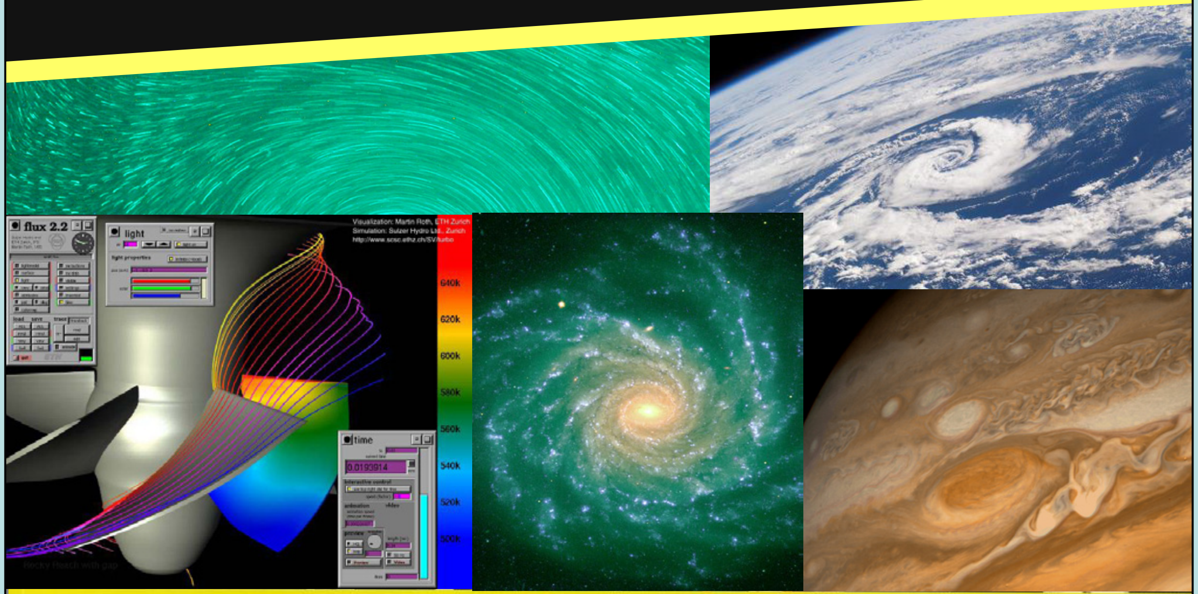
Tout sur les ondes d'inertie !

La turbulence en rotation POUR LES NULS

Frédéric Moisy



À mettre entre toutes les mains !

La turbulence en rotation pour les nuls

F. Moisy

L. Agostini, P.P. Cortet, C. Lamriben, L. Messio,
C. Morize, M. Rabaud, G. Tan, J. Seiwert

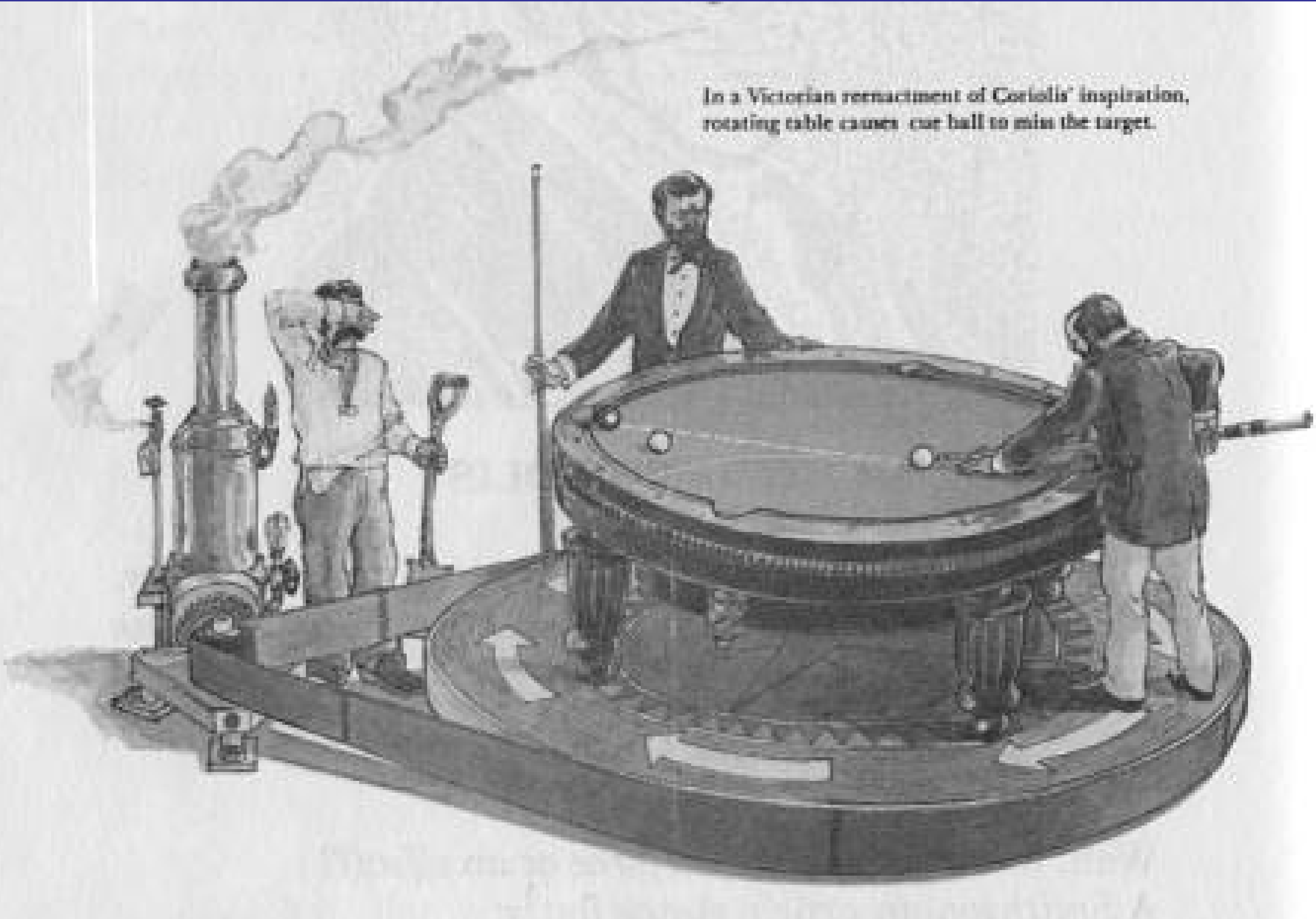
FAST, Univ. Paris-Sud, Univ. Pierre et Marie Curie, Orsay.

J. Sommeria

LEGI / Coriolis, Univ. Joseph Fourier, Grenoble.

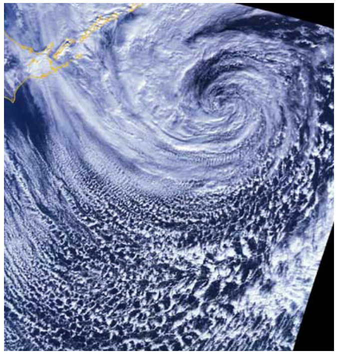
Funded by ANR « HiSpeedPIV » grant no. 06-B-0363

Turbulence en rotation = Turbulence + Coriolis



In a Victorian reenactment of Coriolis' inspiration, rotating table causes cue ball to miss the target.

Rotating turbulence: Where? and Why?



Motivations:

Geophysics (ocean, atmosphere, dynamo…)

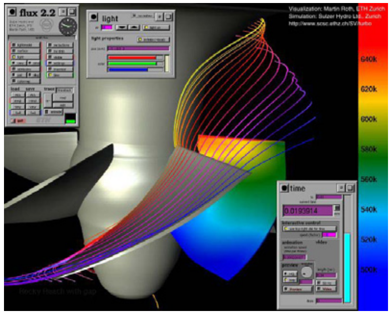
Astrophysics (galaxies, accretion disks)

Industrial flows (turbomachines…)

-> challenge for modelling

Basic effects of the background rotation:

- (partial) two-dimensionnalisation
- Energy transfers inhibited, reduced decay
- Cyclone/Anticyclone symmetry breaking



The Taylor-Proudman theorem

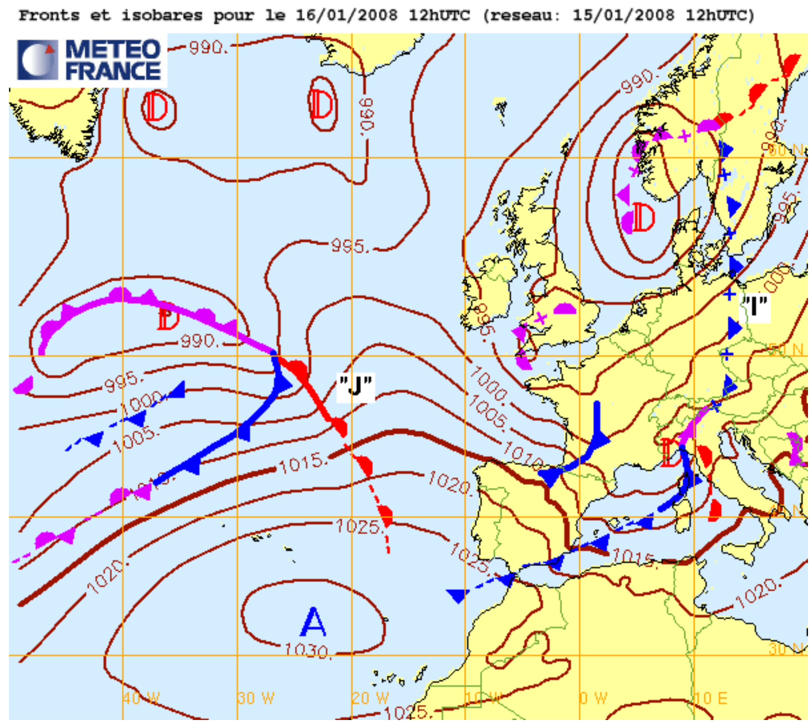
Navier-Stokes in a rotating frame:

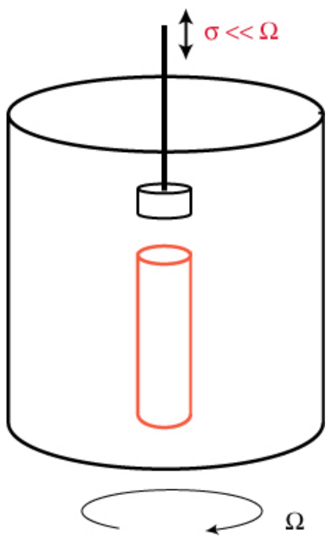
$$\cancel{\frac{\partial}{\partial t}\vec{u}} + \cancel{(\vec{u}\cdot\vec{\nabla})\vec{u}} = -\frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega}\times\vec{u} + \cancel{\nu\nabla^2\vec{u}}$$

Geostrophic equilibrium :

$$\vec{u} \perp \vec{\nabla}p$$

$\partial(.)/\partial z = 0$: **2D (*but 3C !!*)**





Assumptions: $\left|\vec{u}\cdot\vec{\nabla}\vec{u}\right| << \left|2\vec{\Omega}\times\vec{u}\right|$ $\quad (Ro = \frac{\omega}{2\Omega} << 1)$

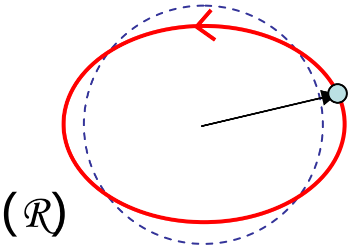
i.e., **non-linearities neglected**

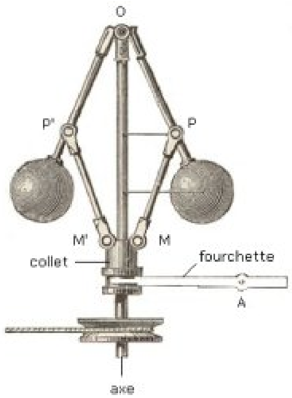
- **Two-dimensionalisation = nonlinear mechanism**
- **Transition 3D-2D ≠ Taylor-Proudman**

Ondes d’inertie

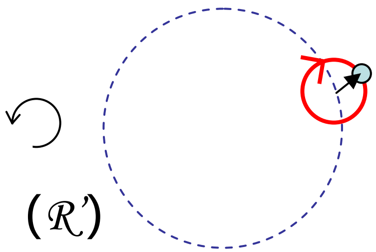
Origine des ondes d’inertie:

- Reférentiel fixe : conservation du moment cinétique



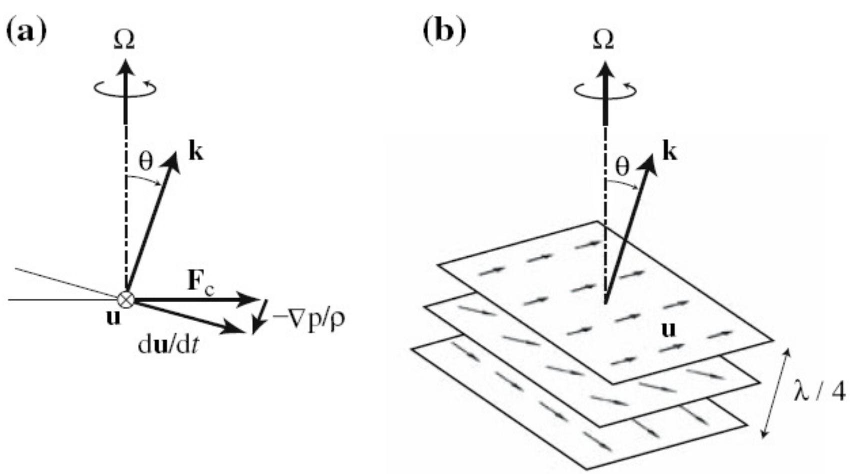


- Référentiel tournant : Force de Coriolis = Force de rappel



$$\frac{\partial \vec{u}}{\partial t} = -2\vec{\Omega} \wedge \vec{u}$$

Fluide incompressible : $\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho}\vec{\nabla} p - 2\vec{\Omega} \wedge \vec{u}$



Ondes forcées à fréquence σ < 2Ω

=> plan d’oscillation penché

Ondes d’inertie

Solutions en onde : $\vec{u}(t) = \vec{u}_0 \exp(\sigma t - \vec{k} \cdot \vec{x})$

Ondes **transverses** : $\vec{\nabla} \cdot \vec{u} = 0 \Rightarrow \vec{u} \perp \vec{k}$

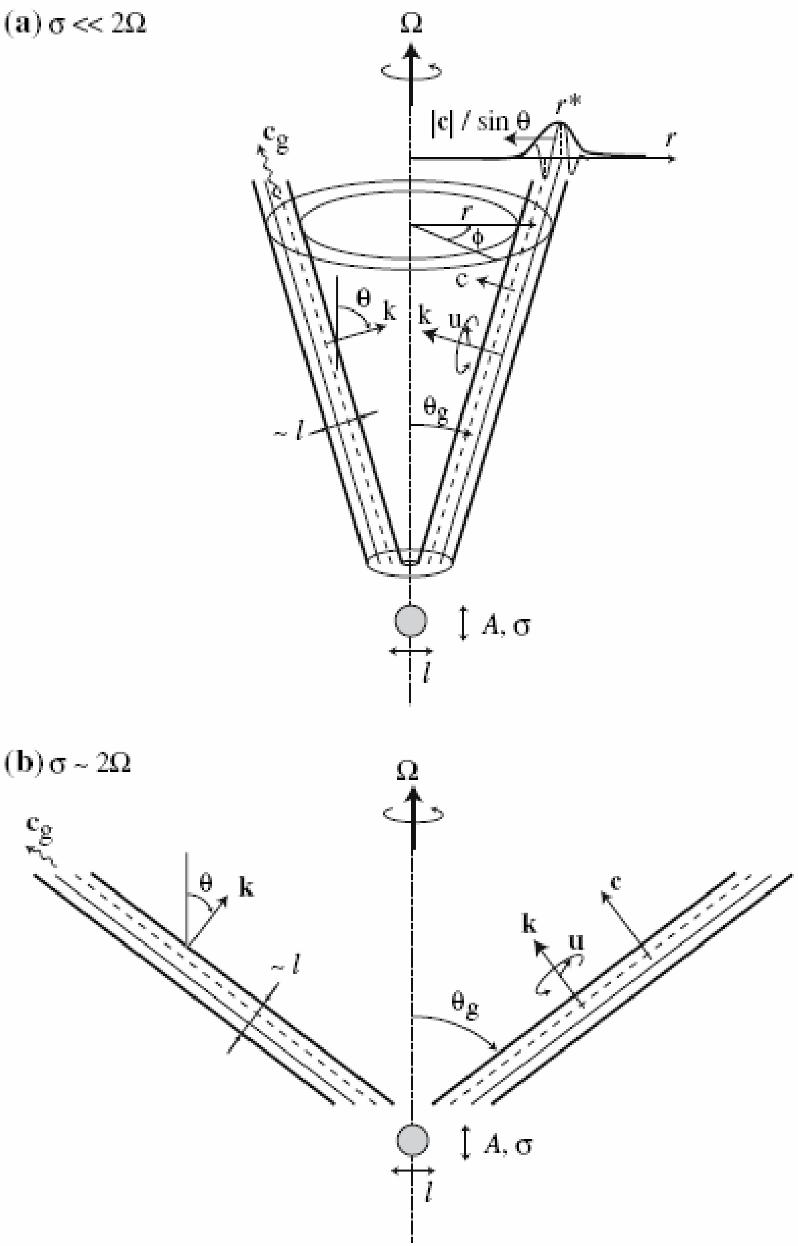
Relation de dispersion : $\sigma(\vec{k}) = 2\Omega \dfrac{k_z}{|\vec{k}|} = 2\Omega \cos\theta$

