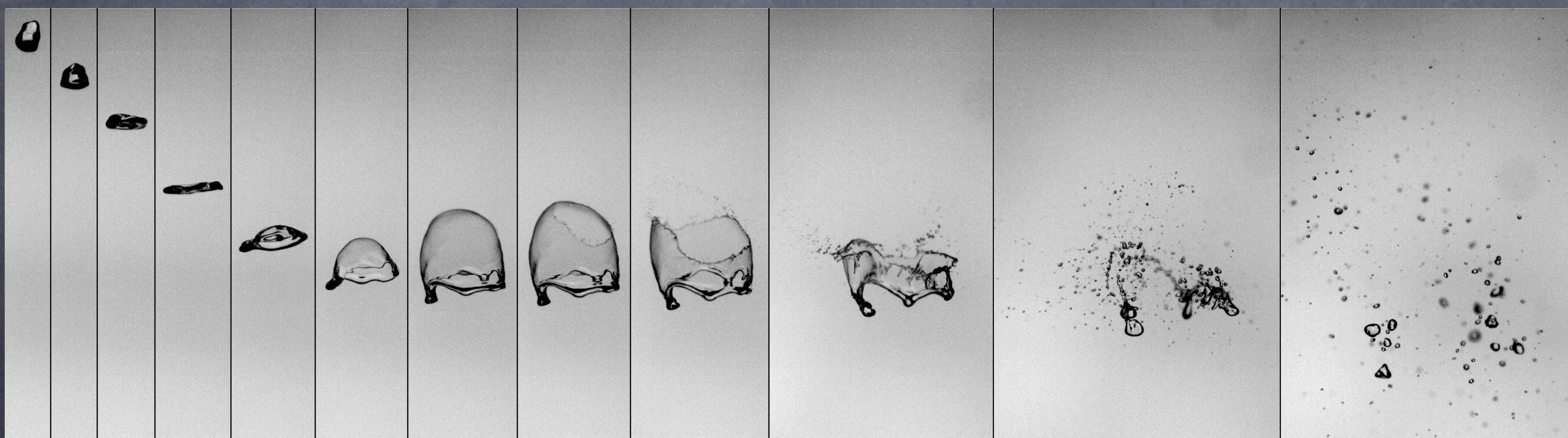
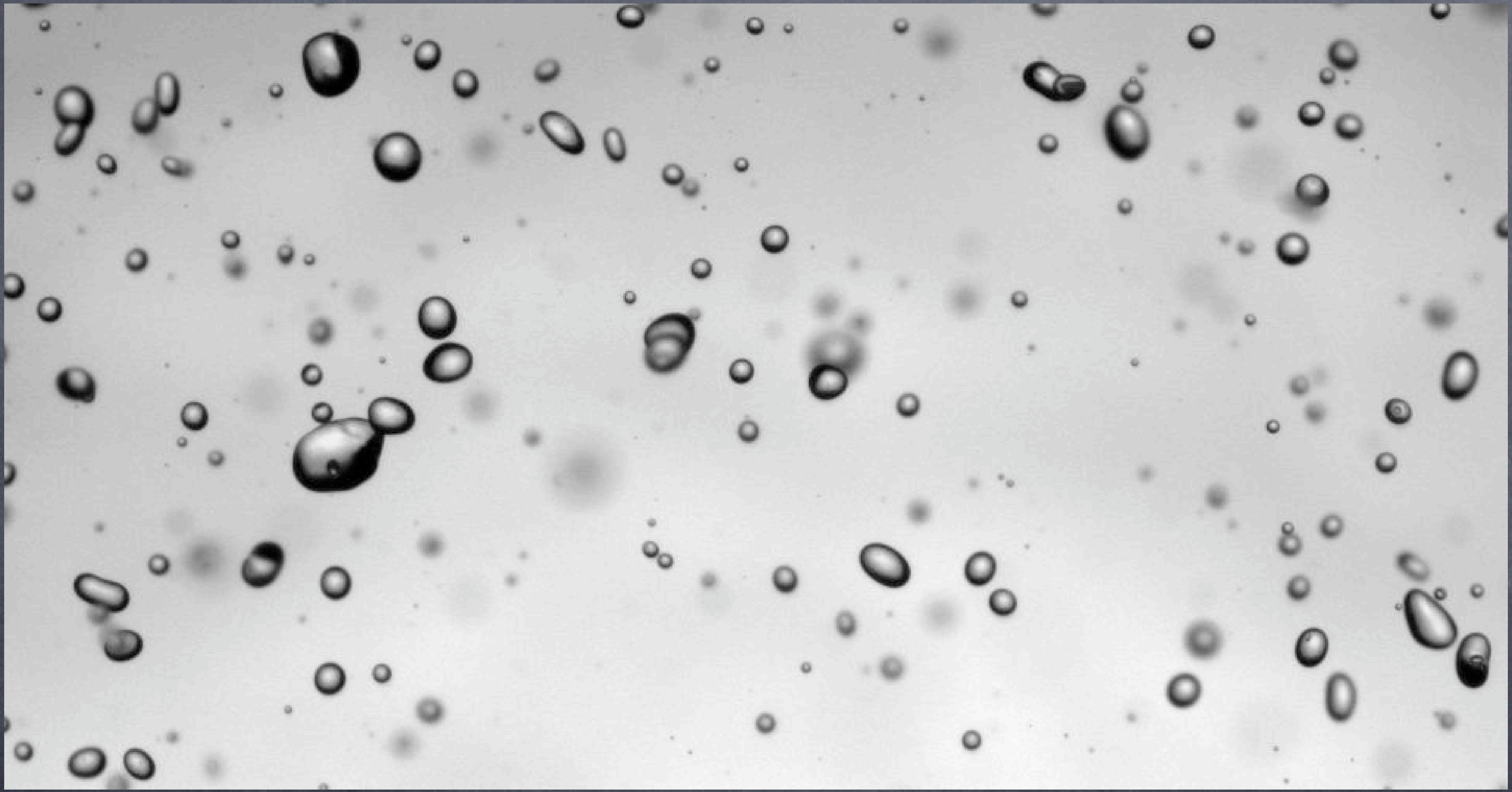
Fragmentation





Marshall-Palmer (1948)

SHORTER CONTRIBUTIONS

THE DISTRIBUTION OF RAINDROPS WITH SIZE

By J. S. Marshall and W. McK. Palmer[1]

McGill University, Montreal

(Manuscript received 26 January 1948)

Measurements of raindrop records on dyed filter papers were made for correlation with radar echoes (Marshall, Langille, and Palmer, 1947). These measurements have been analyzed to give the distribution of drops with size (fig. 1). The distributions are in fair agreement with those of Laws and Parsons (1943).

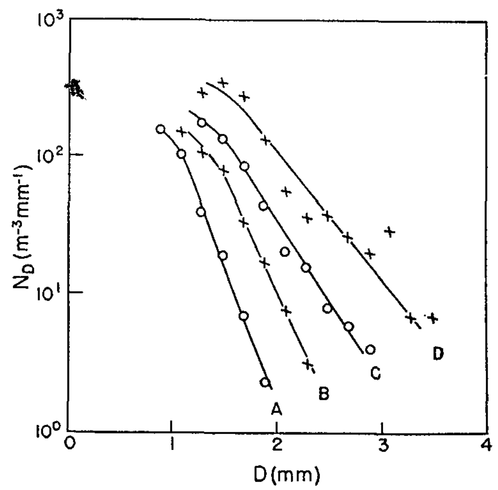


FIG. 1. Distribution of number versus diameter for raindrops recorded at Ottawa, summer 1946. Curve A is for rate of rainfall 1.0 mm hr^{-1}, curves B, C, D, for 2.8, 6.3, 23.0 mm hr^{-1}. $N_D\delta D$ is the number of drops per cubic meter, of diameter between D and $D + \delta D$ mm. Multiplication by 10^{-5} will convert N_D to the units of equation (2).

Except at small diameters, both sets of experimental observations can be fitted (fig. 2) by a general relation,

$$N_D = N_0 e^{-\Lambda D}, \tag{1}$$

where D is the diameter, $N_D\delta D$ is the number of drops of diameter between D and $D + \delta D$ in unit volume of space, and N_0 is the value of N_D for $D = 0$.

It is found that

$$N_0 = 0.08 \text{ cm}^{-4} \tag{2}$$

for any intensity of rainfall, and that

$$\Lambda = 41\, R^{-0.21} \text{ cm}^{-1}, \tag{3}$$

where R is the rate of rainfall in mm hr^{-1}.

For diameters less than about 1.5 mm, both sets of observations fall short of the value for N_D given by equation (1), and they disagree slightly with each other. Laws and Parsons' observations are better in this region, and tend toward a common value of N_0 for all rates of rainfall.

The mass of rain water M per unit volume of space, and the sum Z of sixth powers of drop diameters in unit volume (a radar quantity), can be calculated as functions of Λ from equation (1), and so correlated with the rate of rainfall R by equation (3). It is of interest to compare these correlations with those obtained when M, Z, and R are determined more directly from the experimental records (table 1). The deficit of small drops in the observations, as compared with equation (1), should make the observed value of M, and to a lesser extent that of Z, smaller than those derived from the equations.

TABLE 1. $M = \frac{1}{6}\pi\Sigma N_D D^3\delta D$ and $Z = \Sigma N_D D^6 \delta D$ as functions of the rate of rainfall R.

Reference	M mgm m^{-3}	Z mm^6 m^{-3}
Marshall, Langille and Palmer (1947)	80 $R^{0.83}$	190 $R^{1.72}$
Revision of the above	72 $R^{0.88}$	220 $R^{1.60}$
Z/R correlation by Wexler (1947) (data of Laws and Parsons, 1943)	68 $R^{0.88}$	320 $R^{1.44}$
From equations (1) and (3)	89 $R^{0.84}$	296 $R^{1.47}$

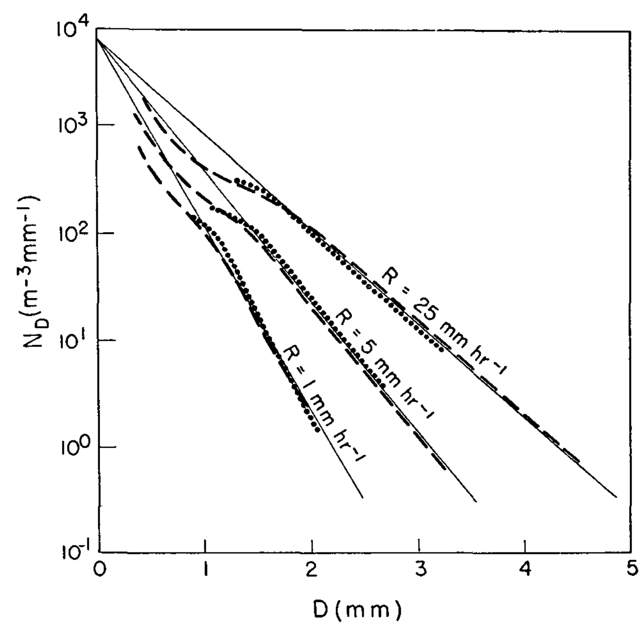
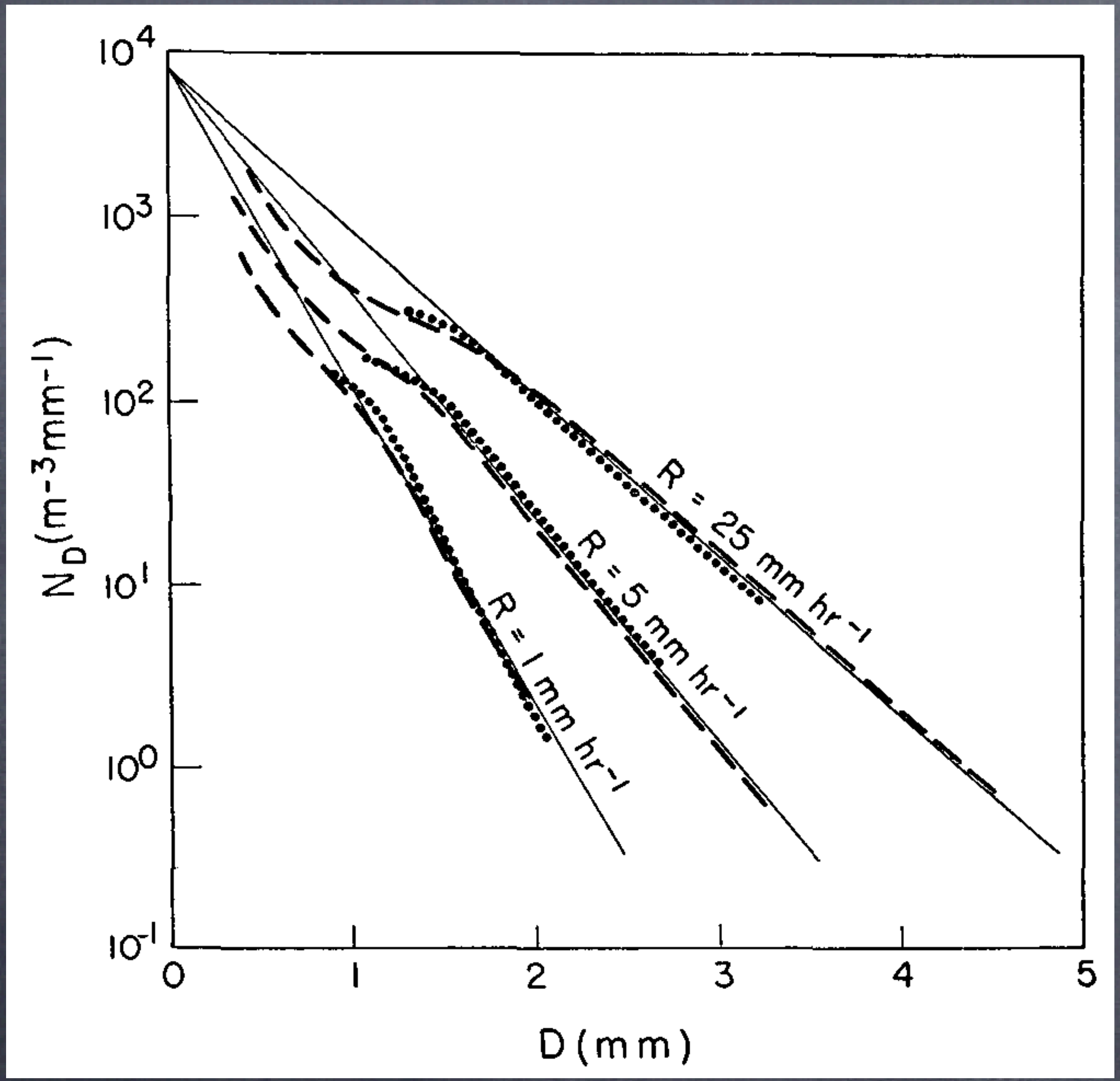
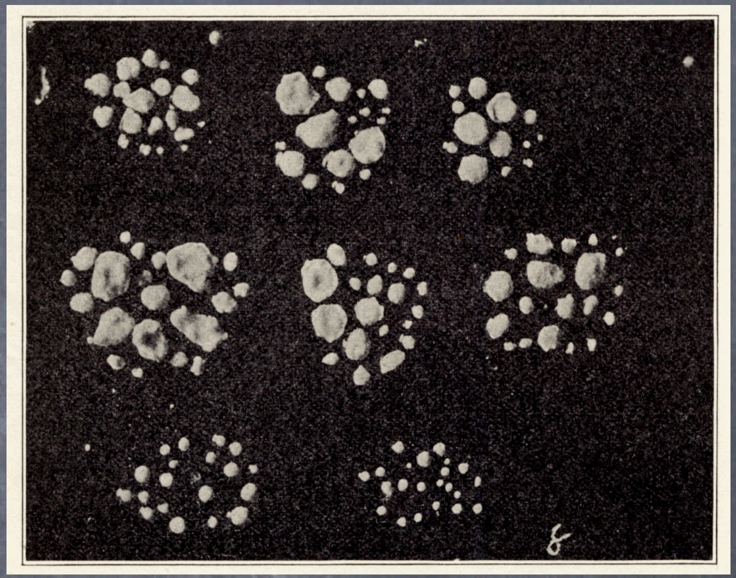


FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

[1] Holding a Bursary of the National Research Council of Canada.

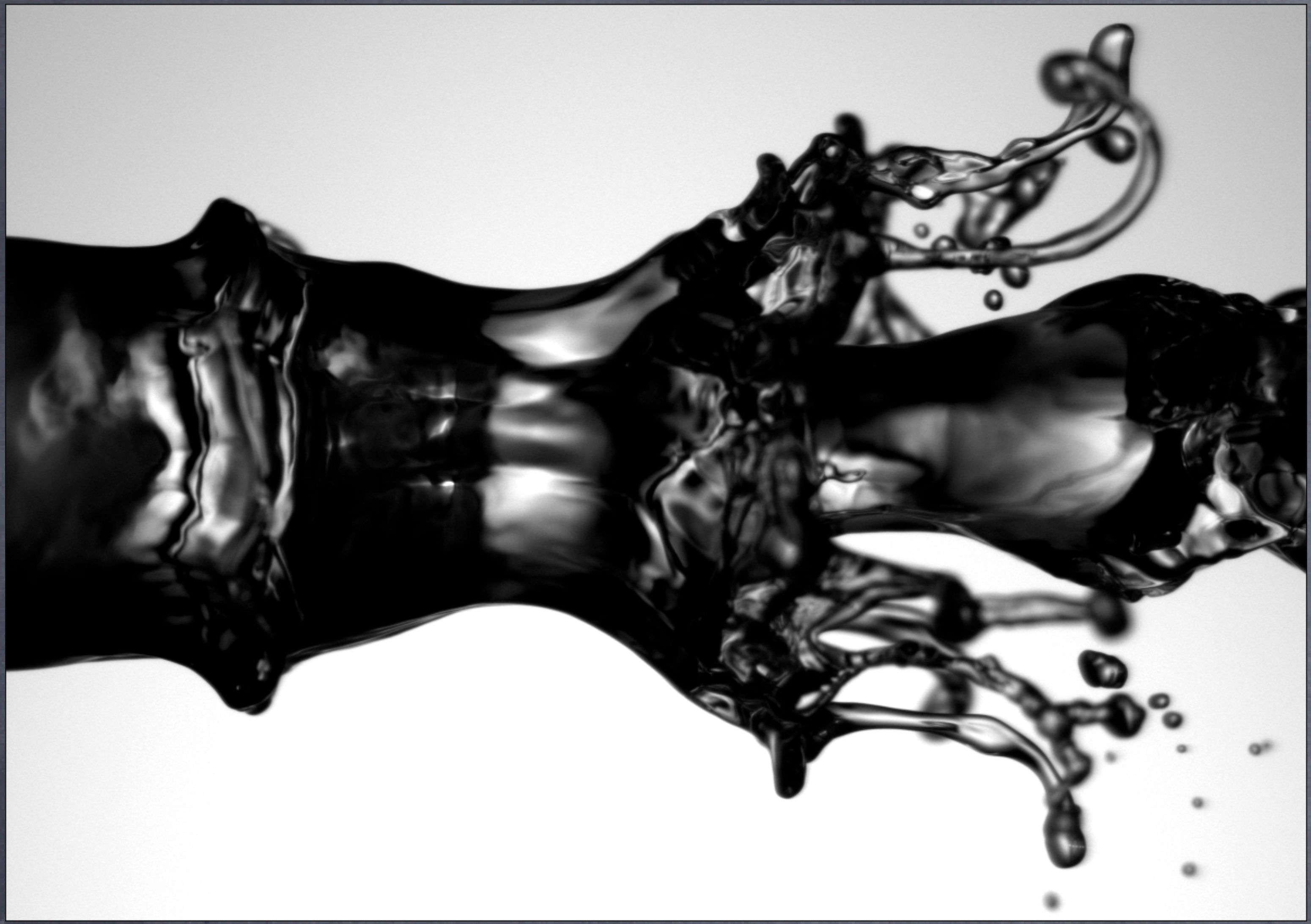
Drop size distribution





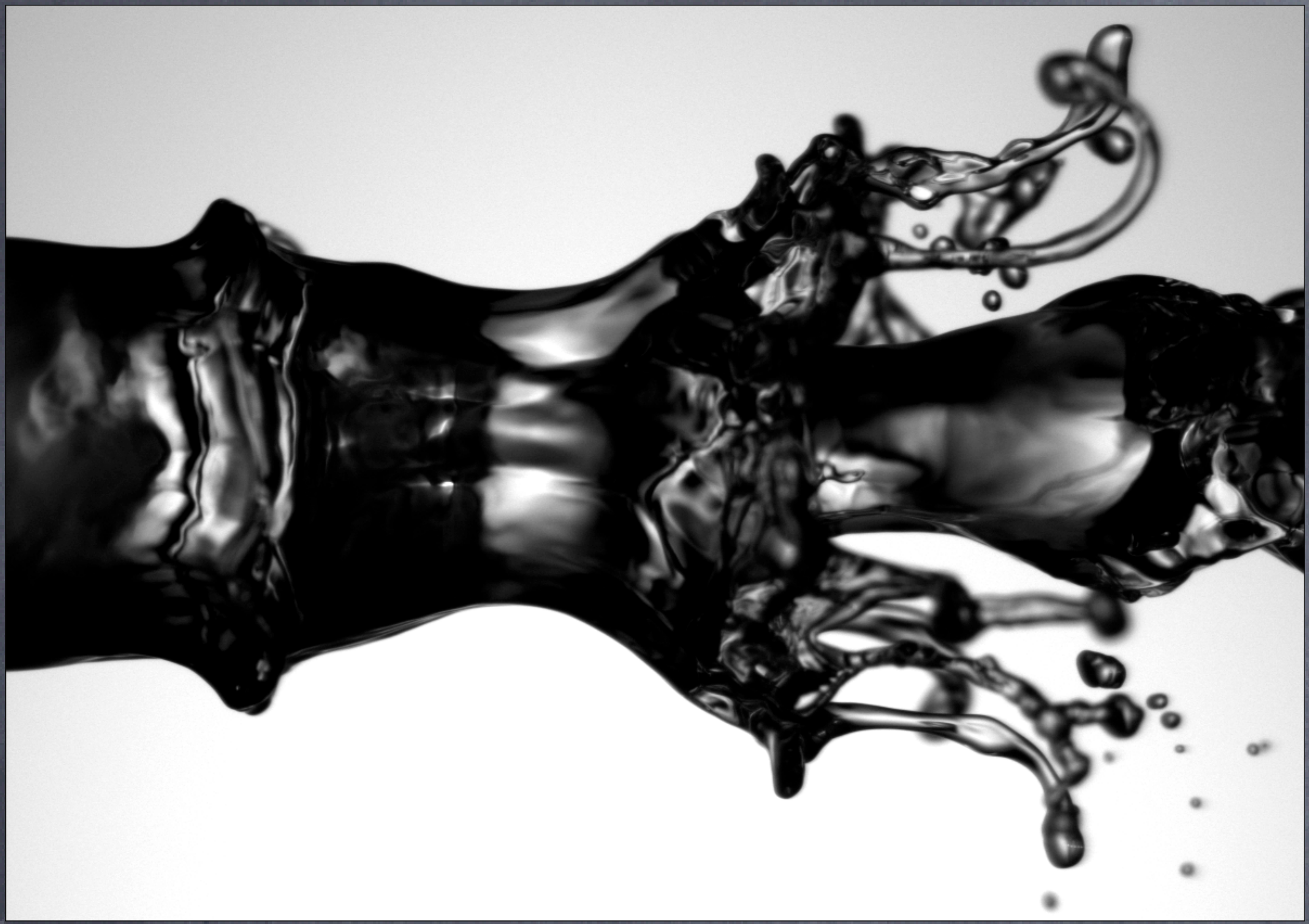
Bentley (1904)

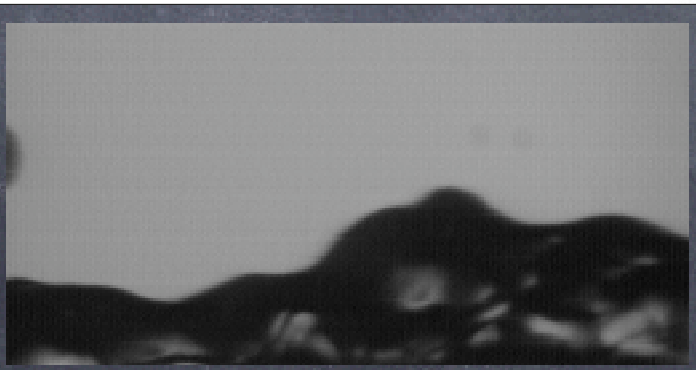
wind induced sprays



Marmottant & Villermaux, JFM (2004)

