Lagrangian acceleration in time-periodic laminar flows

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Motivation

Recently enormous scientific efforts spent on the study of Lagrangian acceleration in turbulence.

But do we know the behavior in laminar flows ?

This study gives preliminary results on the Lagrangian acceleration in chaotic advection.

Chaotic advection

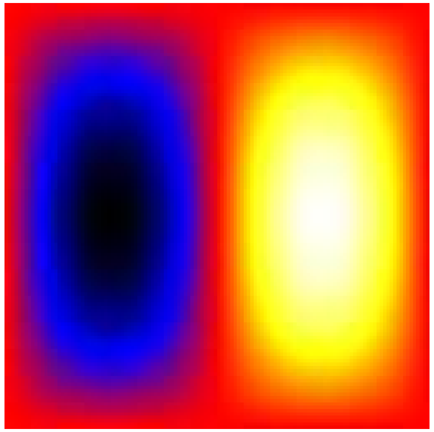
Certain laminar flows exhibit chaotic particle trajectories (cf. Aref JFM ’84). Minimum conditions : 2D, non-stationary or 3D flows. What is the statistic behavior of these trajectories :

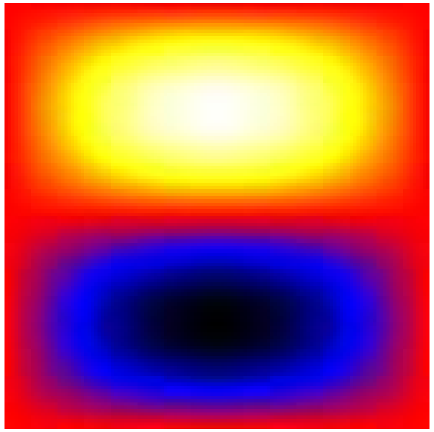
- time correlations ?

- PDFs ?

Flow 1

$\sin(\Omega t)\times \quad + \quad \cos(\Omega t)\times$

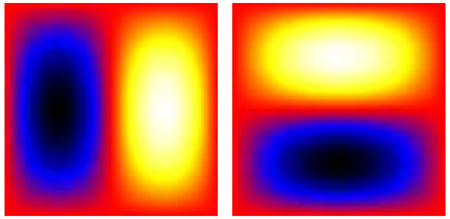




$$\mathbf{u}(x) = -2\sin(\pi x)\cos(2\pi y)\mathbf{e}_x + \cos(\pi x)\sin(2\pi y)\mathbf{e}_y$$