Vitesse de phase : $\vec{c}(\vec{k}) = \sigma(\vec{k}) \dfrac{\vec{k}}{|\vec{k}|^2}$

Vitesse de groupe : $\vec{c}_g(\vec{k}) = \vec{\nabla}_k \sigma$

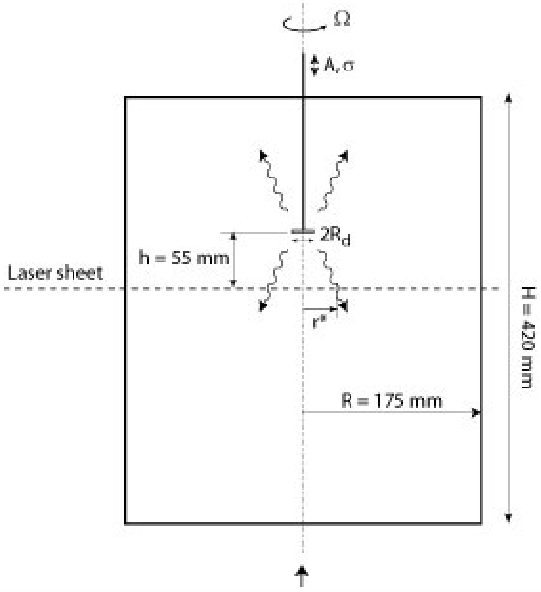
Ondes dispersives (c = $f(k)$) et **anisotropes**

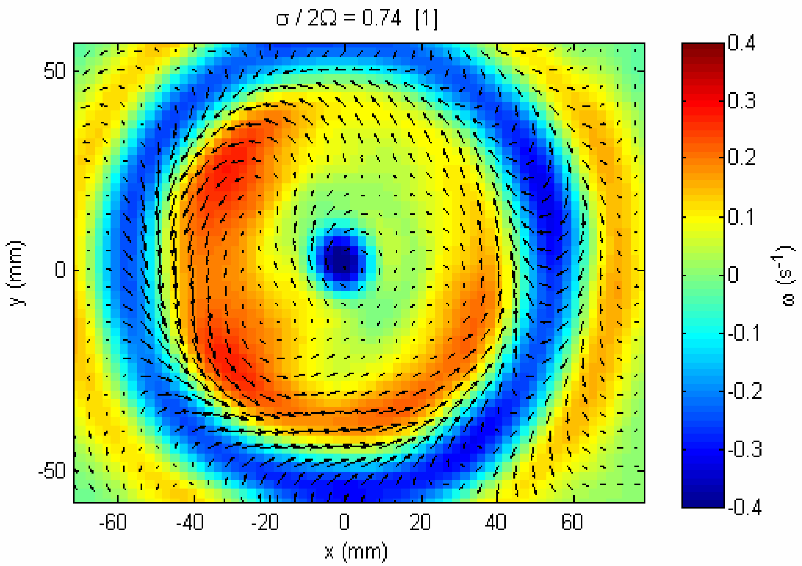
(Pour $\sigma << 2\Omega$, Taylor-Proudman retrouvé)

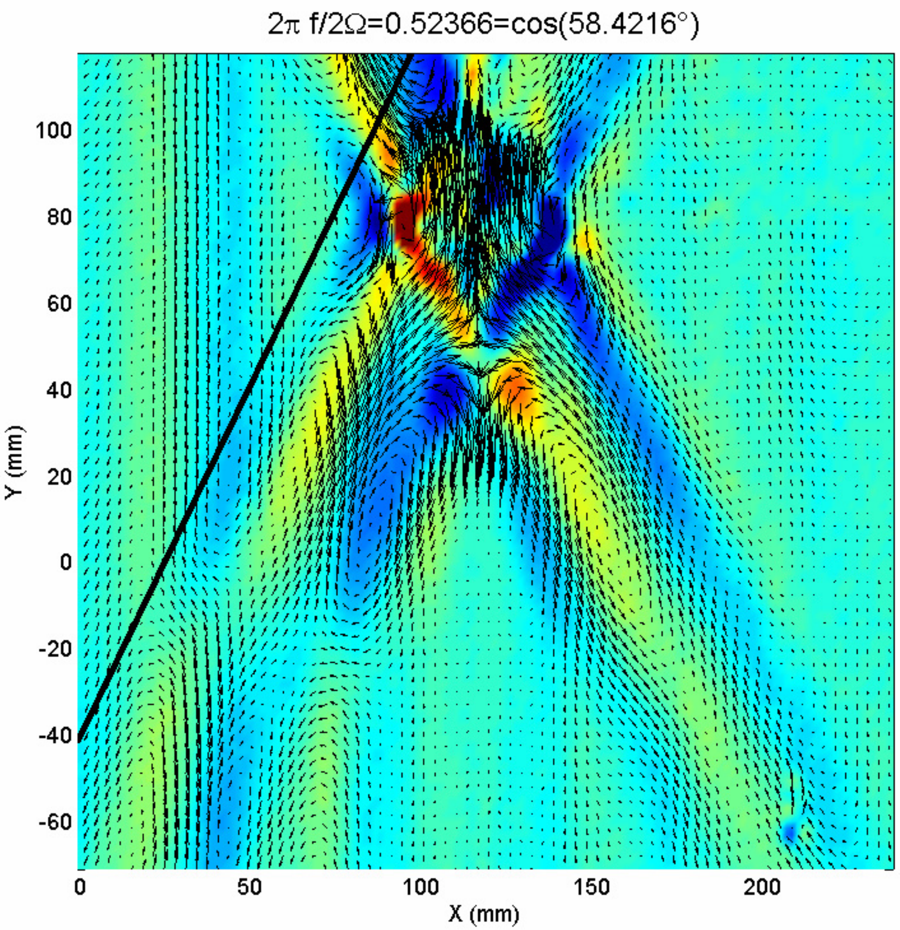


Phillips 1963; Lighthill

Ondes d'inertie : observations par PIV



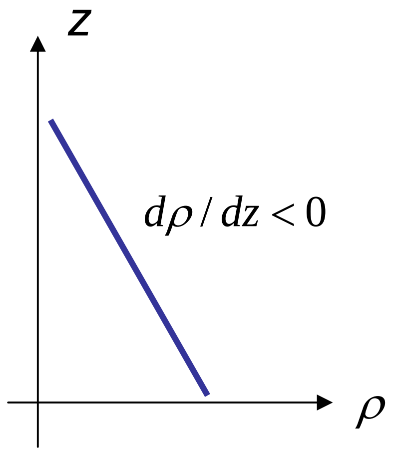




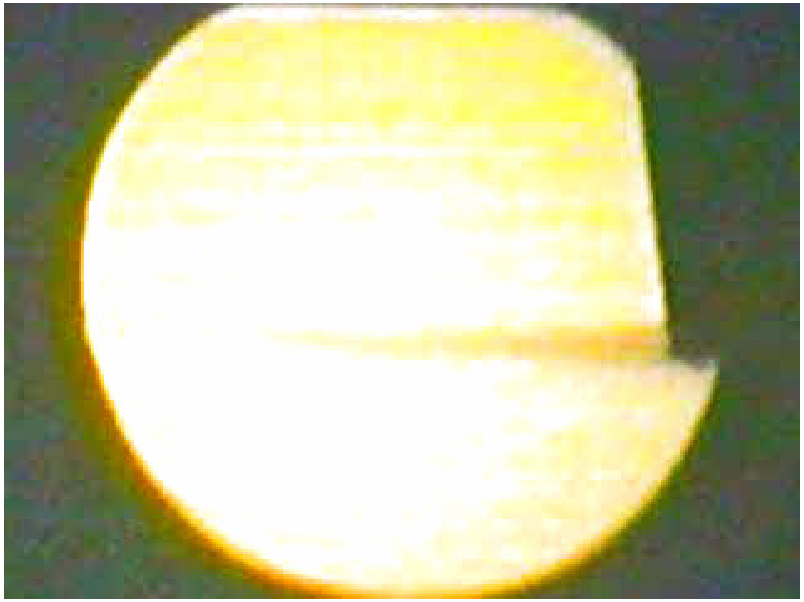
P.P. Cortet, décembre 2009

L. Messio, C. Morize, M. Rabaud,
F. Moisy, *Exp. in Fluids* **44**, 519 (2008)

Ondes d'inertie vs. Ondes internes



Stratification
stable



Physical Oceanography Demo Movies

at University of Rhode Island

Fréquence de Brunt-Väisälä

$$N = \sqrt{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}$$

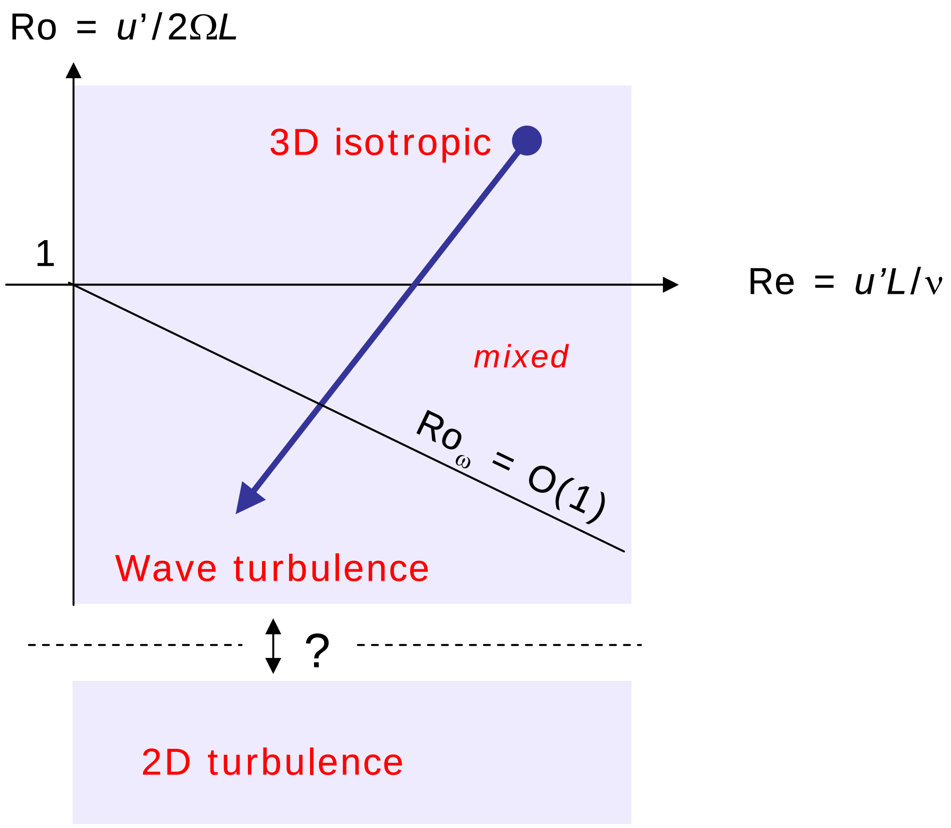
Relations de dispersion

$\sigma = N \sin\theta$ -> « Pancakes »

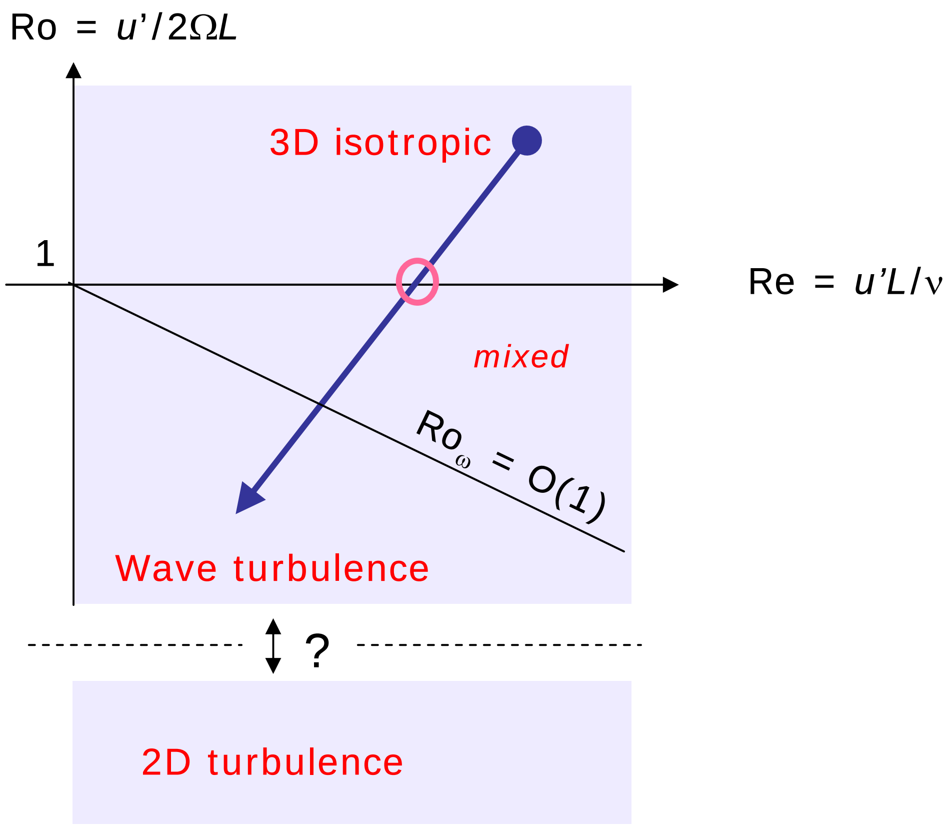
$\sigma = 2\Omega \cos\theta$ -> « Colonnes de Taylor-Proudman »

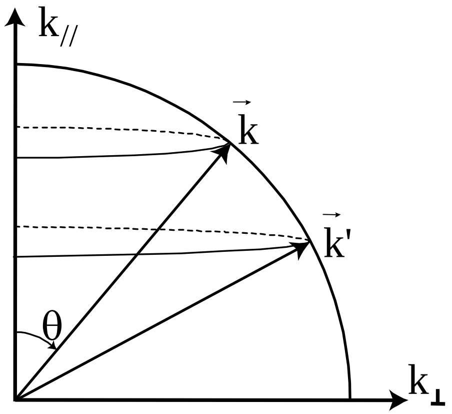
$$\sigma = \sqrt{(2\Omega\cos\theta)^2 + (N\sin\theta)^2}$$

Some background for decaying rotating turbulence



Some background for decaying rotating turbulence





Linear: timescale Ω^{-1}

Energy propagation via inertial waves

Non-linear: timescale *l* / *u'*

Two-dimensionalisation process

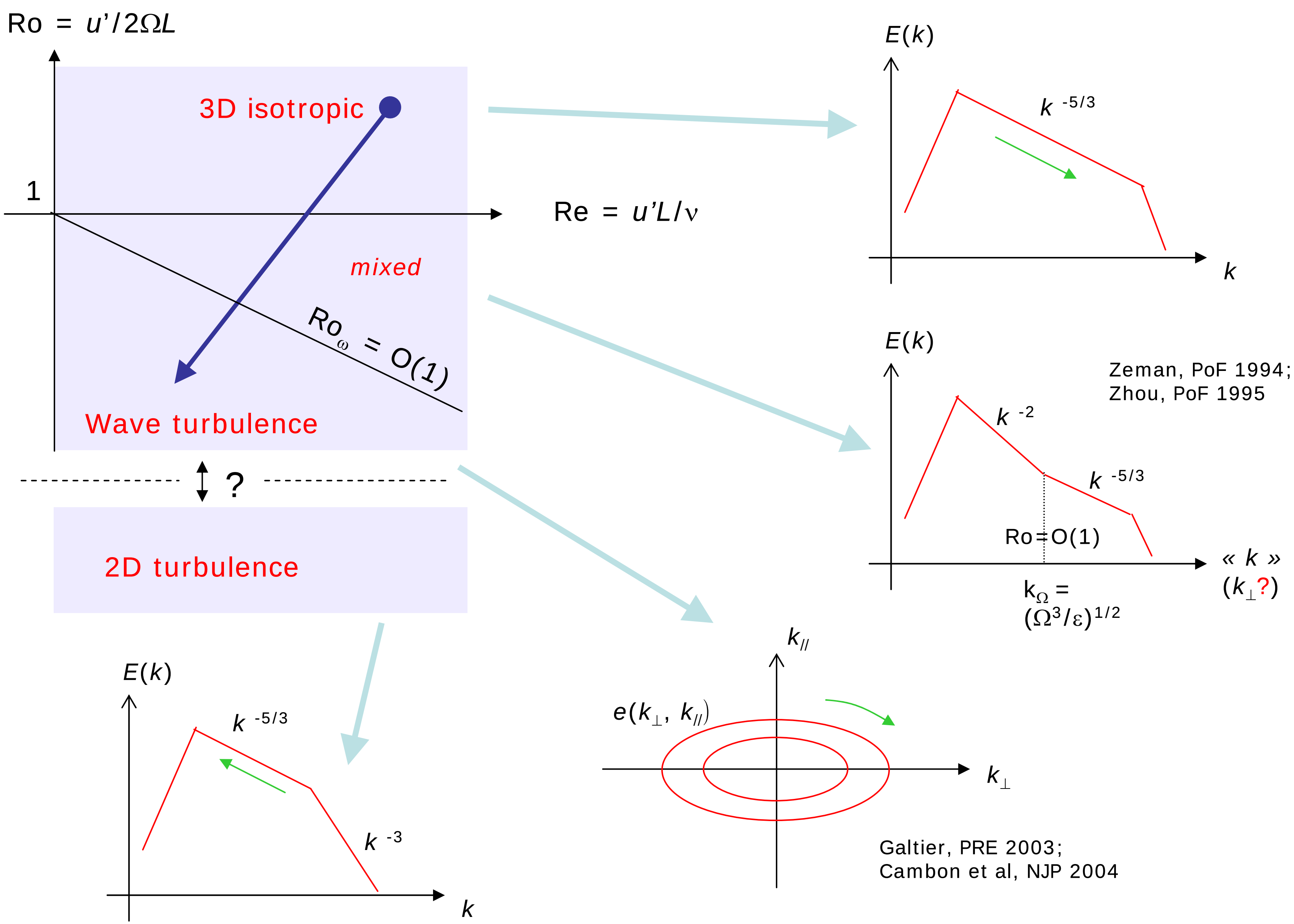
via angular energy transfer,

i.e. nonlinear mode coupling.

