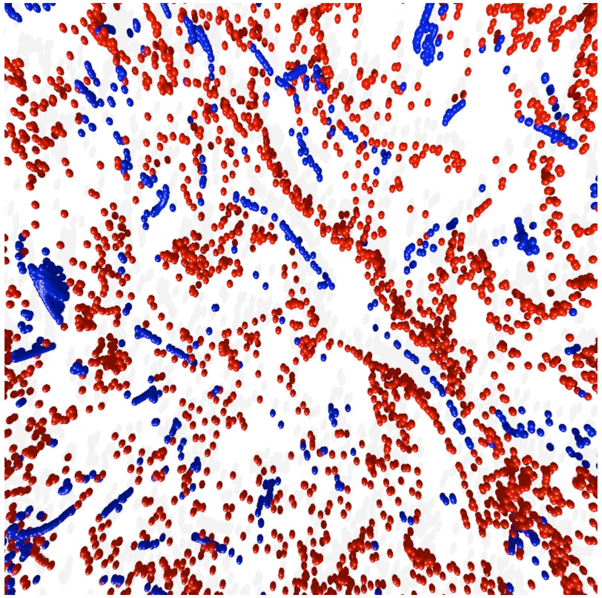
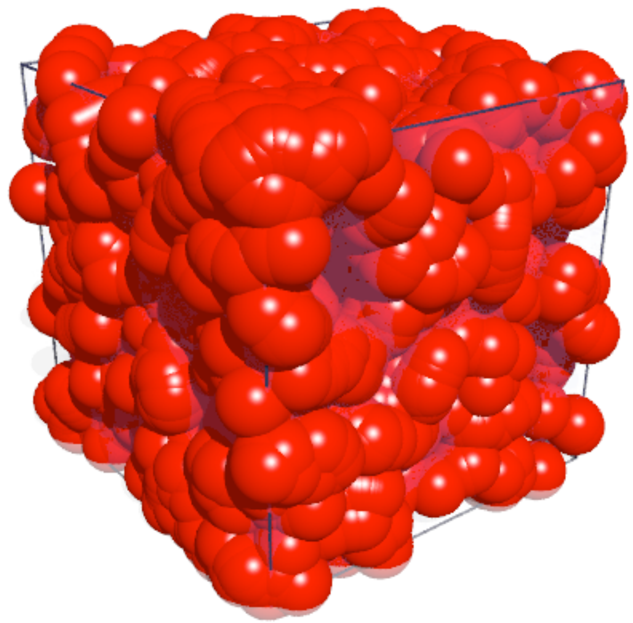
Quantifying clustering and segregation of particles and bubbles in turbulent flow



Enrico Calzavarini

ENS Lyon, Physique

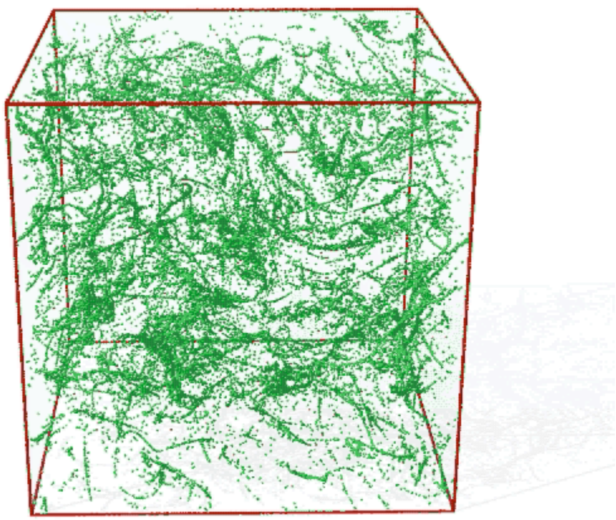
International Collaboration for Turbulence Research



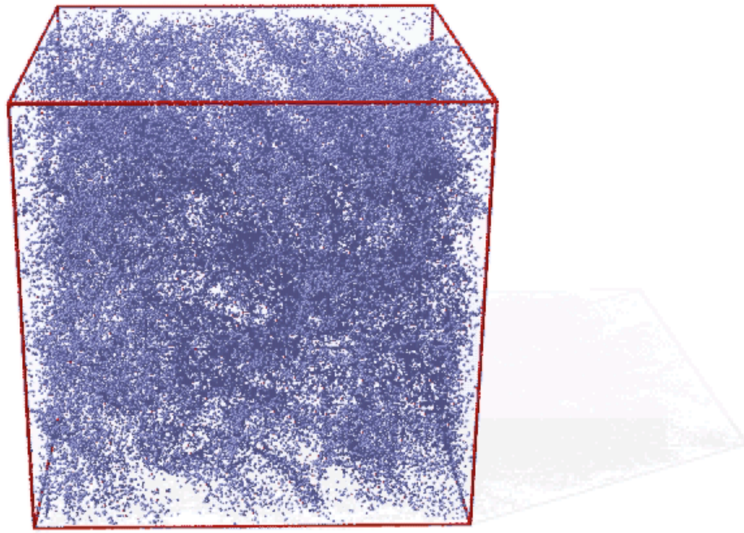
In collaboration with:
M. Cencini, F. Toschi, (Roma, IT), D. Lohse (UTwente, NL), M.Kerscher (Munich,D).

Focus on spatial statistics of particles

bubbles



heavy



Review and propose tools to characterize
clustering and segregation

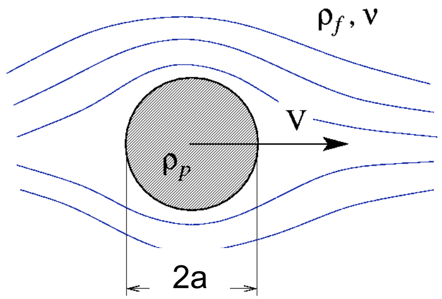
Goal: detailed comparison with experiments
to test/improve particle models

Particle's equation of motion

heavy and light inertial particles: finite response time & added mass

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \beta \frac{D}{Dt}\mathbf{u}(\mathbf{x}(t),t) - \frac{1}{\tau_p}(\mathbf{v} - \mathbf{u}(\mathbf{x}(t),t))$$



$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p} \qquad \tau_p = \frac{1}{3\beta}\frac{a^2}{\nu} \qquad St \equiv \frac{\tau_p}{\tau_\eta} = \frac{1}{3\beta}\frac{a^2}{\eta^2}$$

valid in the limits of $a << \eta$, $\mathrm{Re}_a = a v_a / \nu << 1$

negligible gravity, dilute suspension (no collisions)

see: Maxey, M. & Riley, J. 1983 Equation of motion of a small rigid sphere in a nonuniform flow. Phys. Fluids **26**, 883-889.

