Reynolds number effect on the velocity increment skewness in isotropic turbulence

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The velocity increment skewness of isotropic turbulence is computed using the EDQNM model and compared to results of the multifractal formalism. At the highest Reynolds number available in windtunnel experiments, $R_\lambda = 2500$, both the multifractal model and EDQNM give power-law corrections to the inertial range scaling. For EDQNM, this correction is a finite Reynolds number effect, whereas for the multifractal formalism it should persist at high Reynolds number. Therefore, at $R_\lambda = 2500$, corrections to the inertial range scaling of the skewness are not an adequate measure for intermittency, since the influence of intermittency cannot be distinguished from the influence of the Reynolds number.

The nonlinearity in the Navier-Stokes equations gives rise to an interaction between different length-scales in a turbulent flow. These interactions are the basic mechanism behind the celebrated Kolmogorov-Richardson energy cascade. In the Lin equation for the turbulent energy spectrum $E(k)$,

$$\frac{\partial E(k)}{\partial t} = T(k) - 2\nu k^2 E(k), \tag{1}$$

(k being the wavenumber and ν the kinematic viscosity), these scale interactions are represented by the nonlinear transfer $T(k)$. In physical space, the second-order and third-order longitudinal structure functions, $D_{ll}(r)$ and $D_{lll}(r)$, are related to $E(k)$ and $T(k)$ by the following expressions,

$$D_{ll}(r) = 4\int_0^\infty E(k)\left[\frac{1}{3} - \frac{\sin kr - kr\cos kr}{(kr)^3}\right]dk, \tag{2}$$

$$D_{lll}(r) = 12r\int_0^\infty T(k)f(kr)dk, \tag{3}$$

with

$$f(kr) = \frac{3\left(\sin kr - kr\cos kr\right) - (kr)^2\sin kr}{(kr)^5}. \tag{4}$$

The possibility of corrections to the inertial range scaling of structure functions, due to the intermittent character of the energy dissipation was advanced by Kolmogorov [1]. A phenomenological model which succeeds to give a coherent picture of the influence of intermittency is the multifractal model [2]. This model compares well to measurements and gives non-zero intermittency corrections to the inertial range scaling of the energy spectrum at high Reynolds numbers. For $D_{ll}(r)$ this correction is believed to yield the scaling

$$D_{ll}(r) \sim r^{2/3+\mu}, \tag{5}$$

with μ the intermittency correction, of the order of 0.03. For $D_{lll}(r) \sim r$, there is no correction. This gives for the (longitudinal velocity increment) skewness $Sk(r) = D_{lll}(r)/D_{ll}(r)^{3/2}$, a scaling $Sk(r) \sim r^{-0.045}$. In the present communication we will compare the skewness as computed from spectral closure, with the prediction of the multifractal model and experimental measurements of windtunnel turbulence.

The spectral closure used here is the EDQNM model [3]. This model, which is a simplification of the DIA-closure family developed by Kraichnan (*e.g.* [4, 5]), closes the Lin-equation by giving a closed expression for $T(k)$ as a function of the energy spectrum. It is known to be a valuable tool to study the influence of the Reynolds number on some statistical properties of turbulent flows [6]. It is however not able to take into account intermittency in the sense that at asymptotically high Reynolds number no corrections to the inertial range scaling of the energy spectrum remain.

In figure 1, the results of EDQNM computations of freely decaying turbulence, the multifractal formalism as described in [7] and experimental measurements of windtunnel turbulence [8] are compared, all at a Taylor-scale-based Reynolds R_λ number of approximately 2500. A good qualitative agreement is observed at small scales. At large scales, the statistics of the experiment are not fully converged. Both EDQNM and the multifractal prediction follow a powerlaw with an exponent close to -0.045. At very high Reynolds number this exponent should tend to zero for the EDQNM results. Therefore, at physically relevant Reynolds numbers ($R_\lambda = 2500$ is the highest value obtained in windtunnels until now), the inertial range scaling of the skewness is not an adequate measure for intermittency, since the influence of intermittency cannot be distinguished from the influence of the Reynolds number. Other quantities, such as the flatness, would perhaps give a better measure, since their corrections due to intermittency should be larger.

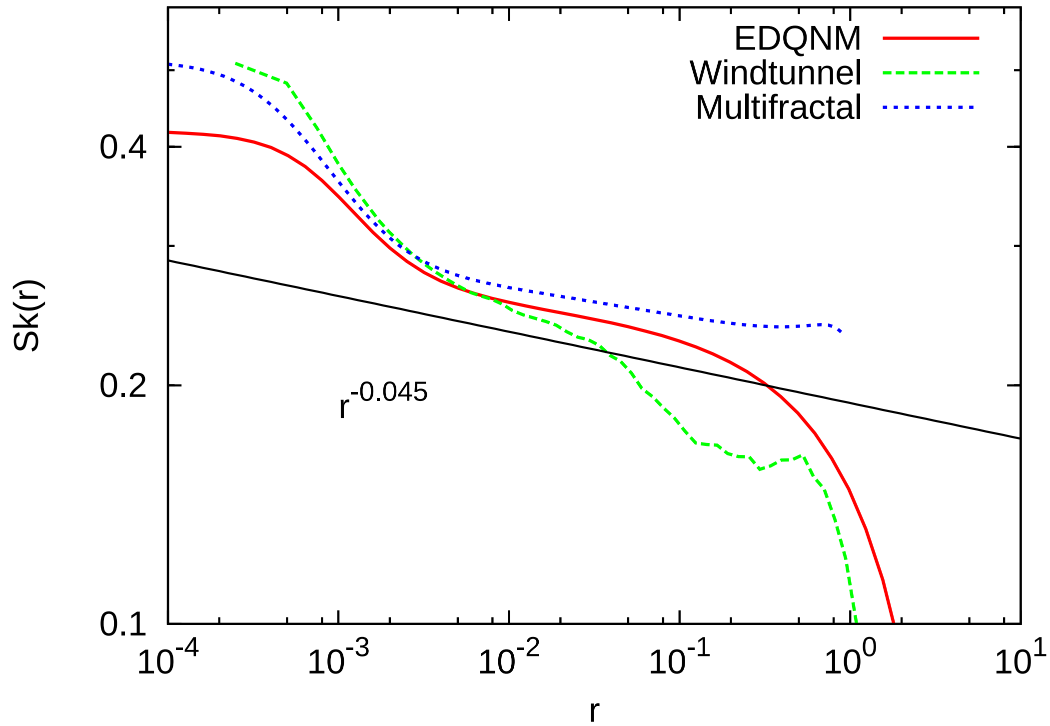

FIG. 1: The velocity increment skewness at $R_\lambda \approx 2500$. Results from EDQNM computations, the multifractal formalism and experimental results.

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