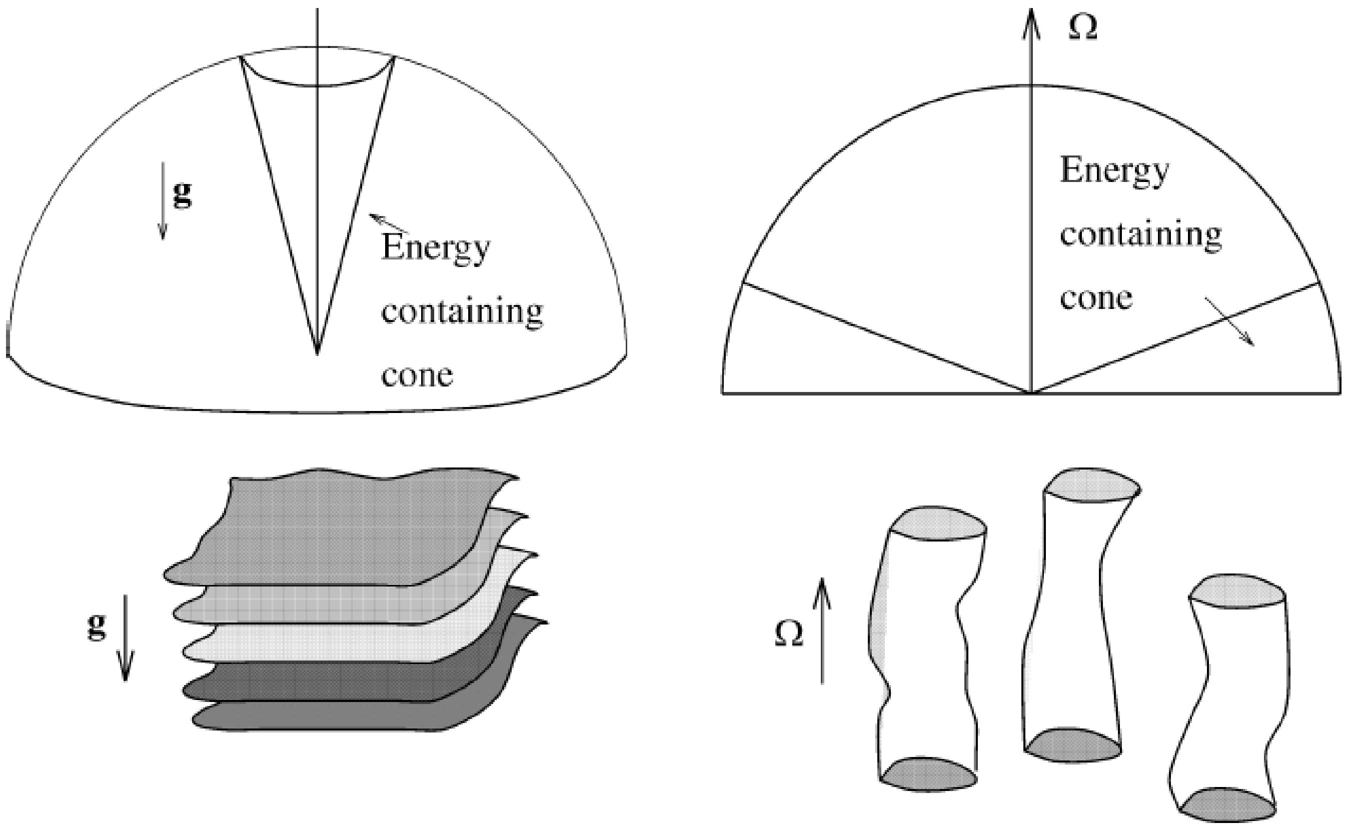
Coupling between linear and non-linear effects

at $t = t^*$, when Ro $(t^*) = \tau_{lin} / \tau_{nl} = O(1)$

Some background for decaying rotating turbulence



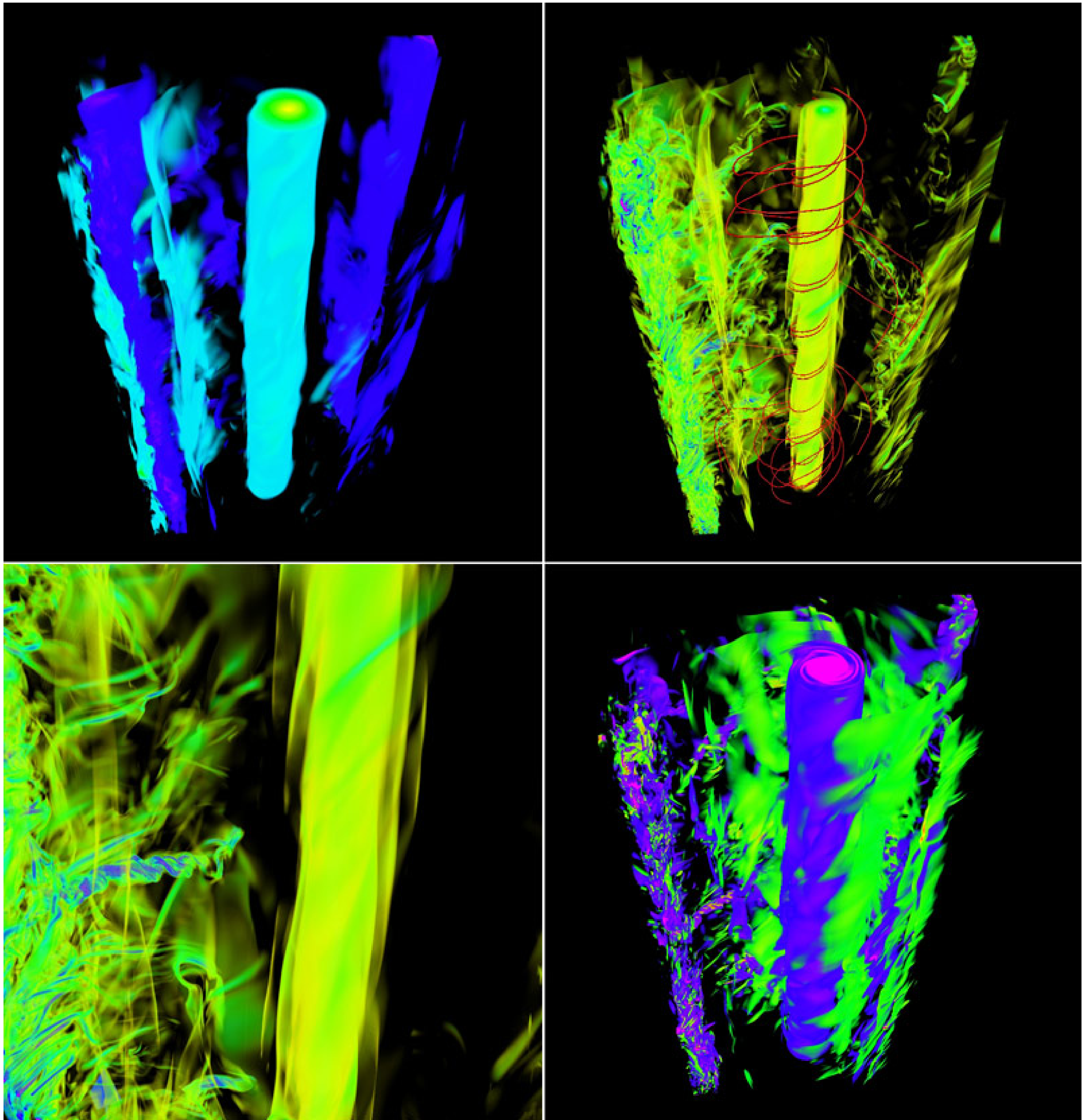
Energy transfers



From Cambon, *Eur. J. Mech. B - Fluids* ***20*** *(2001)*

Découplage du mode « lent » 2D ? (*3C !*)

Simulations



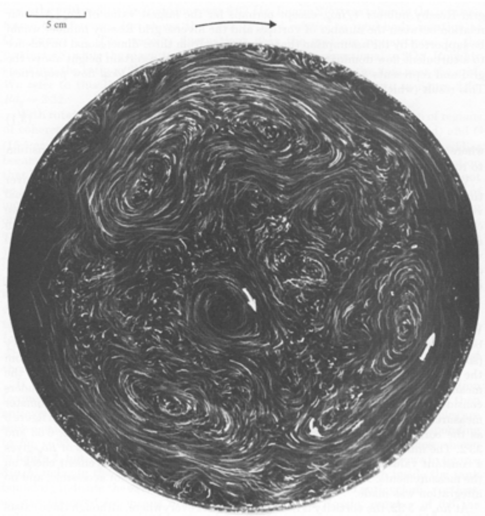
DNS 1596^3

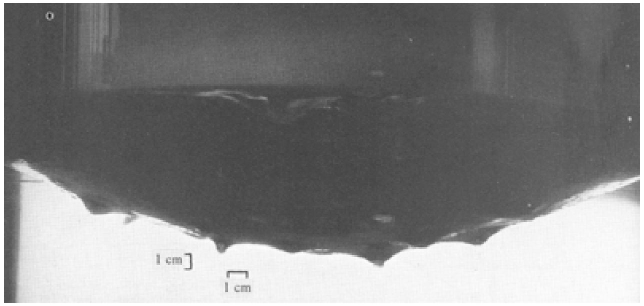
Pouquet,
Mininni
(2009)

Some passed and recent experiments (XXth century)

Oscillated grid in a rotating tank

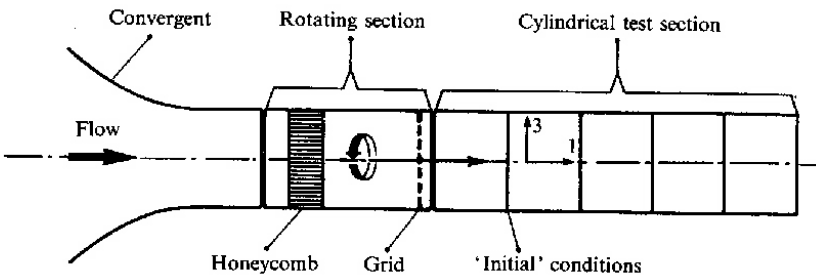
Hopfinger, Browand & Gagne, JFM (1982)





Hot wire meas. in wind tunnel

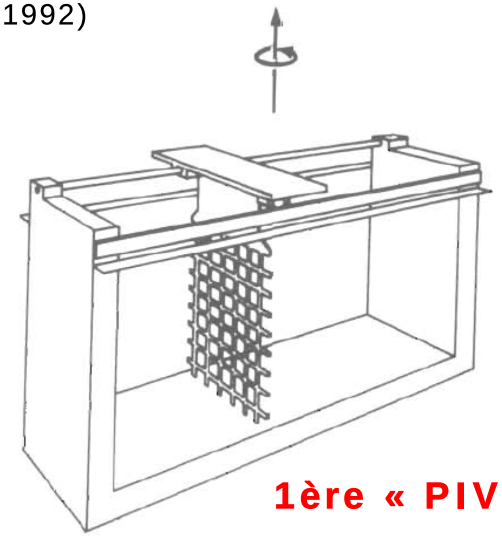
Jacquin *et al*, JFM (1990)



1-point measurement

Grid in a rotating channel

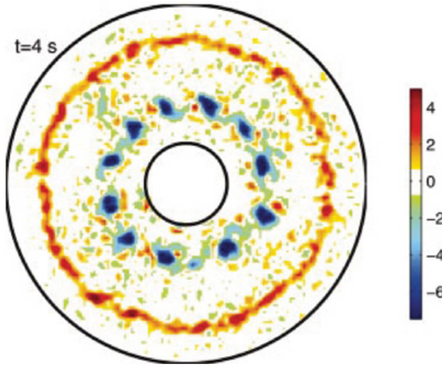
Dalziel (1992)



1ère « PIV » !

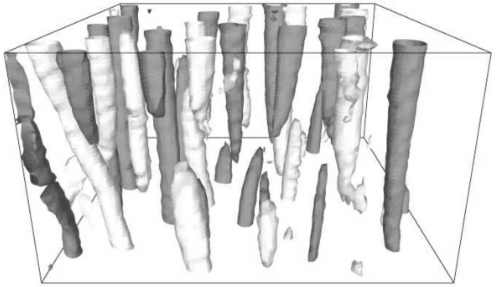
Some passed and recent experiments (XXIst century)

Baroud, Plapp, She
and Swinney, PRL (2002)

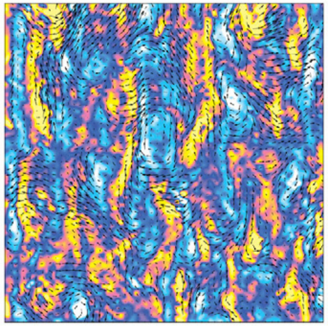


Morize, Moisy and
Rabaud, POF (2005)

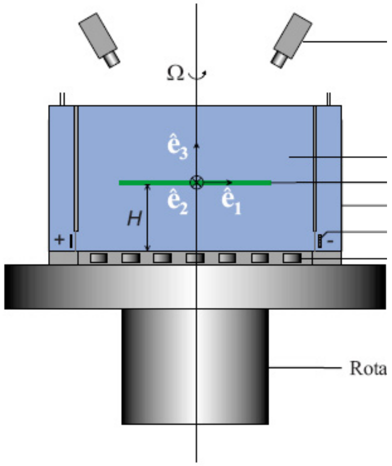
Praud, Sommeria
& Fincham, JFM (2006)



Staplehurst,
Davidson and Dalziel
JFM (2008)

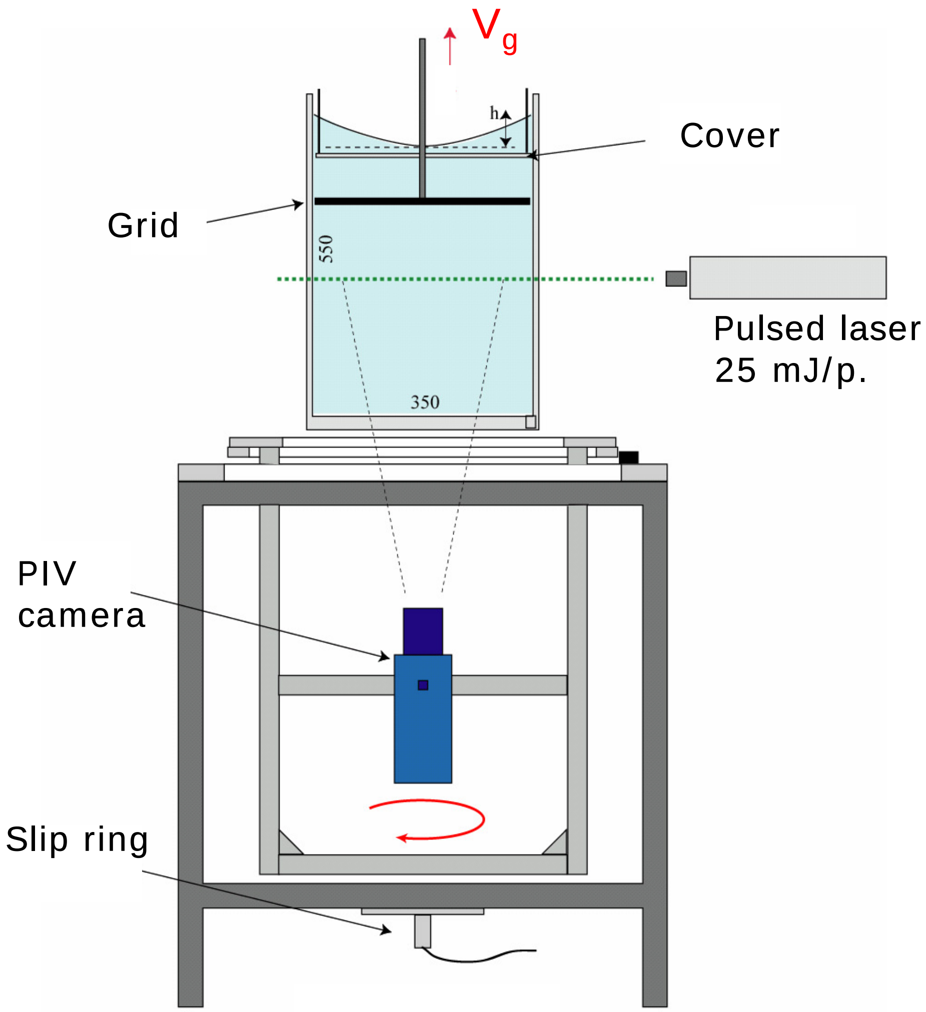


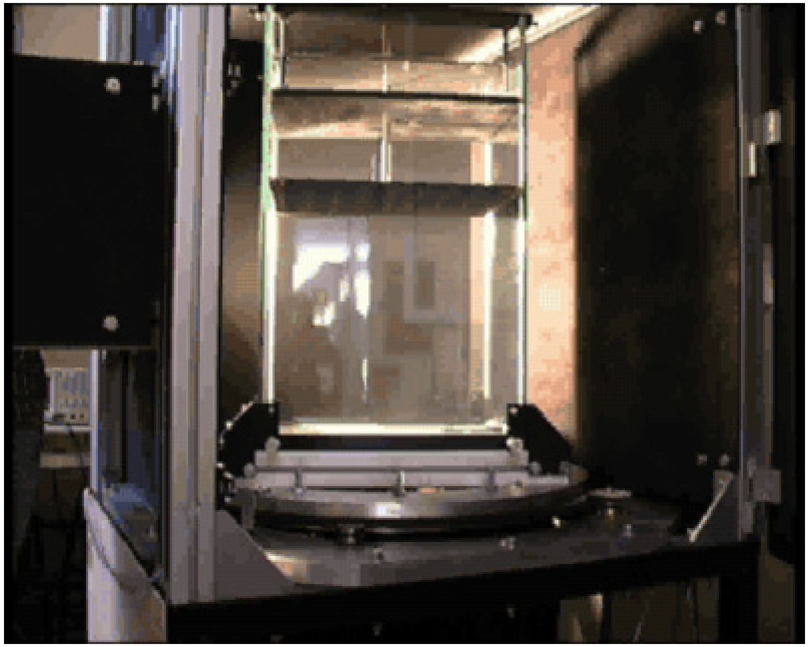
Van Bokhoven, Clercx,
van Heijst and Trieling,
POF (2009)



Lamriben, Cortet
and Moisy...