Eulerian dynamics:

Homogeneous isotropic turbulent flow at $\mathrm{Re}_\lambda \approx O(10^2)$

spectral simulation in periodic box

DNS:

1. *SARA* 128^3 Re_λ =75, tracers & heavy & light (504 families)

2. *CASPUR* 512^3 Re_λ =180 tracers & heavy & light (64 families)

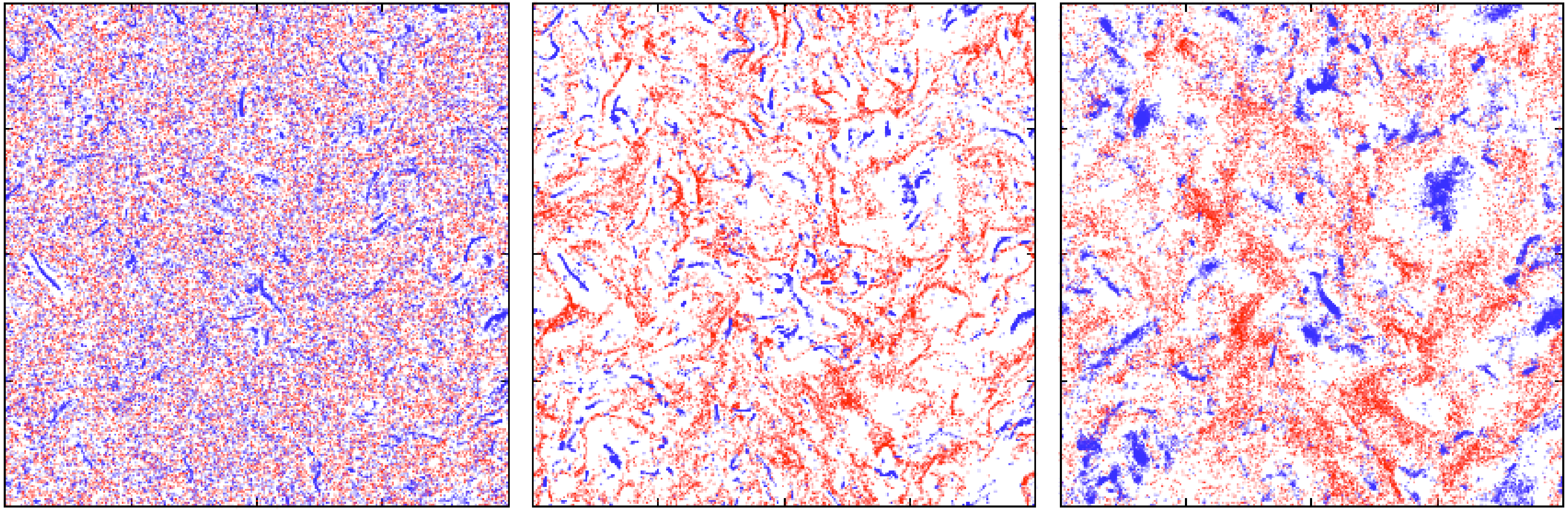
3. *DEISA* 2048^3 Re_λ =400 with tracers & heavy (20 families)

Different particles classes (ß,St) evolved in the same flow

Red color: heavy particles $\rho_p >> \rho_f$

Blue color: light particles $\rho_p << \rho_f$

Slices ~512x512x8 η from DNS at $Re_\lambda \approx 180$



St=0.1 St=1.0 St=4.1

Characterizing particle clusters

1. Kaplan-Yorke dimension: D_{KY}

Particle equations of motion defines a dissipative dynamical system
Attractor's dimension in the (x,v) space: D_{KY}

$$D_{KY} \equiv J - \frac{\lambda_1 + \cdots + \lambda_J}{\lambda_{J+1}}$$

$$\lambda_1 + \cdots + \lambda_J \geq 0$$
$$\lambda_1 + \cdots + \lambda_{J+1} < 0$$

6 Lyapunov exponents computed by tracking

$$\mathbf{R}(t) \equiv (\delta\mathbf{x}(t), \delta\mathbf{v}(t))$$

$$\frac{d\mathbf{R}}{dt} = \mathcal{M}_t \mathbf{R}$$

$$\lambda_i = \lim_{T\to\infty} \gamma_i(T)$$

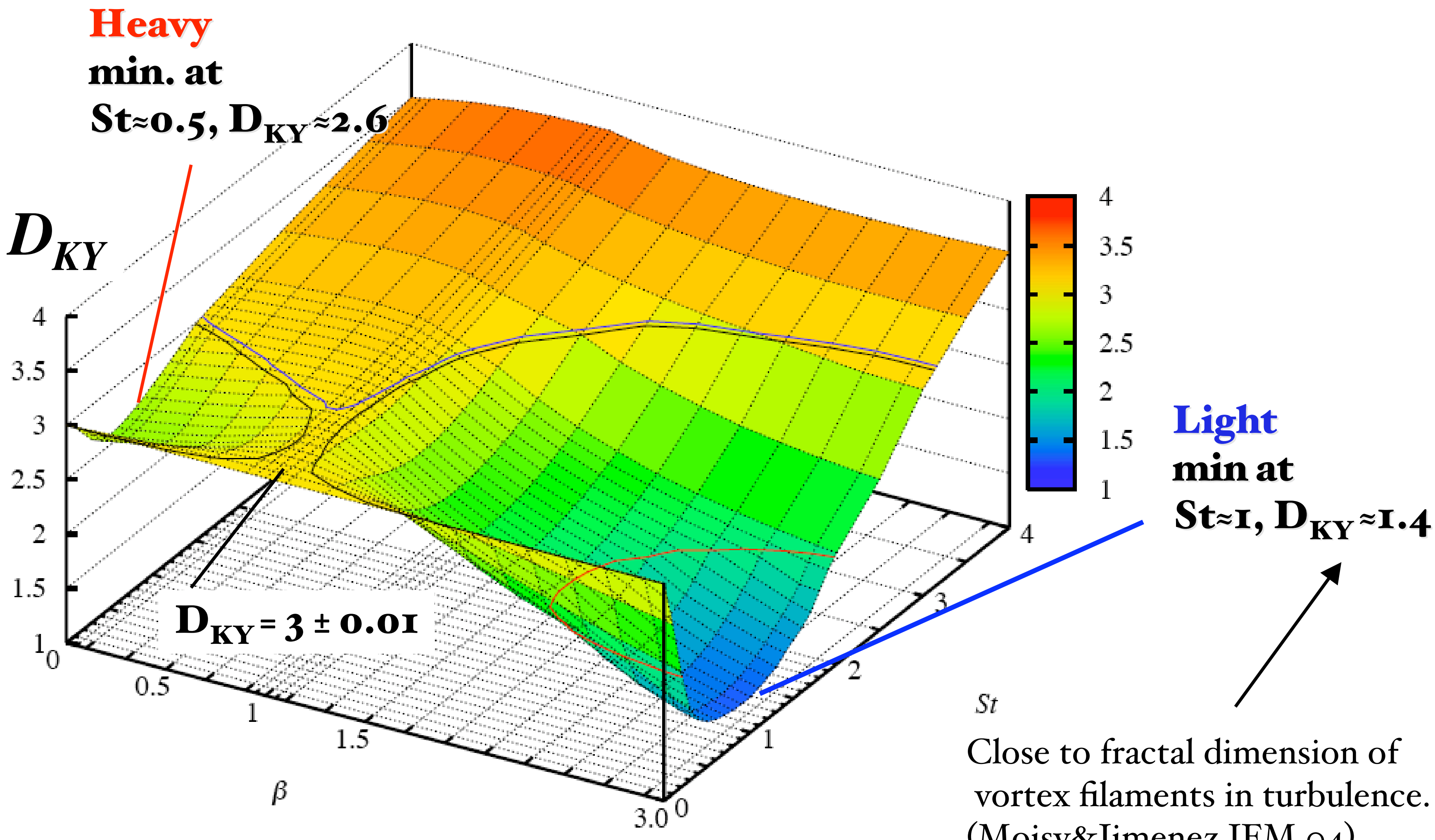
← stretching rates

Standard orthonormalization
Gram-Schmidt procedure adopted

As in J. Bec *Phys. Fluids* (2003) & *JFM* (2005), J. Bec et al. *JFM* (2006)