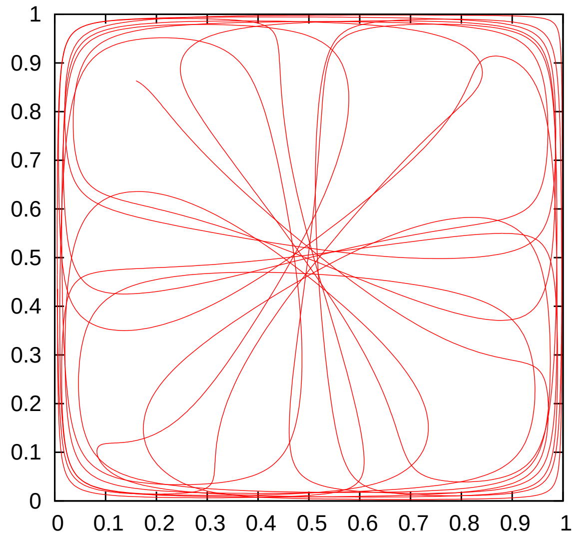
The kinetic energy is constant

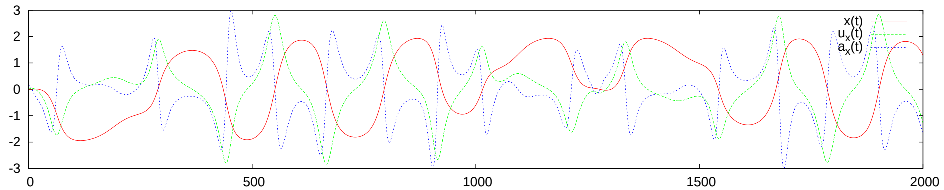
$\sin(\Omega t) + \cos(\Omega t)$



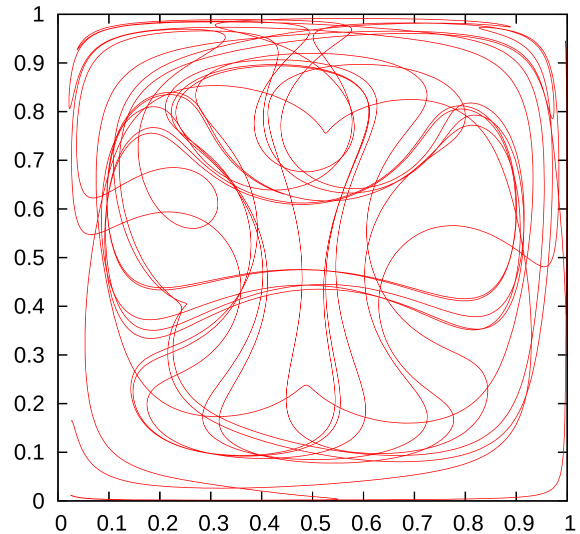
Flow 1, trajectories and time signals

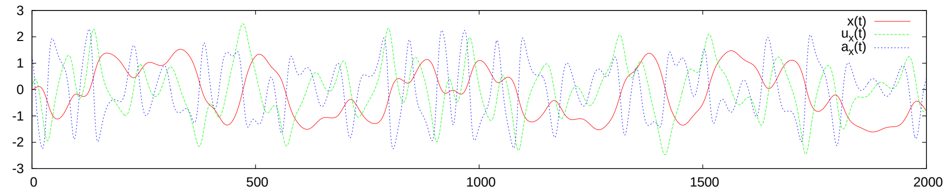
$$\Omega/2\pi = 0.04$$





$$\Omega/2\pi = 0.5$$





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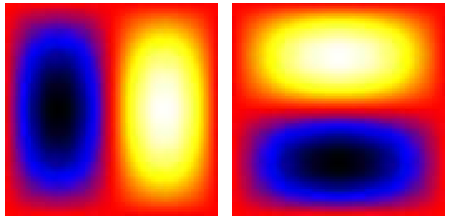
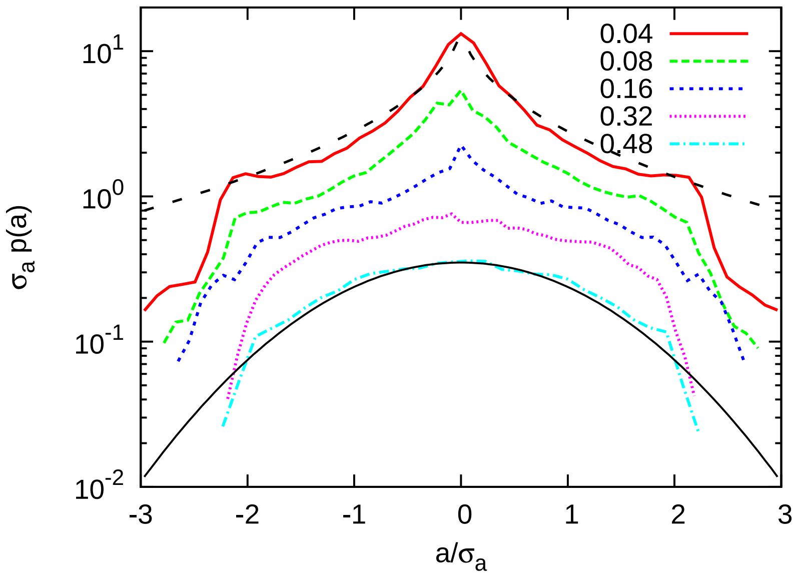
$\sin(\Omega t) + \cos(\Omega t)$



Flow 1, pdfs

The pdfs of the acceleration show both Gaussian and exponential ($\exp(-|a|^{1/2})$) characteristics, depending on Ω.