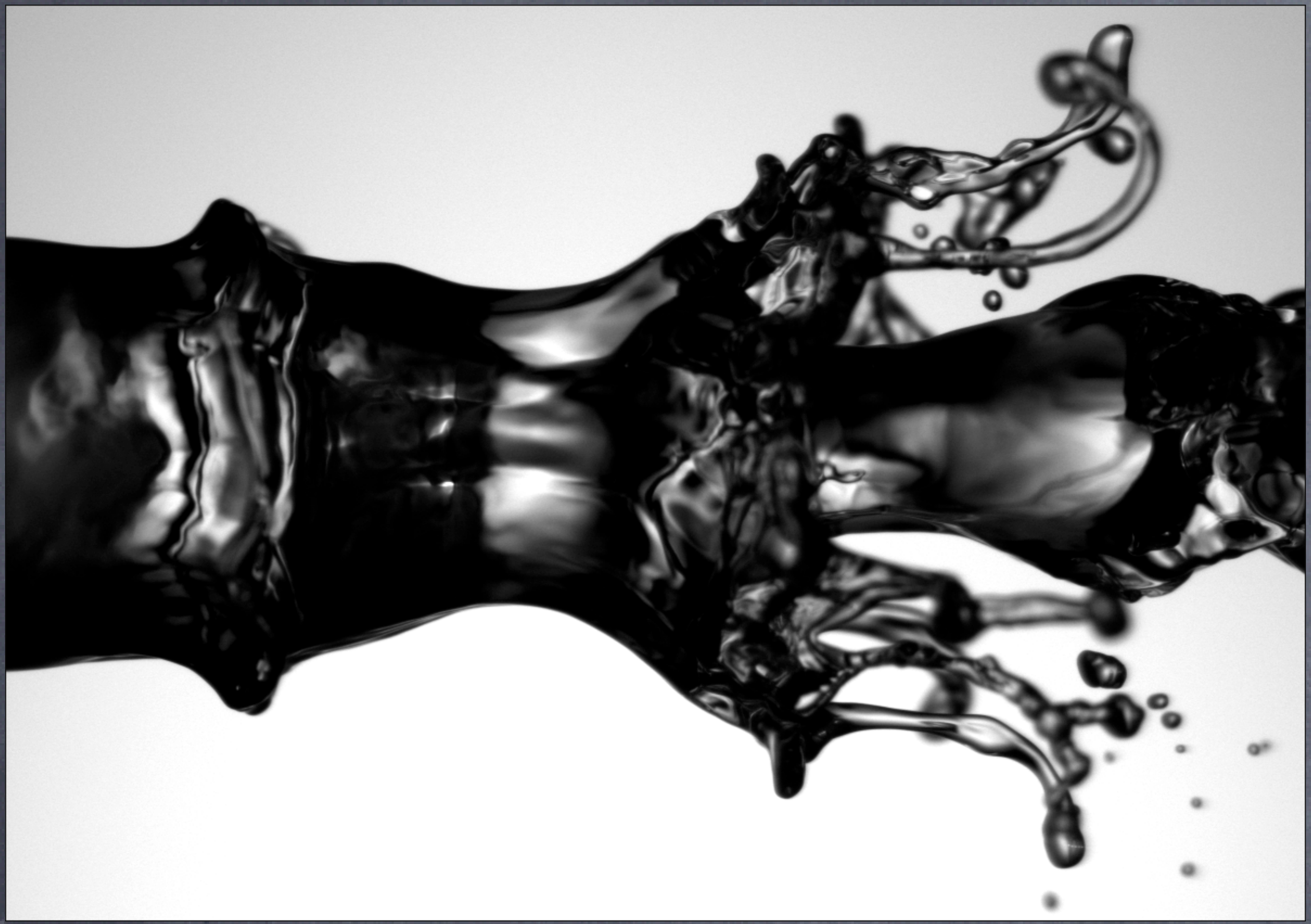
wind induced sprays

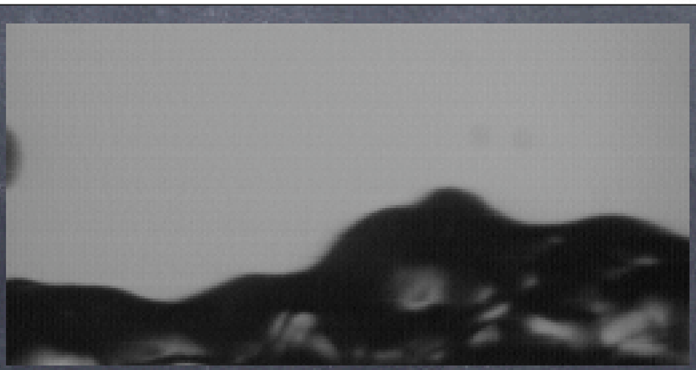


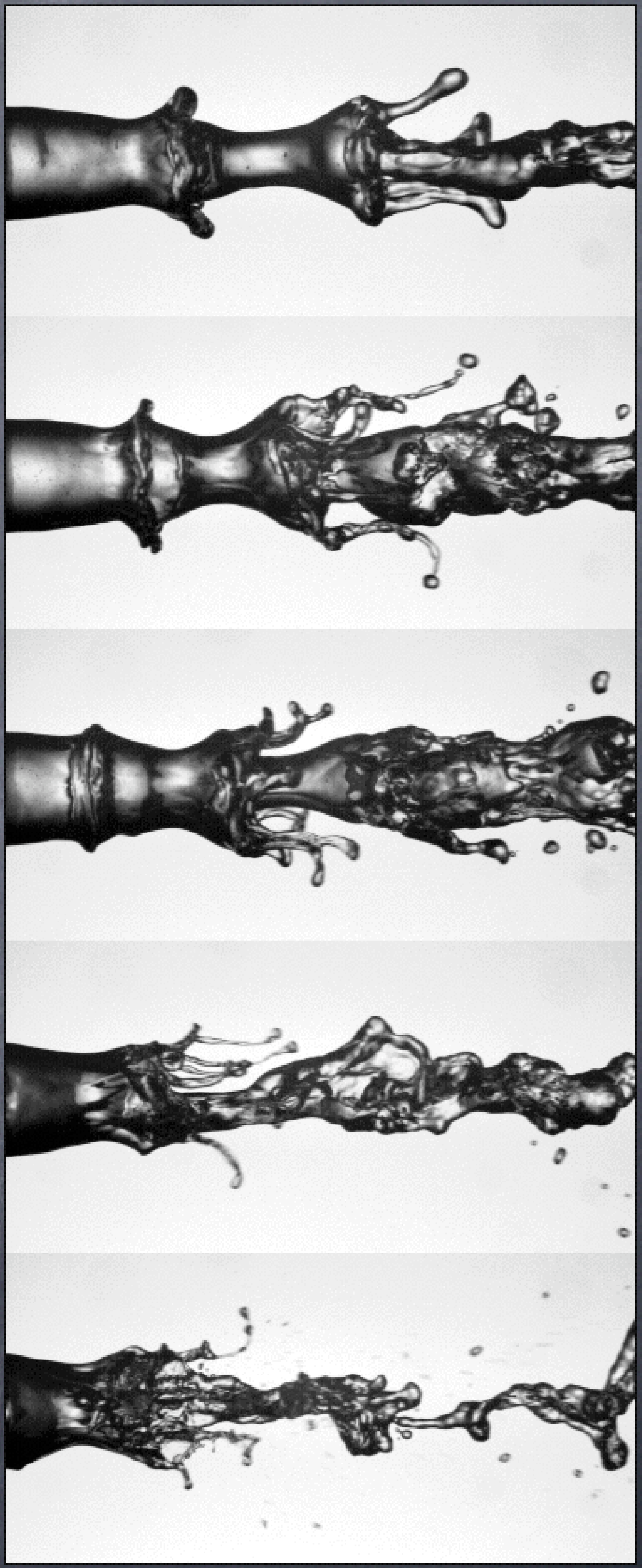


Marmottant & Villermaux, JFM (2004)

wind induced sprays







Marmottant & Villermaux, JFM (2004)

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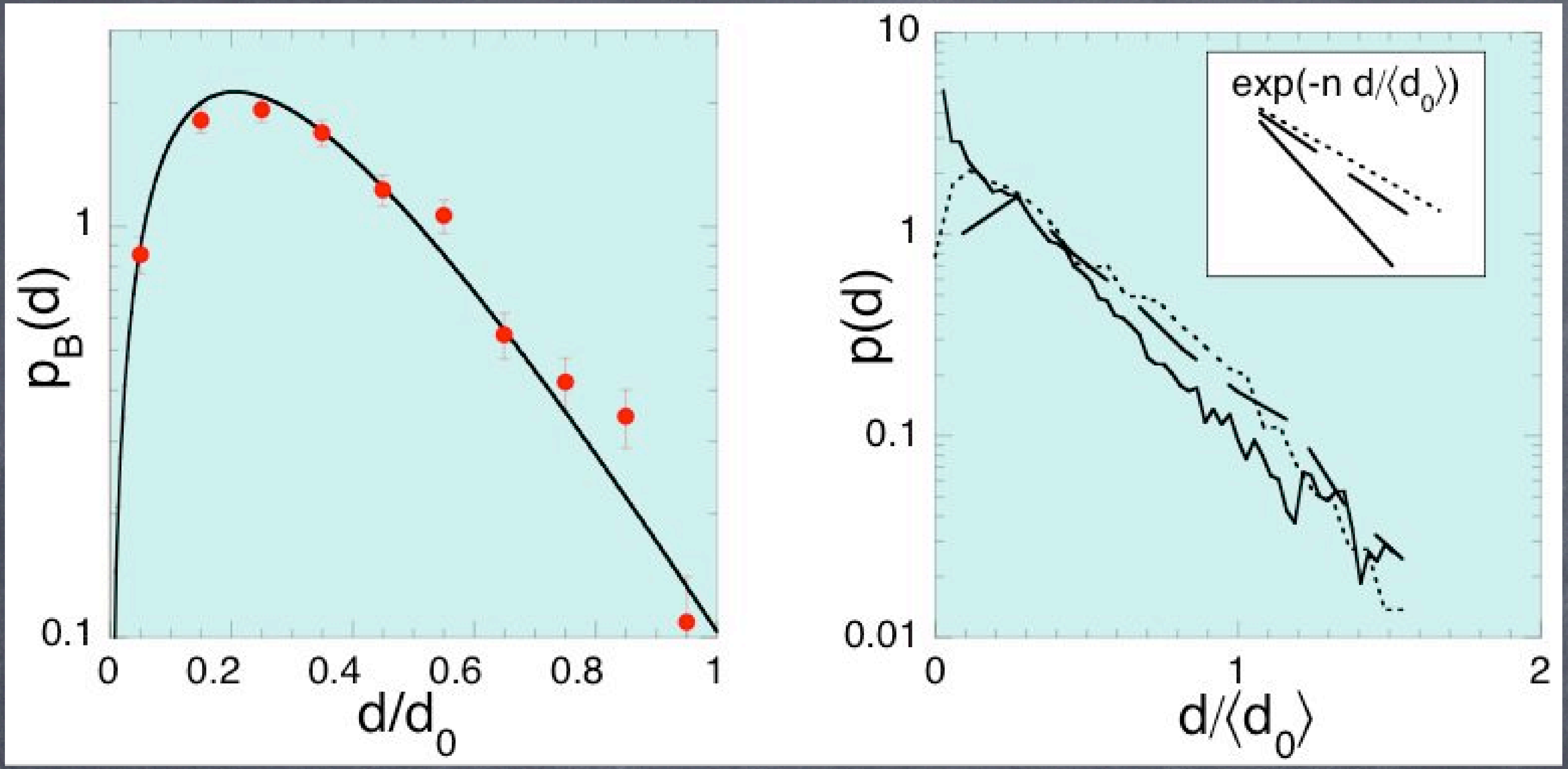
Dynamic capillarity

Drop size distribution



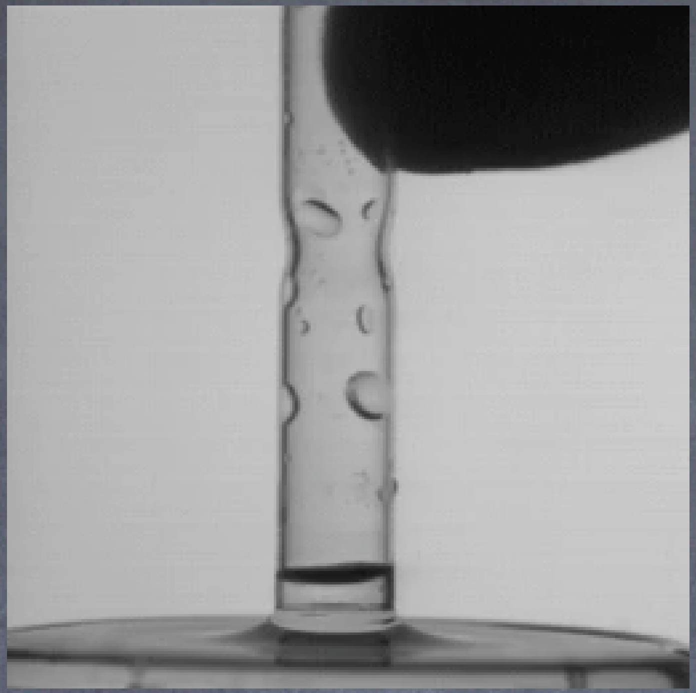
from ligaments

in the spray

$$P(X = d/\langle d\rangle) = \frac{n^n}{\Gamma(n)}\, X^{n-1}\, e^{-nX}$$

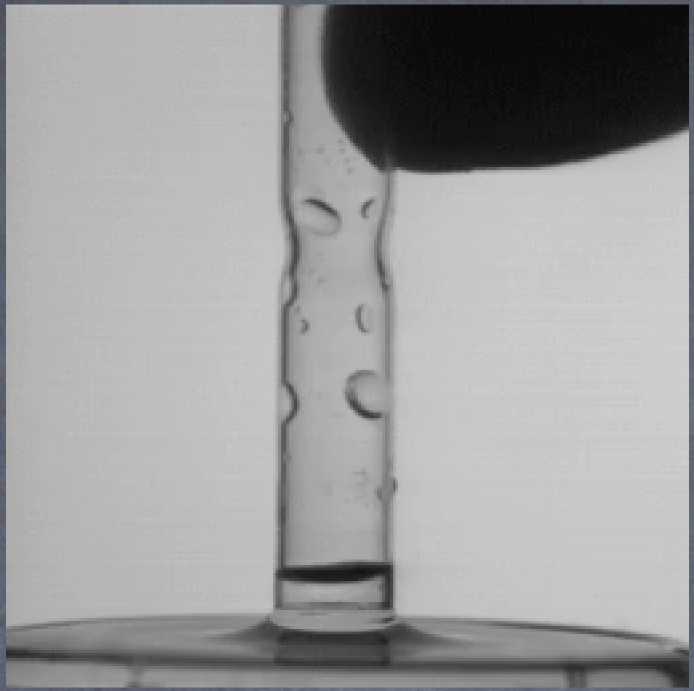
$$n \simeq 4$$

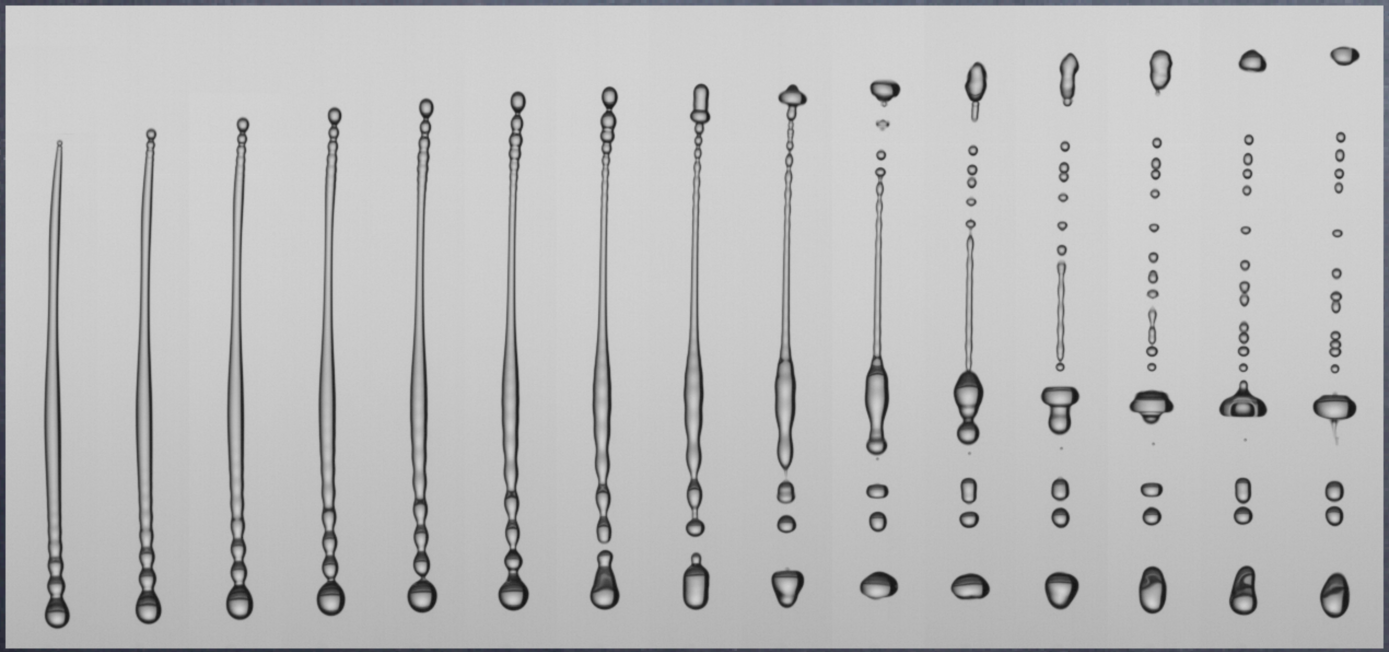
Fast withdrawal



Marmottant & Villermaux, Phys. Fluids (2004)

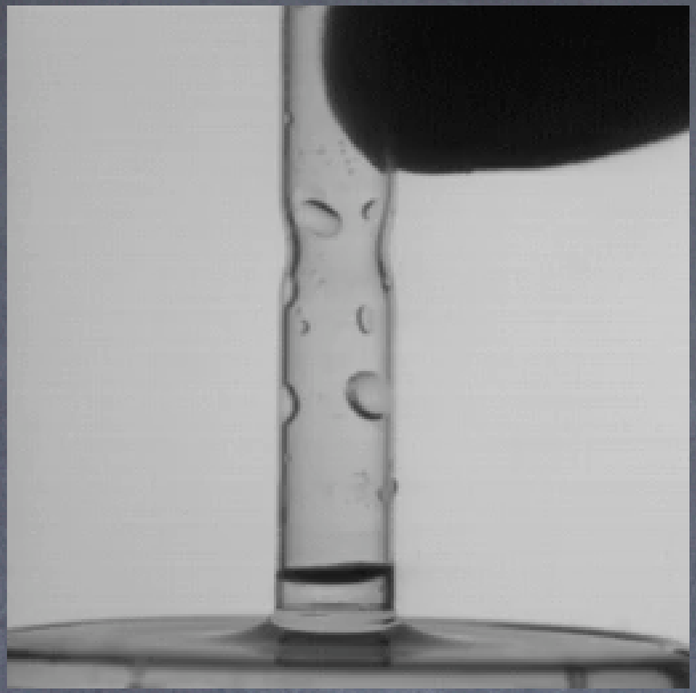
Fast withdrawal

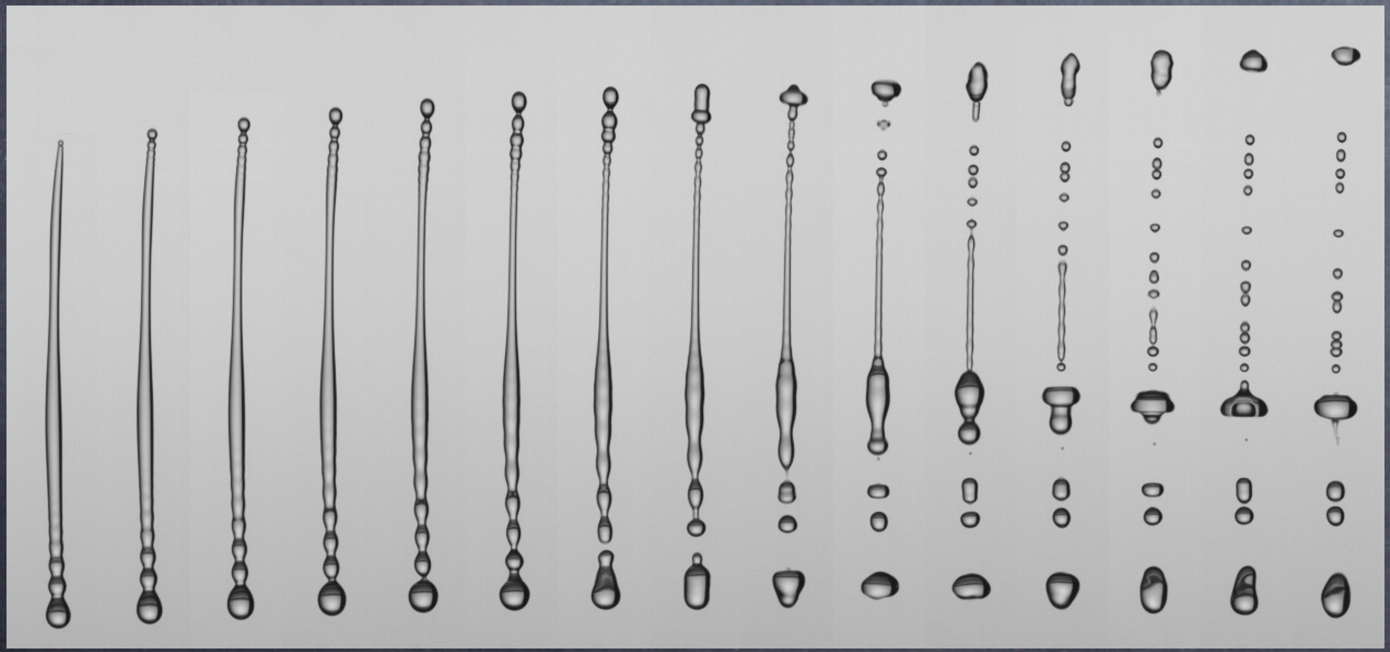


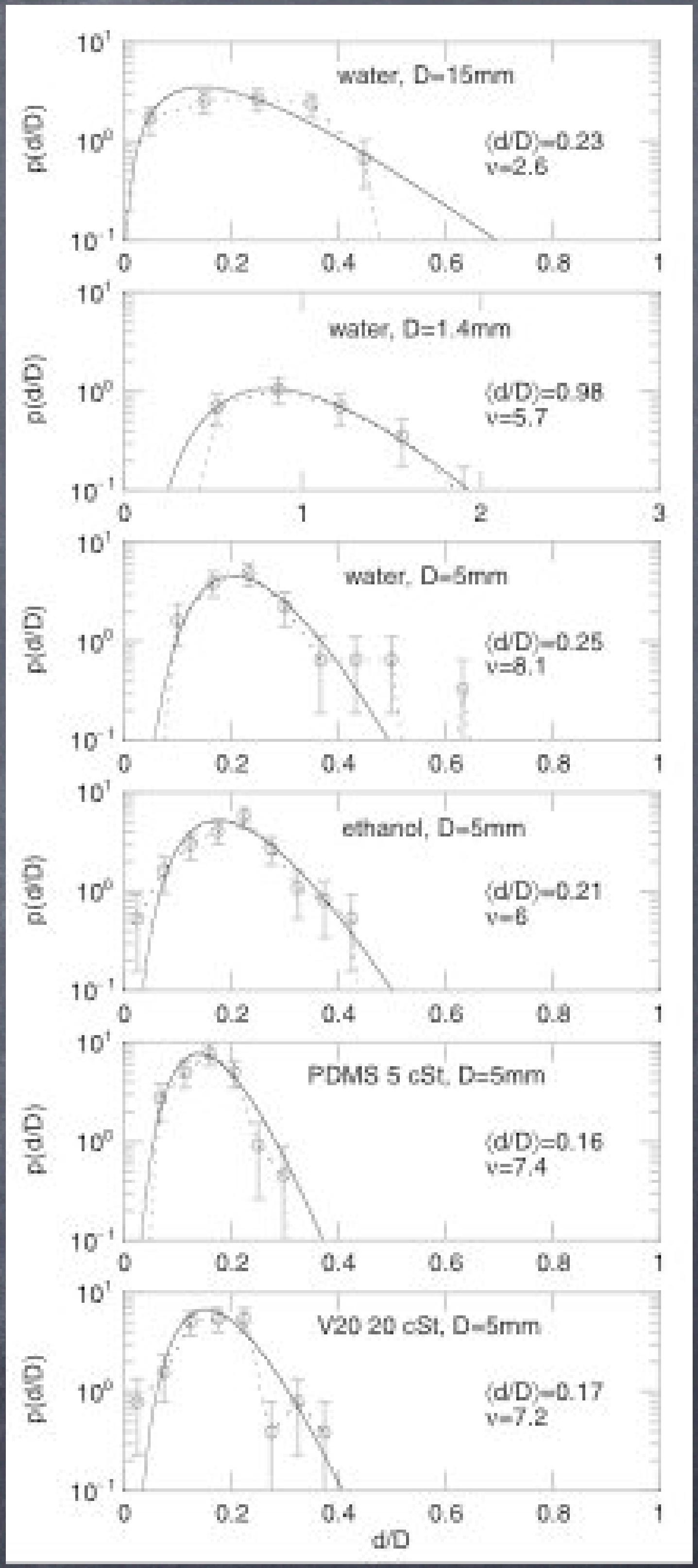


Marmottant & Villermaux, Phys. Fluids (2004)

Fast withdrawal

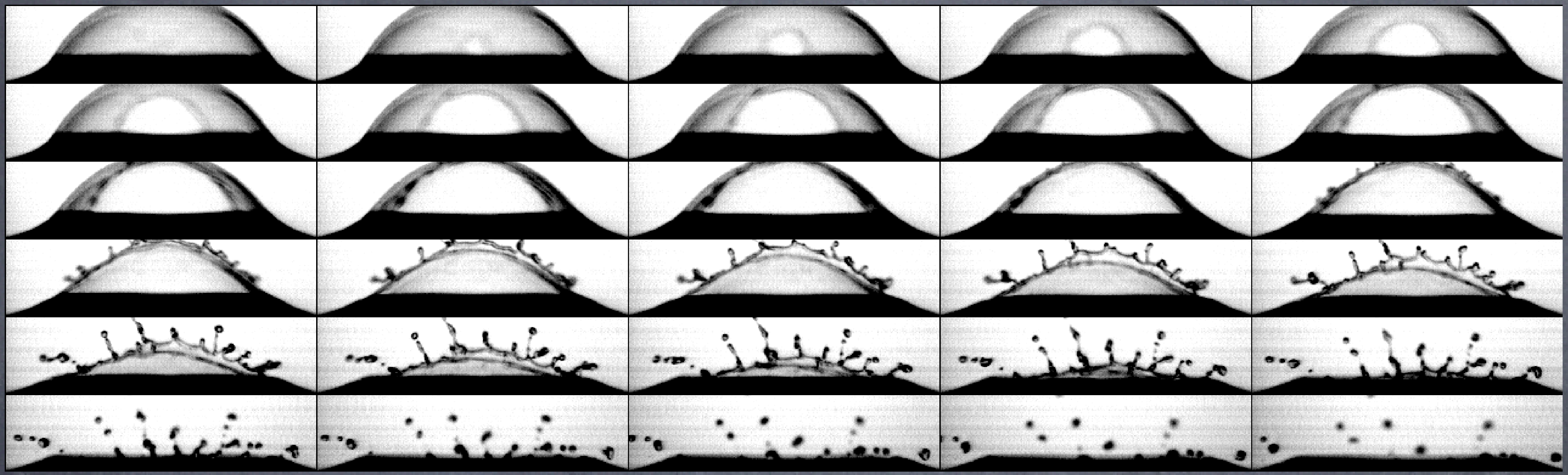




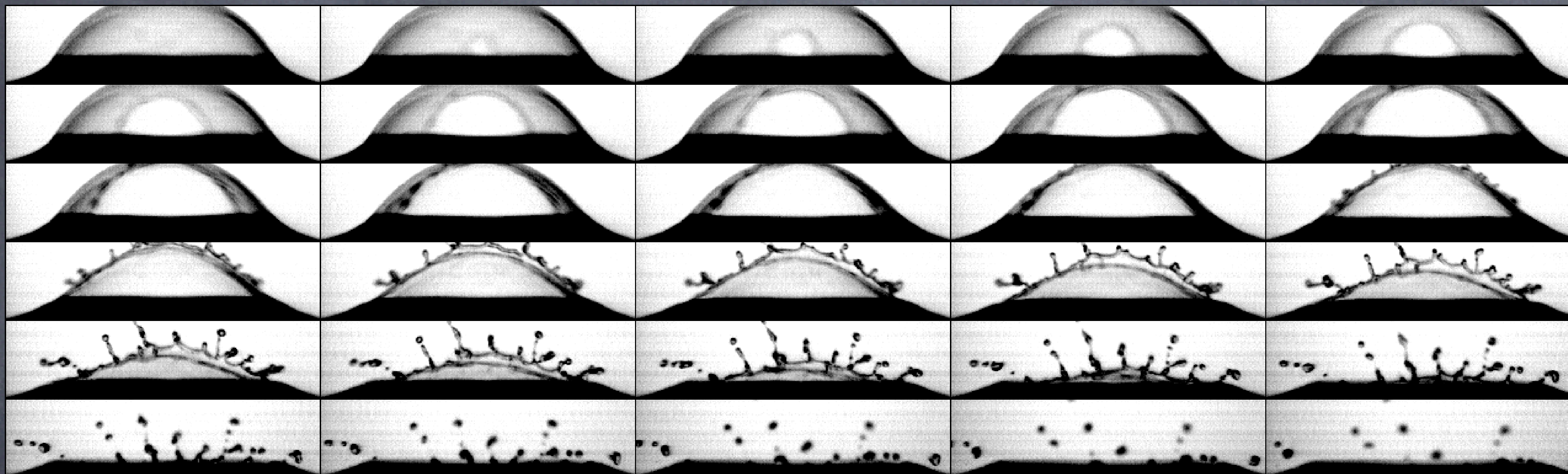


Marmottant & Villermaux, Phys. Fluids (2004)