D_{KY} dimension

balance between contraction and expansion in phase space

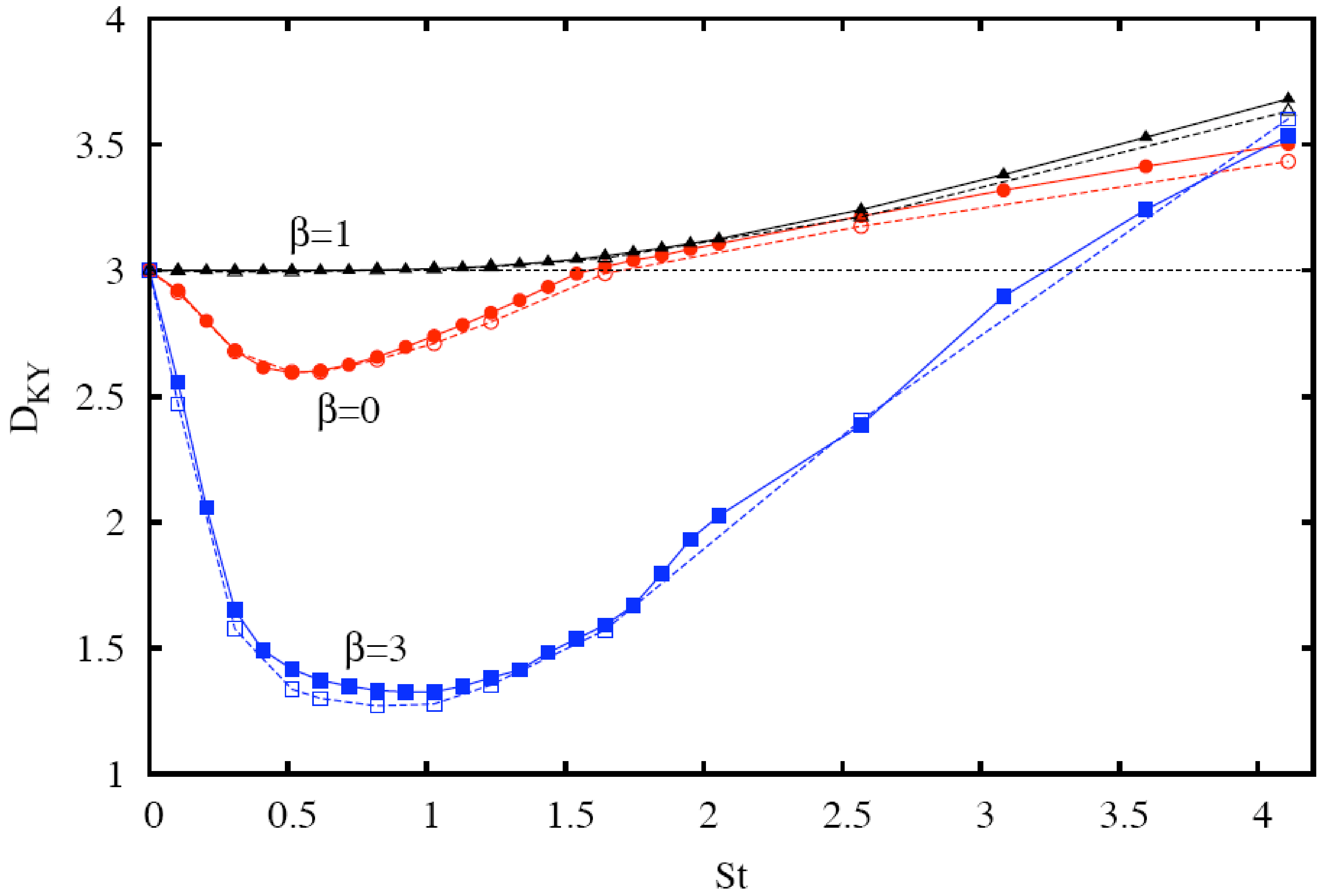


Close to fractal dimension of vortex filaments in turbulence. (Moisy&Jimenez JFM 04)

$$D_\omega \to 1.1 \pm 0.1$$

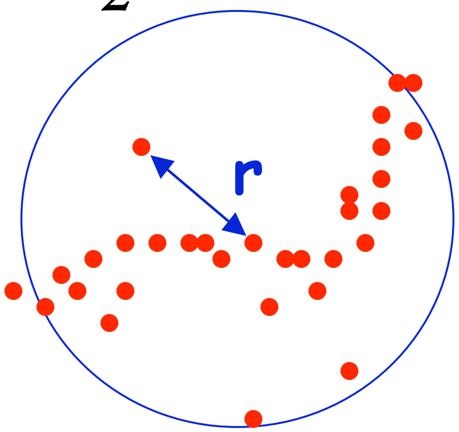
Re_λ dependence?

Comparison: Re_λ =75 (filled sybmols) and Re_λ =180 (empty sybmols)