Experimental setup (laboratory FAST)





Grid velocity: V_g = 0.65 m/s (Re_g = 2.5 10^4)

Rotation rate: 0 - 0.7 Hz (Ro_g < 15)

Re' = u'M/ν ~ 4000 down to 100

Ro' = u'/2ΩM ~ 10 down to 0.01

Morize et al, Phys. Fluids (2005, 2006)

PIV measurements

PIV meas. in horizontal plane

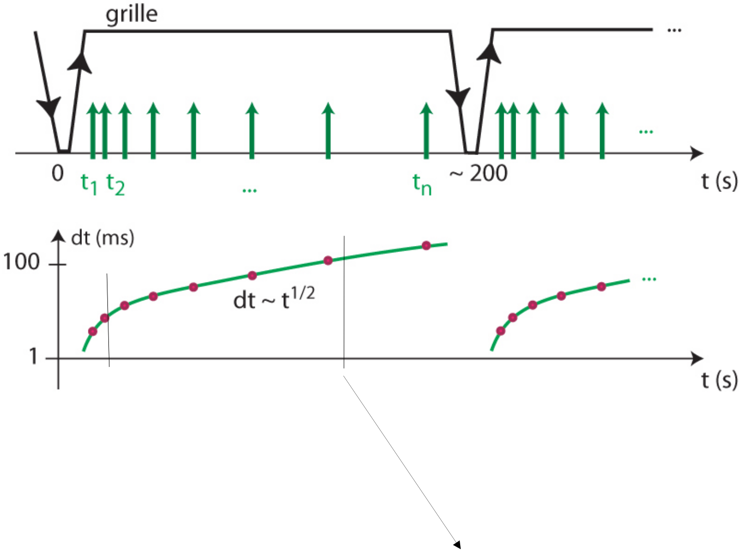
$v_{max} / v_{min} \sim 10^{-2}$

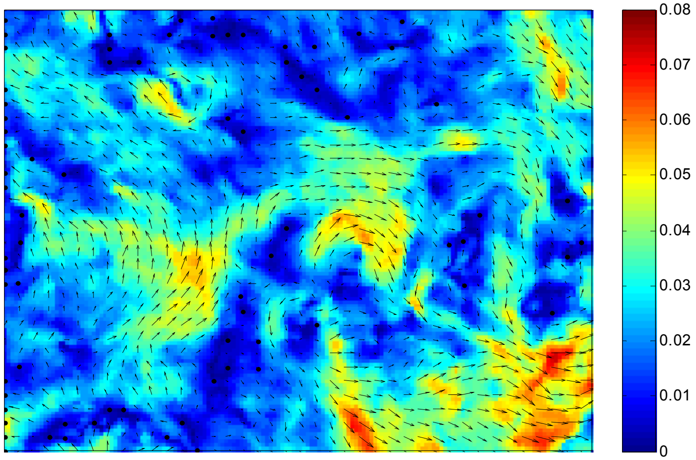
Spatial resolution: ~ 1 mm

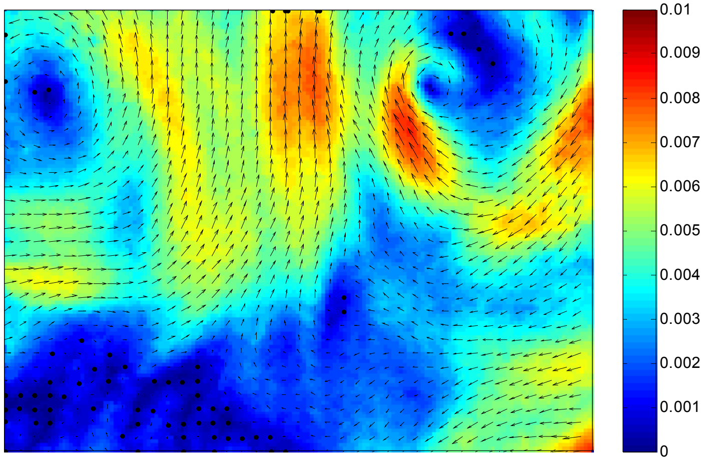
Ensemble averages

600 decays (independant realizations)

20 hours of experiment





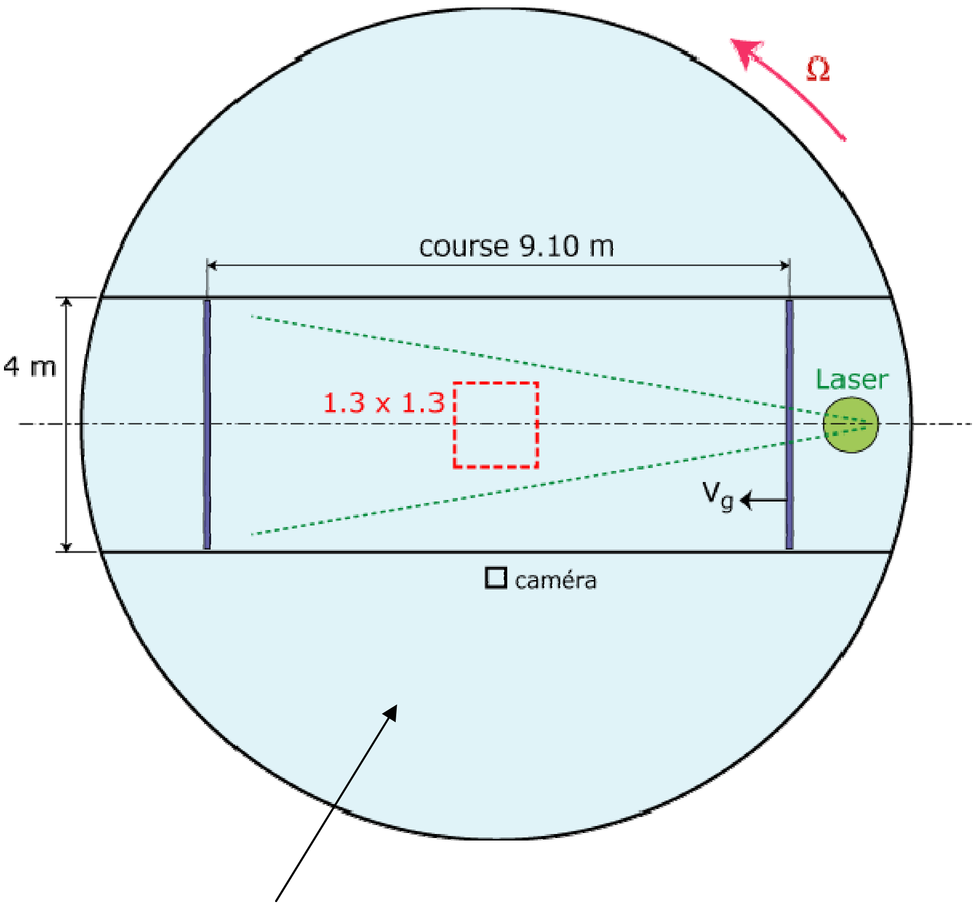


Experimental setup: 'Coriolis' Rotating Plateform (LEGI, Grenoble)

In collaboration with J. Sommeria, H. Didelle, S. Viboud

Moisy et al, subm. J. Fluid Mech (2009)

Experimental setup: ‘Coriolis’ Rotating Plateform (LEGI, Grenoble)



150 tons of water

9 m x 4 m x 1 m channel

Grid (of square mesh M = 14 cm), translated at V_g = 0.3 m s^{-1}

mounted on the **13 m** diameter ‘Coriolis’ rotating plateform

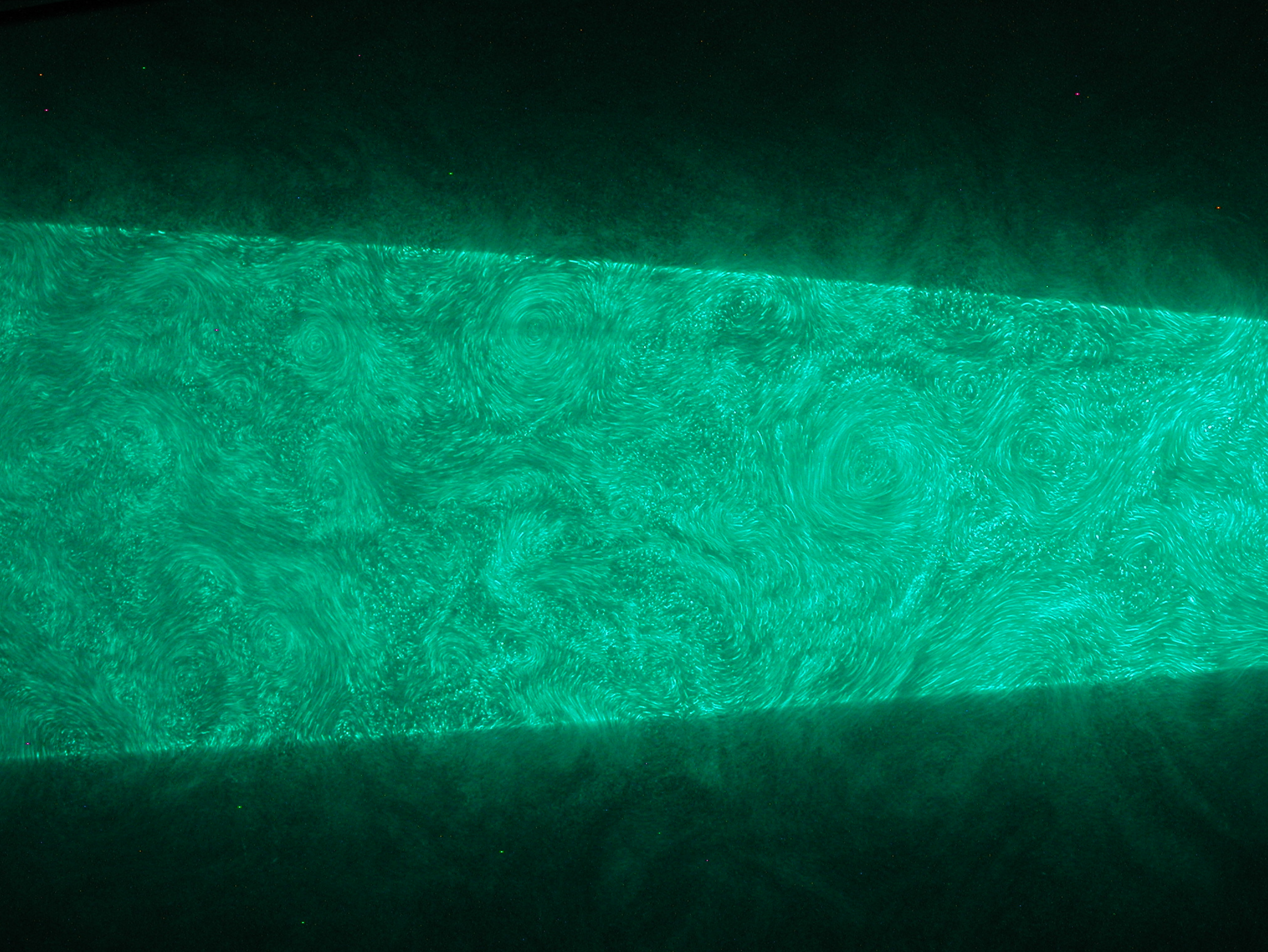
Rotation periods: T = 30, 60, 120 s

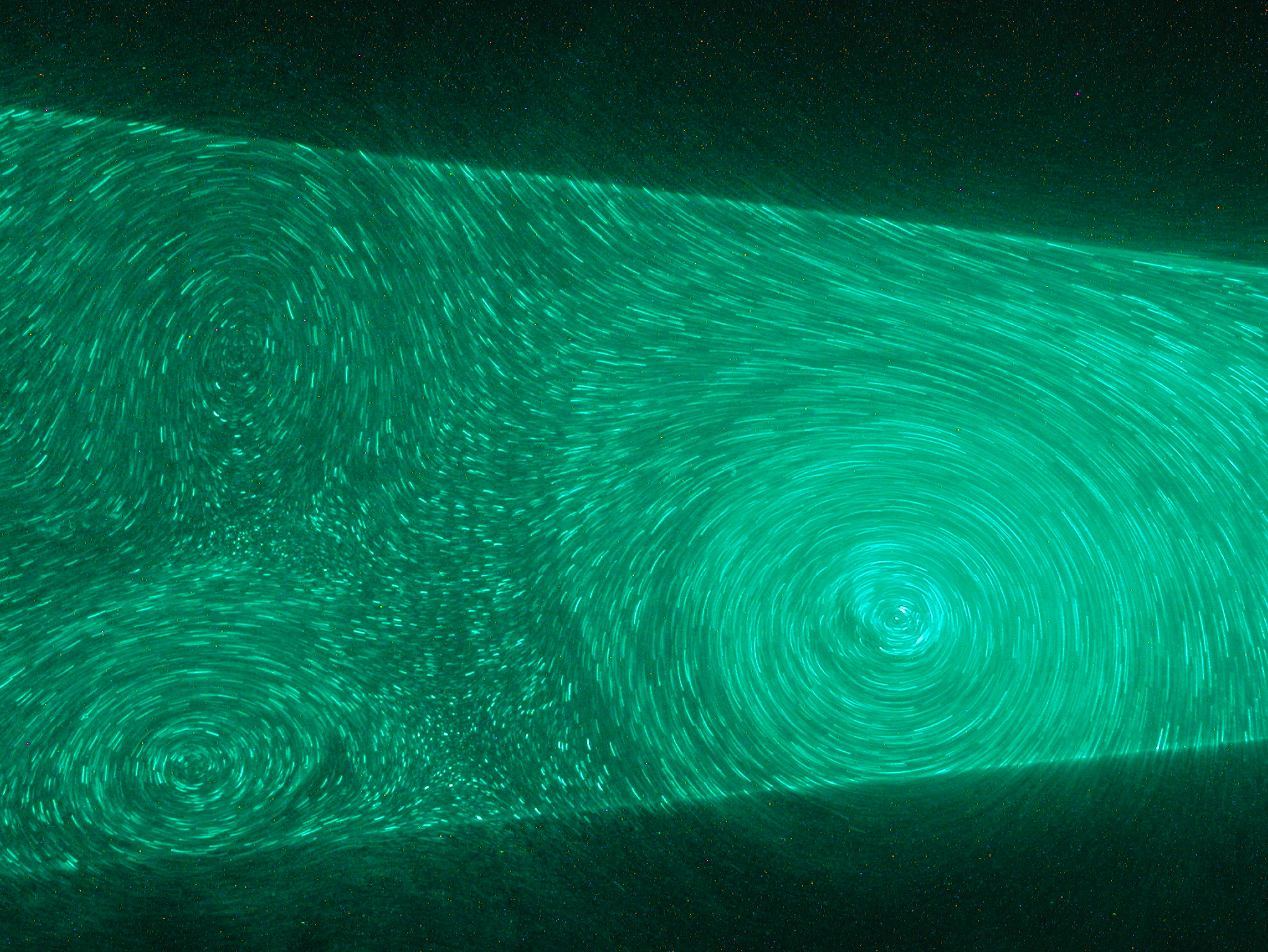
1 decay ~ 1 hour ~ 10^4 M/V_g

PIV measurements in horizontal and vertical planes

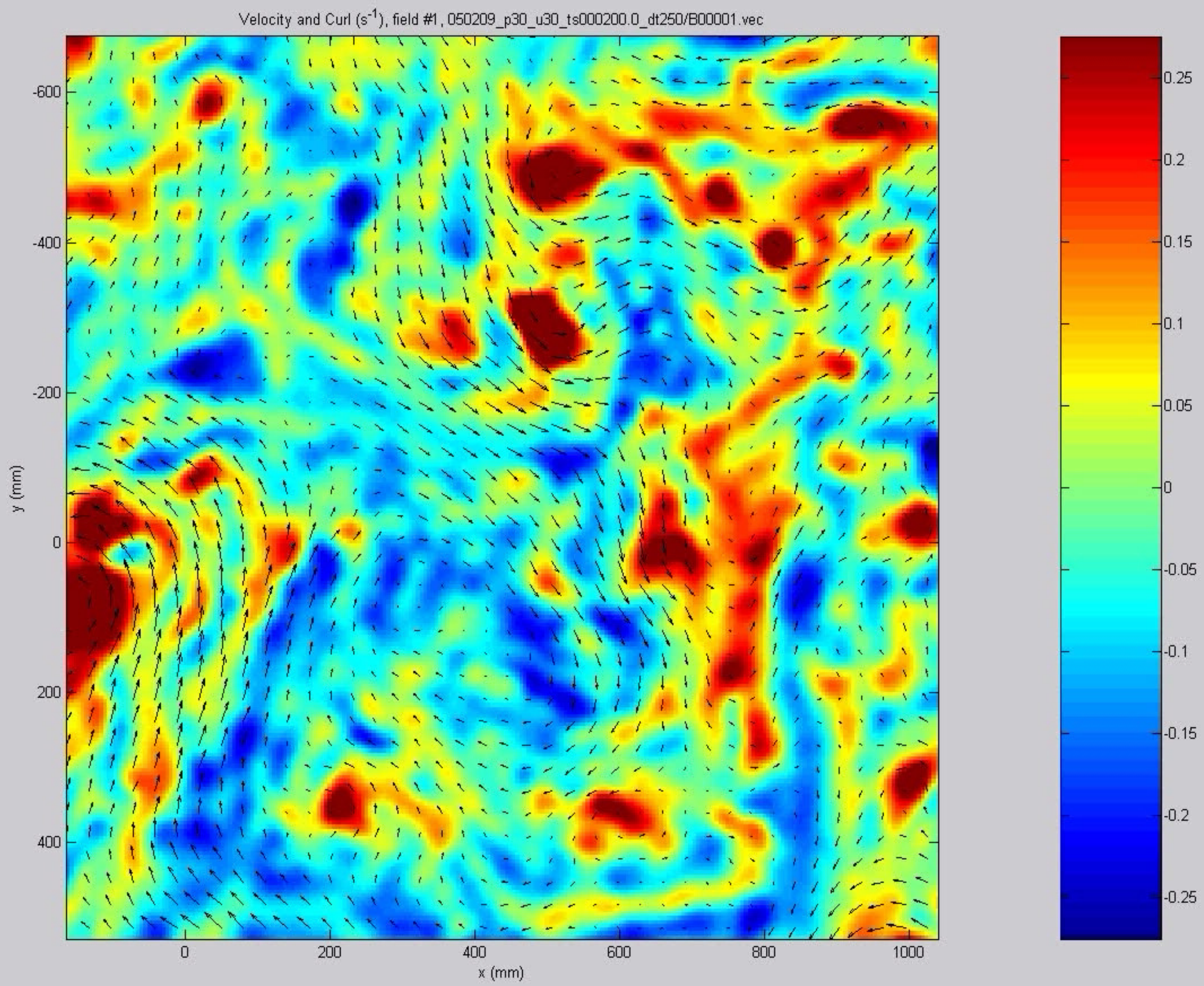
2000x2000 HR camera

Experimental setup: ‘Coriolis’ Rotating Plateform (LEGI, Grenoble)