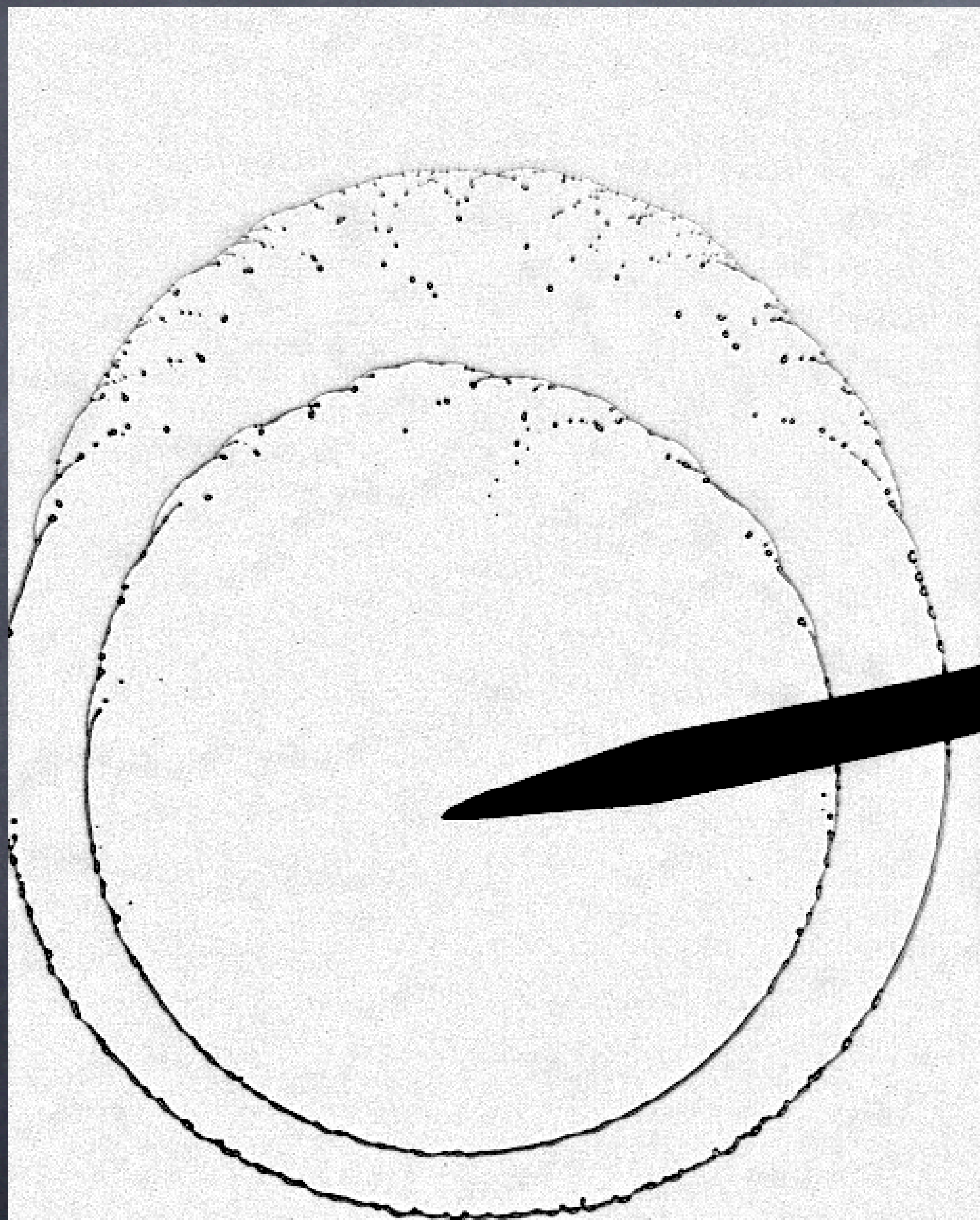
Bursting bubble

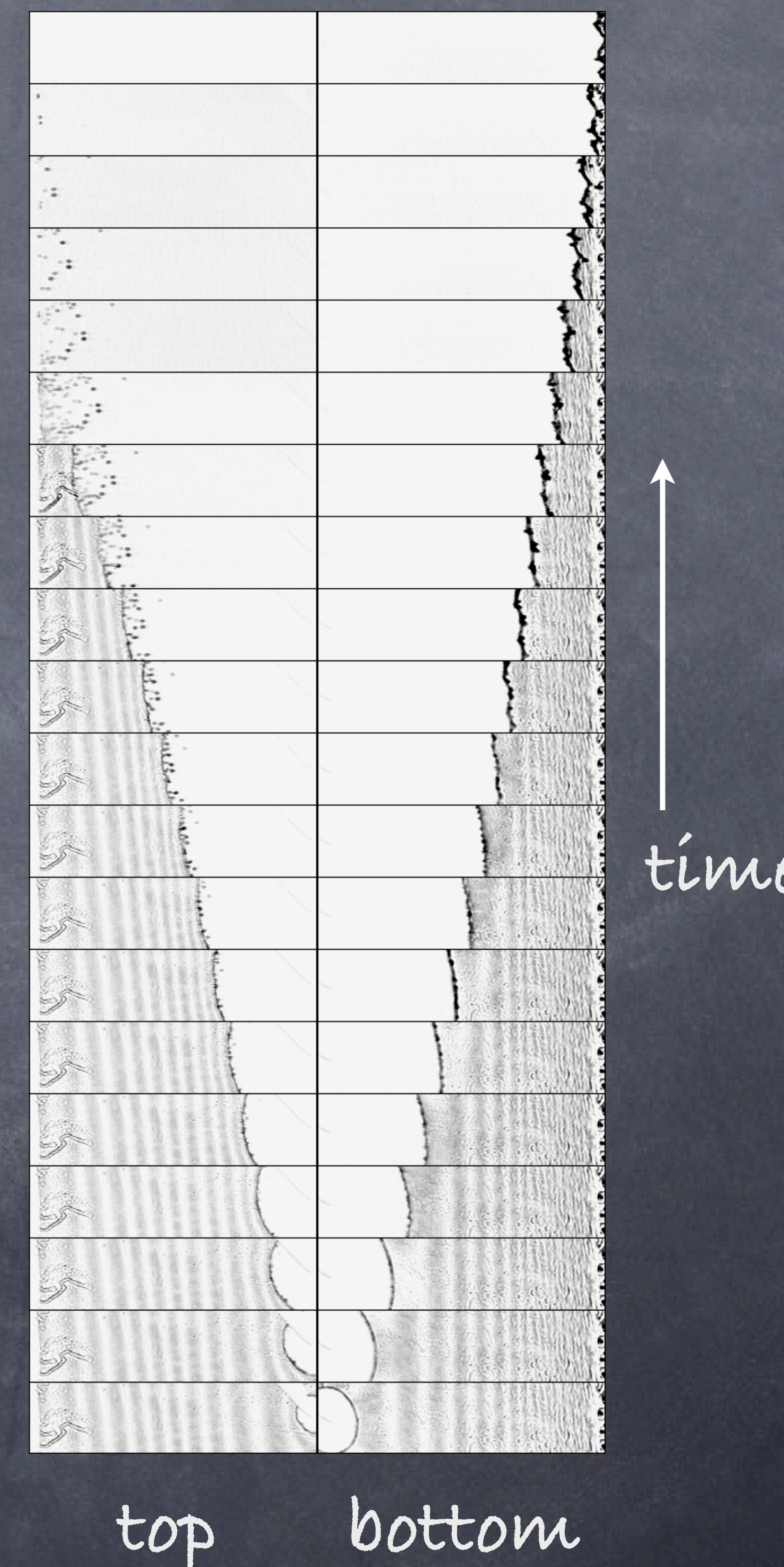


Bursting bubble

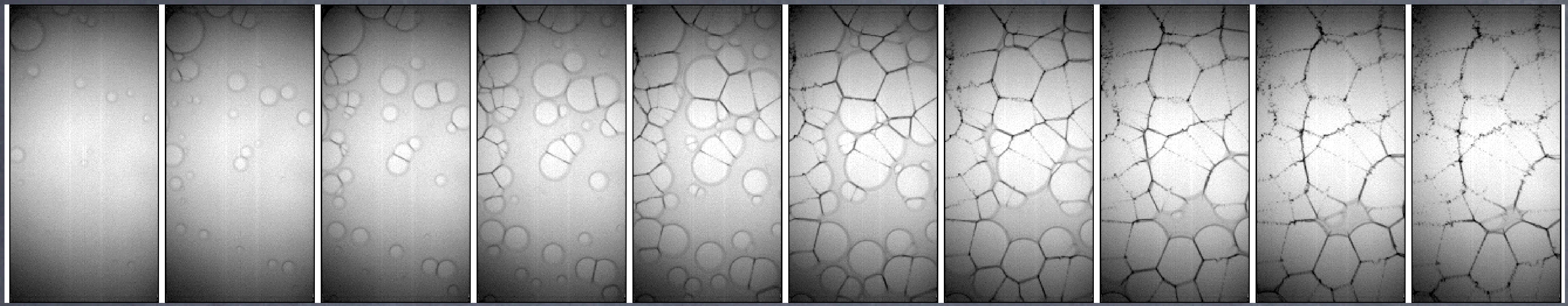


Soap film

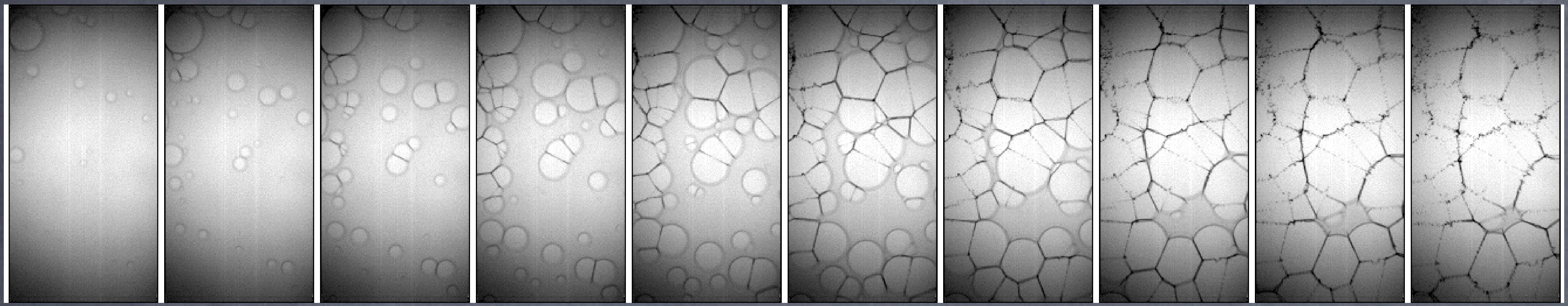




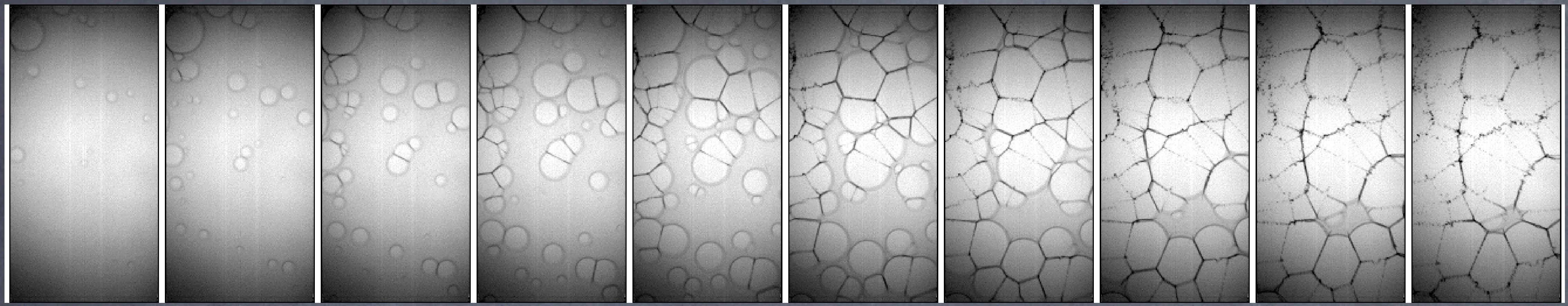
Accelerated thin film



Accelerated thin film

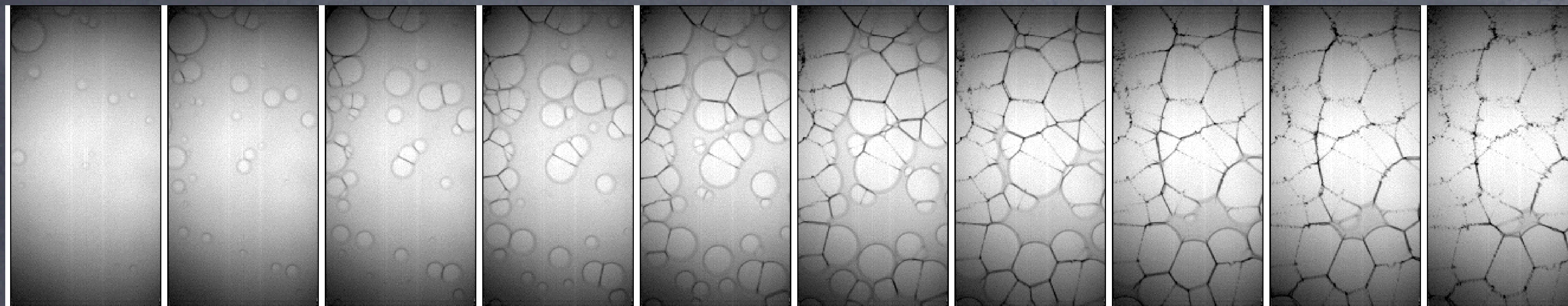


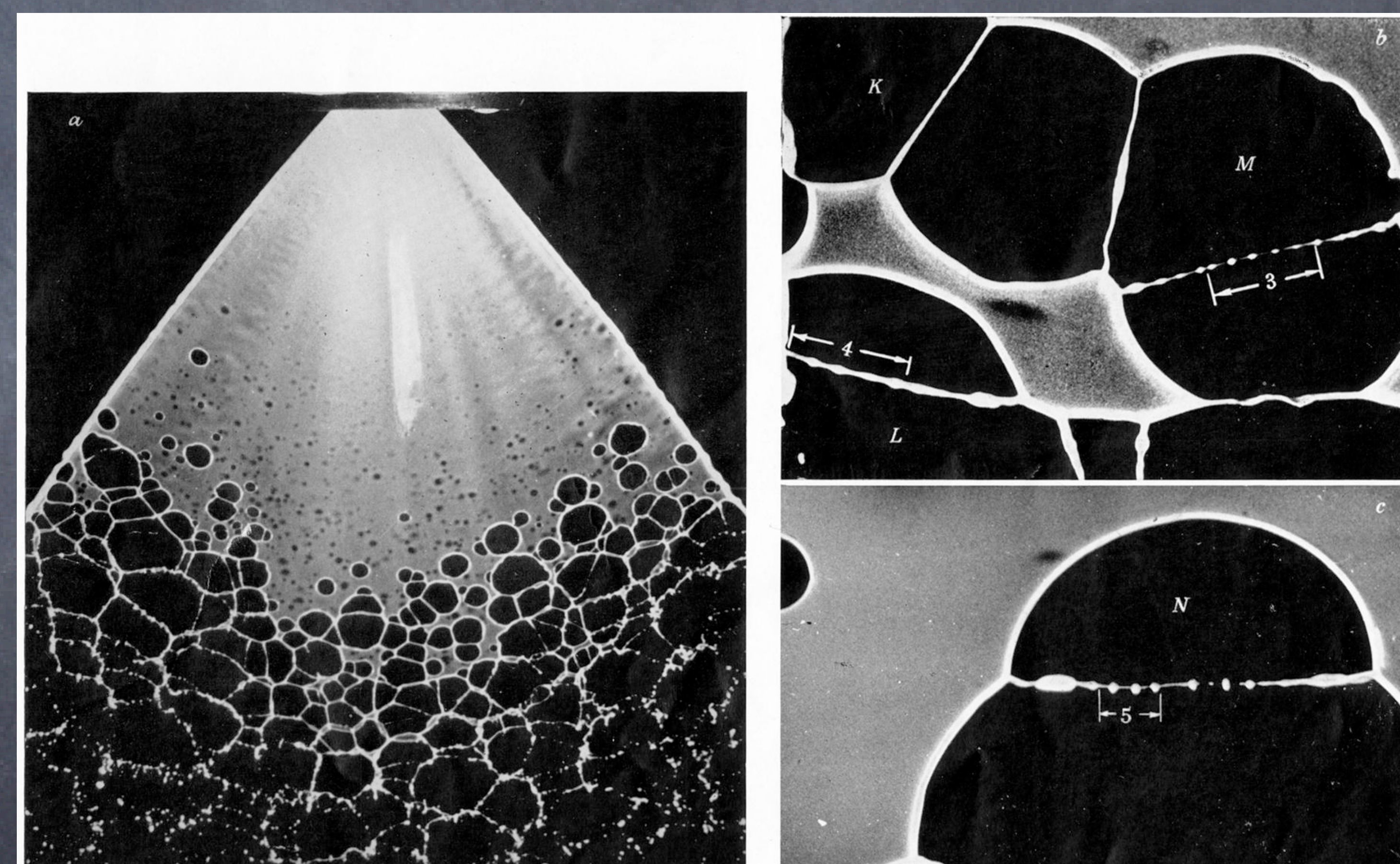
Accelerated thin film



Bremond & Villermaux, JFM (2005)

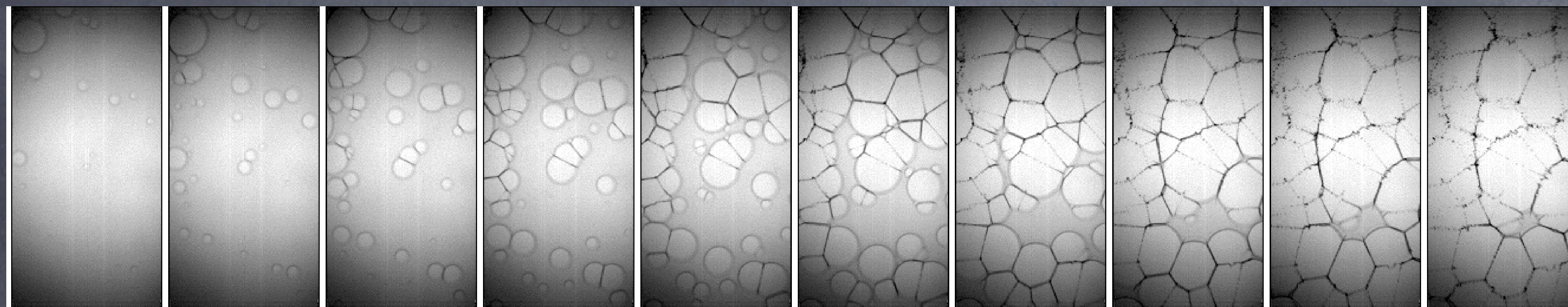
Accelerated thin film

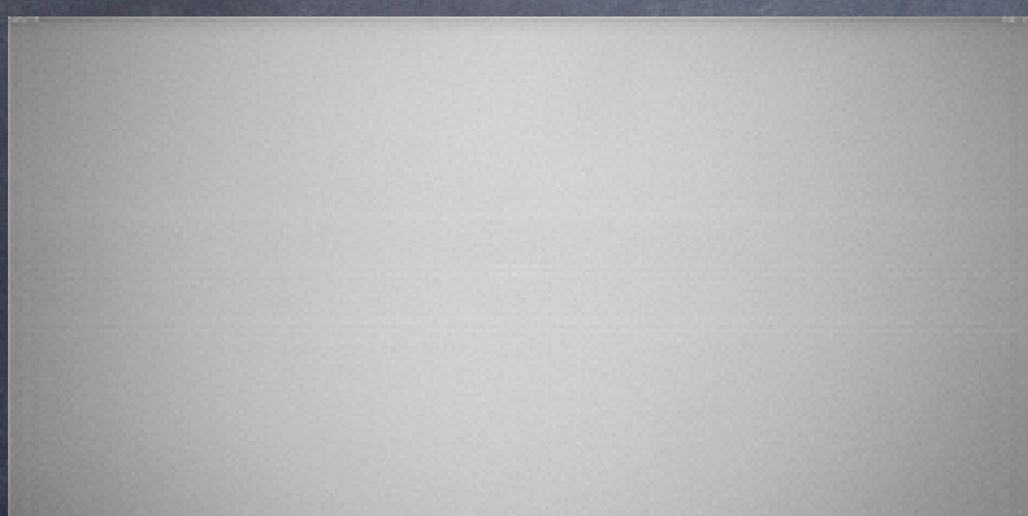


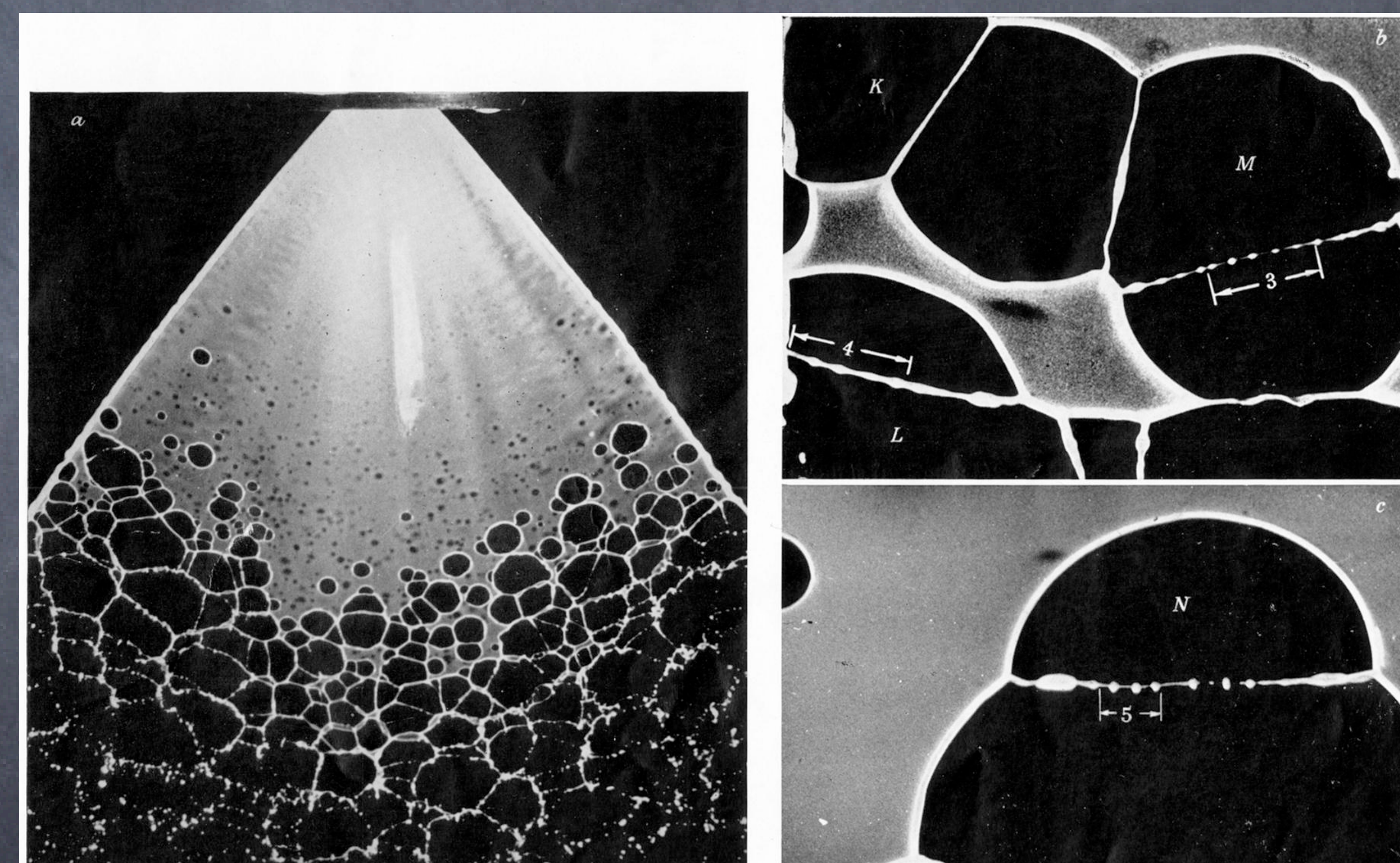


Bremond & Villermaux, JFM (2005)

Accelerated thin film



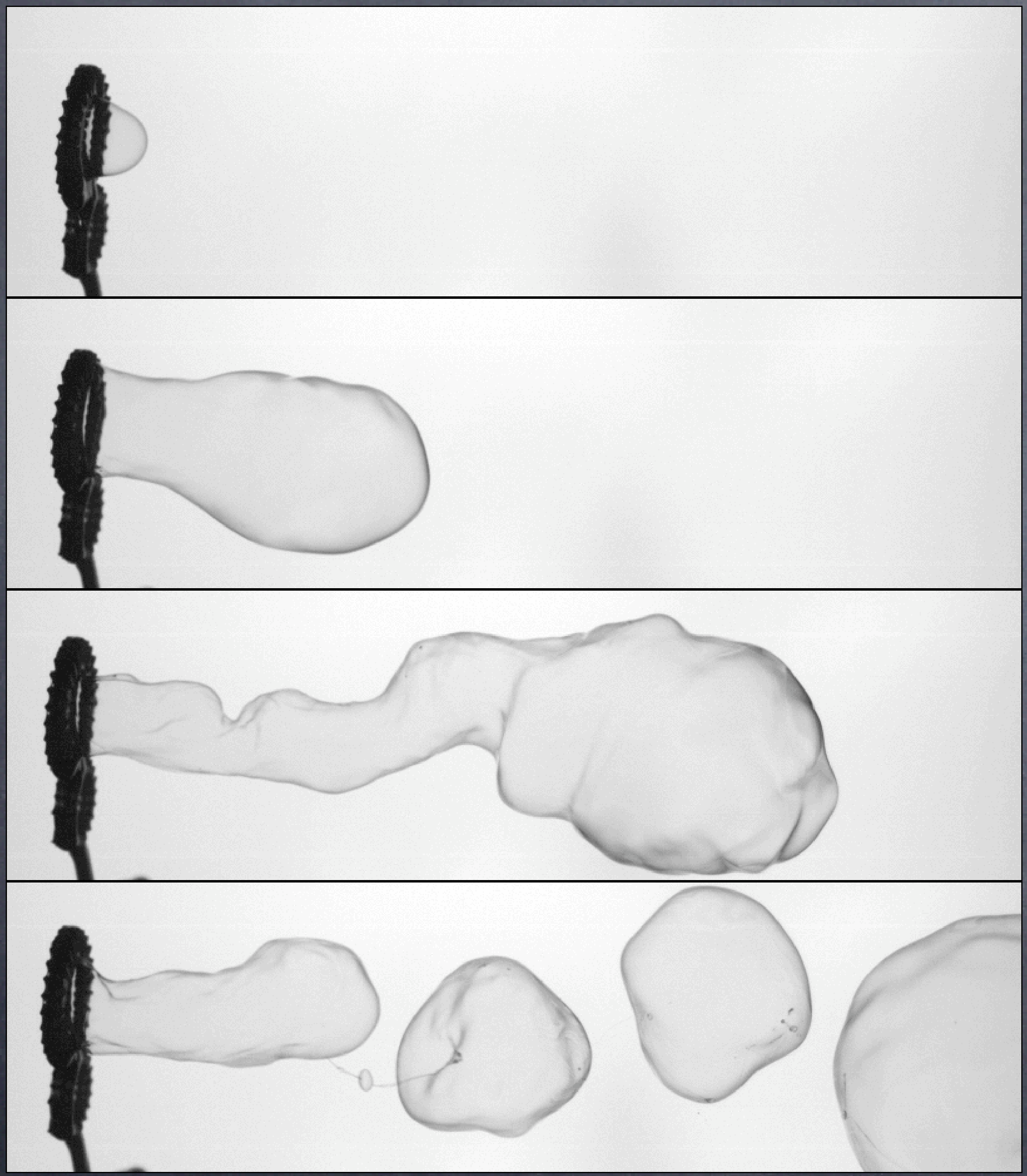




Bremond & Villermaux, JFM (2005)