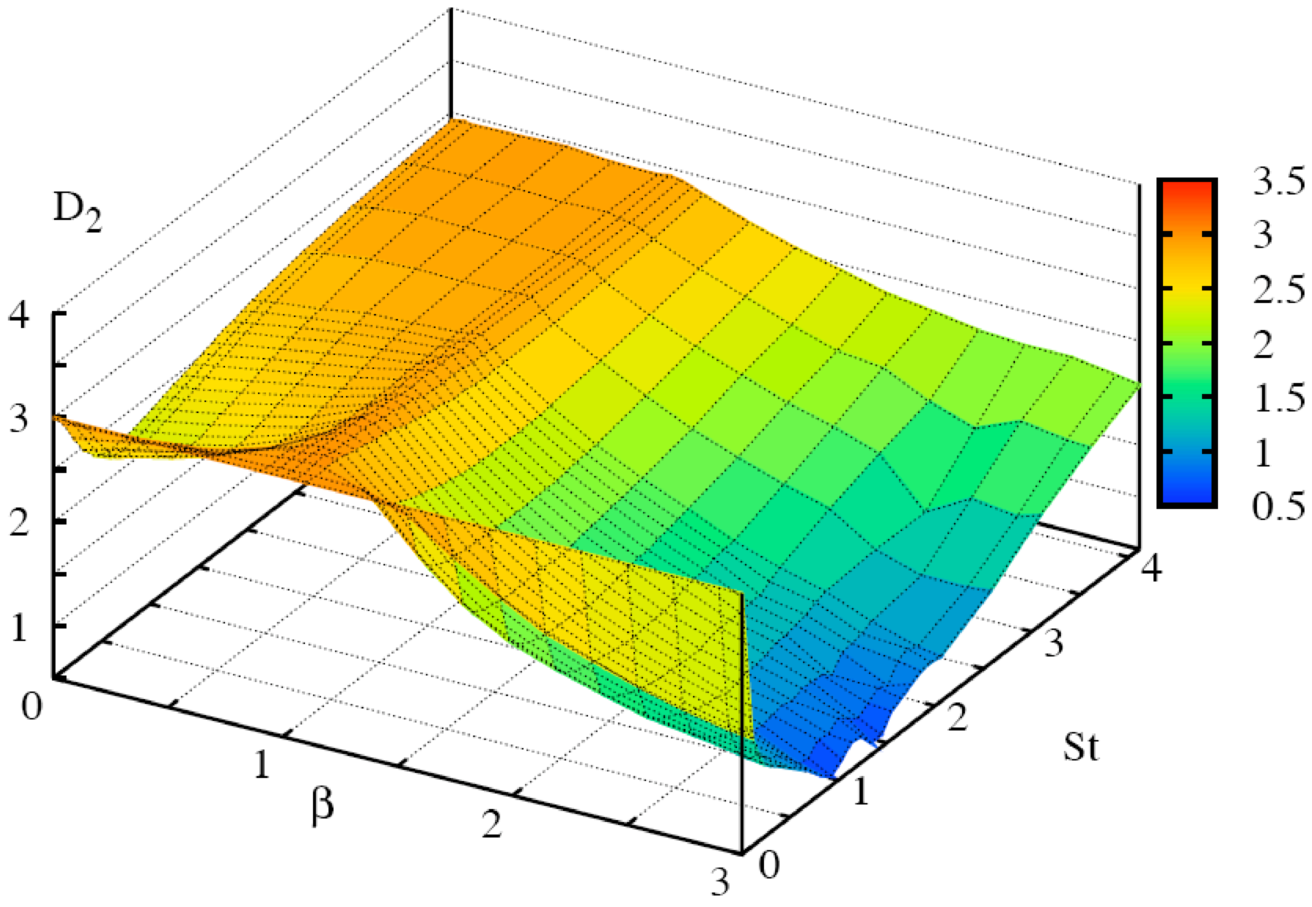


2.Correlation dimension D_2

$P_2(r)$ Probability to find a couple of particle whose distance is below r.



At $r \ll \eta$ $P_2(r) = A\, r^{D_2}$



Fractal dimension hierarchy:
$D_2 \leq D_1 = D_{KY}$

Same features as D_{KY} and *easily accessible to experiments*

3. A morphological characterization: Minkowski functionals of point clouds

1. Put balls $\mathcal{B}_r(\mathbf{x}_i)$ with radius ***r*** around each particle ***i***

2. Let ***r*** increase

3. Consider $\mathcal{A}_r = \bigcup_{i=0}^{N} \mathcal{B}_r(\mathbf{x}_i)$

4. Measure: total volume, surface, mean curvature and Euler characteristic of A_r

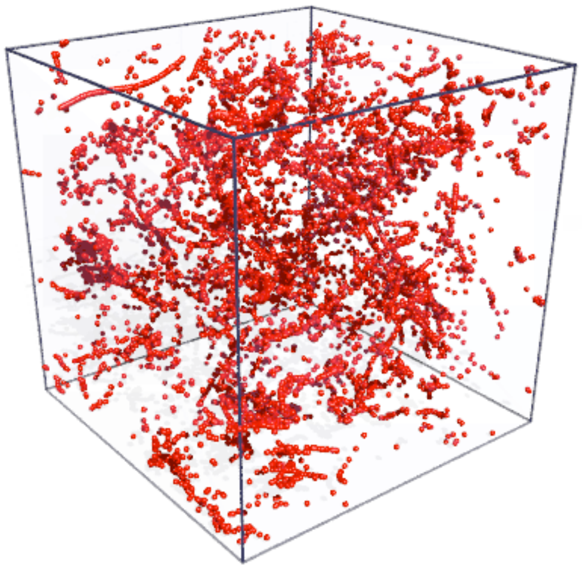
Complete morphological characterization of 3D point clouds through 4 $\mathbf{V_\mu(r)}$ →

geometric quantity	μ	$V_\mu(r)$
V Volume	0	V
A Surface	1	$A/6$
H Mean curvature	2	$H/(3\pi)$
χ Euler characteristics	3	χ

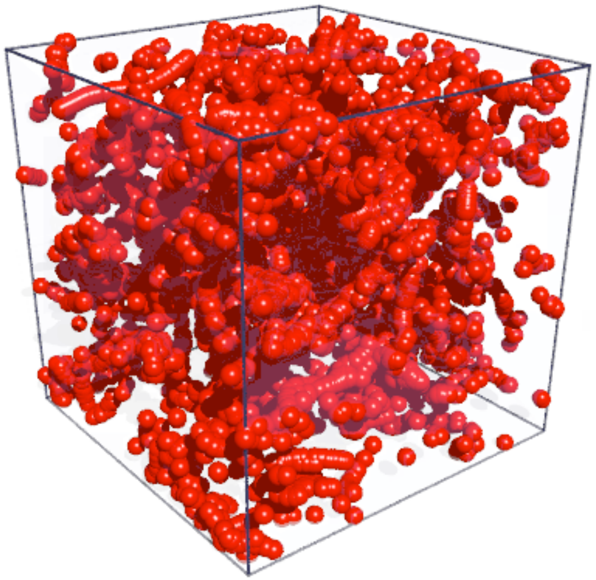
See: Mecke, K. R. et al. 1994 Robust morphological measures for large-scale structure in the Universe. *Astron. Astrophys.* 288, 697–704 and ref. therein.