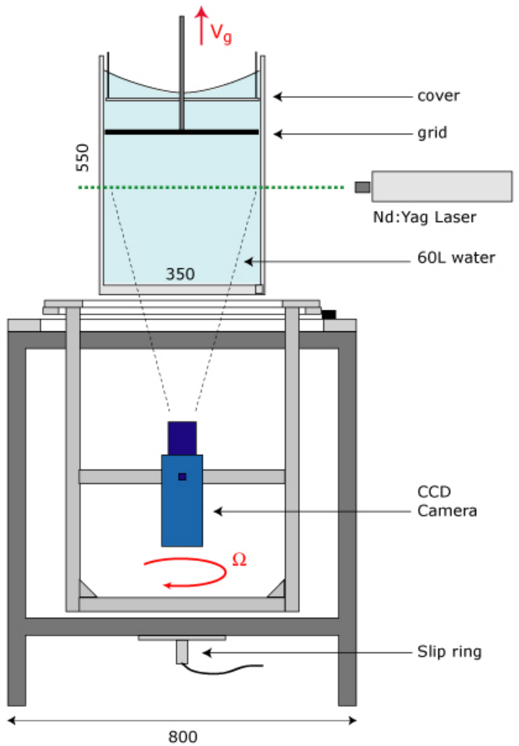


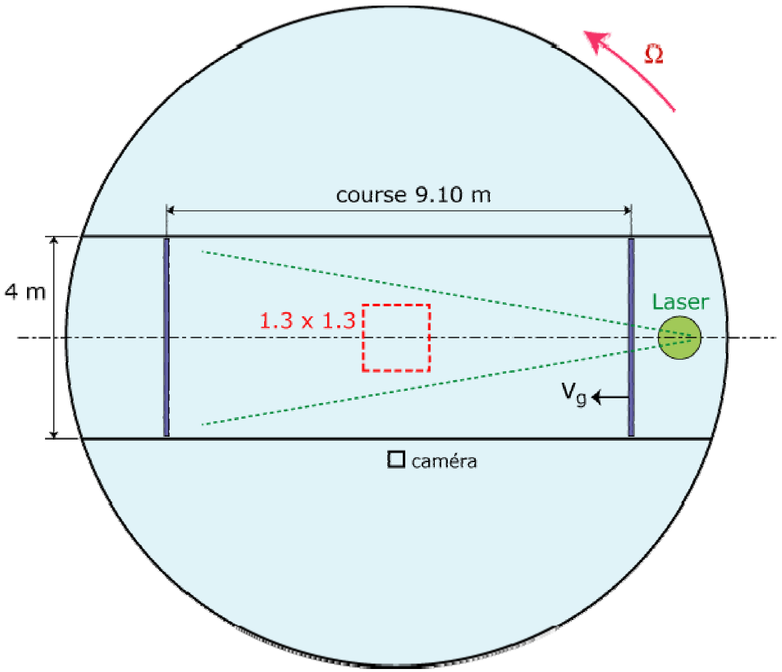


Vorticity field



Experimental setups: FAST vs Coriolis

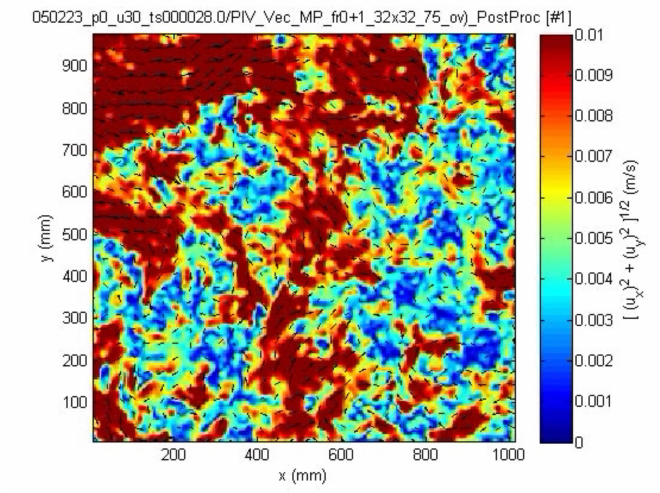




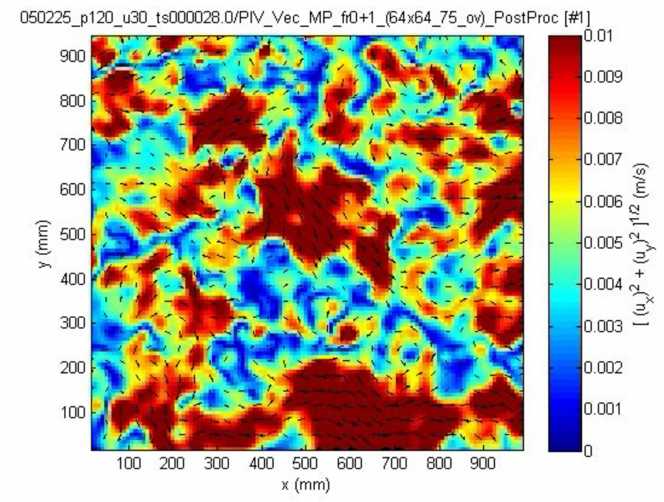
	FAST (0.44 m)	**Coriolis** (9.1 m)
	$V_g // \Omega$	$V_g \perp \Omega$
$Re_g = V_g M / \nu$	$(3 - 6) \times 10^4$	4×10^4
$Ro_g = V_g / 2\Omega M$	2 .. 20	4, 8, 16
Aspect ratios L_1 / h ; L_2 / h	(0.8 ; 0.8)	(4 ; 10)
	Ensemble average	Time resolved

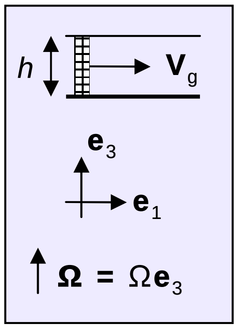
Velocity (u_1, u_3) $(x_1, x_3\ ;\ x_2 = L/2)$, in a 1 x 1 m^2 vertical plane

$T = \infty$
$Ro_g = \infty$

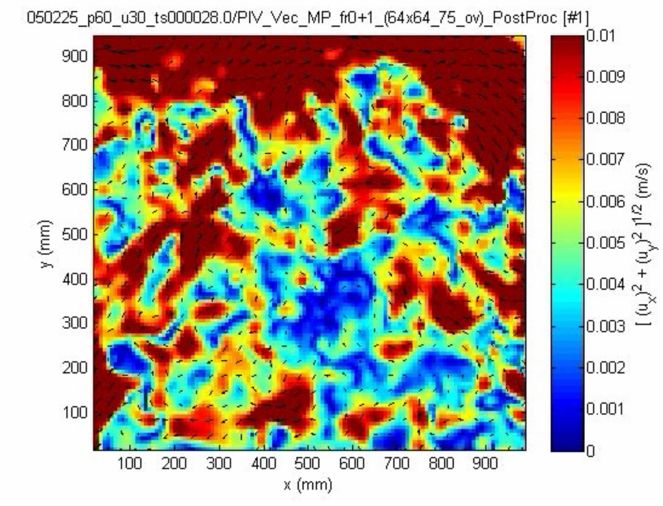


$T = 120$ s
$Ro_g = 16$

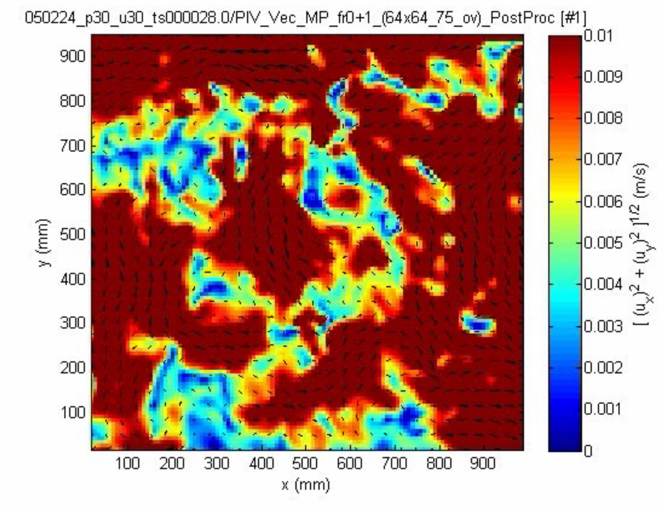




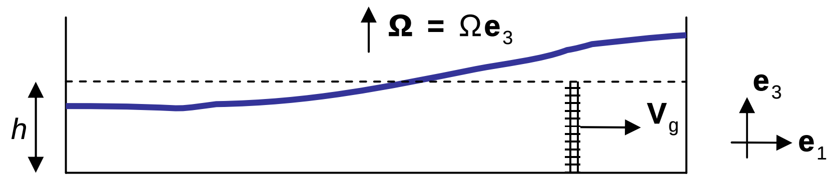
$T = 60$ s
$Ro_g = 8$

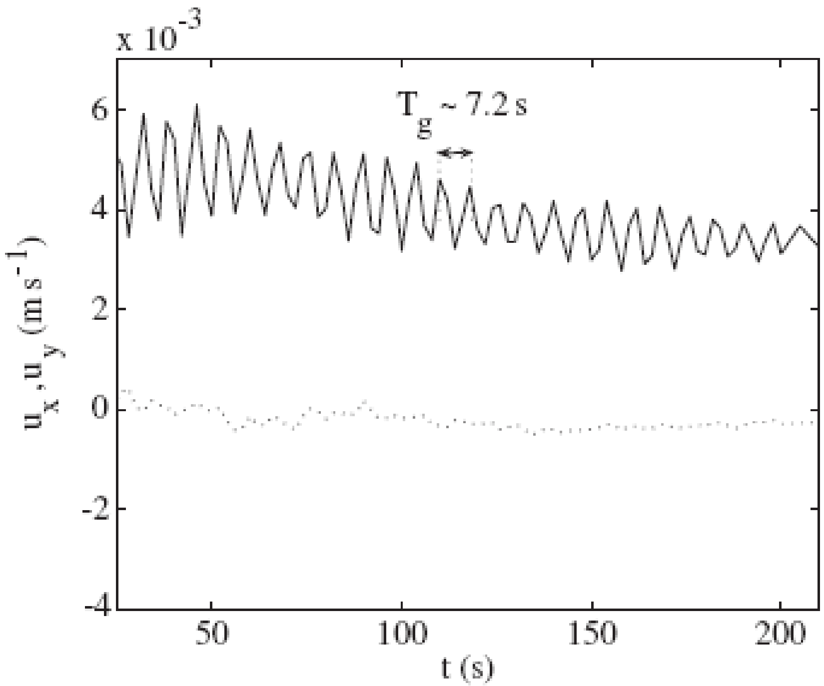


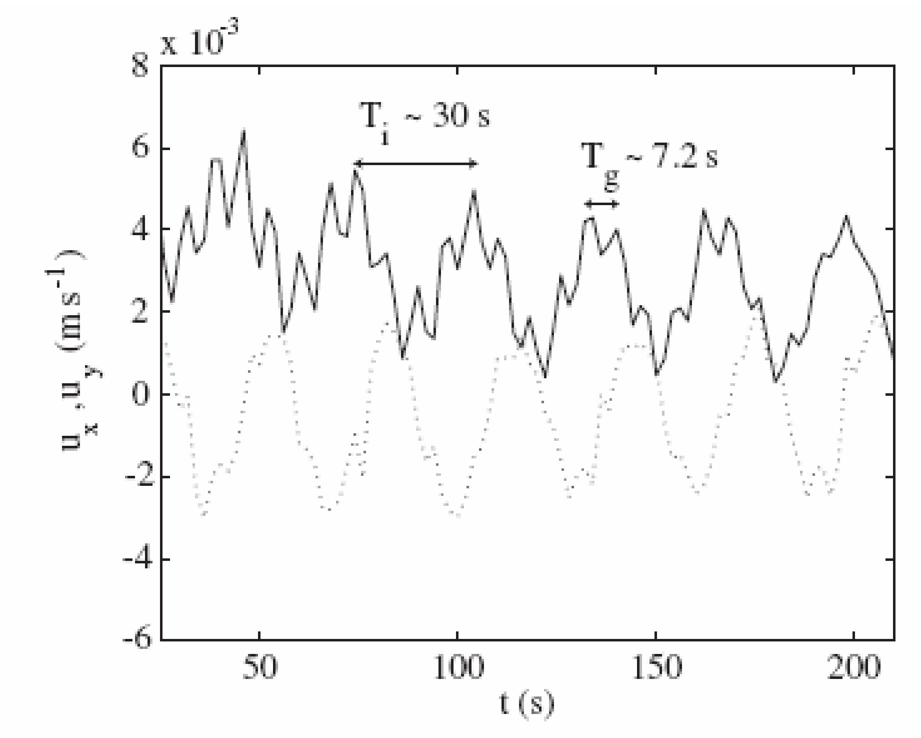
$T = 30$ s
$Ro_g = 4$



Large scale flow: gravity and inertia-gravity waves







$\Omega = 0$

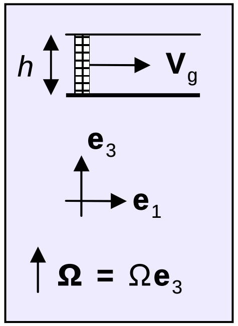
gravity wave, $T_g = (L/2) / (gh)^{1/2}$

$\Omega \neq 0$

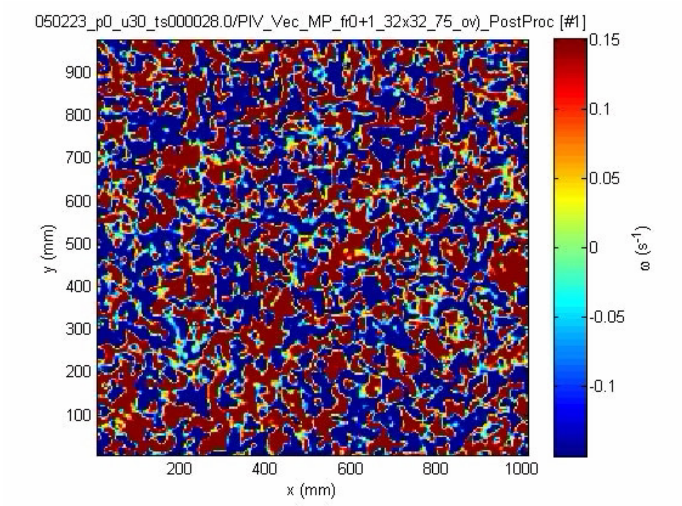
gravity wave T_g

+ inertia-gravity wave, $T_i = \pi / \Omega$

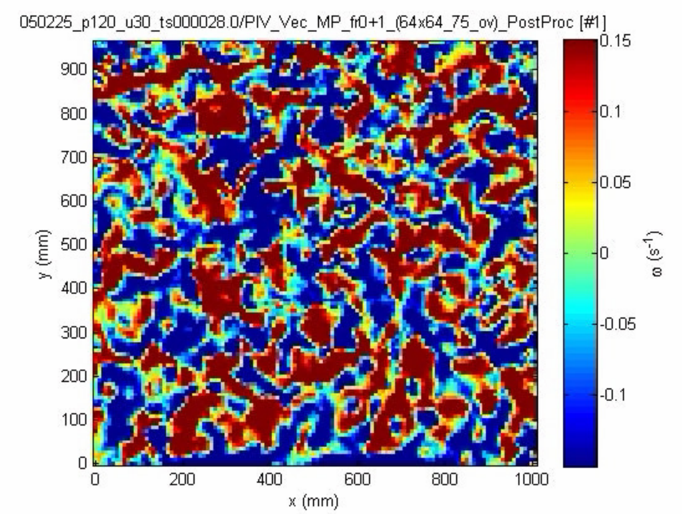
Vorticity ω_2 (x_1, x_3 ; $x_2 = L/2$), in a 1 x 1 m^2 vertical plane