Dombrowski & Frazer (1954)

Blowing on Soap Films



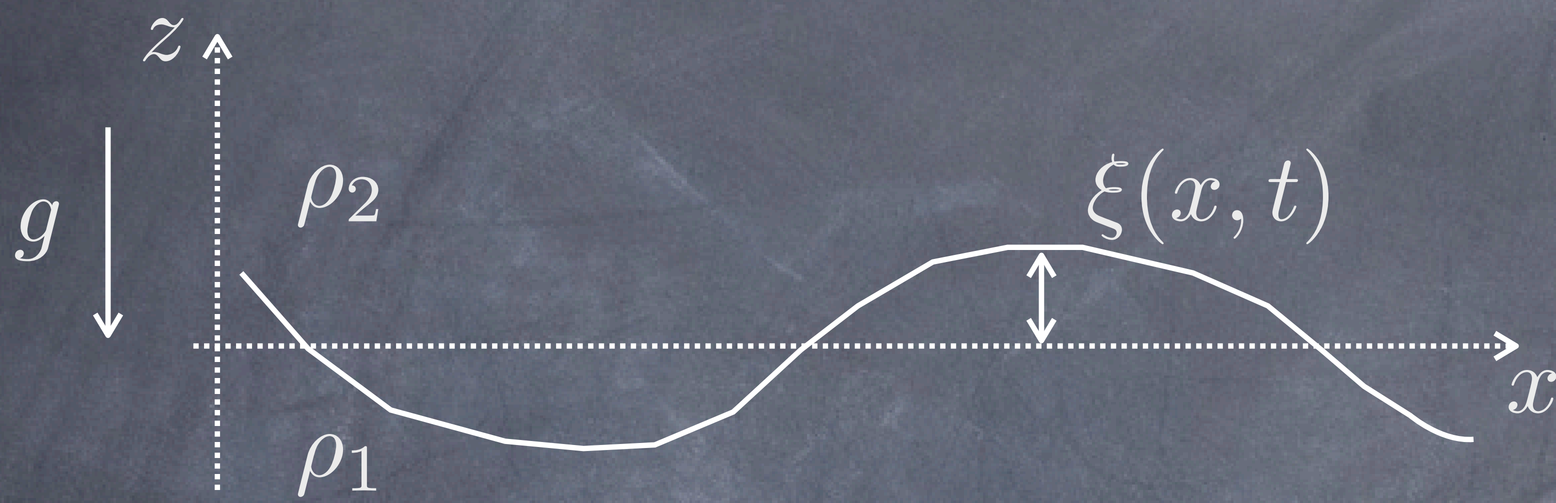
$$\text{Acceleration} \sim \frac{\ell}{\tau^2}$$

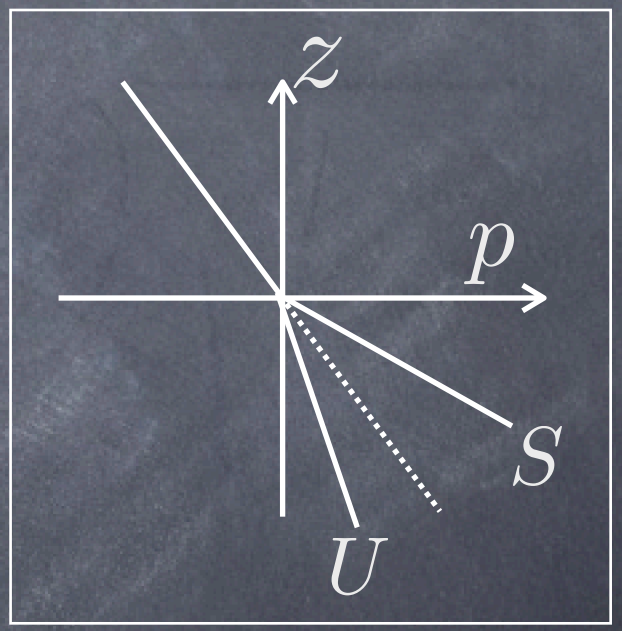
$$\frac{10^{-2}\text{m}}{(10^{-2}\text{s})^2} = 10^2\,\text{m}\,\text{s}^{-2}$$

$$= 10\,g\,!$$

Mechanism:

$$p_i = -\rho_i g z\big|_{i=1,2}$$





Energy:

$$\begin{aligned} E &= -\int_0^{\xi} (p_1 - p_2)\mathrm{d}z \\ &= \int_0^{\xi} (\rho_1 - \rho_2) g z \mathrm{d}z \\ &= \frac{1}{2}(\rho_1 - \rho_2) g \xi^2 < 0 \quad \text{if} \quad \rho_2 > \rho_1 \end{aligned}$$

with surface tension:

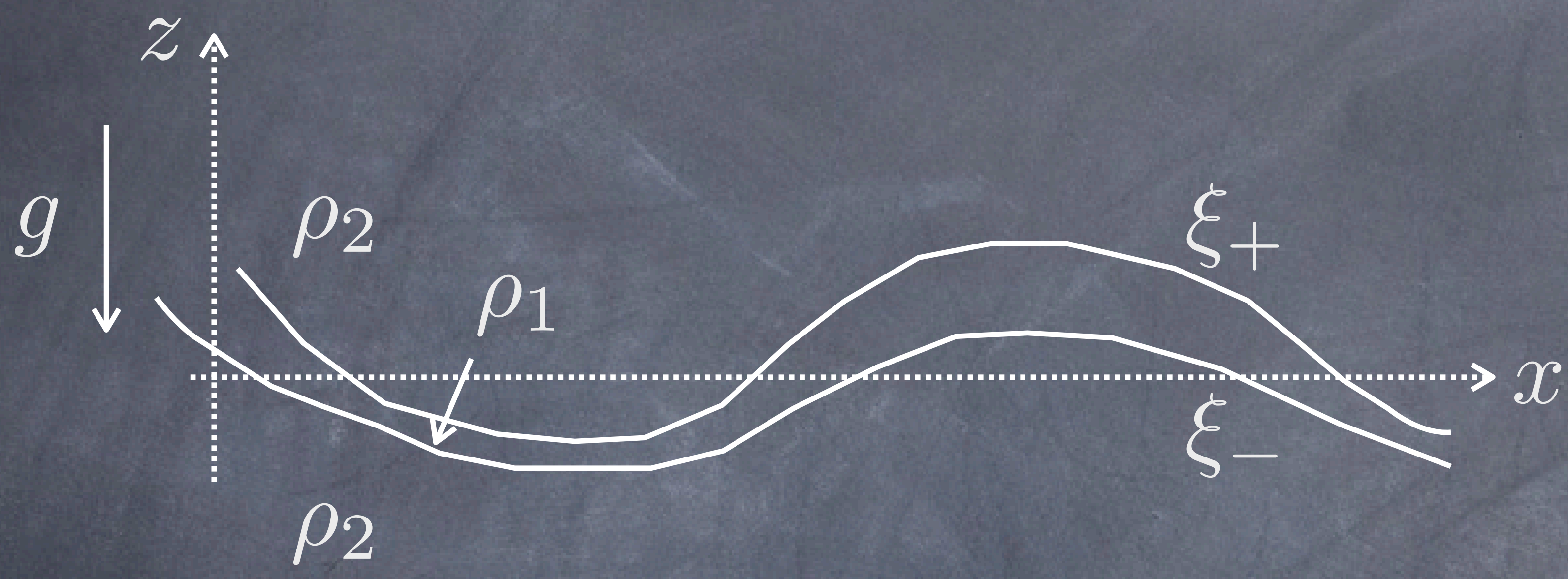
$$
\begin{aligned}
E &= \frac{1}{2}(\rho_1 - \rho_2) g \xi^2 + \sigma\{\sqrt{1+\xi'^2} - 1\} \\
&\approx \frac{1}{2}(\rho_1 - \rho_2) g \xi^2 + \frac{1}{2}\sigma\xi'^2
\end{aligned}
$$

$$\xi(x,t) \sim e^{ikx - i\omega t}$$

Mode selection: $k_c = \sqrt{\frac{\Delta\rho g}{\sigma}}$

Growth rate: $\mathrm{Re}(-i\omega) \sim \sqrt{g k_c}$

Layer with finite thickness



$$\xi_\pm \sim e^{ikx-i\omega t}$$

Energy:
$(\rho_2 = 0\,; \rho_1 \equiv \rho)$

$$E = \int \mathrm{d}x \Big\{ \int_{-h/2+\xi_-}^{h/2+\xi_+} \rho g z \mathrm{d}z$$
$$+\sigma \left(\sqrt{1+|\xi_+'^2|} - 1 + \sqrt{1+|\xi_-'^2|} - 1 \right) \Big\}$$

E<0 if:

$$\xi_-^2 - \xi_+^2 > (k/k_\mathrm{c})^2(\xi_-^2 + \xi_+^2)$$

if: $kh \gg 1$ then no coupling (Taylor 1950)

if: $kh \ll 1$ let $\xi_+/\xi_- \sim 1 - kh$

then $k_\mathrm{m} \sim k_\mathrm{c}^2 h$

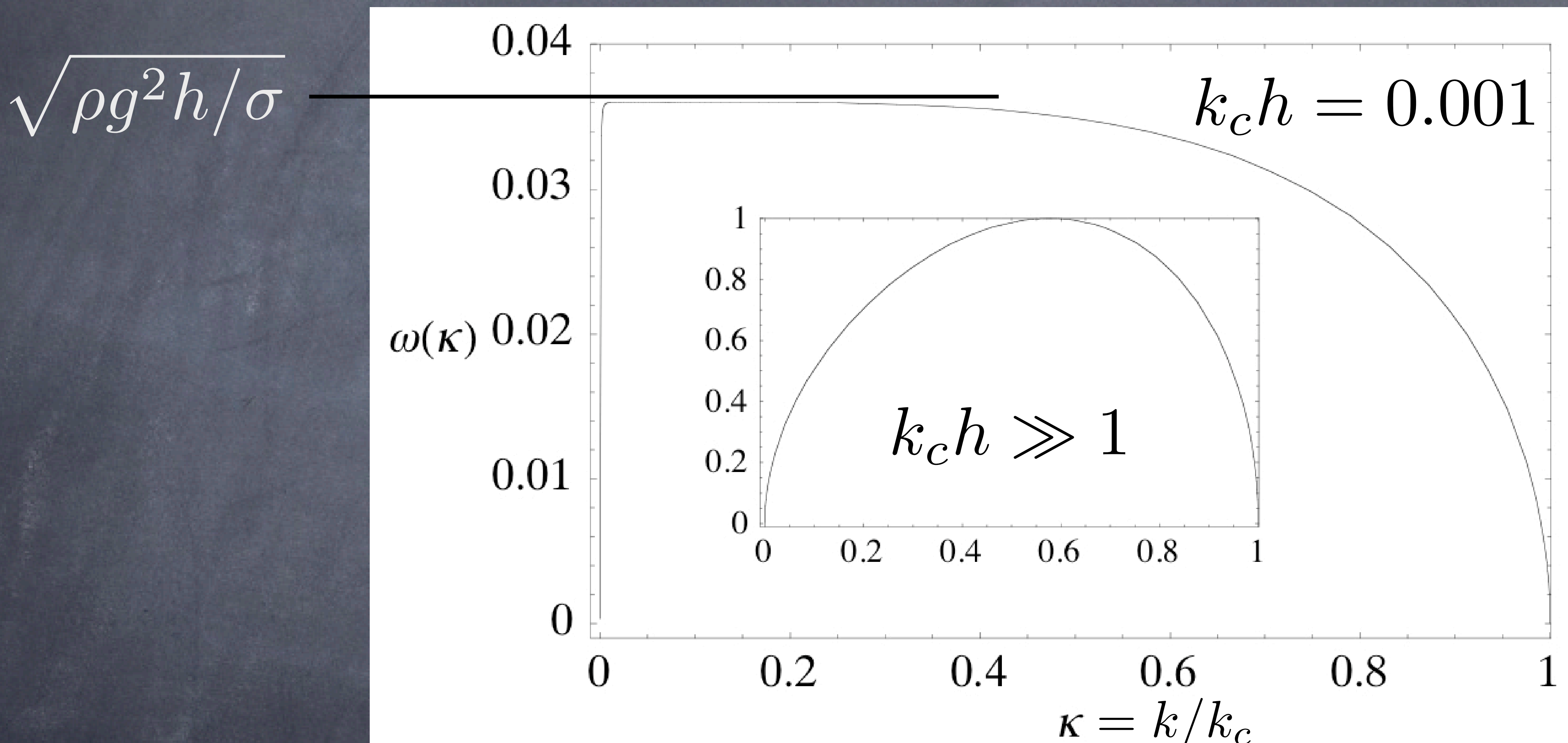
Growth rate $\sqrt{g k_\mathrm{m}} \sim \sqrt{\rho g^2 h/\sigma}$

Full dispersion equation
(Keller-Kolodner (1954))

$$
\begin{aligned}
\omega^2 &= k^3 \coth(k) \left\{ 1 \pm \left[1 - \left(1 - \left(\frac{k_c}{k} \right)^4 \right) \tanh^2(k) \right]^{1/2} \right\} \\
k &\equiv kh \\
\omega &\equiv \omega \sqrt{\rho h^3 / \sigma}
\end{aligned}
$$

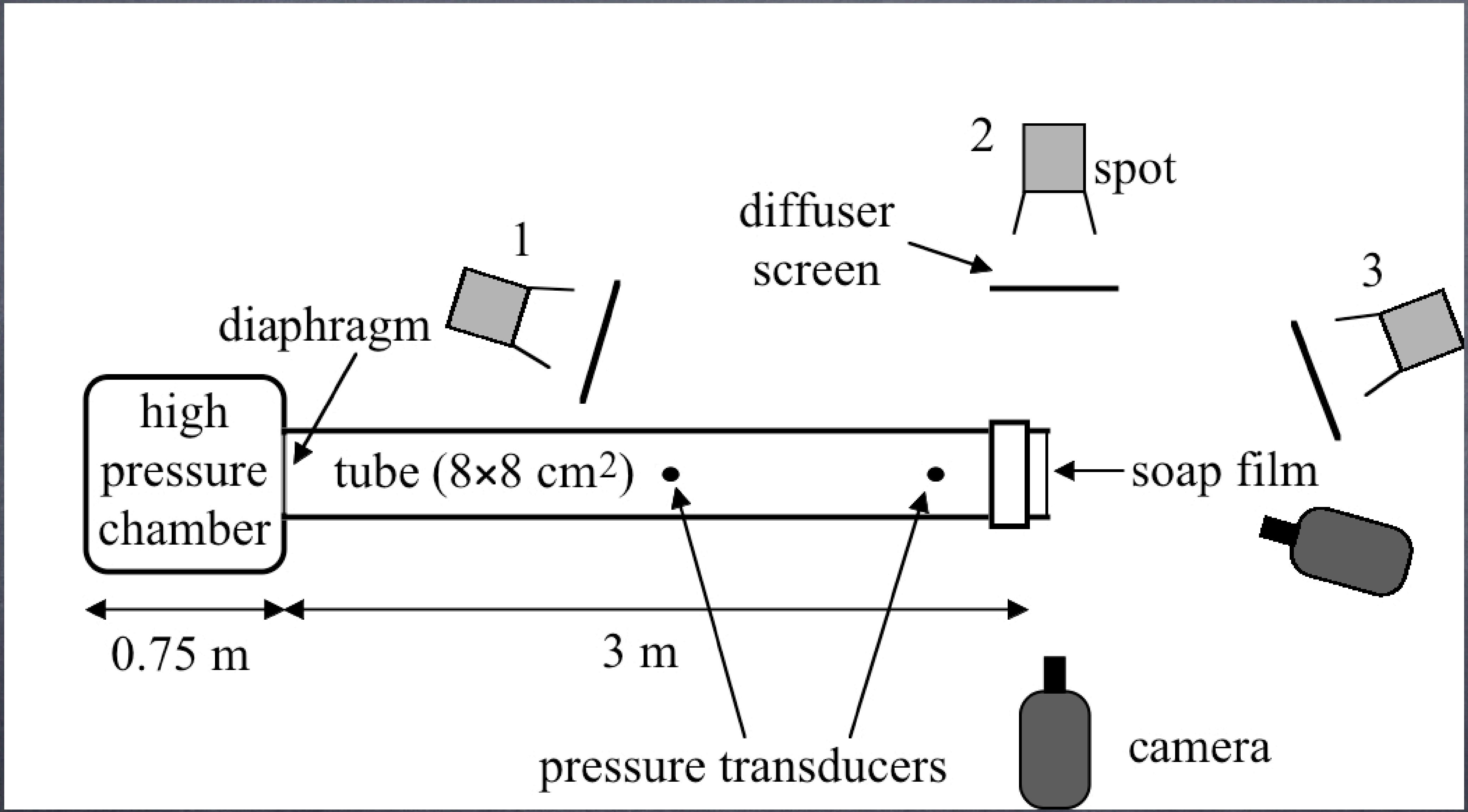
For $k_c \gg 1$, then $\omega^2 = k^3\{1 - (k_c/k)^2\}$

For $k_c \ll 1$, then $\omega^2 = k^4/2\{1 - (k_c/k)^4\}$

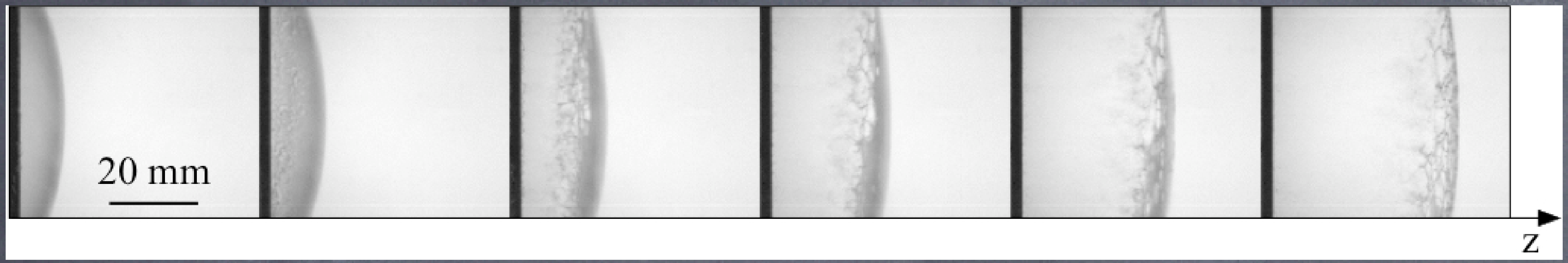


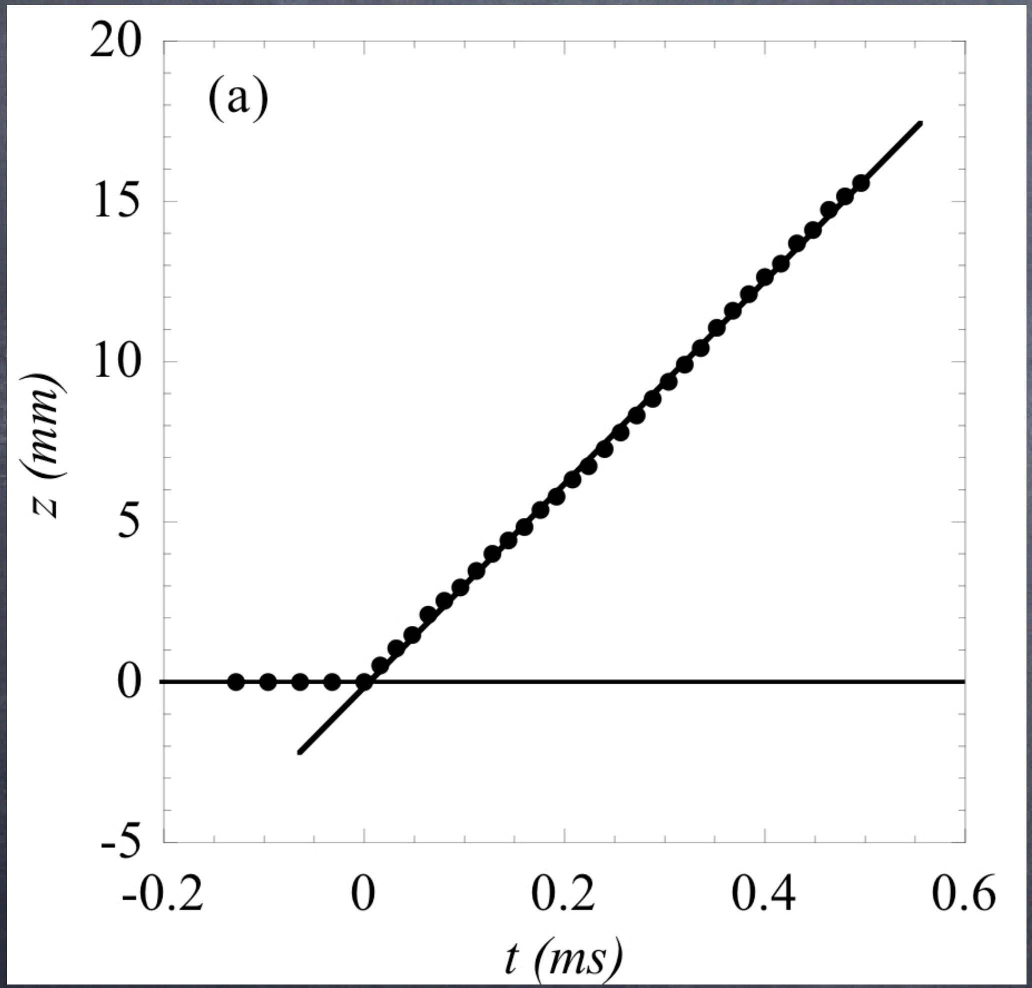
$\sqrt{\rho g^2 h/\sigma}$
$k_c h = 0.001$
$\omega(\kappa)$
0.04
0.03
0.02
0.01
0
0.2
0.4
0.6
0.8
1
$\kappa = k/k_c$
$k_c h \gg 1$

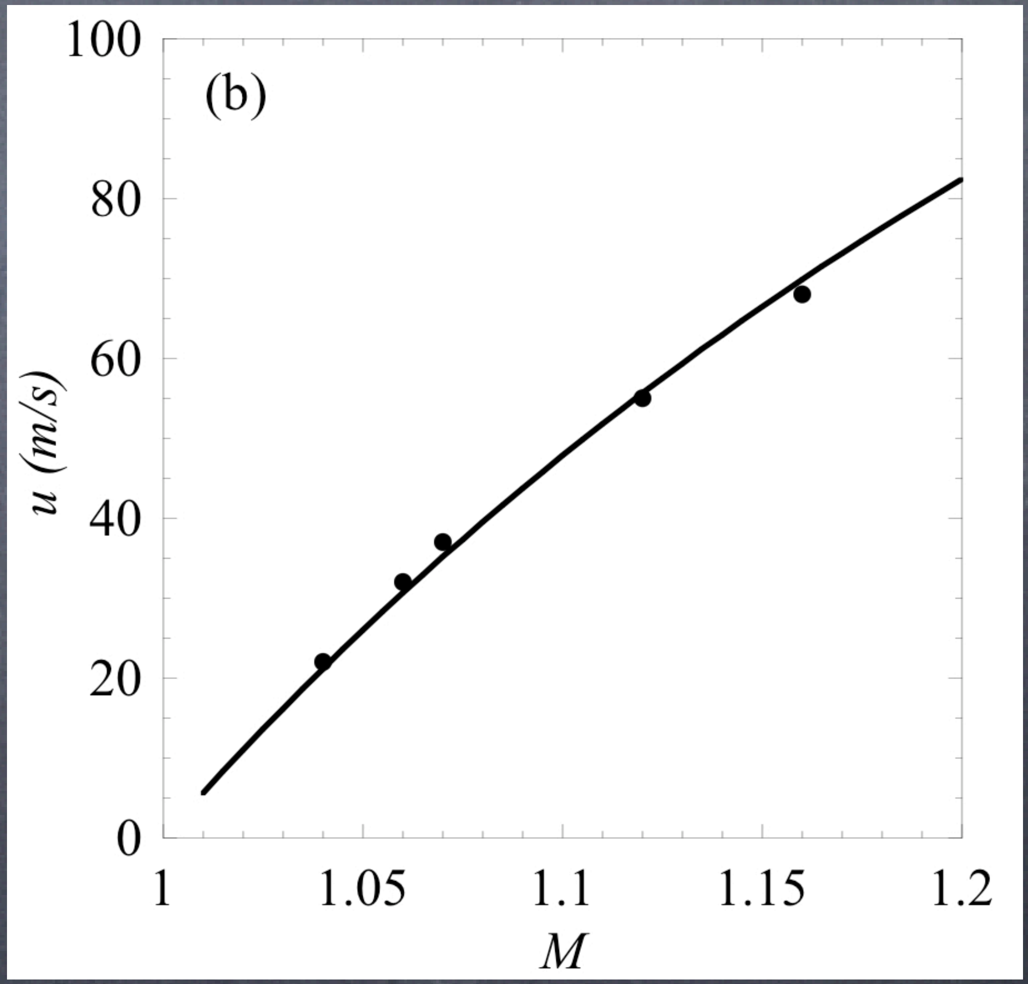
Set-up



2
spot
diffuser
screen
1
3
diaphragm
high
pressure
chamber
tube (8×8 cm2)
soap film
0.75 m
3 m
pressure transducers
camera