Visualization of A_r

$2 \cdot 10^4$ particles with $\beta=3$ and $St=1$

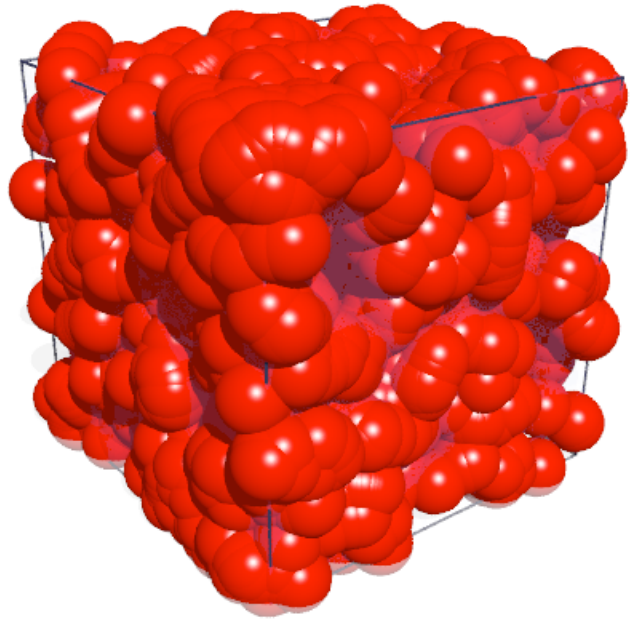


$r = 0.5\ \eta$



$r = 3\ \eta$

$r = 10\ \eta$



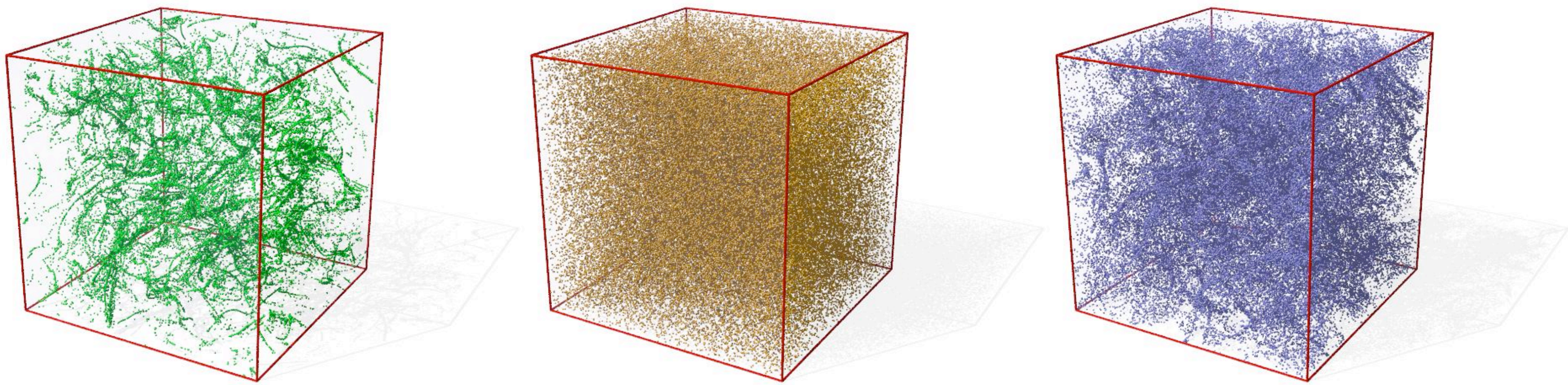
Comparison of three extreme cases

$\beta = 0$, heavy

$\beta = 1$, tracer

$\beta = 3$, bubble

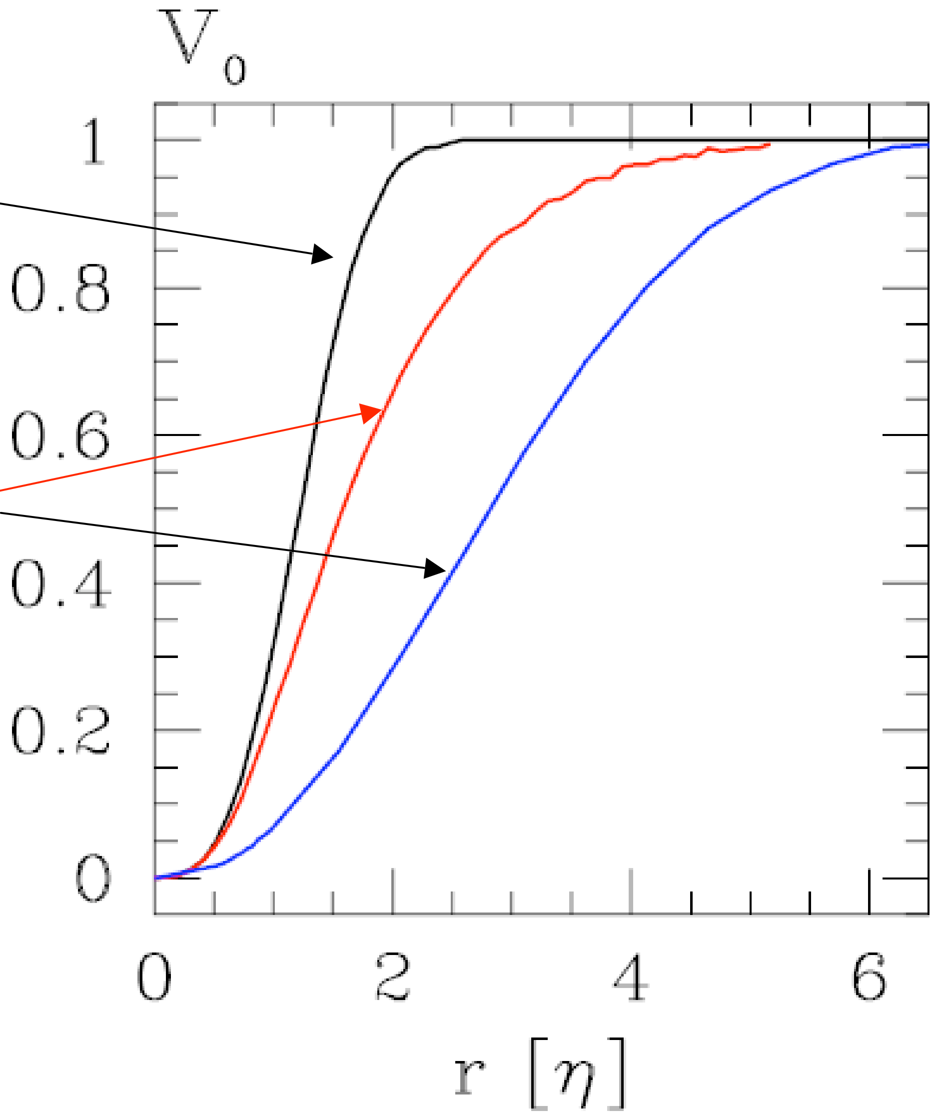
bubble tracer heavy