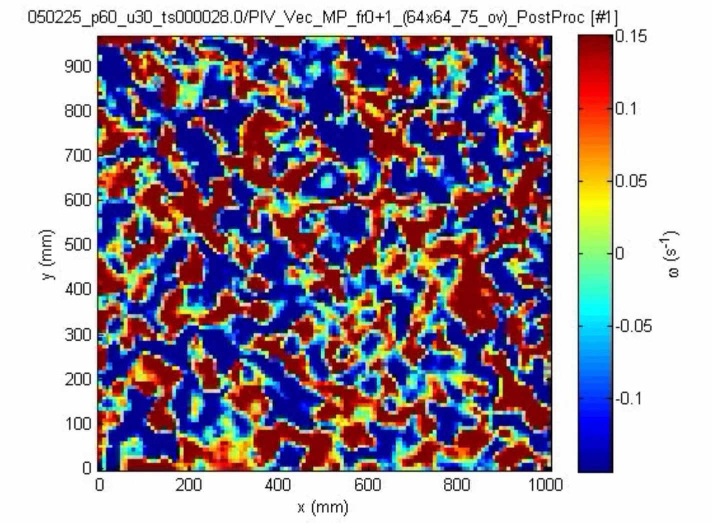
$T = \infty$
$Ro_g = \infty$



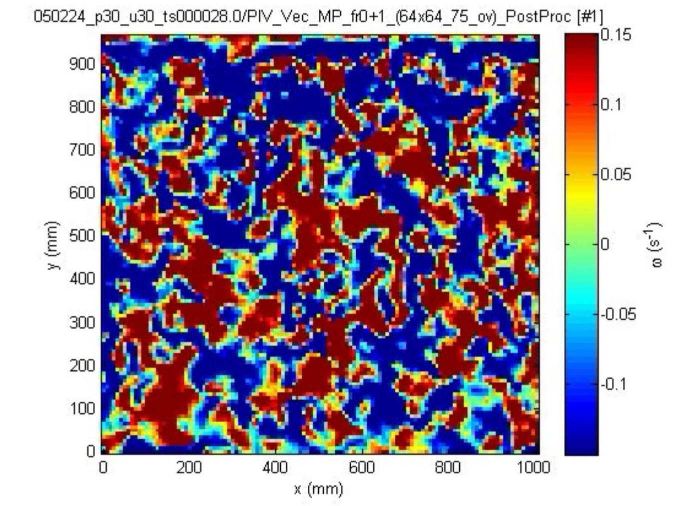
T = 120 s
Ro_g = 16



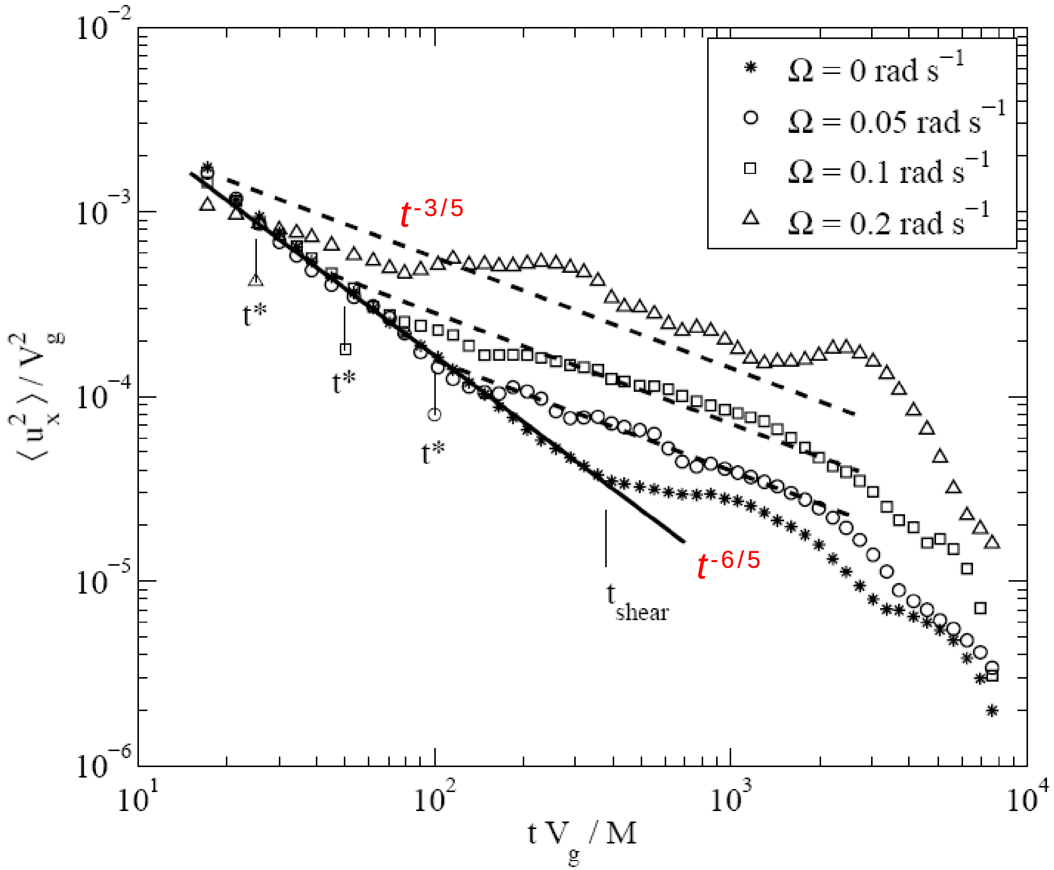
T = 60 s
Ro_g = 8



T = 30 s
Ro_g = 4



Energy decay



Small time:

$$\frac{\langle u_x^2 \rangle}{V_g^2} \simeq A \left(\frac{tV_g}{M} \right)^{-6/5}$$

(Saffman 1967)

Large time:

$$\frac{\langle u_x^2 \rangle}{V_g^2} \simeq A_\Omega Ro_g^{-3/5} \left(\frac{tV_g}{M} \right)^{-3/5}$$

(Squires et al 1994)

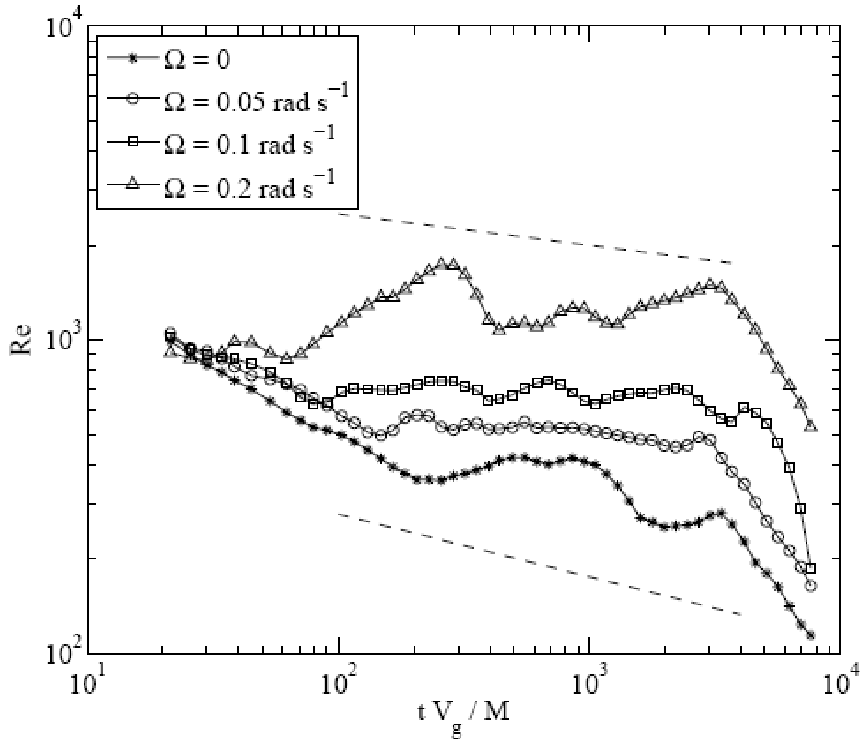
Nonlinear time $\tau_{nl}(t) \sim t$ in decaying isotropic turbulence

=> Ro = $\tau_{lin} / \tau_{nl} \sim O(1)$ reached at ***fixed*** number of tank rotations !

$\Omega t^* / 2\pi = 0.4$ tank rotations

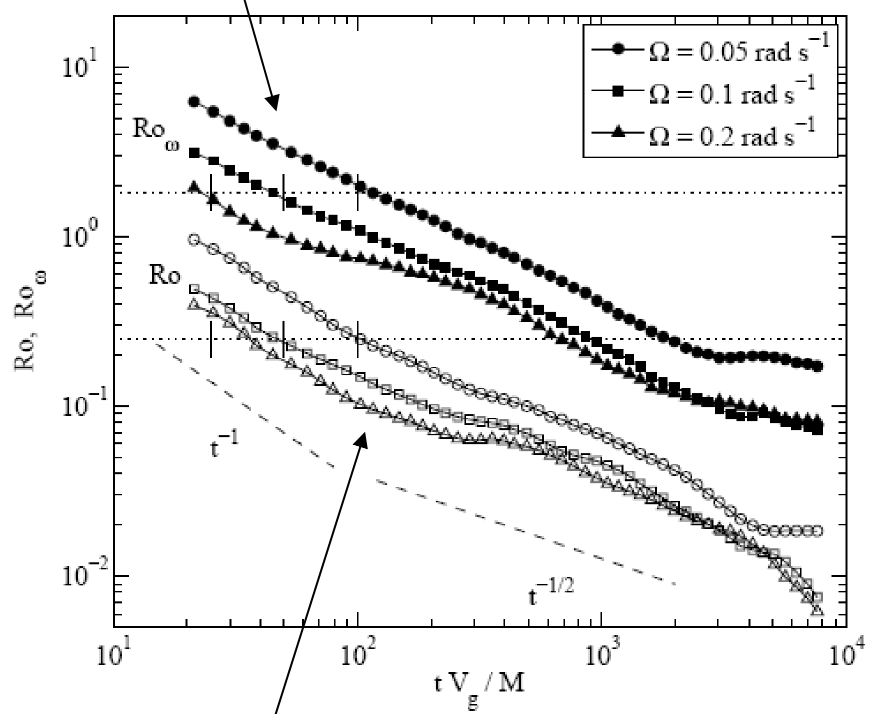
Reynolds and Rossby numbers

Re’ = u’ L / ν



Micro-Rossby number

$Ro_\omega' = \omega' / 2\Omega$



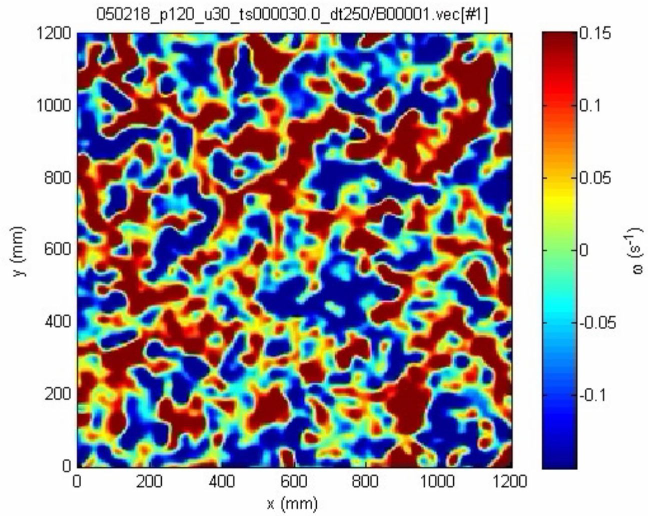
Macro-Rossby number

$Ro' = u' / 2\Omega L$

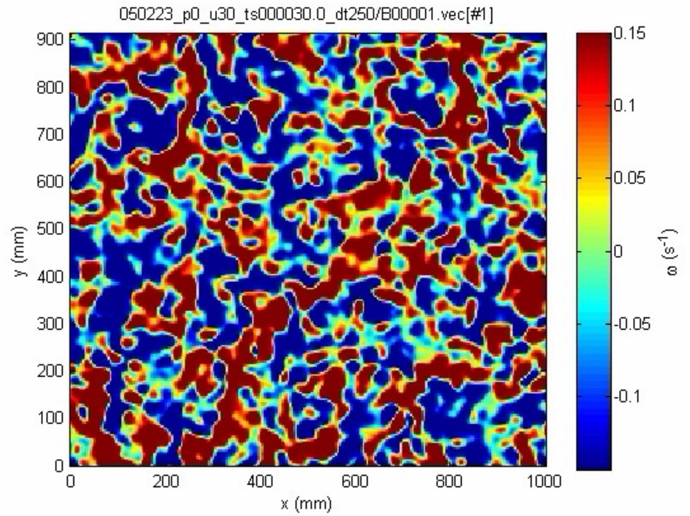
Transition at Ro (t^*) ~ 0.25

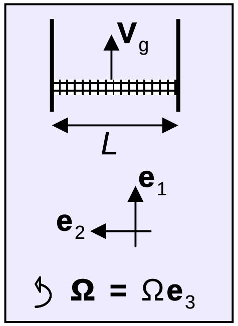
Vorticity ω_3 (x_1, x_2 ; $x_3 = h/2$), in a 1,3 x 1,3 m^2 centered horizontal square

T = ∞
Ro_g = ∞
$2\Omega = 0\ s^{-1}$

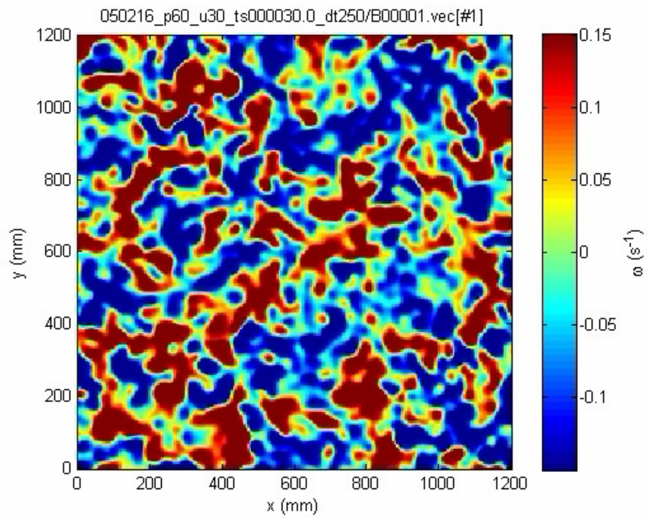


T = 120 s
Ro_g = 16
$2\Omega = 0.1\ s^{-1}$

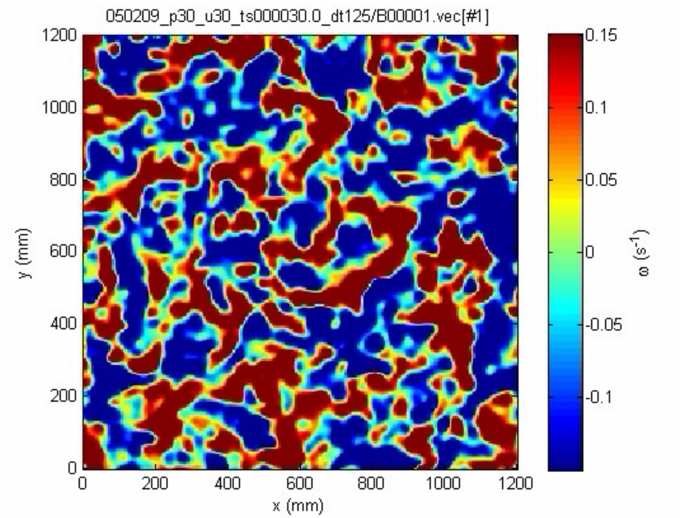




T = 60 s
Ro_g = 8
$2\Omega = 0.2\ s^{-1}$

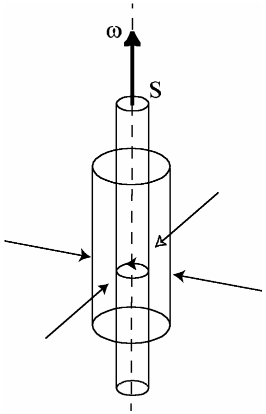


T = 30 s
Ro_g = 4
$2\Omega = 0.4\ s^{-1}$



Vortices in rotating fluids

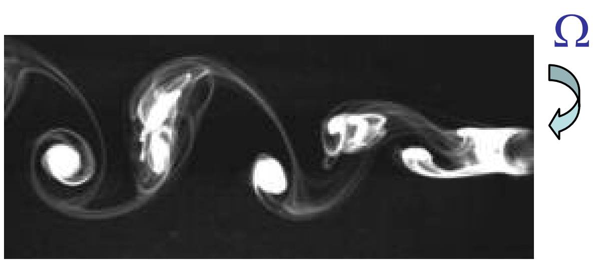
Cyclone / anticyclone asymmetry is a generic feature for rotating systems:



- Vortex stretching acts on <u>absolute</u> vorticity, $\omega_a = \omega + 2\Omega$

$$d\omega_a/dt = (du_z/dz)\,\omega_a$$

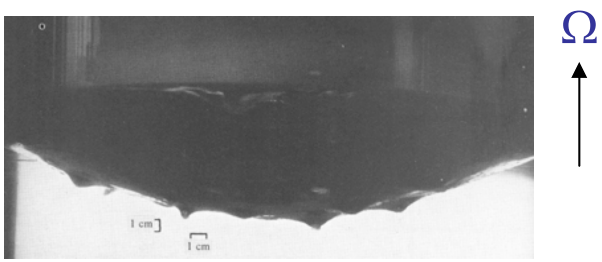
- Inertial instabilities for $Ro = \omega_z / 2\Omega \sim O(-1)$