20 mm
z


(a)
z (mm)
t (ms)
20
15
10
5
0
-5
-0.2
0
0.2
0.4
0.6


(b)
u (m/s)
M
100
80
60
40
20
0
1
1.05
1.1
1.15
1.2

Accelerated thin film

Accelerated thin film

Accelerated thin film

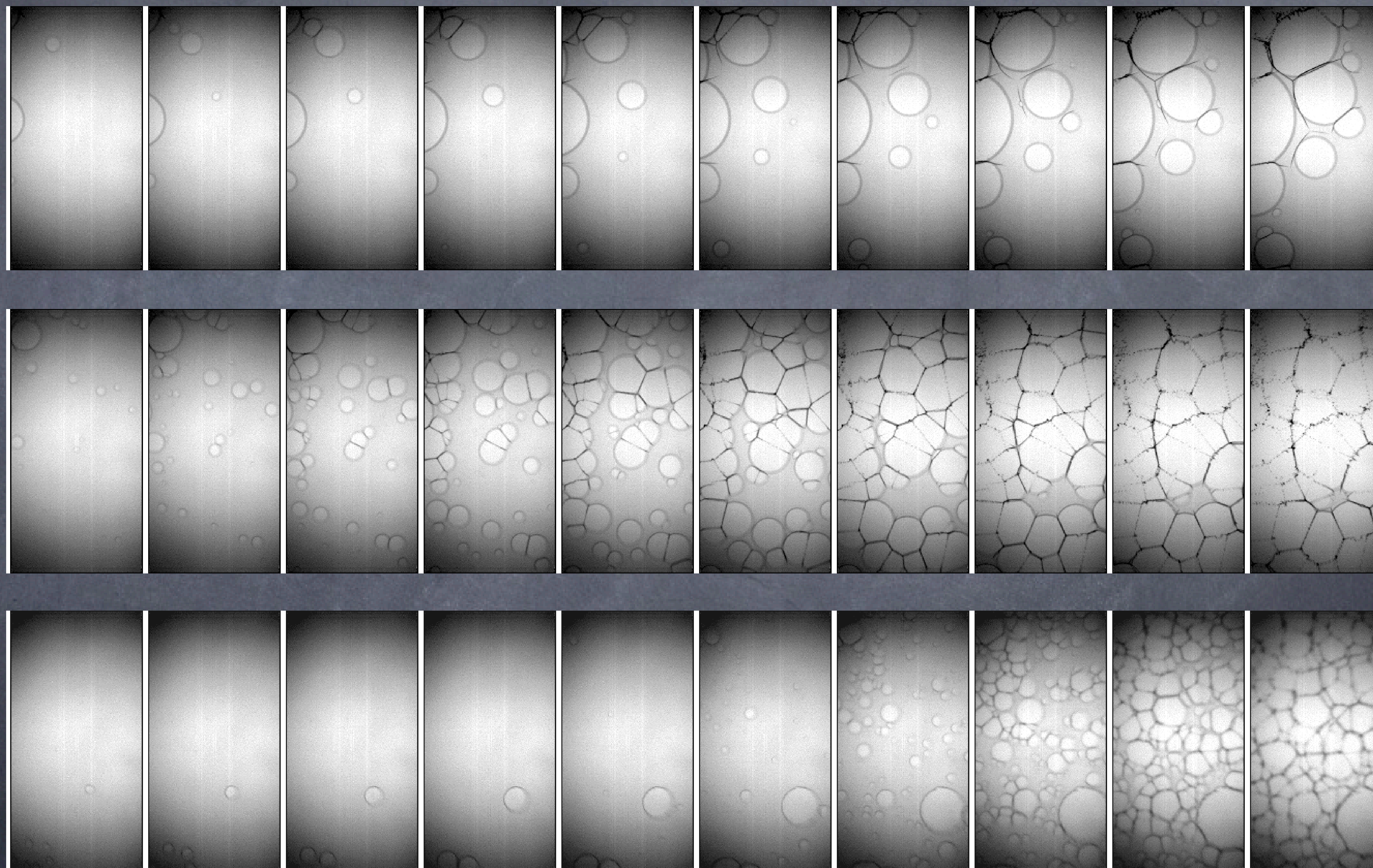
Bremond & Villermaux, JFM (2005)

