Split energy cascade in quasi-2D turbulence

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By means of numerical simulations we investigate the transition from two-dimensional to three-dimensional turbulence which occurs in the turbulent flow of a thin layer of incompressible fluid as the thickness of the layer is increased. Coexistence of 2D and 3D turbulence is observed when the thickness is larger than viscous scale, but smaller than the forcing correlation length.

Two-dimensional Navier-Stokes equation is a prototype model for geophysical flow, in which the combined effects of rotation and stratification suppress vertical motions and allows to describe mesoscale dynamics in terms of two-dimensional equations. In these applications the two-dimensional flow is thought as the limit of vanishing ratio between the thickness of the fluid layer and the horizontal scales of interest.

The phenomenology of two-dimensional turbulent flows remarkably differs from the behavior of three-dimensional turbulence. In the classical picture of the Richardson cascade kinetic energy injected at large scale by an external forcing is transferred to smaller and smaller eddies until it reaches the viscous scale where it is dissipated by the viscosity. Conversely, in two dimensions, the simultaneous conservation of kinetic energy and enstrophy results in an inverse energy cascade, i.e. the power injected by the forcing feeds large-scale structures in the flow.

In thin layers of fluid one expects to observe a transition from two-dimensional to three-dimensional turbulence characterized by an inversion of the direction of the energy cascade, as the thickness of the layer is increased. In this paper we investigate this transition by means of numerical simulations. Three-dimensional Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P/\rho + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f} \qquad (1)$$

with the incompressibility constraint $\nabla \cdot \boldsymbol{u} = 0$, are solved in a periodic box of sizes $L_x = L_y = 2\pi$, $L_z = rL_x$ at resolution $N_x = N_y = 512, 1024$, $N_z = rN_x$ for various aspect ratio r. The flow is sustained by a two-dimensional random force which excites only the horizontal components of the velocity field. The forcing is active on wavenumbers $|\boldsymbol{k}| \sim k_f$ with $k_z = 0$, and has correlation length $L_f = 2\pi/k_f$. The behavior of the resulting turbulent flow is strongly dependent on the thickness of the layer L_z.

When the thickness of the fluid is smaller than the viscous lengthscale L_η, vertical motions are suppressed by the viscosity. In this regime the fluid recovers the two-dimensional behavior, characterized by an inverse energy cascade with a constant flux of kinetic energy toward

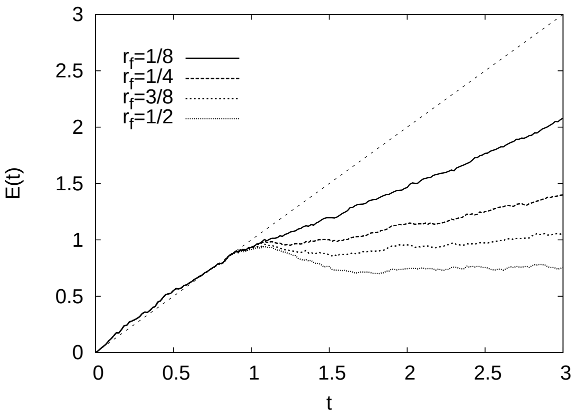

FIG. 1: Growth of kinetic energy for various values of $r_f = L_z/L_f$.

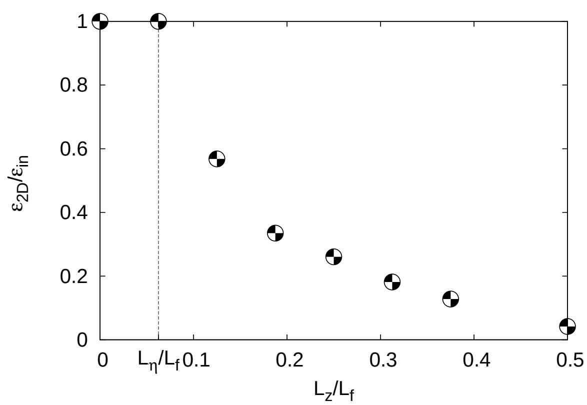

FIG. 2: Kinetic energy growth rate ϵ_{2D} normailzed with the power injected as a function of r_f.

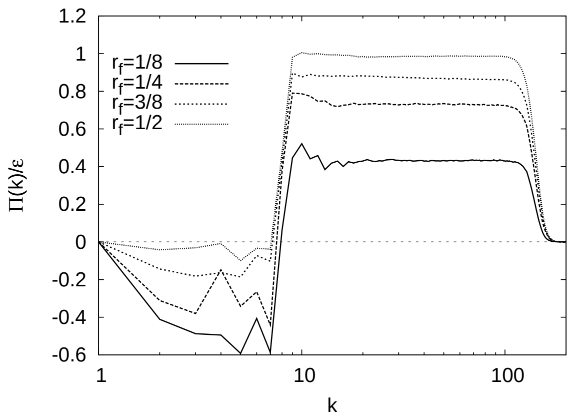

FIG. 3: Spectral flux of kinetic energy for various values of $r_f = L_z/L_f$.

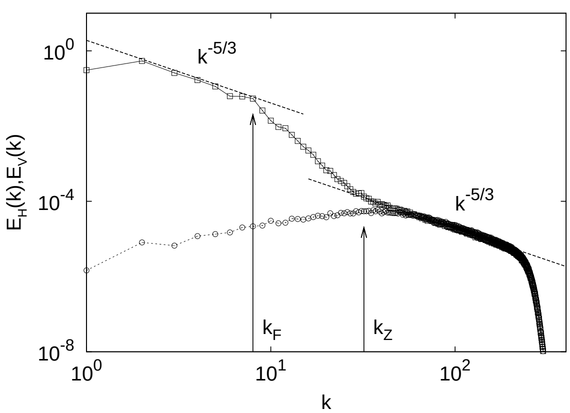

FIG. 4: Kinetic energy spectrum of horizontal (squares) and vertical (circles) velocities. Here $r_f = 1/4$

large scales $\ell > L_f$. In absence of large-scale dissipations the kinetic energy grows linearly in time, with a growth rate equal to the power injected by the forcing.

Conversely, when $L_z > L_\eta$, small three-dimensional perturbations can be amplified by the vortex stretching mechanism . The amount of small-scale vorticity produced is able to dissipate a substantial part of the energy injected. As a consequence the fraction of the energy that is transferred toward large scale reduces, and the energy growth rate decreases, as shown in Figure 1. The ratio between the energy growth rate and the power injected is dependent on the aspect ratio $r_f = L_z/L_f$ between the thickness and the forcing correlation length (see Fig. 2).

When the thickness of the fluid is much larger than viscous scale, but smaller than the forcing correlation length L_f , 2D and 3D turbulence can coexist. In this regime we observe a splitting of the energy cascade. Part of the energy is still transferred toward large scale, feeding the inverse cascade. The remnant energy gives rise to a direct energy cascade with constant flux toward small scale (see Figure 3).

Coexistence of 2D and 3D turbulence is clearly visible in the energy spectra (see Figure 4). At small wavenumbers $k < k_f$ the energy spectrm of horizontal velocities has a scaling region $E(k) \sim \epsilon_{2D}^{2/3} k^{-5/3}$ which is the signature of the two dimensional inverse energy cascade. At high wavenumbers $k > k_z$ Kolmogorov spectra are observed both for horizontal and vertical velocities, signaling the presence of 3D turbulence at small scales.