Stegner *et al*, Phys. Fluids **17** (2005)

Influence of the boundaries:

- Stabilizing vortices (of both sign) normal to the walls: 'blocking effect'
- Viscous damping at large time (Ekman pumping)

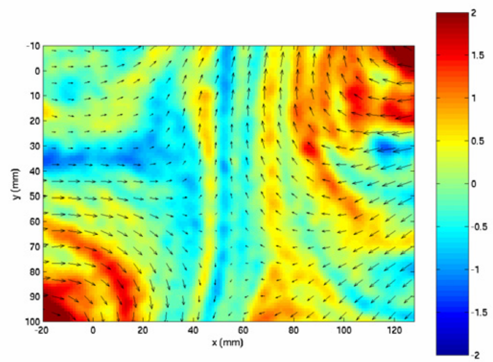


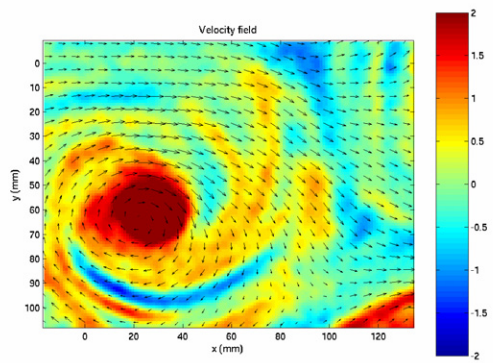
Hopfinger *et al*, JFM **125**, 505 (1982)

Vorticity distributions

Weak vorticity ($|\omega|$ ~ 1-2 ω') :

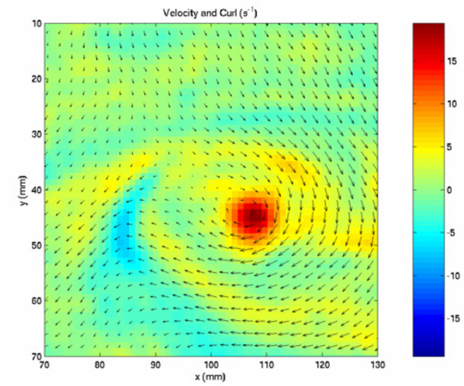
Stretching / wrapping of shear layers



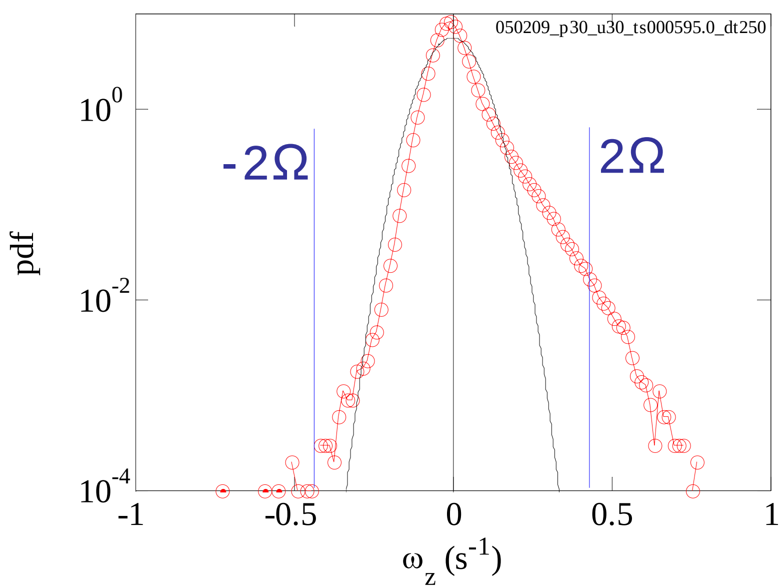


Intense vorticity ($|\omega|$ >> ω') :

Mostly cyclonic vorticity

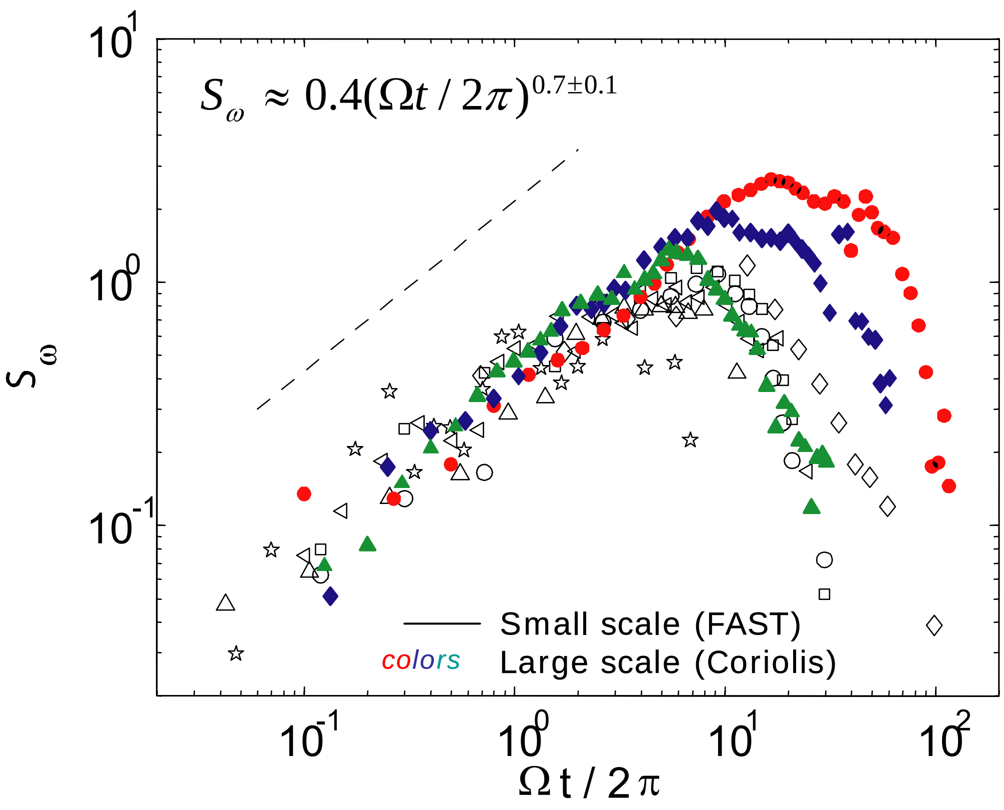


Anticyclones **Cyclones**



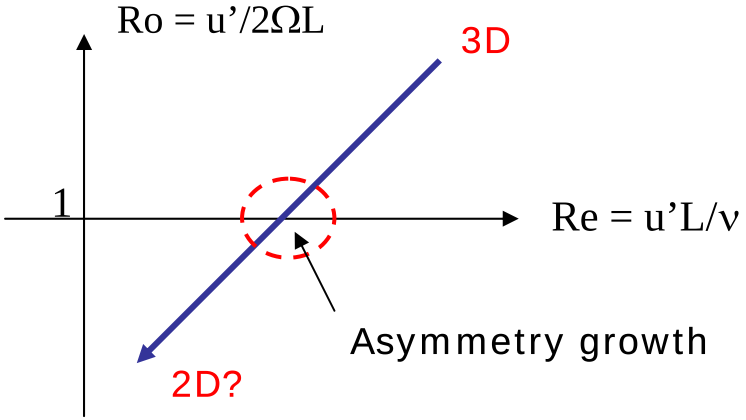
Vorticity skewness: cyclone / anticyclone asymmetry

$$S_\omega = \frac{\langle \omega^3 \rangle}{\langle \omega^2 \rangle^{3/2}}$$ (0 for symmetric fluctuations)



Self-similar growth
of the vorticity asymmetry

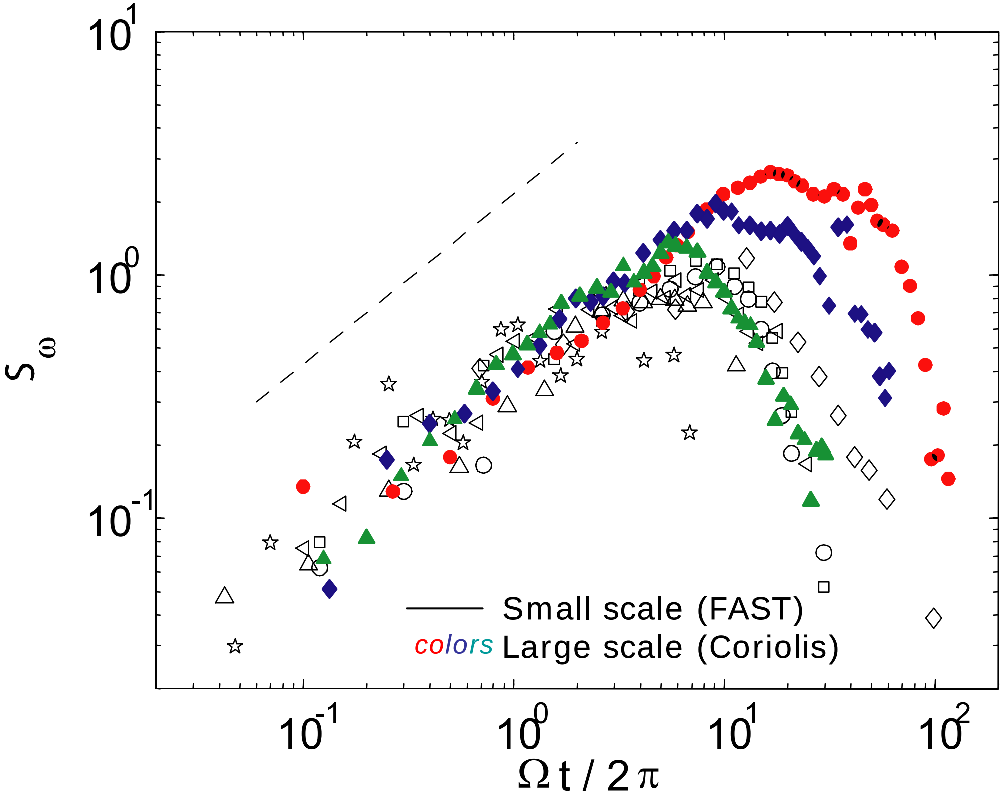
For $t > t_c$, re-symmetrization
Why?



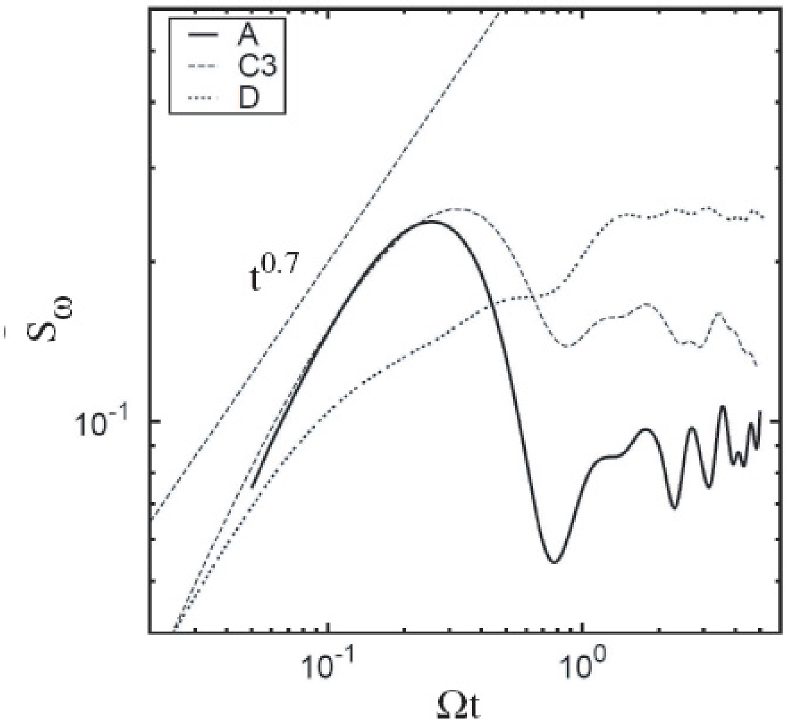
Morize, Moisy & Rabaud, Phys. Fluids (2005)
Moisy, Morize, Rabaud & Sommeria, subm JFM (2009)

Vorticity skewness

$$S_\omega = \frac{\langle \omega^3 \rangle}{\langle \omega^2 \rangle^{3/2}}$$ (0 for symmetric fluctuations)

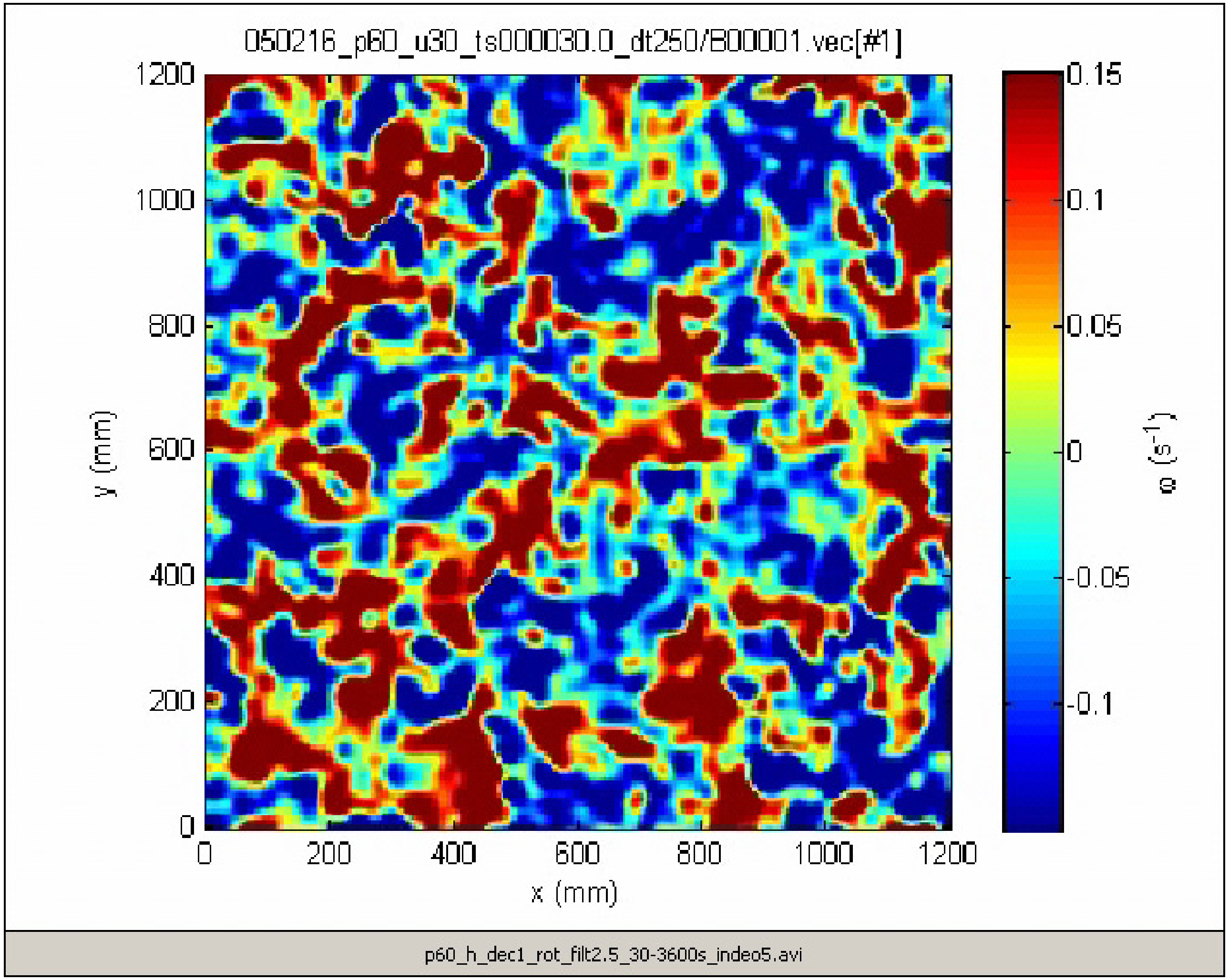


Decrease also observed in DNS with no Ekman Pumping!



Van Bokhoven, Cambon, Liechtenstein, Godeferd & Clercx, J. Turb (2008).

Cyclone-anticyclone re-symmetrisation



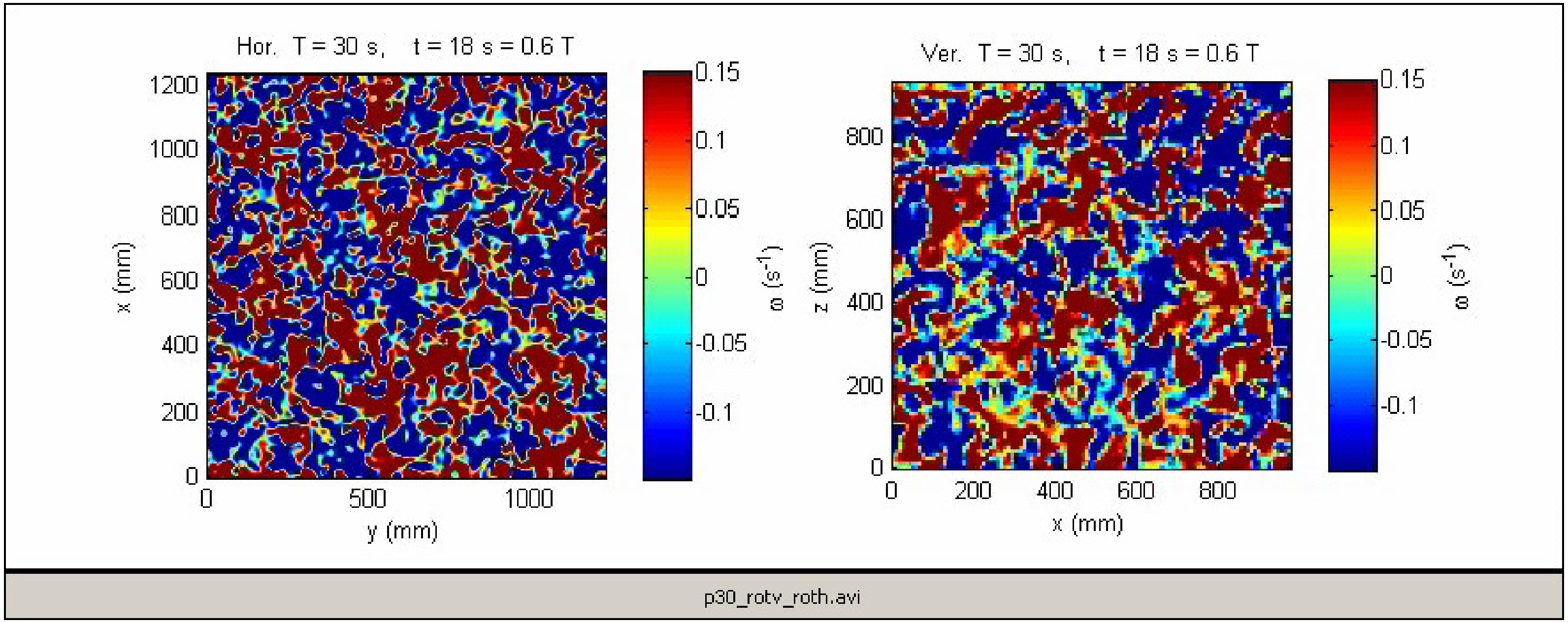
Explanations for asymmetry growth?