Different incoming wave Mach numbers

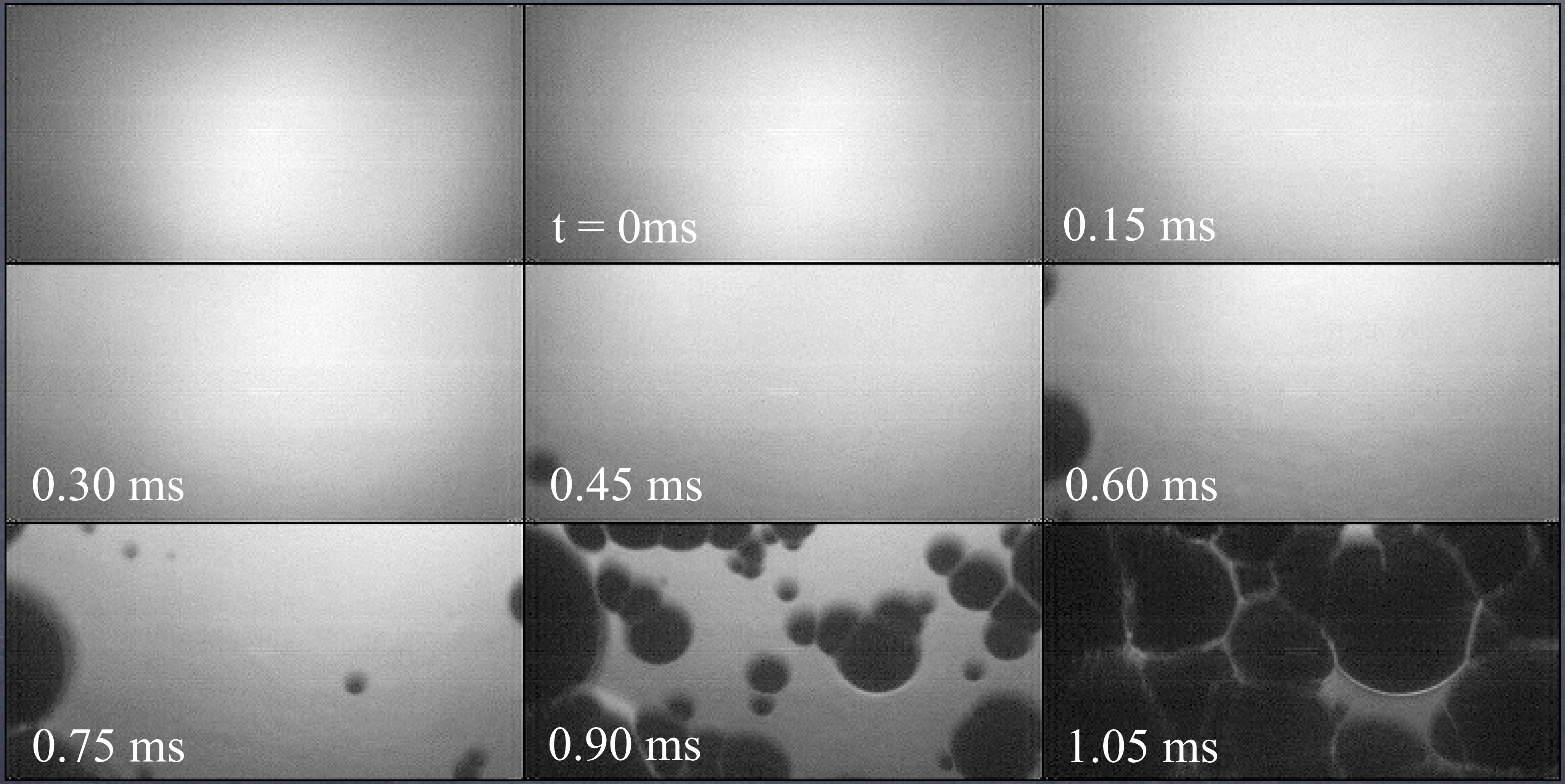
1.03

1.07

1.21

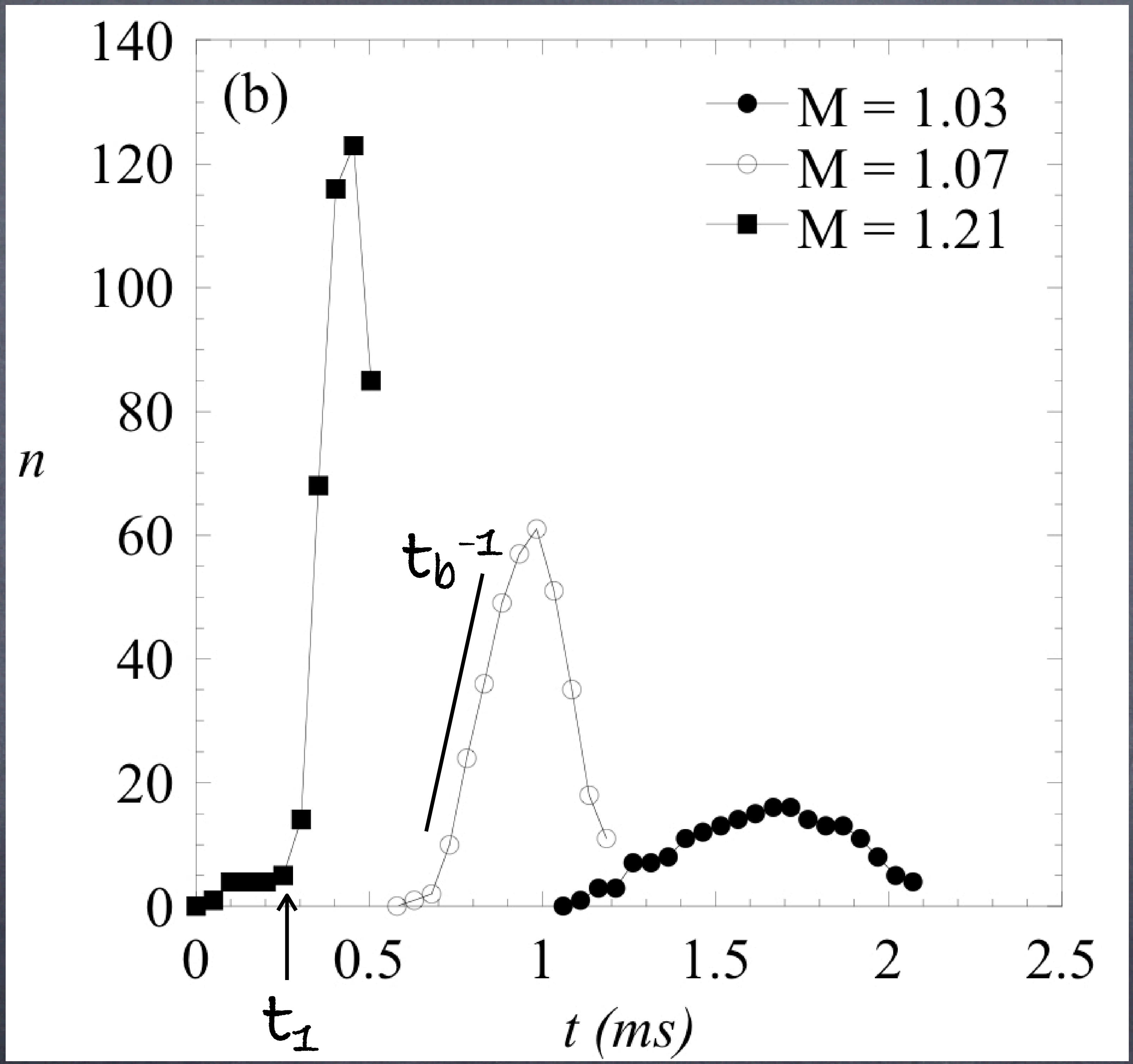


Front view, reflection

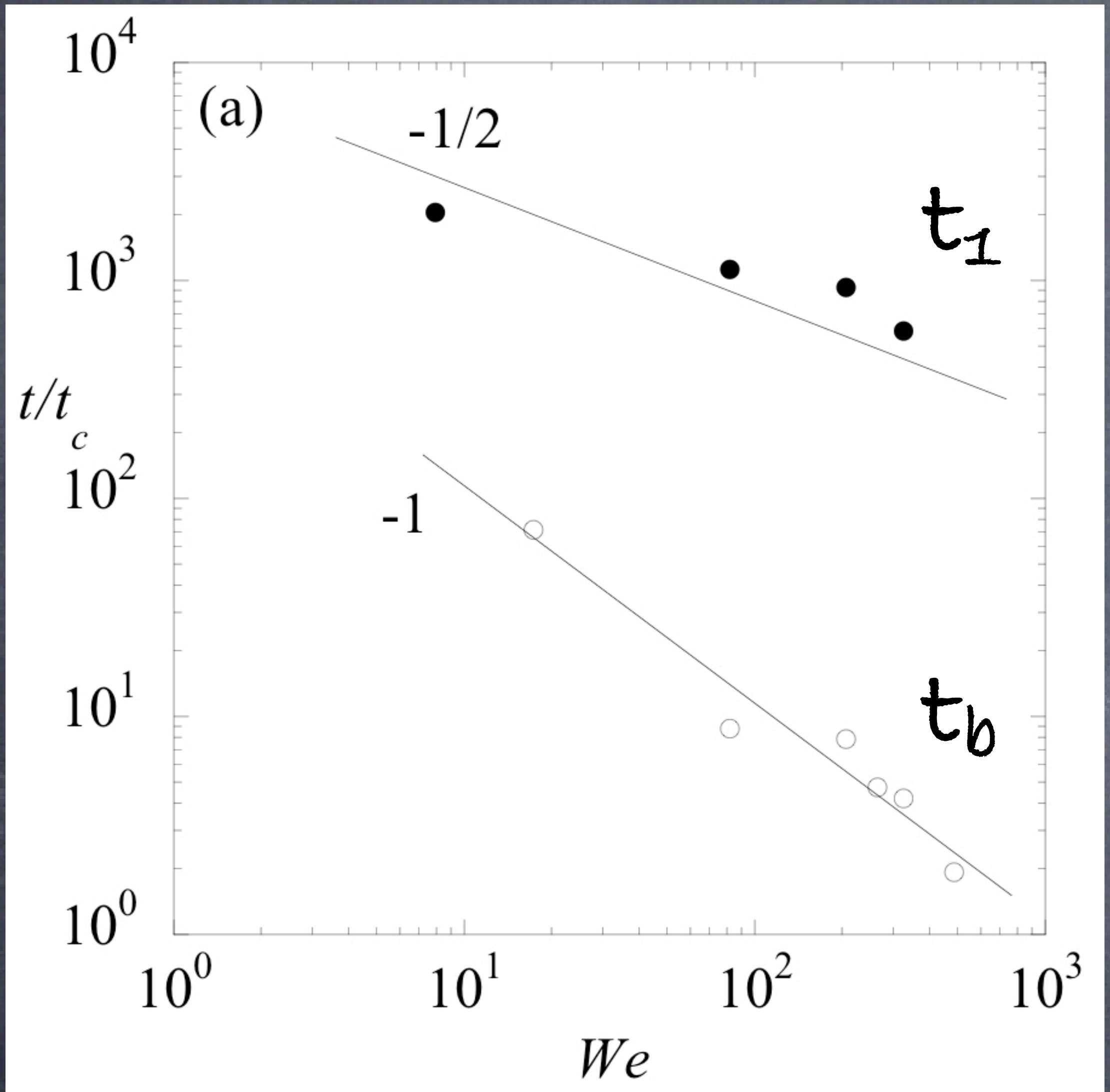


M=1.07

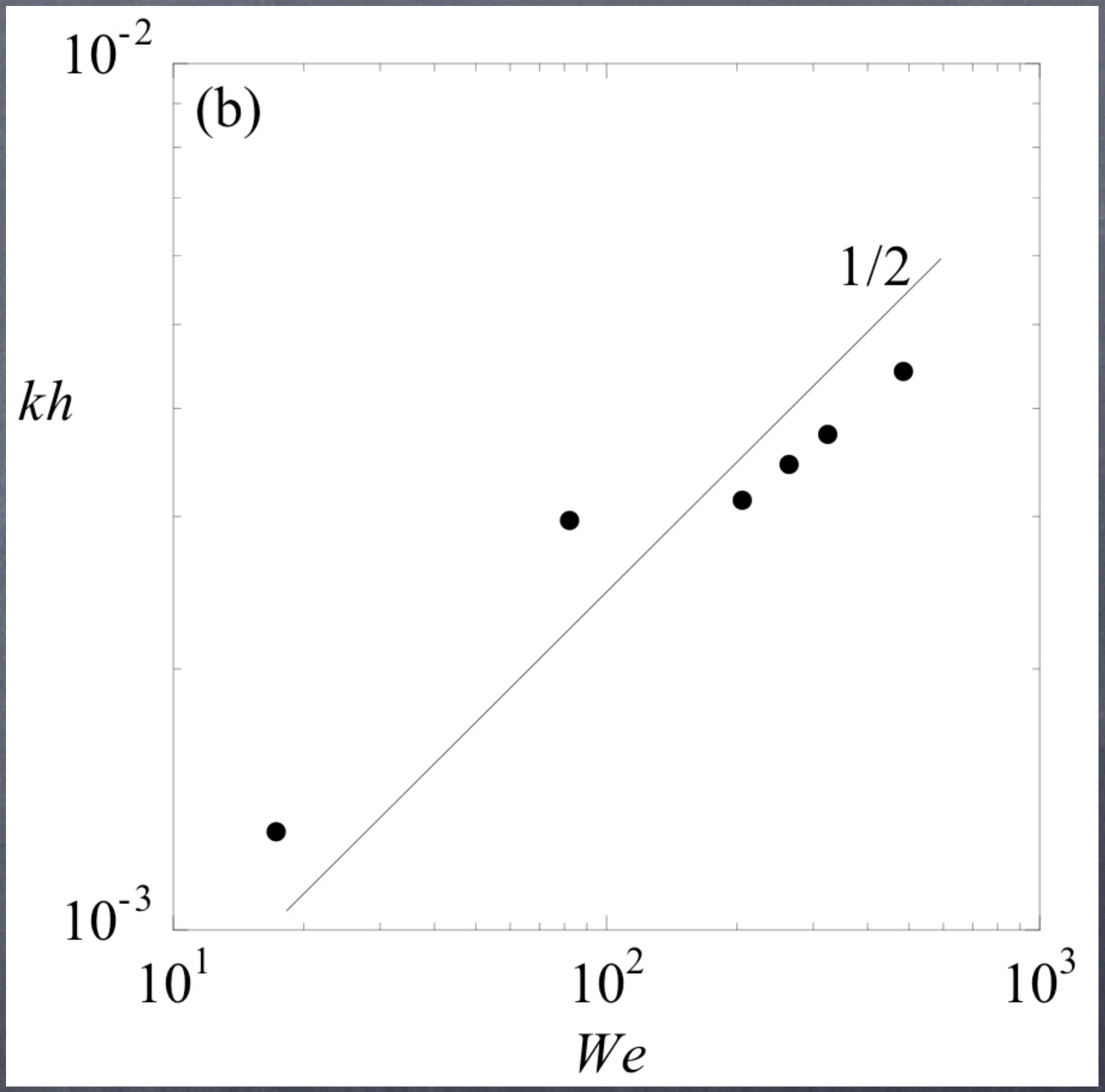
Number of holes n versus time



timescales



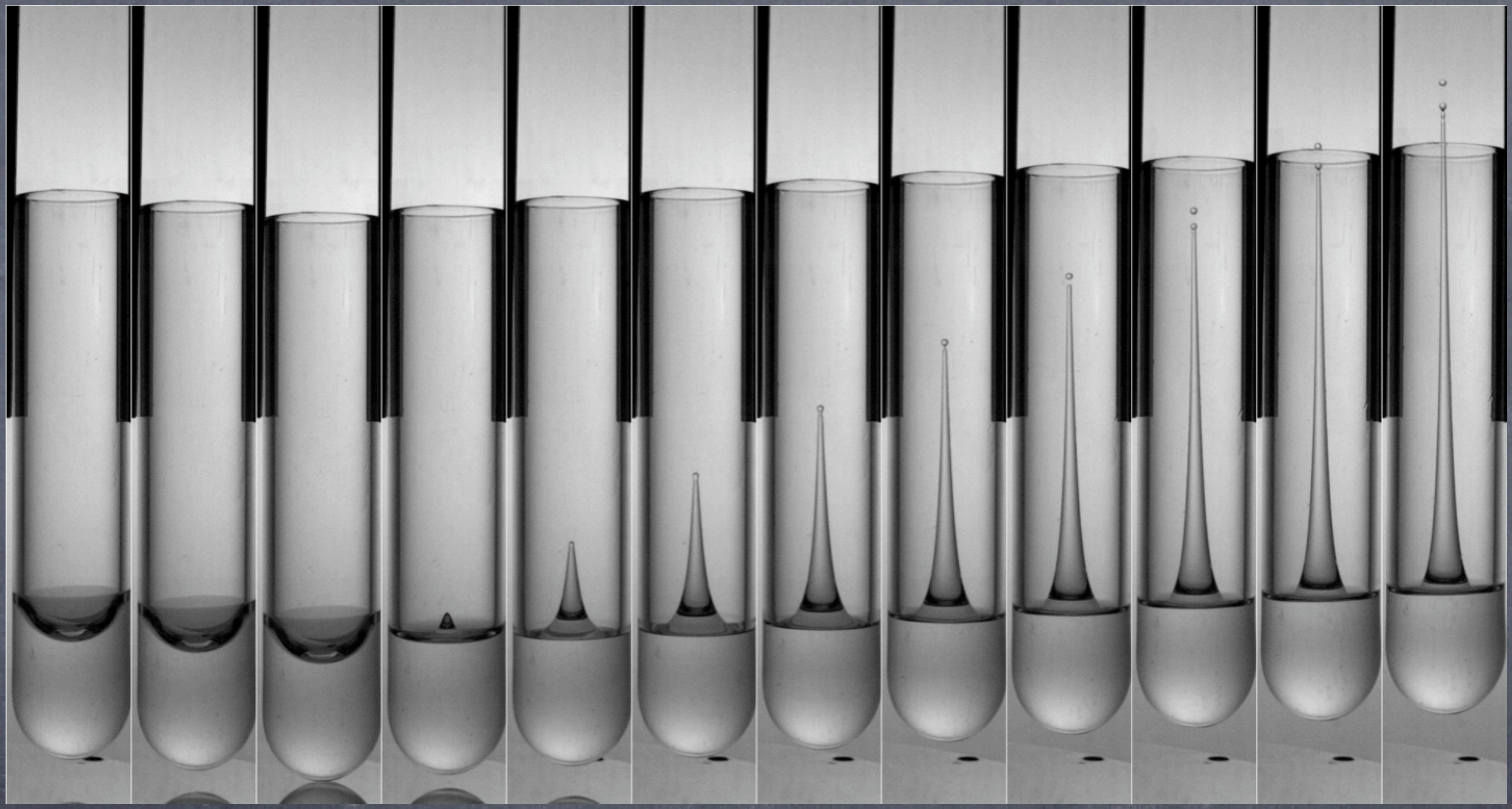
wavenumber



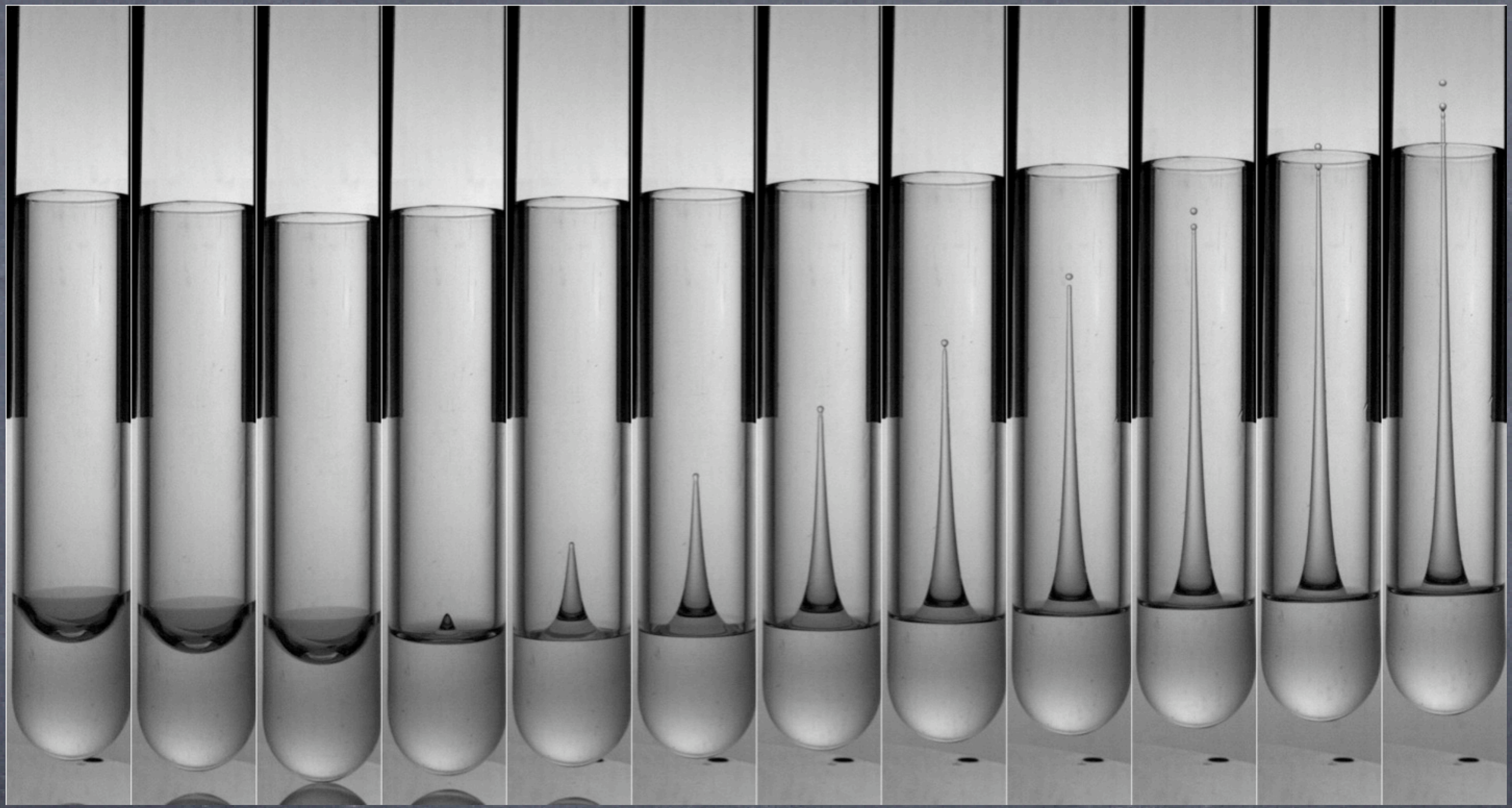
Impacted cavities

Impacted cavities

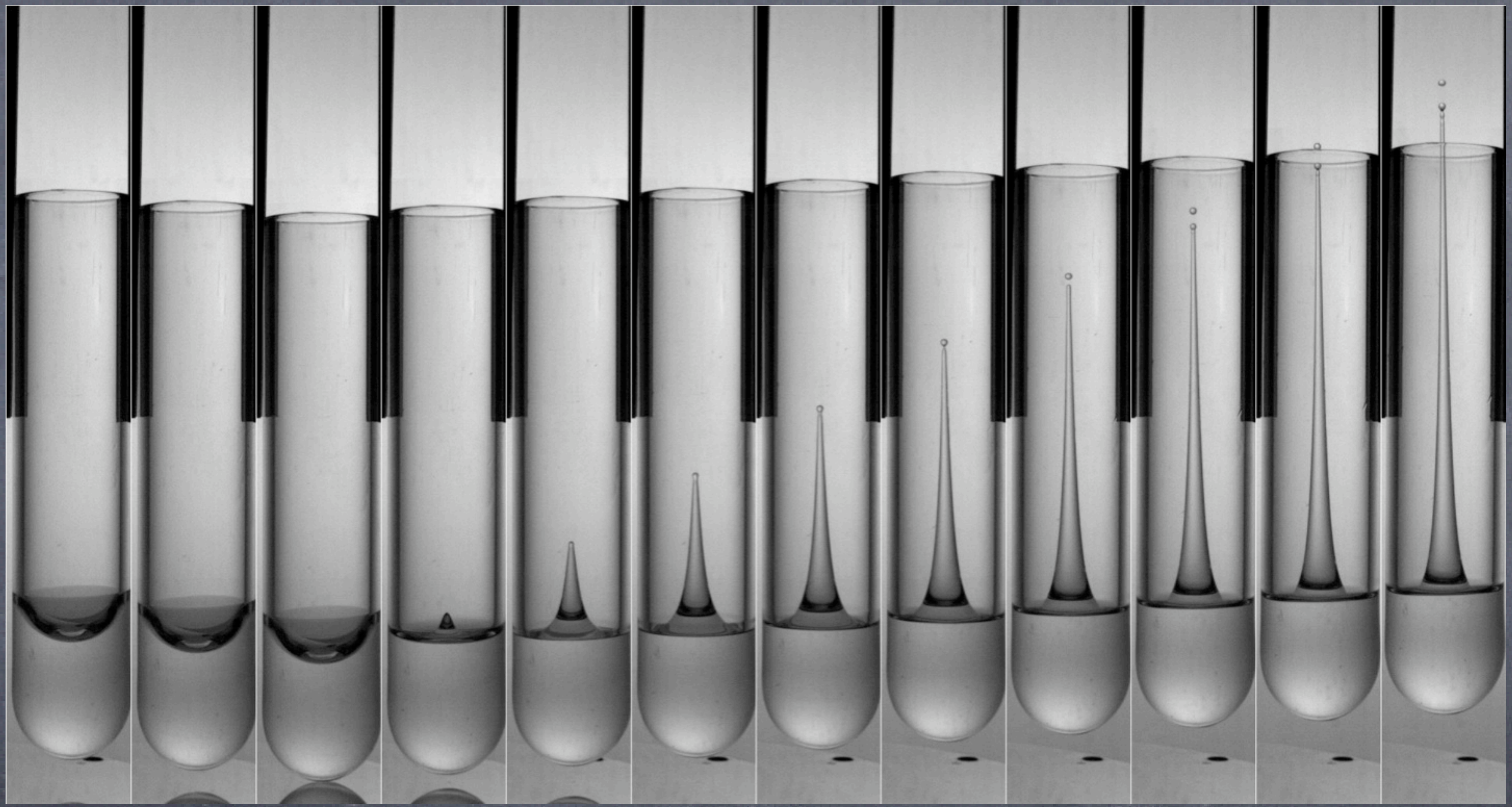
Impacted cavities



Impacted cavities

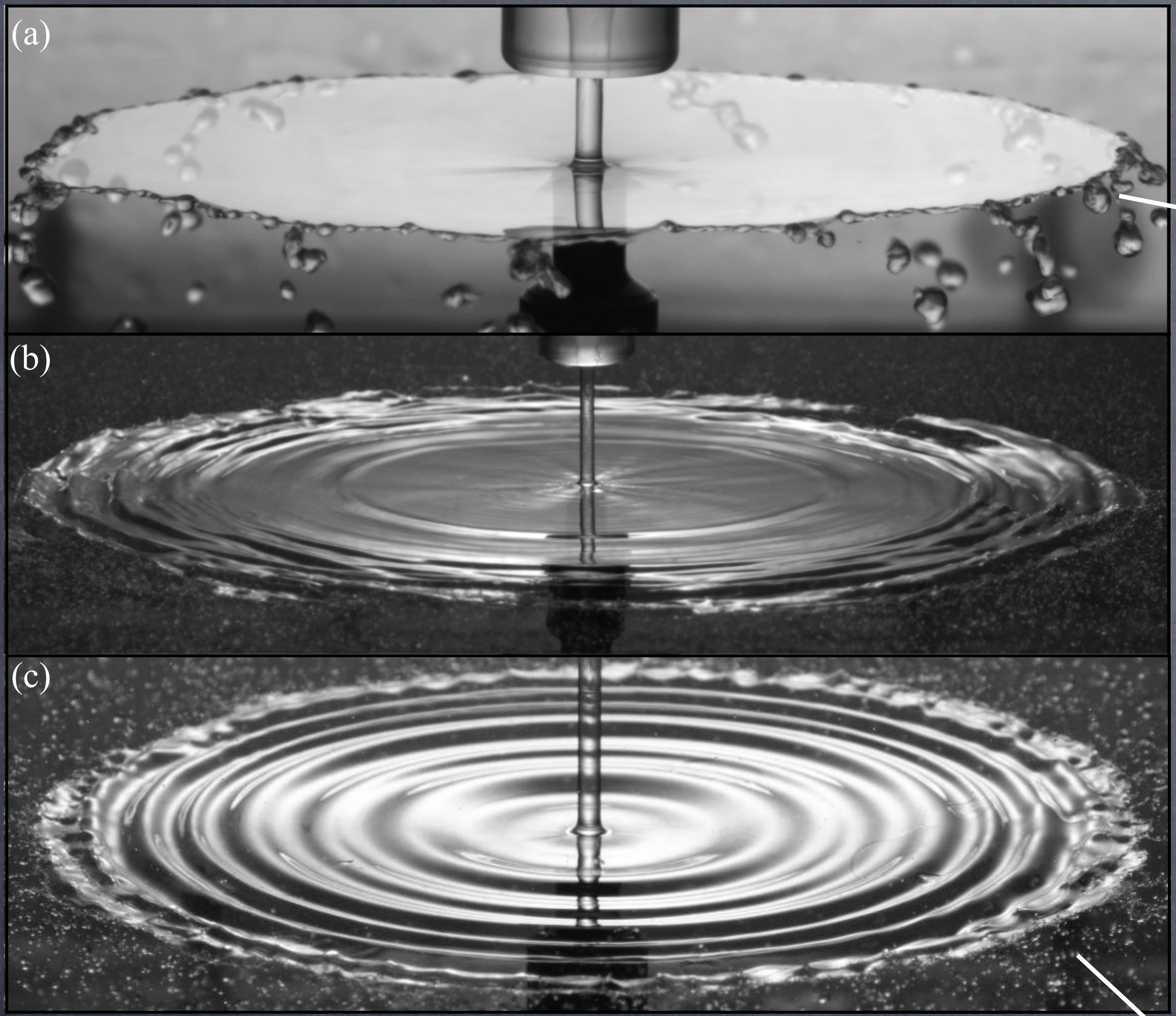


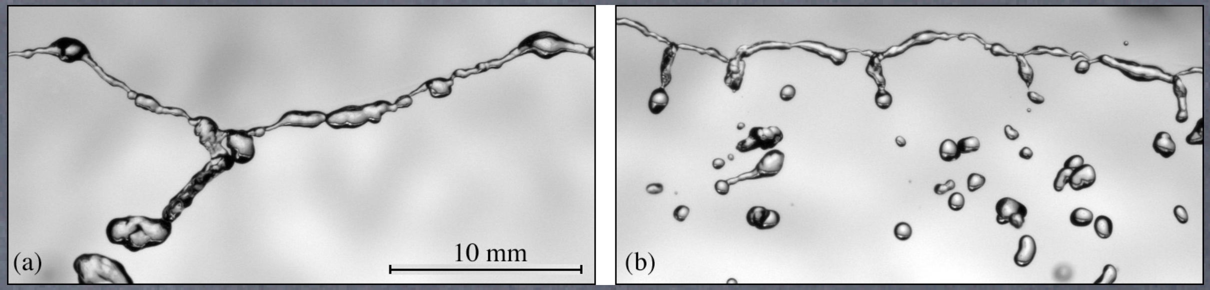
Impacted cavities



Antkowiak & al., JFM (2007)

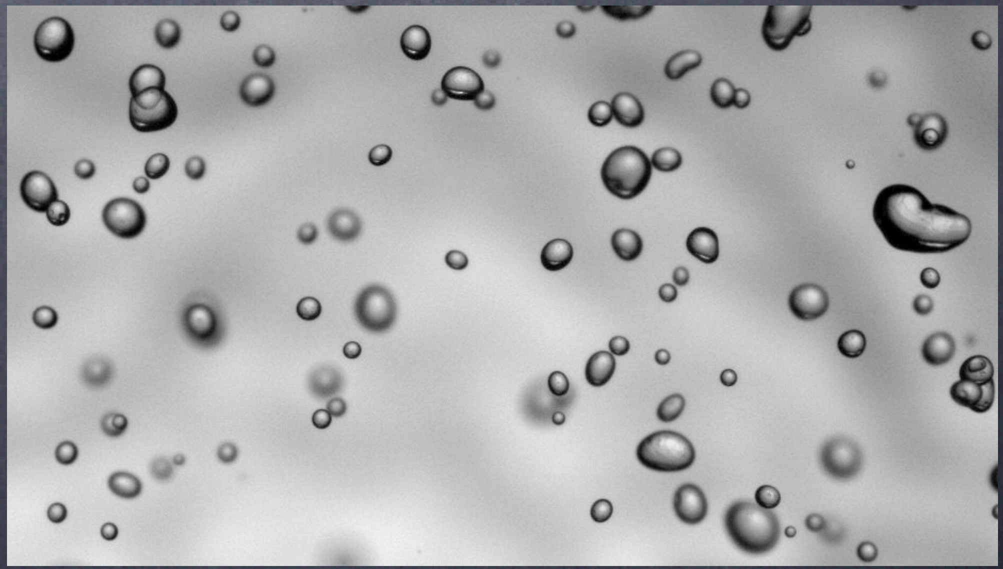
Liquid sheets (Savart 1833)





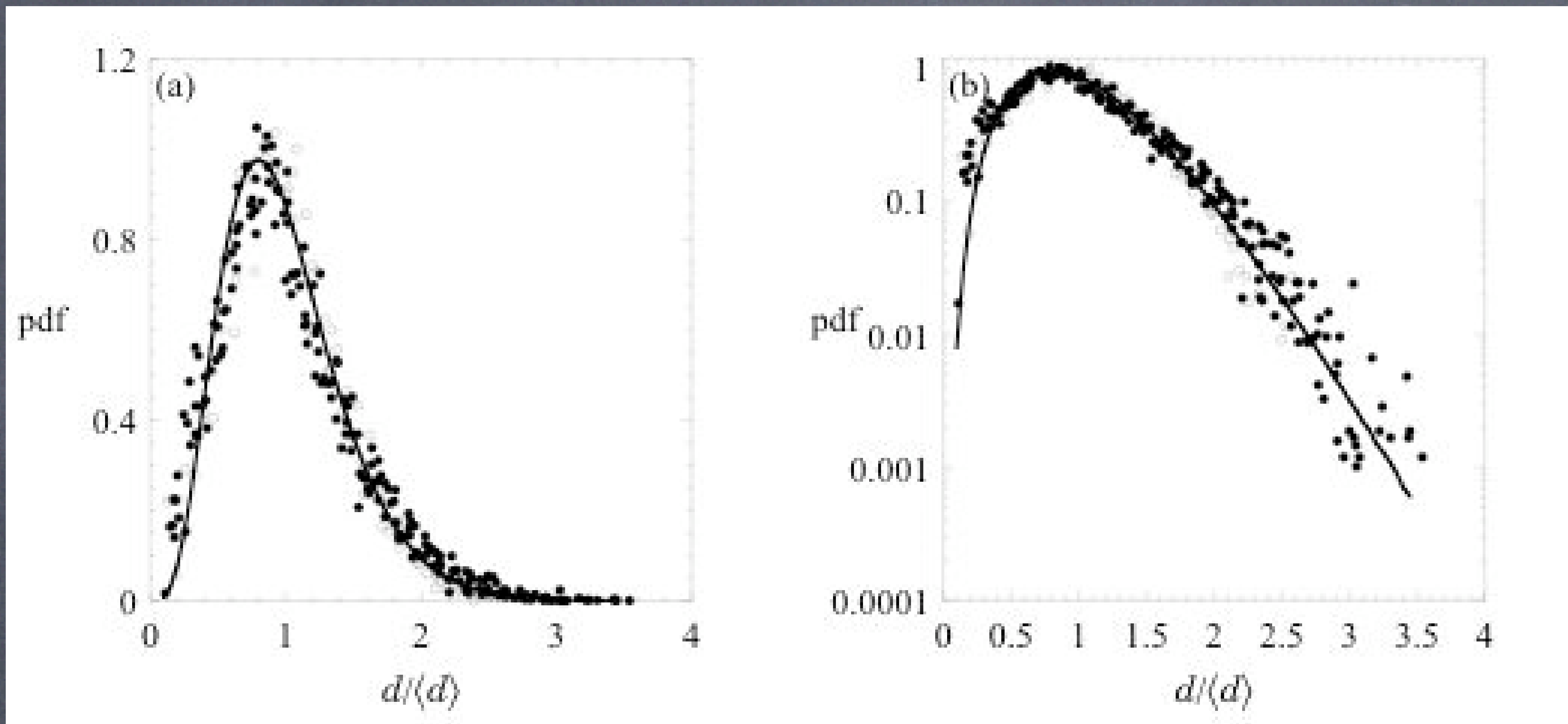
cusped rim

spray



Bremond & al., JFM (2007)

Drop size distribution

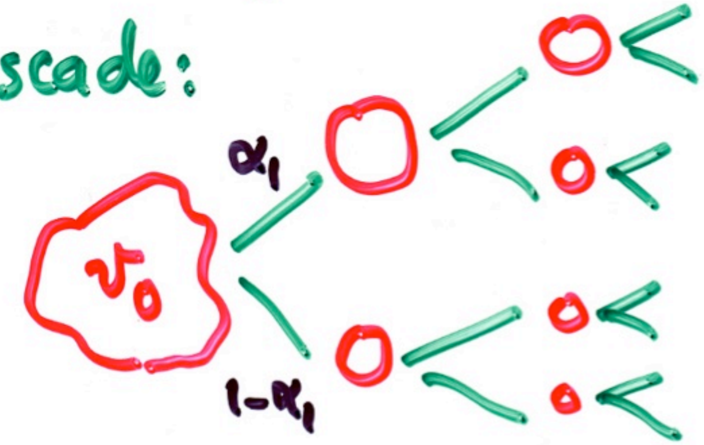


$$P(X = d/\langle d \rangle) = \frac{n^n}{\Gamma(n)}\, X^{n-1}\, e^{-nX}$$

$n \simeq 5$

Fragmentation : usual views

1) **Cascade:**



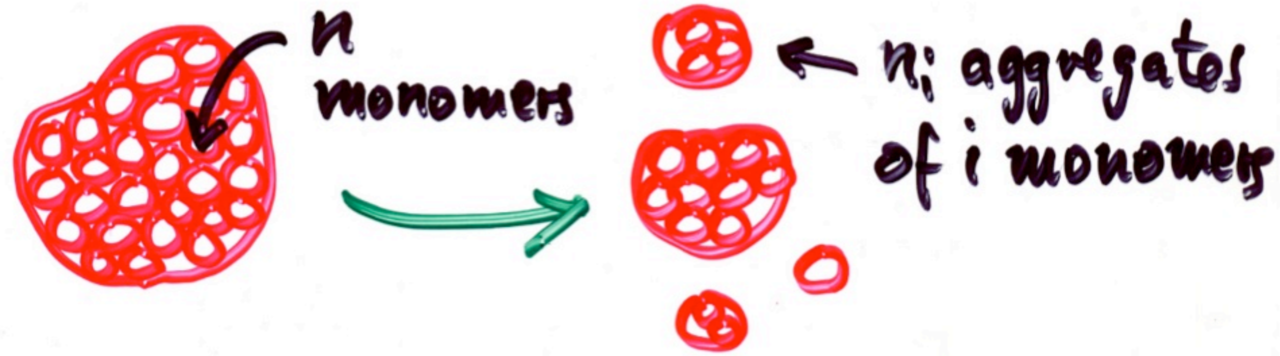
α_i's = random multipliers

Kolmogorov

$$v_n = \prod_{i=1}^{n} \alpha_i \cdot v_0$$

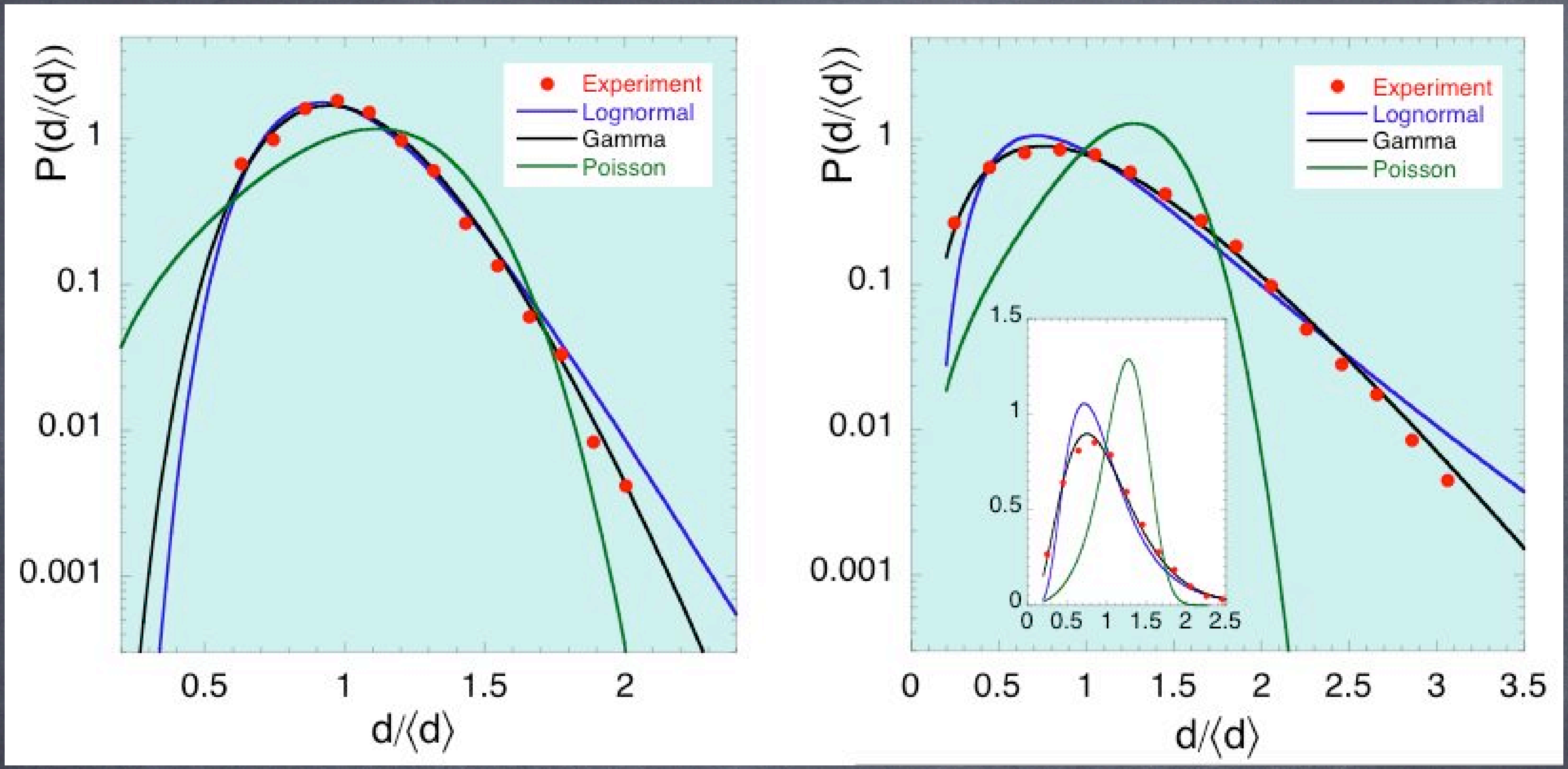
$\ln(v)$ Normally distributed

2) **Thermodynamics:** (Stockmayer, Longet.Higgins)

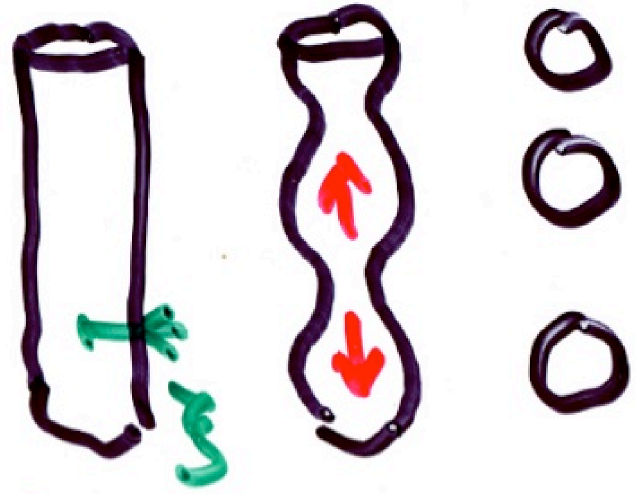


$$W = \frac{n!}{\prod_i n_i! \, (i!)^{n_i}}$$

$$\delta W = 0 \rightarrow \quad p(i) = e^{-z} \cdot \frac{z^i}{i!}$$



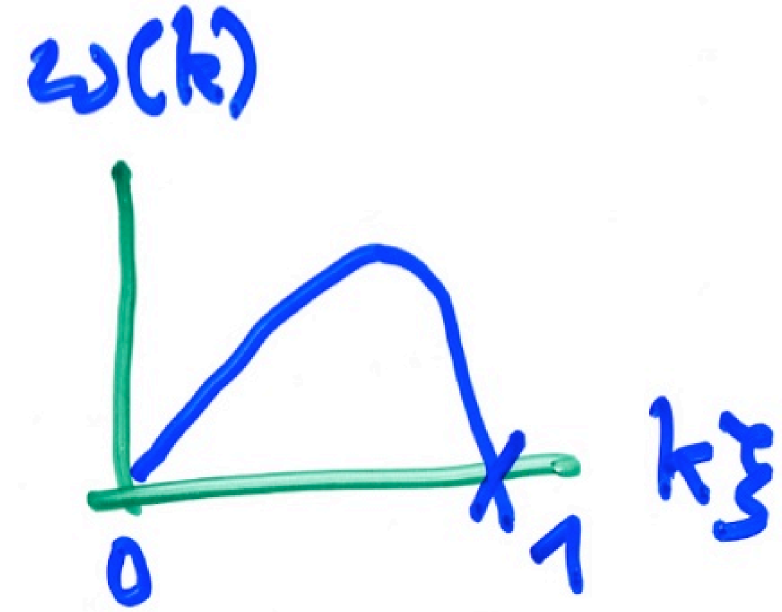
Villermaux, Annu. Rev. Fluid Mech. (2007)



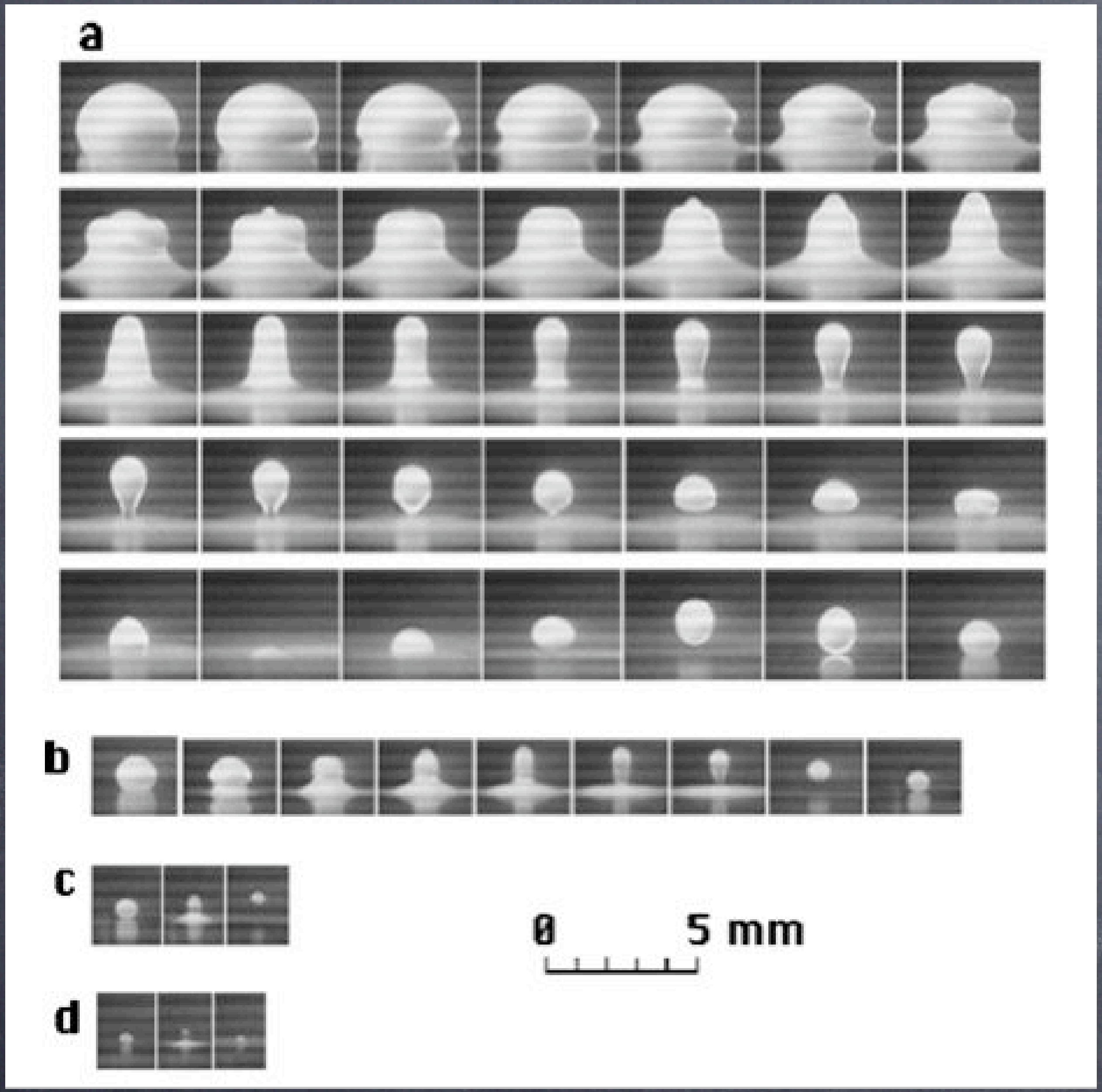
$$\partial_{t}^{2} h = \frac{1}{\tau^2}\left\{(k\xi)^2 - (k\xi)^4\right\} h$$