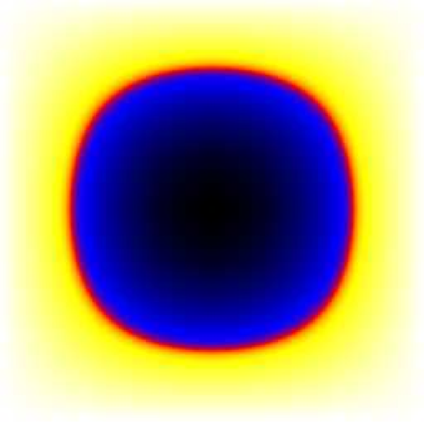


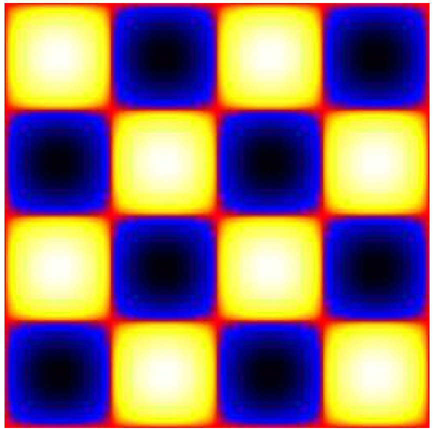
However, the range of the pdf is small. Can we generate a flow with a larger variance ? Increase the ’Reynolds number’ ? But in this purely laminar flow, how do we define Reynolds ?

Flow 2 'Reynolds number'

Change 'Reynolds' $\sim$ change the range of lengthscales

$$a\ \sin(\Omega t)\times \qquad + \qquad b\ \cos(\Omega t)\times$$



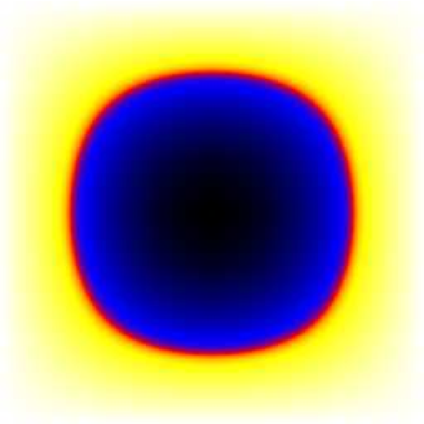


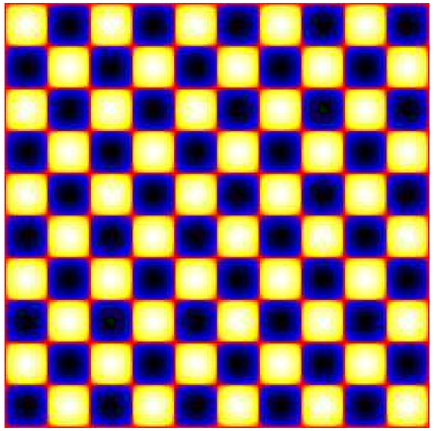
$$\mathbf{u}(x) = \sin(n\pi x)\cos(n\pi y)\mathbf{e}_x + \cos(n\pi x)\sin(n\pi y)\mathbf{e}_y$$

Flow 2

Change 'Reynolds' $\sim$ change the range of lengthscales

$$a\ \sin(\Omega t)\times \qquad + \qquad b\ \cos(\Omega t)\times$$

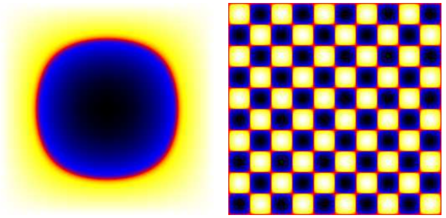




$$\mathbf{u}(x) = \sin(n\pi x)\cos(n\pi y)\mathbf{e}_x + \cos(n\pi x)\sin(n\pi y)\mathbf{e}_y$$