St=0.6 3 snapshots of 10^5 particles in the same velocity field

Volume $V_o(r)$

space-filling



space-filling
less fast due to
overlaps:
remote clusters

heavy

passive

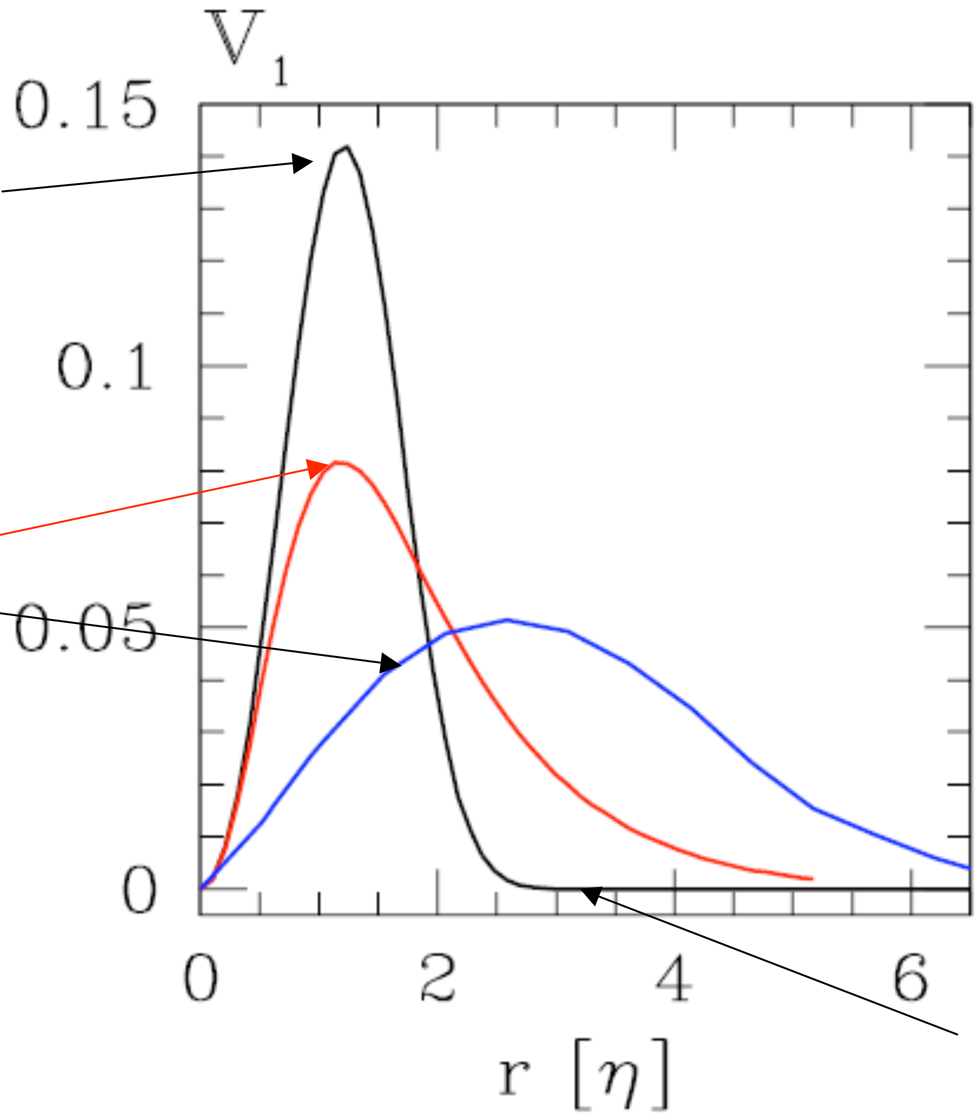
(=Poisson)

bubble

Surface $V_I(r)$

Mainly isolated balls

Smaller maximal surface due to clustering



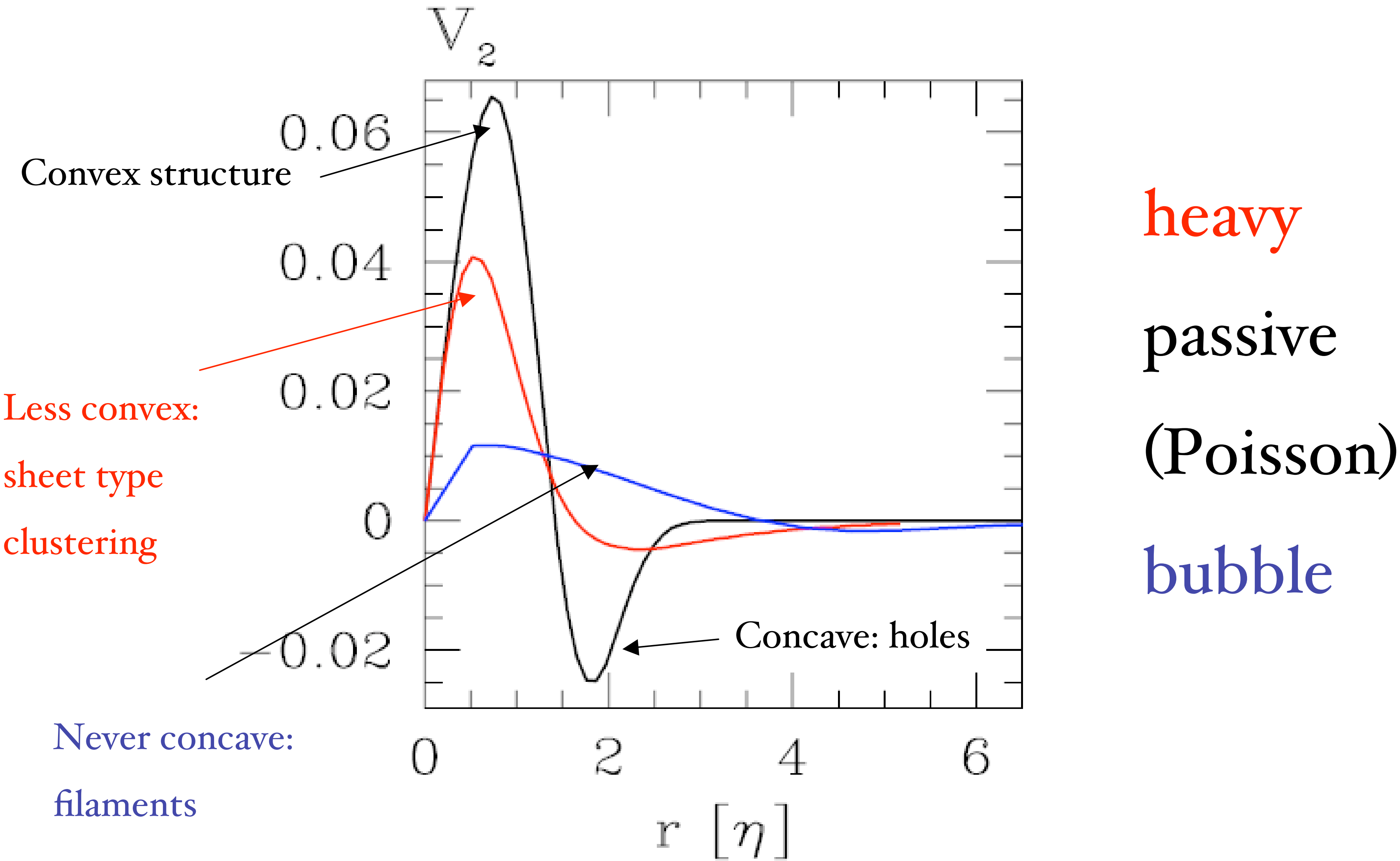
heavy

passive (Poisson)