- instabilities of anticyclones?
- enhanced vortex stretching?

Explanations for re-symmetrisation?

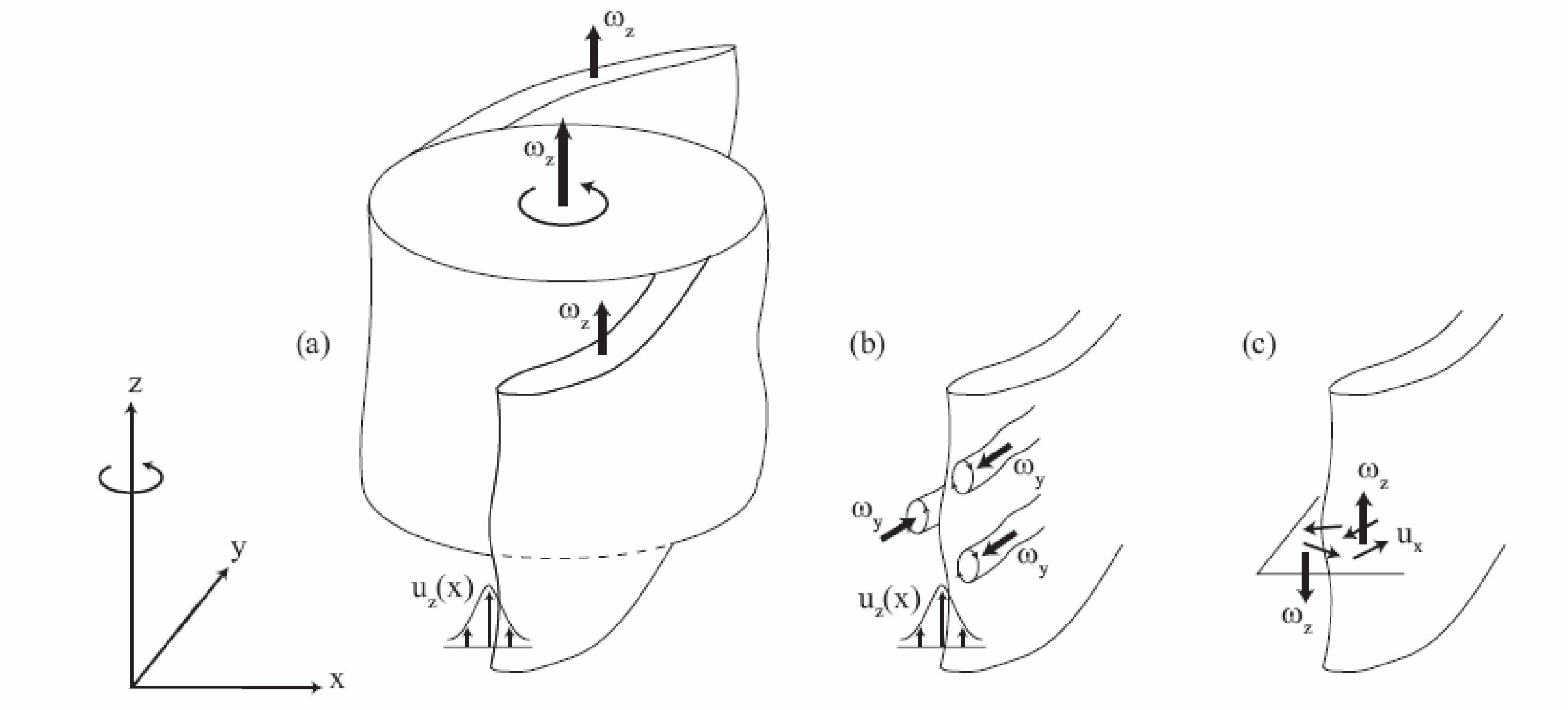
- cyclone merging
- diffusion
- **… Small scale « noise »?**

Cyclone-anticyclone re-symmetrisation



At large time, small scale vorticity « noise » produced by instabilities of the vertical shear layers advected by the large-scale quasi-2D flow

Mechanism for the vorticity asymmetry decrease



Vertical shear layers becomes thinner

Production of horizontal vorticity

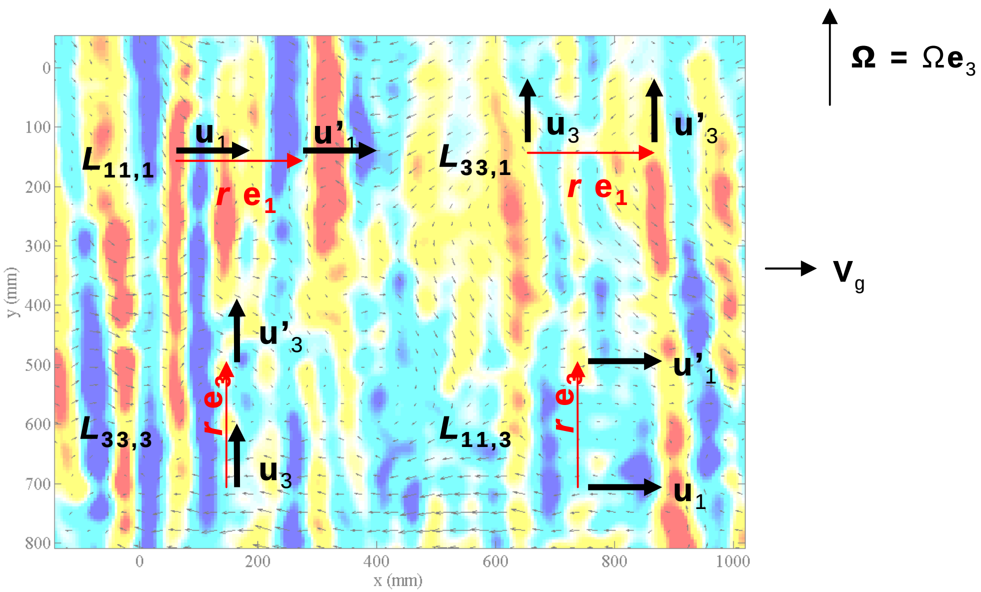
Re-injection of Small-scale Vertical vorticity with random sign

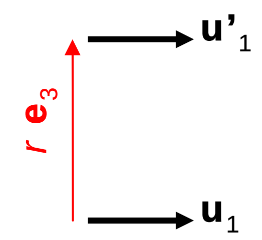
Cascade shortcut:

Vertical velocity (1/3 of energy) is « stored »

as a passive scalar, and reinjected directly at small scale

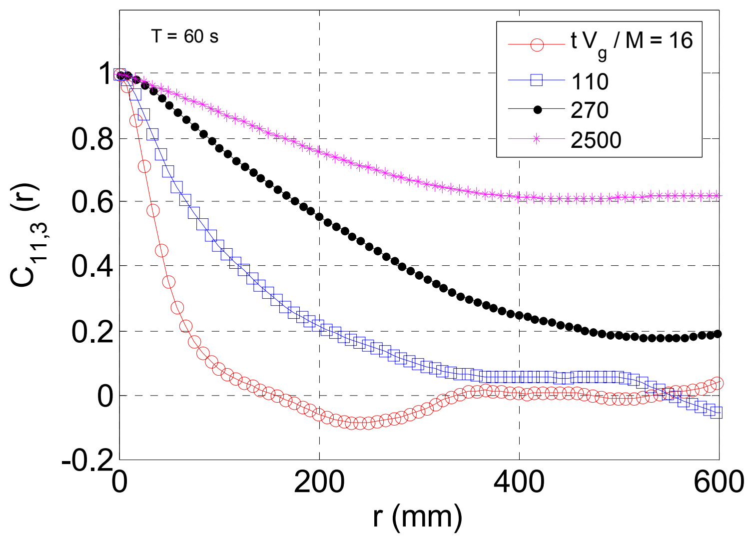
Anisotropy growth: vertical organization of the flow





Two-point correlation functions:

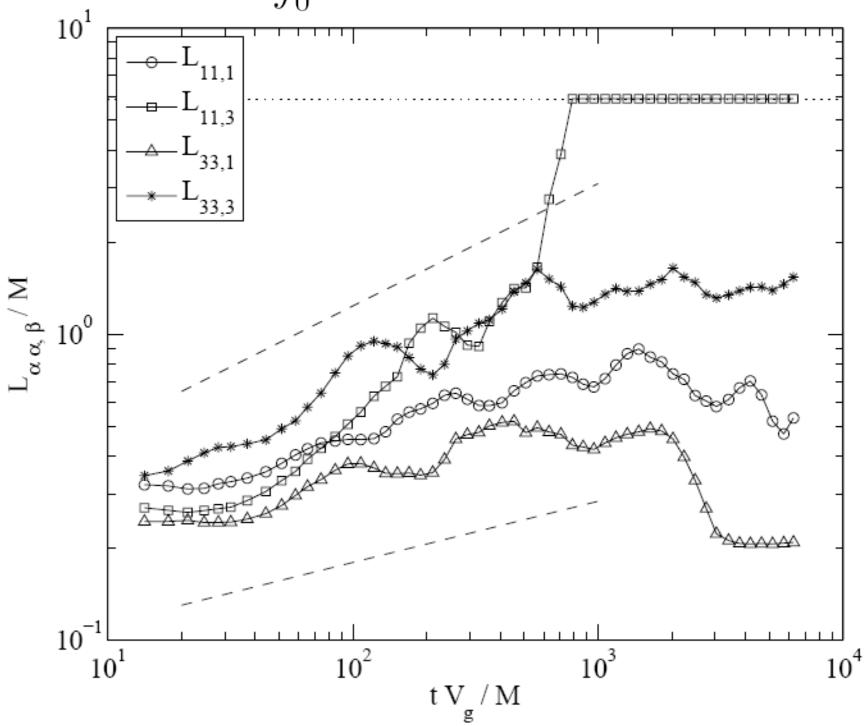
$$C_{\alpha\alpha,\beta}(r,t) = \frac{\langle u_\alpha(\mathbf{x},t)u_\alpha(\mathbf{x}+r\mathbf{e}_\beta,t)\rangle}{\langle u_\alpha^2\rangle}.$$



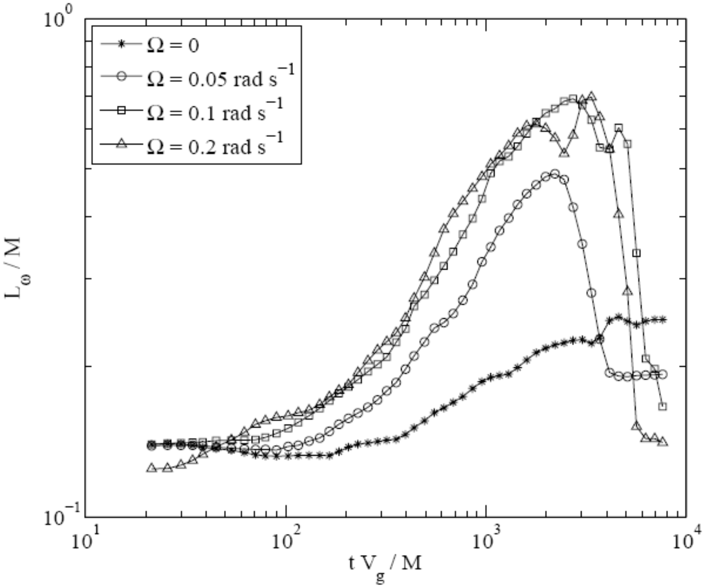
Anisotropy growth: vertical organization of the flow

Velocity correlation scale

$$L_{\alpha\alpha,\beta} = \int_0^{r^*} C_{\alpha\alpha,\beta}(r)\, dr \quad {}_1$$



Vorticity correlation scale



Fast saturation of $L_{11,3}$:

***z*-invariance of *horizontal* velocity**

Simultaneous

fall-off

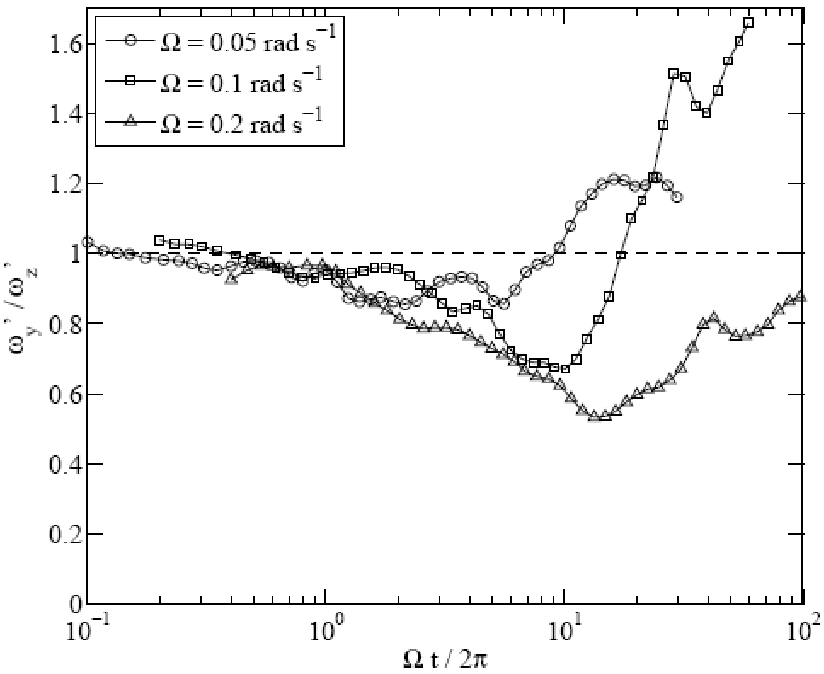
of L_{331} and $L_{\omega 3}$

Nontrivial ordering

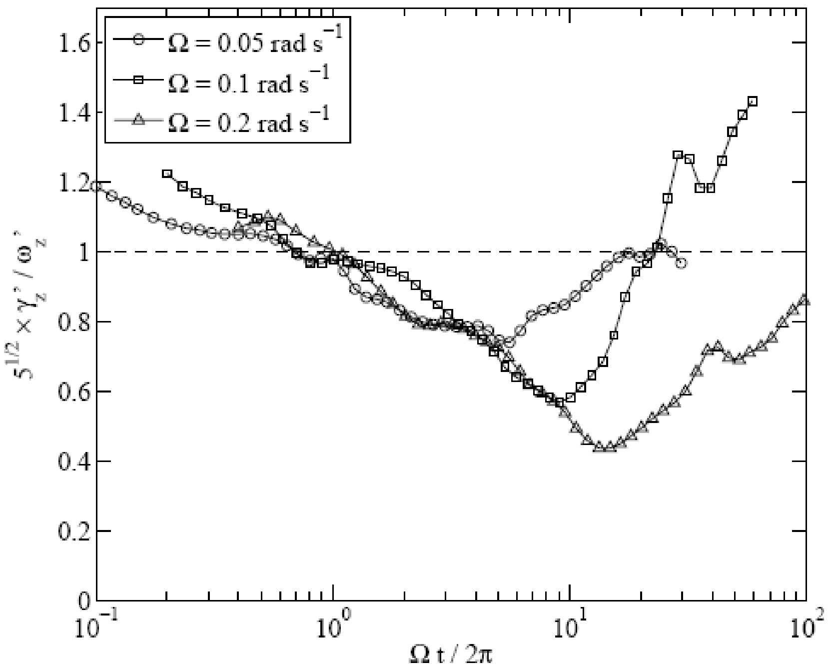
$L_{33,1} << L_{11,1} < L_{33,3}$ **$<< \quad L_{11,3}$**

Return to isotropy at small scale

Ratio of vorticity components



Ratio strain / vorticity



Large scales are strongly anisotropic $L_{33,1} << L_{11,1} < L_{33,3}$ **$<< \; L_{11,3}$**

But small scales remain close to isotropy

3C 2D flow

Summary, conclusions

- Transition at $t^* \sim 0.4$ tank rotation, $Ro(t^*) = 0.25$

- Cyclone-anticyclone asymmetry growth, $S_\omega \sim (\Omega\, t)^{0.7}$ by preferrential cyclonic vortex stretching

- Resymmetrisation of S_ω at large time

- 3D « noisy » small scales induced by instabilities of vertical layers originating from the initiation conditions

- 2D 3C flow at large time

- Open questions
  - Inversion of energy flux?
  - Generalisation of 4/5's law for rotating turbulence ?