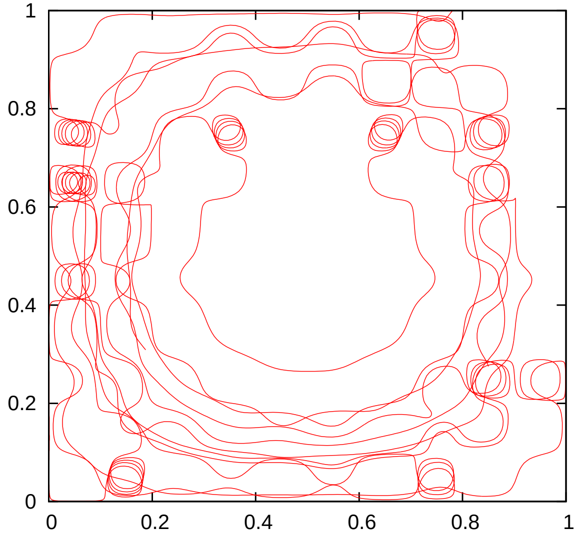
The kinetic energy is, again, constant

$a\sin(\Omega t) + b\cos(\Omega t)$

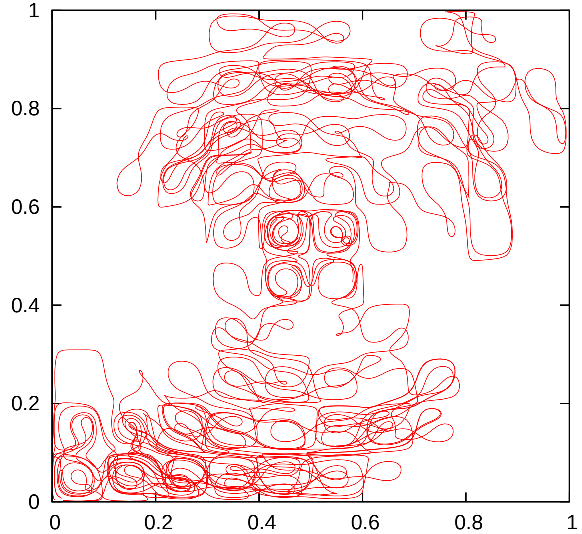


Flow 2, trajectories

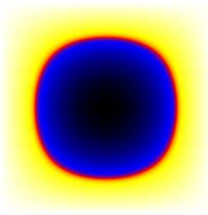
$\Omega/2\pi = 0.16$

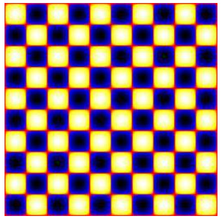


$\Omega/2\pi = 1.6$



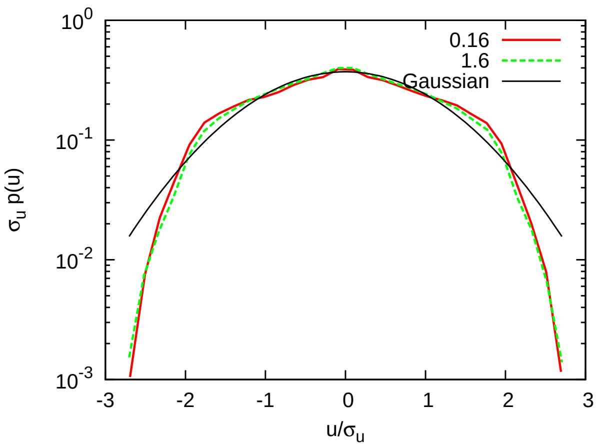
$$a\sin(\Omega t) + b\cos(\Omega t)$$



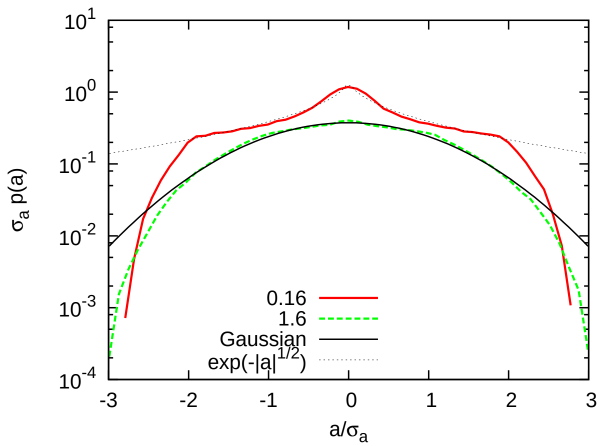


Flow 2, pdfs

pdf(u)

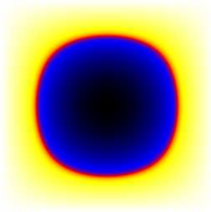


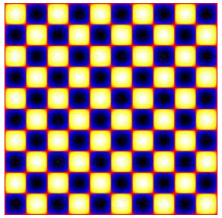
pdf(a)