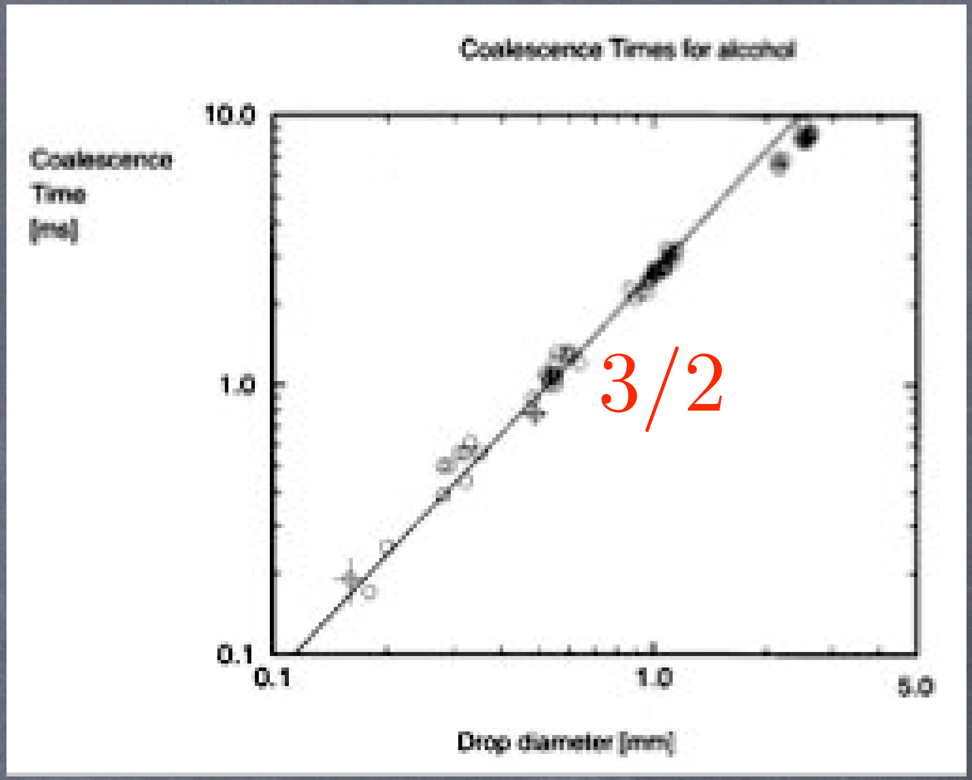
$$\tau = \left(\frac{\rho \xi^3}{\gamma}\right)^{1/2}$$

Savart
Plateau
Rayleigh
Weber
Eggers



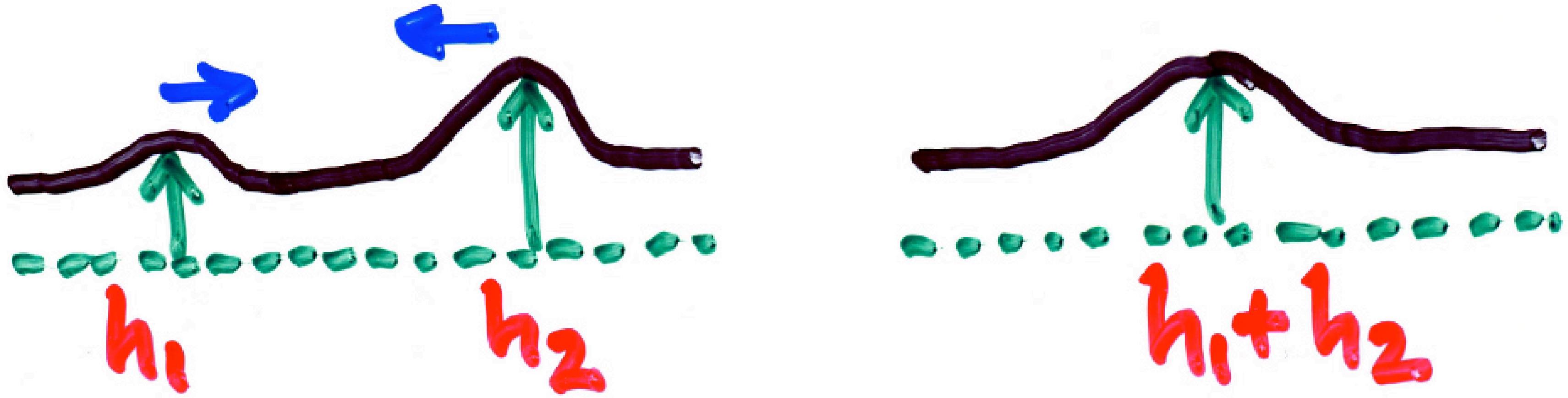
Coalescence cascade of a drop





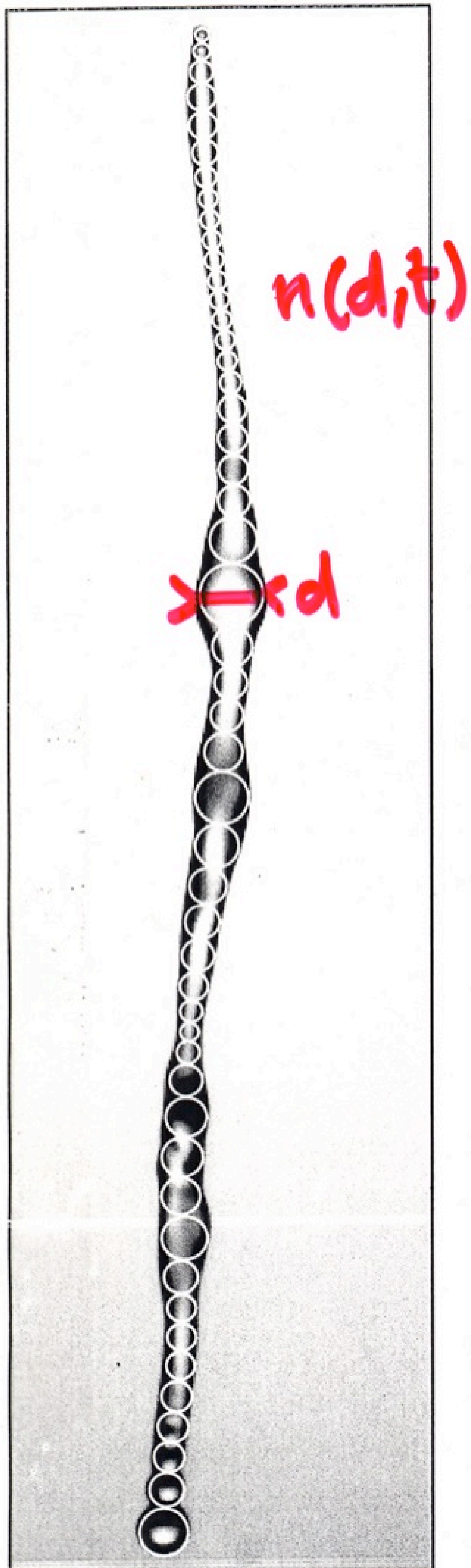
Thoroddsen & Takehara, Phys. Fluids (2000)

Linear dynamics:



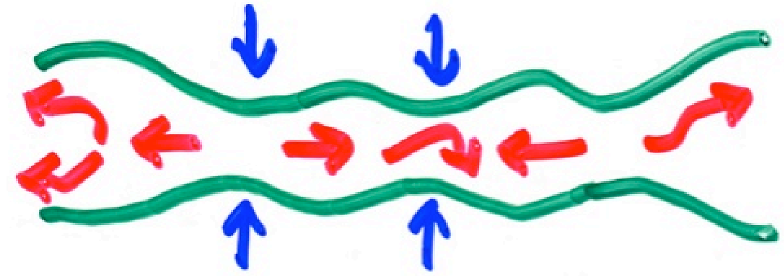
$$\partial_t^2 h = \omega^2(k) \cdot h$$

Ligament dynamics



- $n(d,t)$ = number of blobs needed to cover the ligament at time t
- $\xi = \langle d \rangle_0$ = initial average size
- $t_\xi = \sqrt{\frac{\rho \xi^3}{\sigma}}$ = interaction period
- $p(d) = \frac{n(d, t = t_\xi)}{N(t_\xi)}$
- $N = \int n(d,t)\, d(d)$ = total number of blobs

Motions along the ligament



A surface tension driven process + noize

$\Longrightarrow$ self convolution

sketch:

$$q(d', t+\Delta t) = \int q(d'-d_1', t)\, q(d_1', t)\, d(d')$$
$$= q^{\otimes 2}$$

independent layers: $n(d,t) = q(d',t)^{\otimes \nu}$

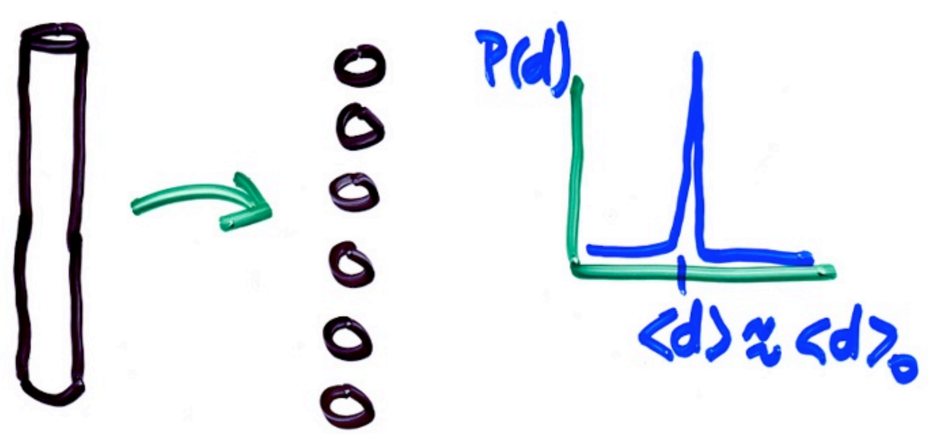
Then:

$$\partial_t n = -n(d,t)\, N(t)^{\gamma-1} + \frac{1}{3\gamma-2}\, n(d,t)^{\otimes\gamma}$$

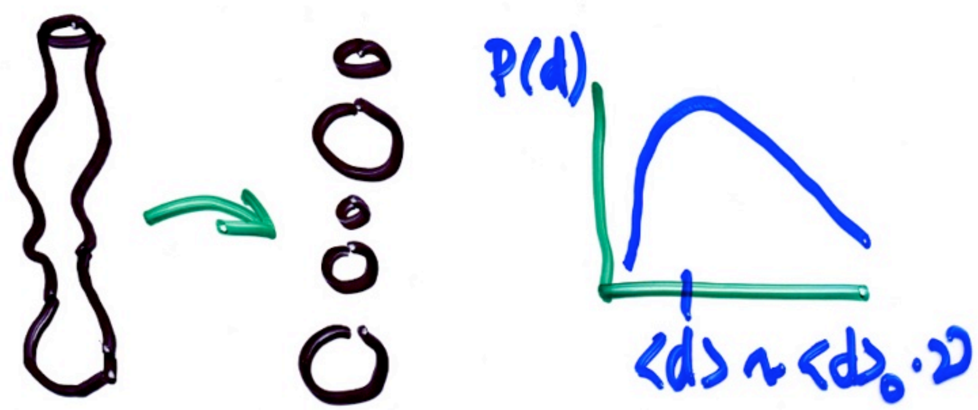
$$\gamma = 1 + \frac{1}{\nu} = \frac{\langle d^2\rangle_0}{\langle d\rangle_0^2}$$

Net volume conserved: $\int d^3\, n(d,t)\, d(d) = c^{st}$

$\gamma \approx 1$



$\gamma > 1$



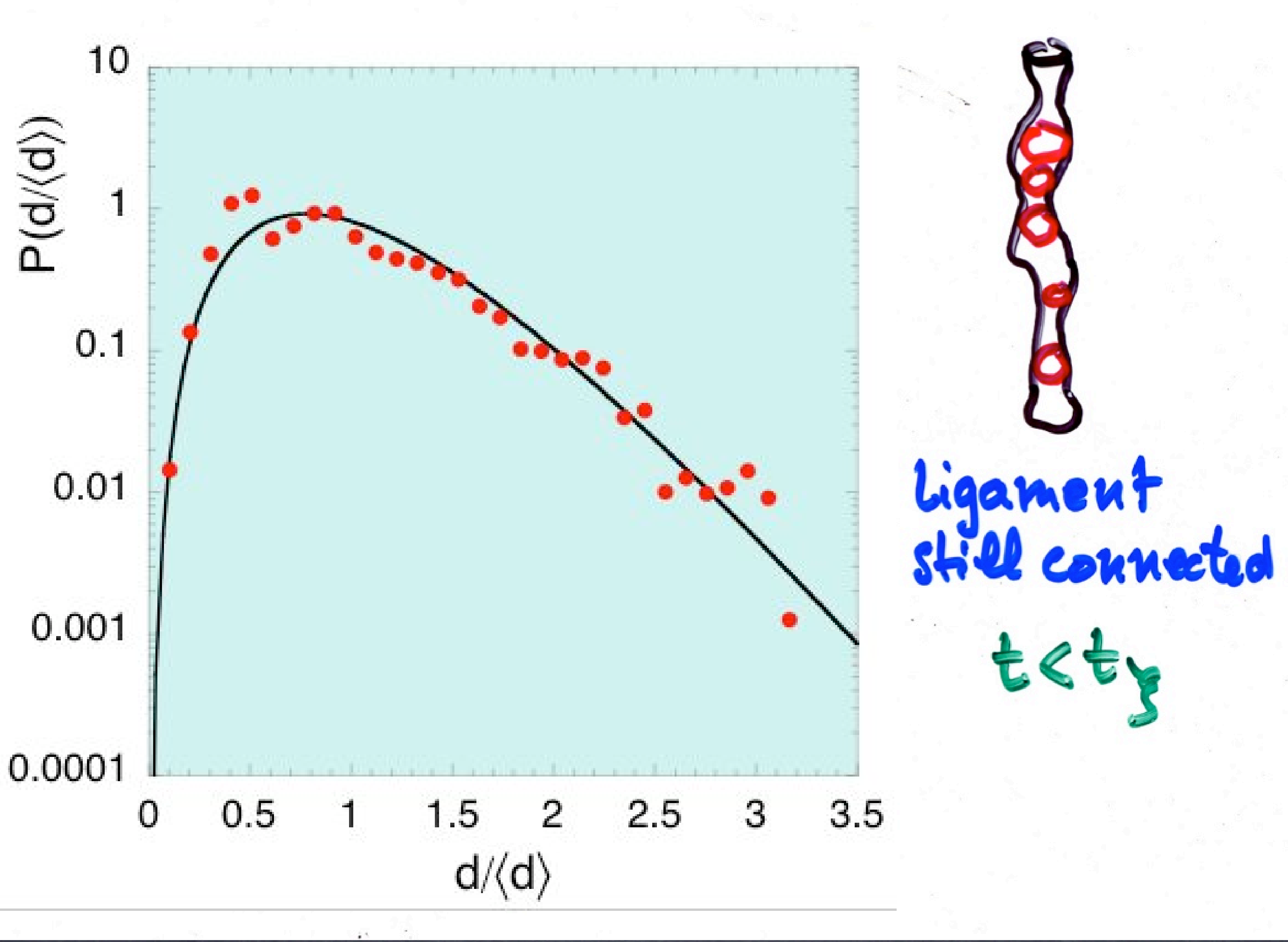
Drop size distribution :

$$p\left(x=\frac{d}{\langle d\rangle}\right) = \frac{n(d,t_\xi)}{N(t_\xi)}$$

$$= \frac{\nu^\nu}{\Gamma(\nu)}\, x^{\nu-1} \exp\{-\nu x\} \qquad (\nu \equiv n)$$

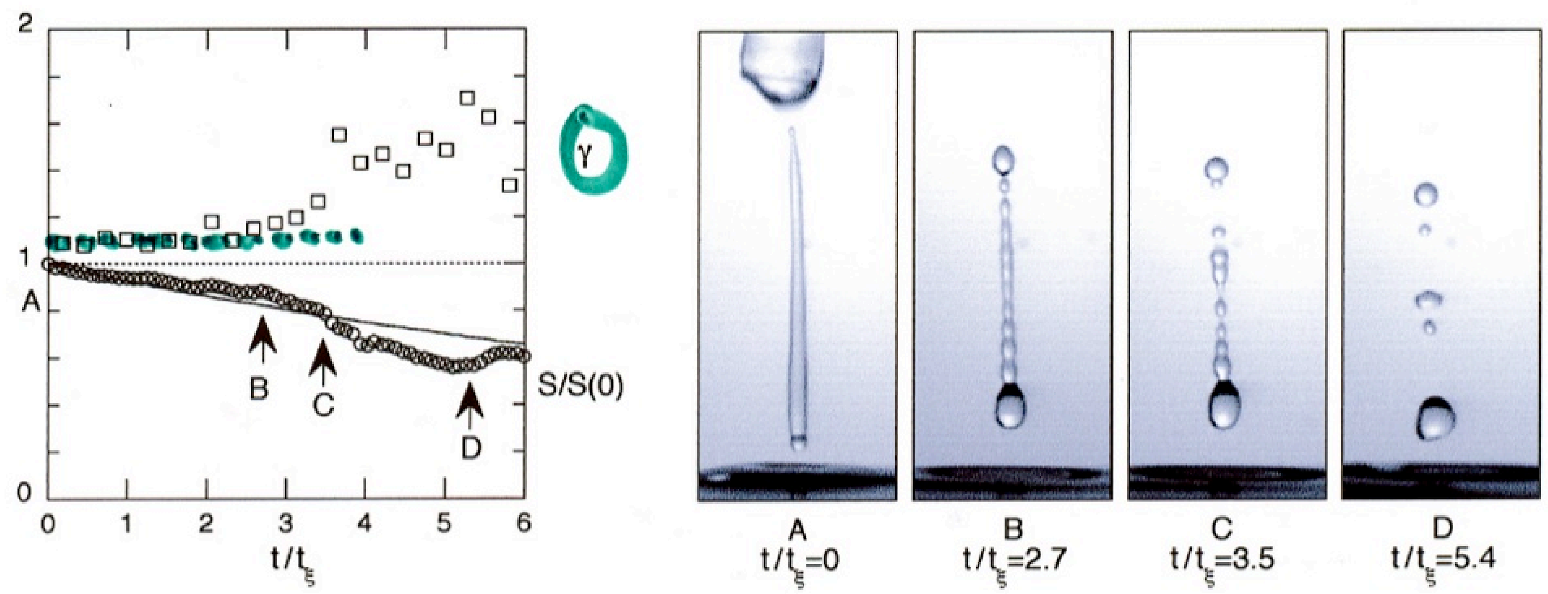
Gamma distribution

Villermaux & al., PRL (2004)



10
1
0.1
0.01
0.001
0.0001
P(d/⟨d⟩)
0
0.5
1
1.5
2
2.5
3
3.5
d/⟨d⟩
Ligament still connected
t<tξ

$\gamma = 1 + \frac{1}{2\nu} = C^{ste}$ during coalescence



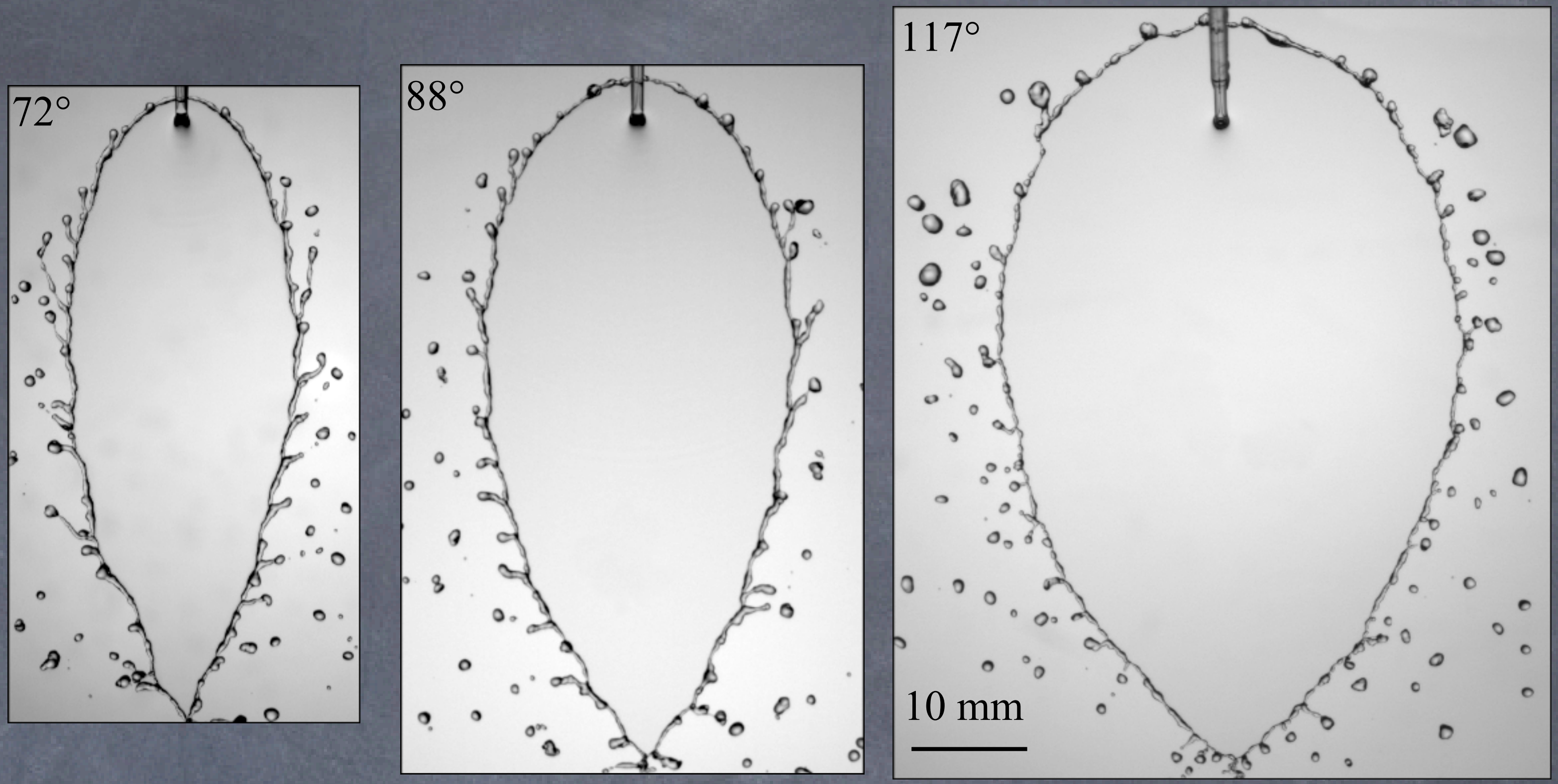
projected surface:

$$S \sim \int d^2 \, n(d,t) \, d(d)$$

decreases in time

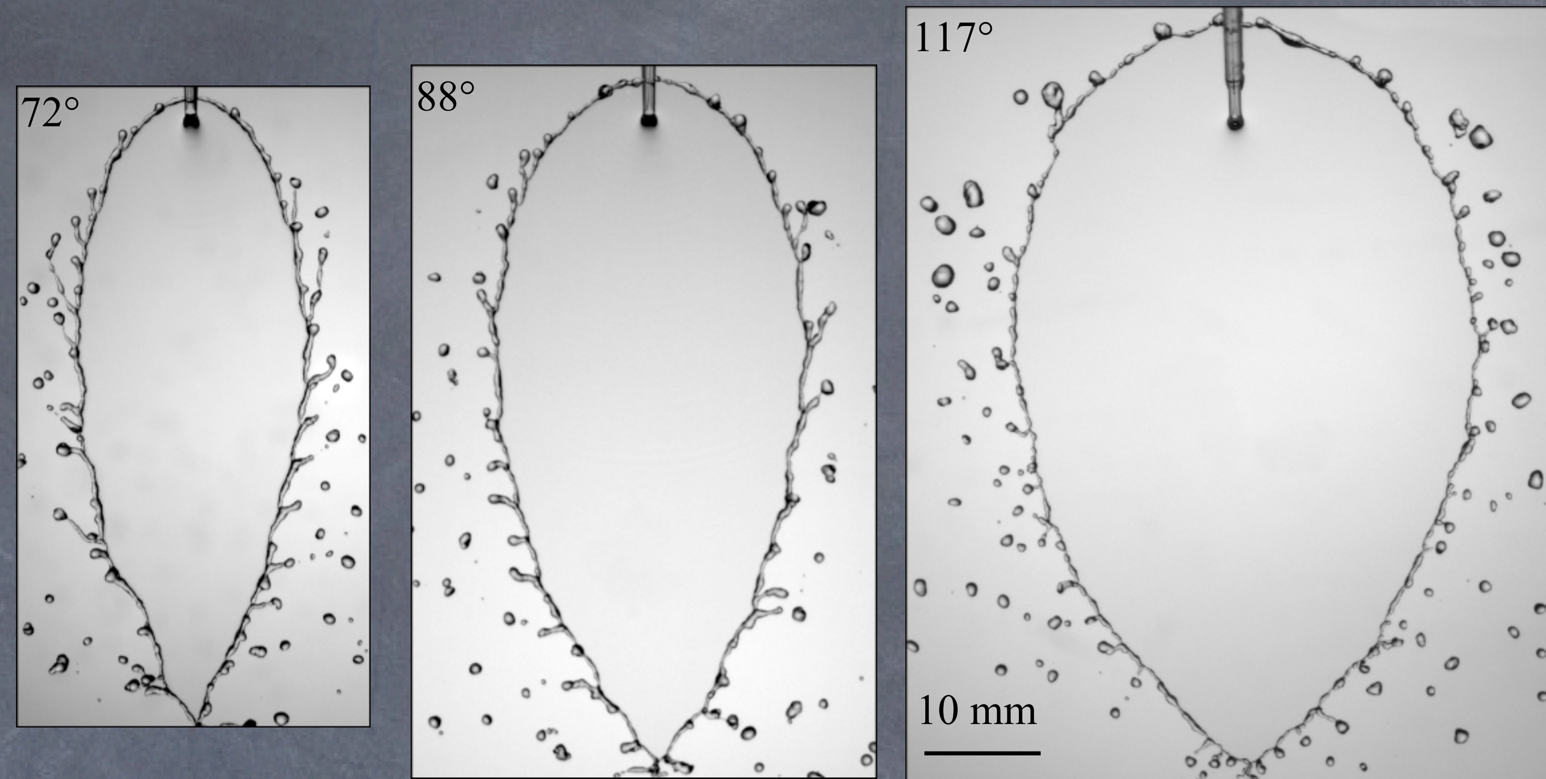
→ aggregtion

Impacted jets



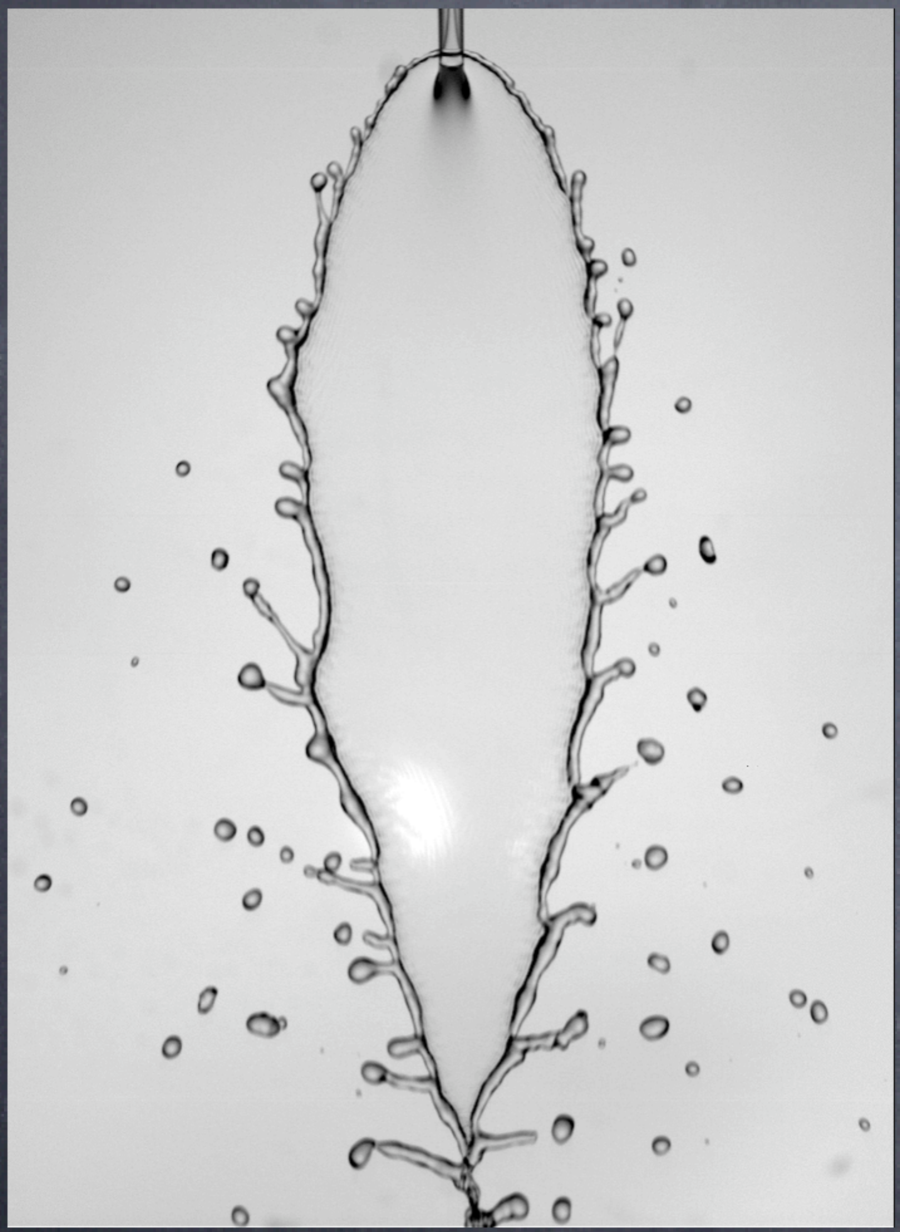
Bremond & Villermaux, JFM (2006)