bubble

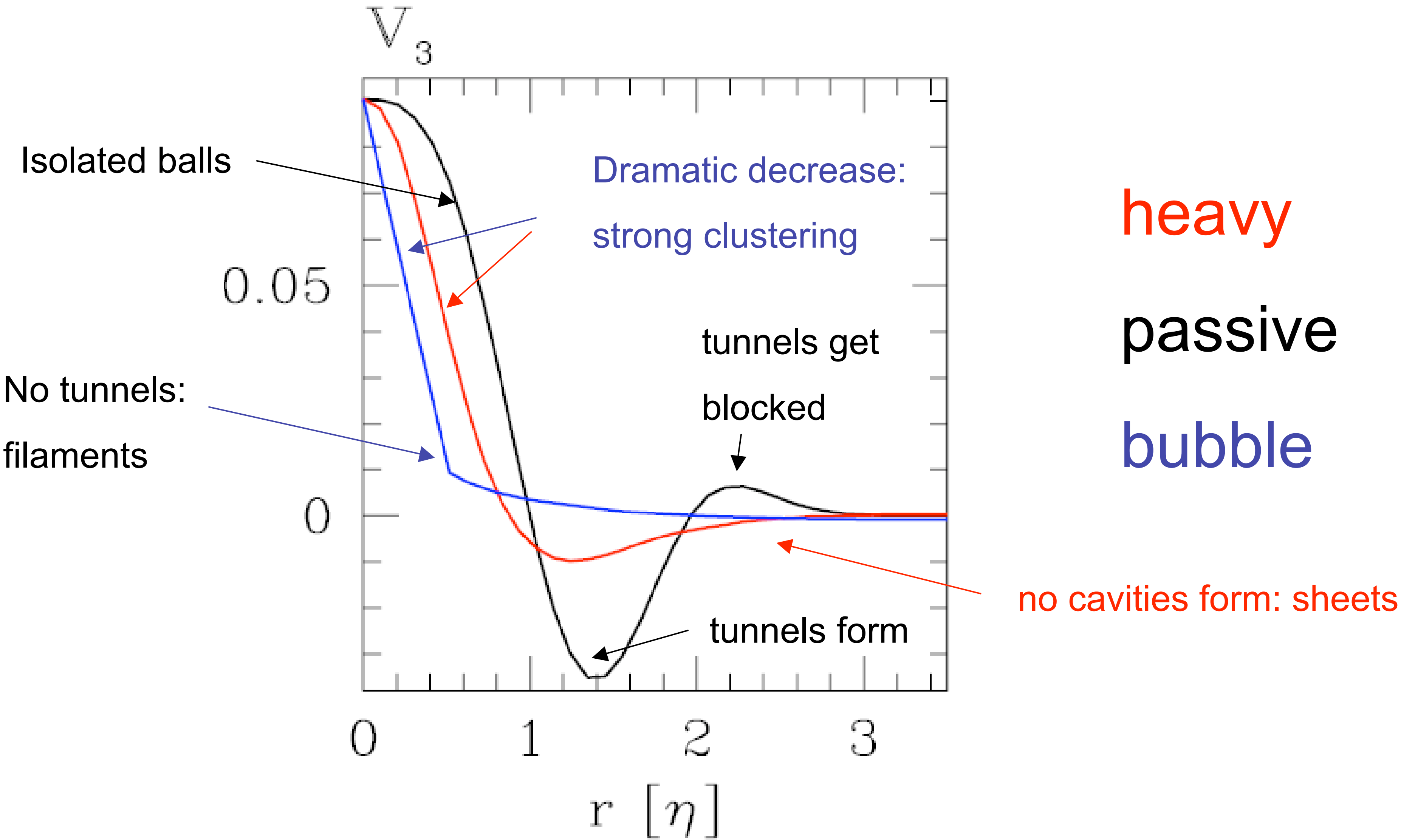
space-filling: no surface left

Mean curvature $V_2(r)$



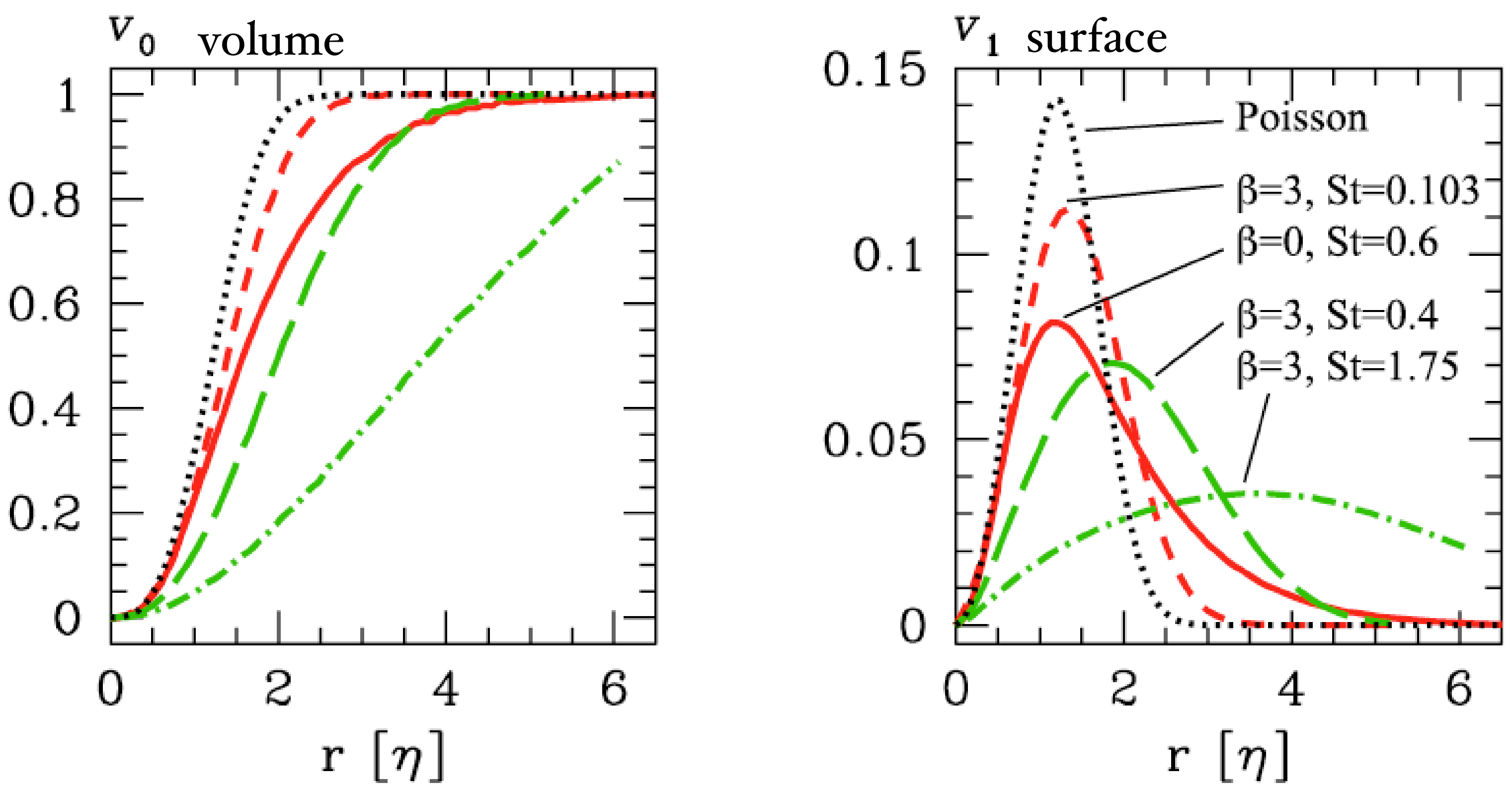
Euler characteristics $V_3(r)$

$$\chi = \#(\text{isolated bodies}) - \#(\text{tunnels}) + \#(\text{completely enclosed cavities})$$