Acceleration shows (sub)-Gaussian or exponential behavior

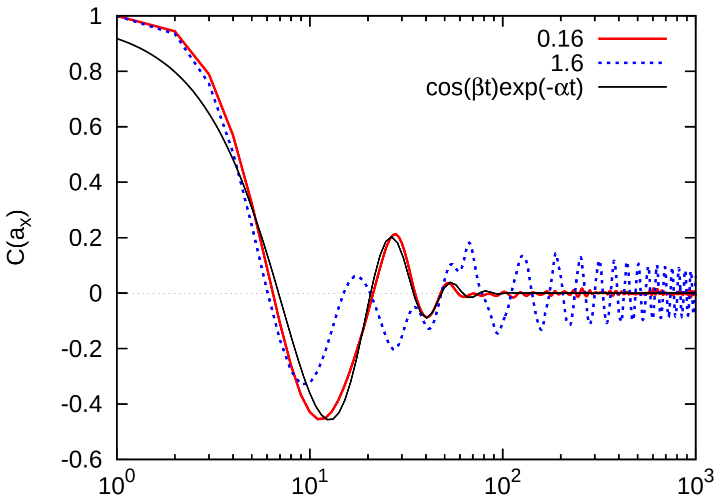
$a\sin(\Omega t) + b\cos(\Omega t)$



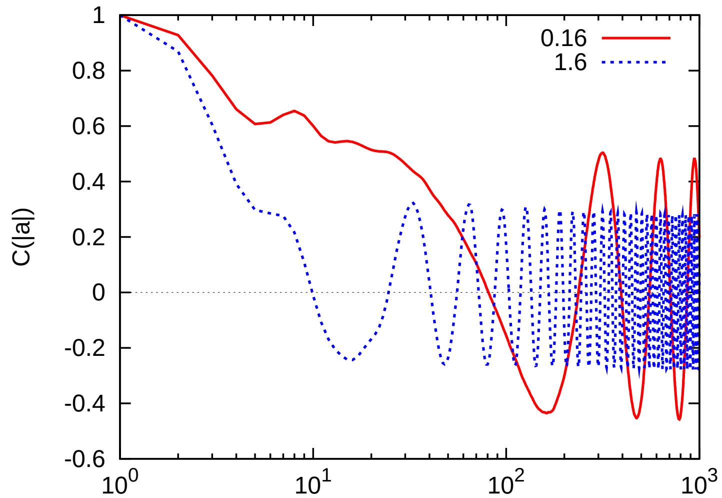


Flow 2, time correlations

$$< a_i(t)a_i(t+\tau) > / < a_i^2 >$$

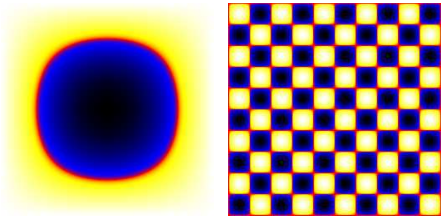


$$< |a(t)||a(t+\tau)| > / < |a|^2 >$$

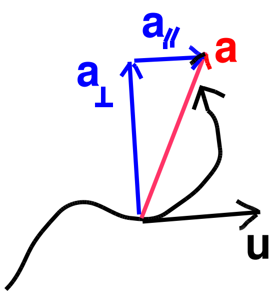


Link between Lyapunov exponents of the flow and value of a in $\exp(-ax)\cos(bx)$?

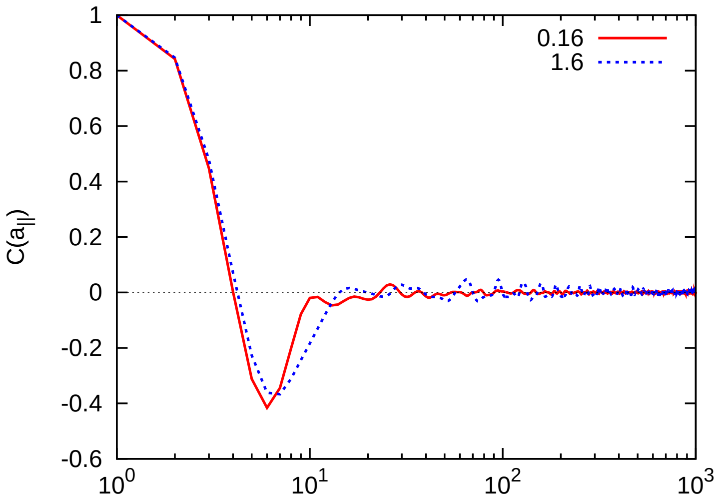
$a\sin(\Omega t) + b\cos(\Omega t)$



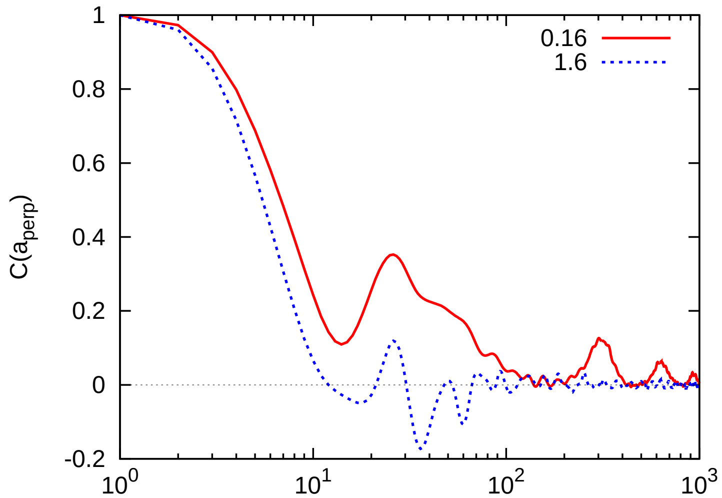
Flow 2, time correlations



$$< a_{||}(t)a_{||}(t+\tau) > / < a_{||}^2 >$$



$$< a_{\perp}(t)a_{\perp}(t+\tau) > / < a_{\perp}^2 >$$



$C(a_{||})$ almost similar, $C(a_{\perp})$ different : the time correlation of $a_{\perp}$ is responsible for the form of the pdf ?

Conclusions, Perspectives

- In laminar flows the pdf of the Lagrangian acceleration can be Gaussian or exponential depending on amount of chaos.
- Time-correlations of the form $\exp(-\alpha t)\cos(bt)$. $\alpha = f($lyapunov$)$?
- Perpendicular acceleration linked to form of the pdf.
- what about more intermittent distributions of vorticity ?