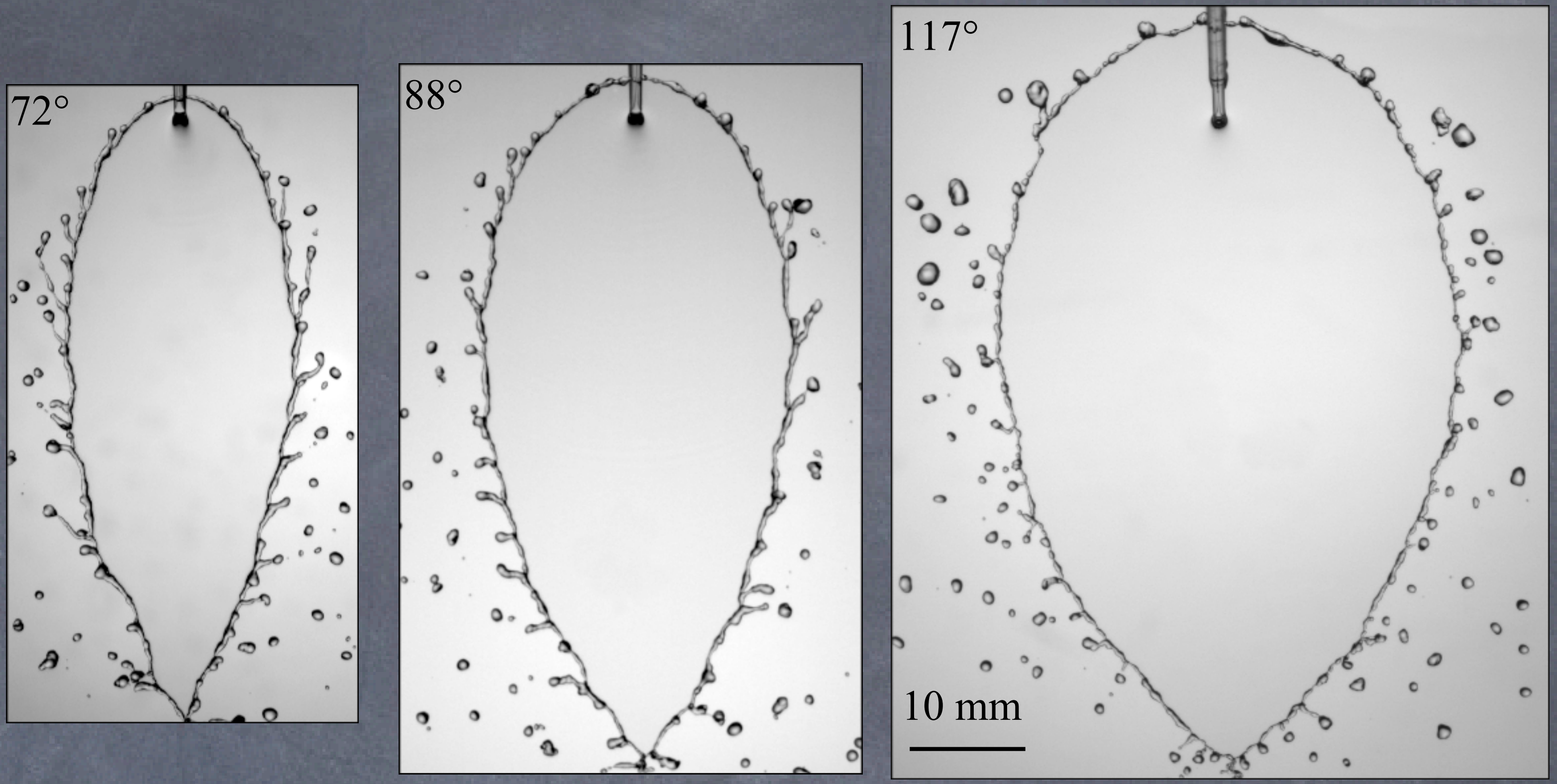
Impacted jets

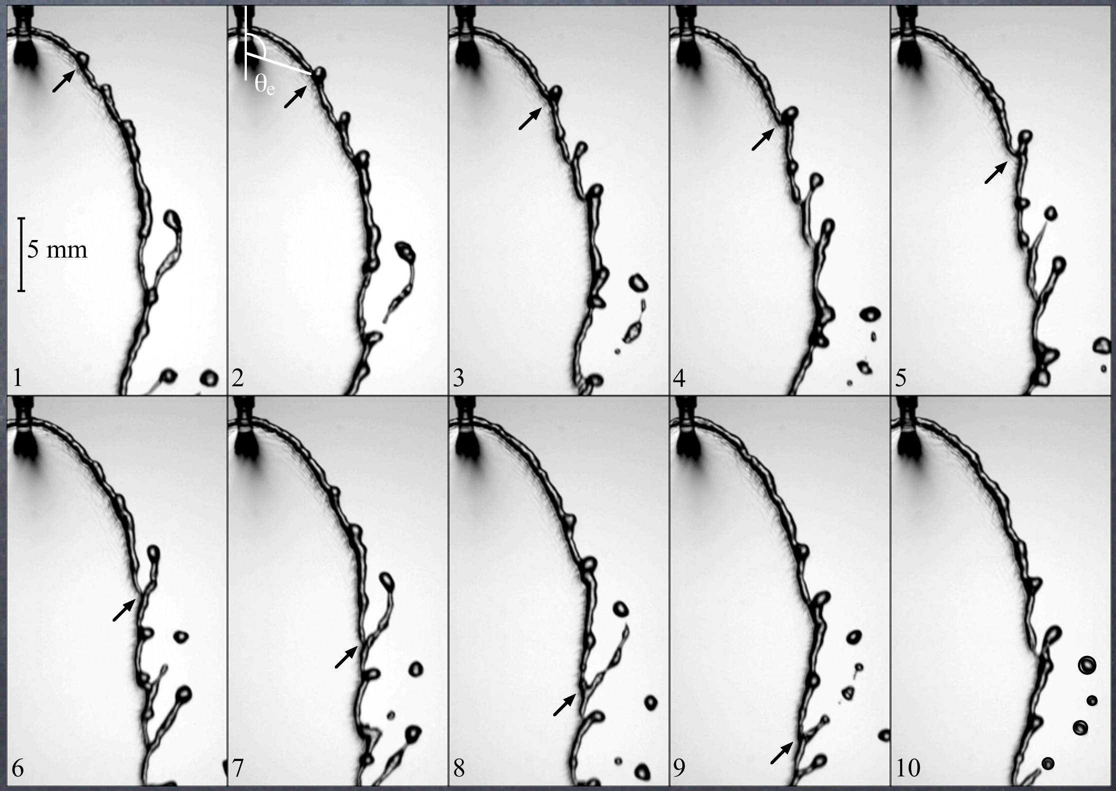


Bremond & Villermaux, JFM (2006)

Impacted jets

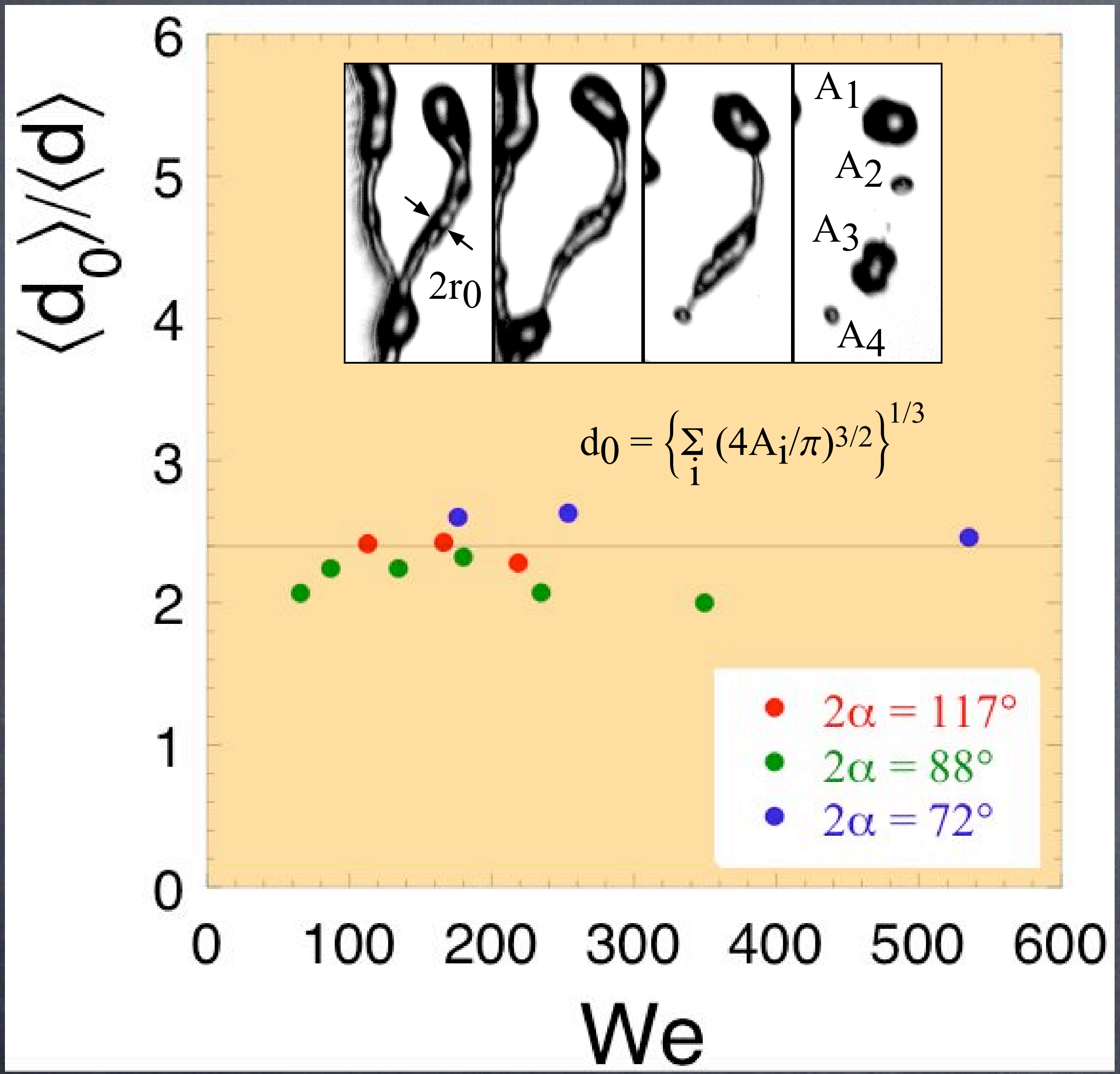


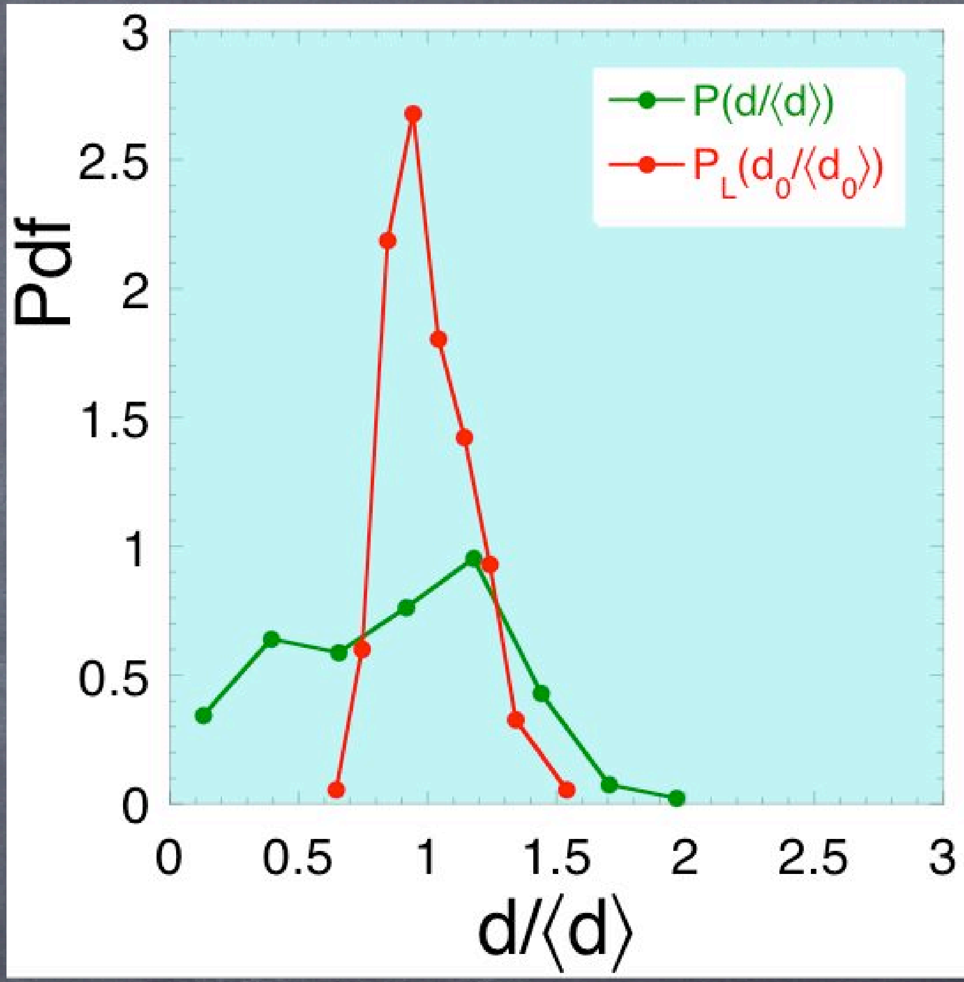


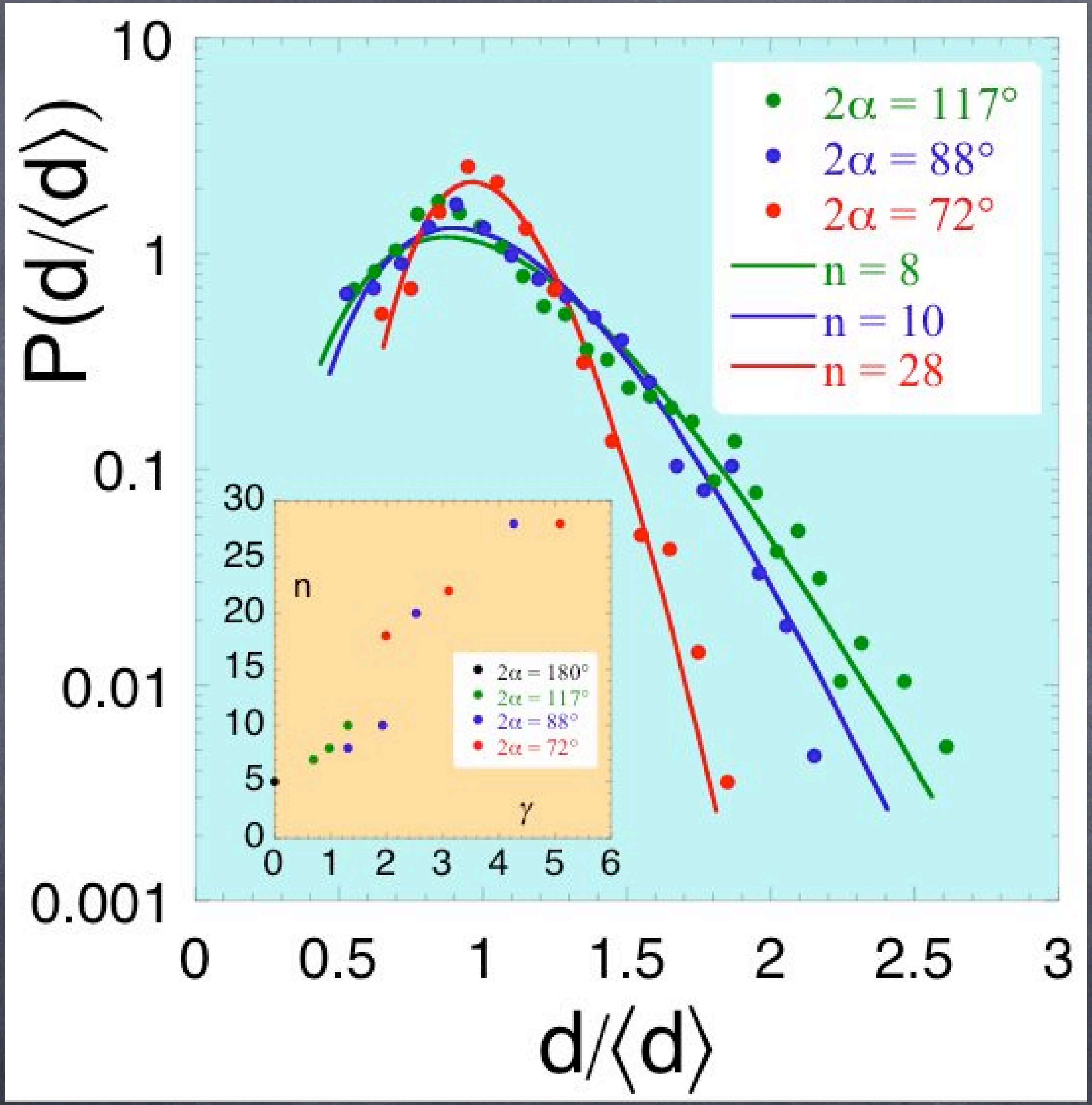


Bremond & Villermaux, JFM (2006)

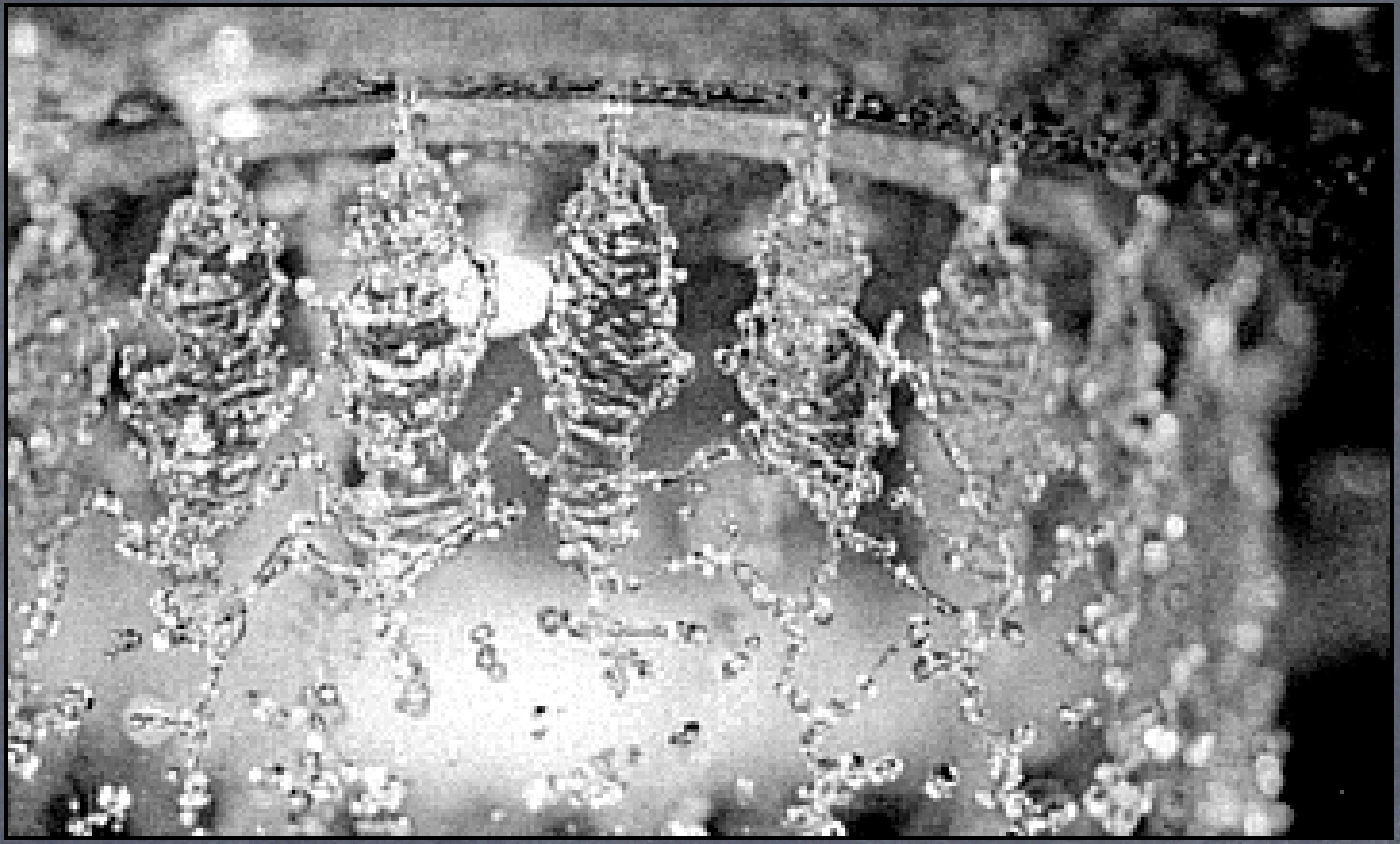
Volume ($\sim d_0^3$) of the ligaments



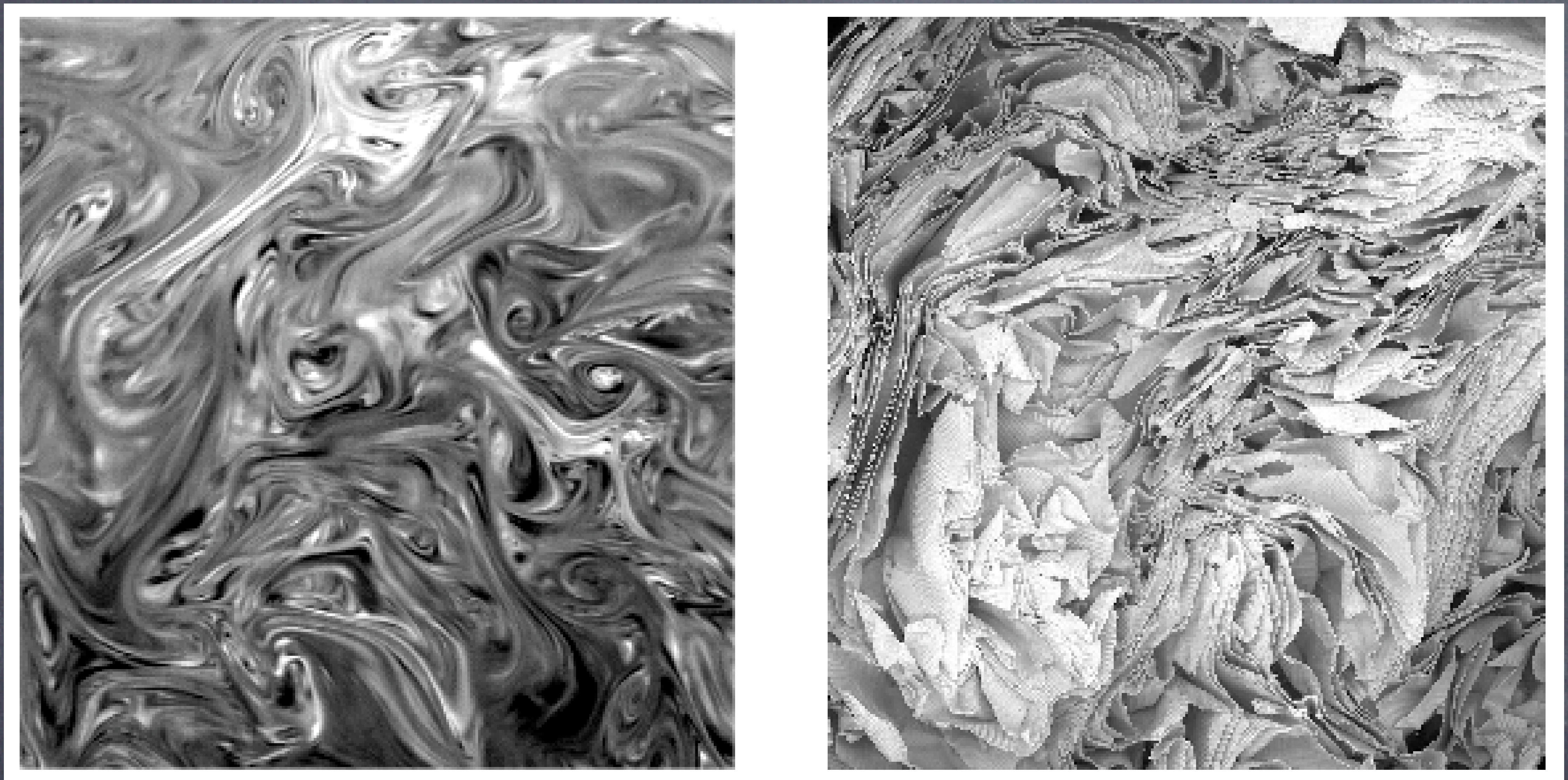




P(d/⟨d⟩)
10
1
0.1
0.01
0.001
0
0.5
1
1.5
2
2.5
3
d/⟨d⟩
2α = 117°
2α = 88°
2α = 72°
n = 8
n = 10
n = 28
30
25
20
15
10
5
0
n
γ
0 1 2 3 4 5 6
2α = 180°
2α = 117°
2α = 88°
2α = 72°



Rocket engine (SNECMA)



Sultan & Boudaoud, PRL (2006)