"Degeneracy" of fractal dimension removed:

particle species with same D_{KY} may have very different morphologies



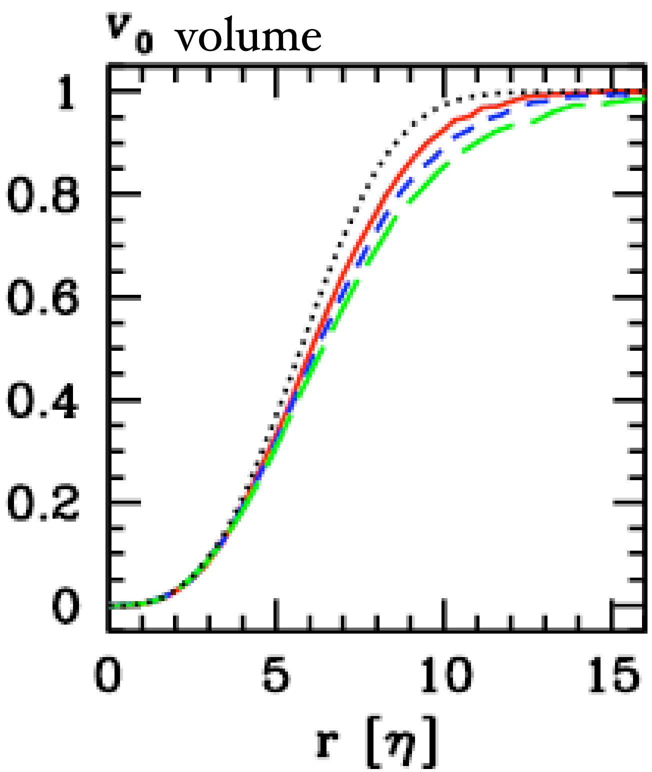
Red color: bubbles and heavy with same $D_{KY} \approx 2.6$

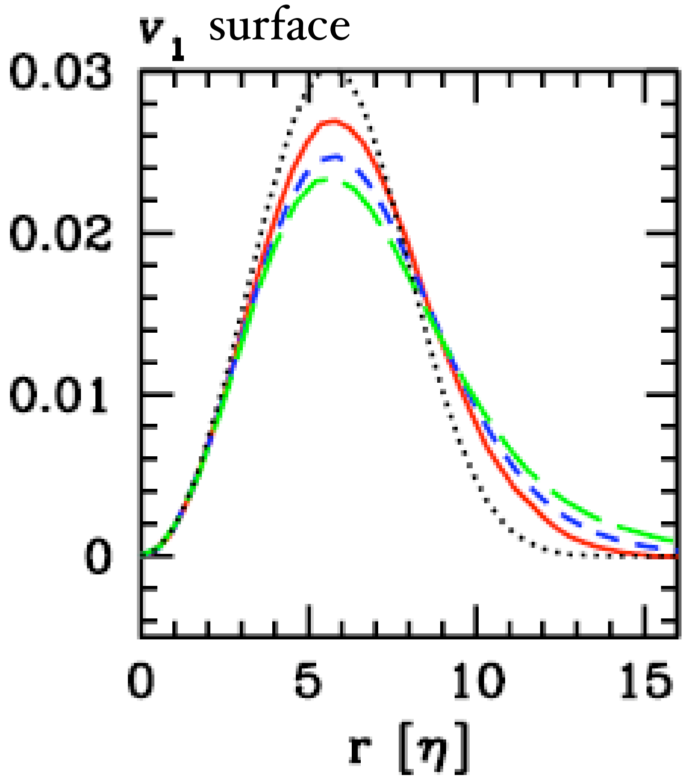
Green color: bubbles and heavy with same $D_{KY} \approx 1.65$

Dotted line: Poisson sample

Re_λ dependence

for heavy particle (β=0, St=0.6)





Red color: Re=75

Blue color: Re=180

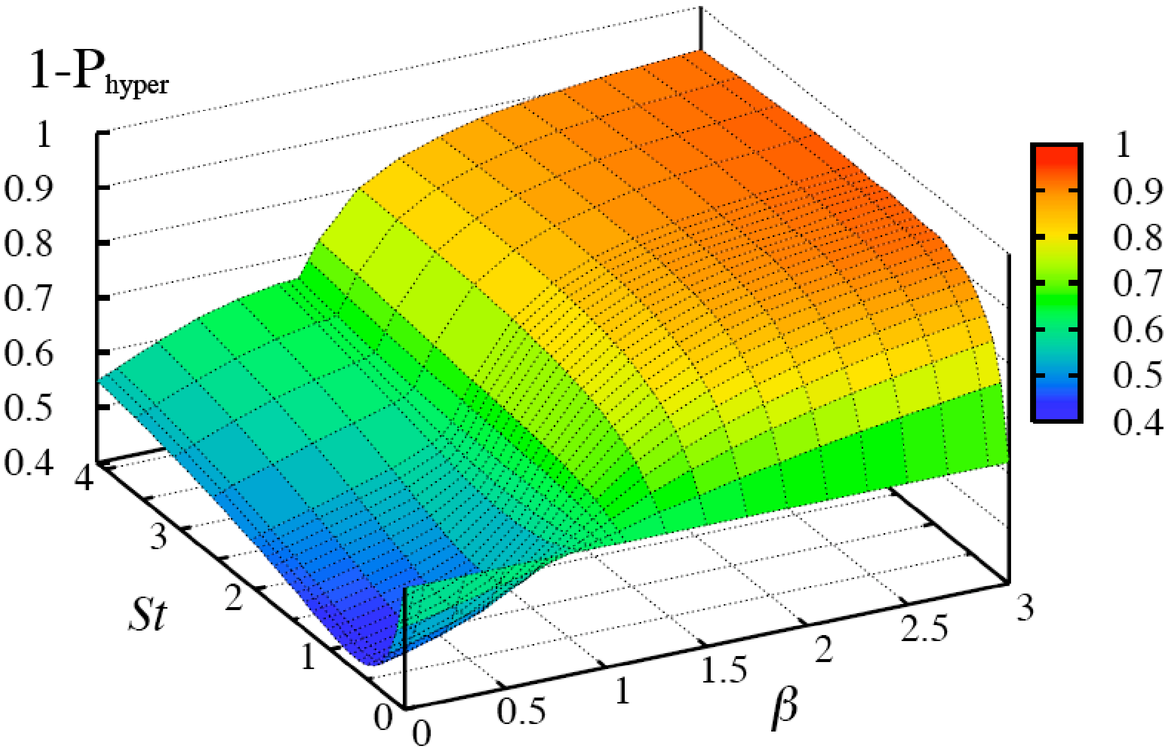
Green color: Re=400

Dotted line: Poisson sample

detectable and weak dependence

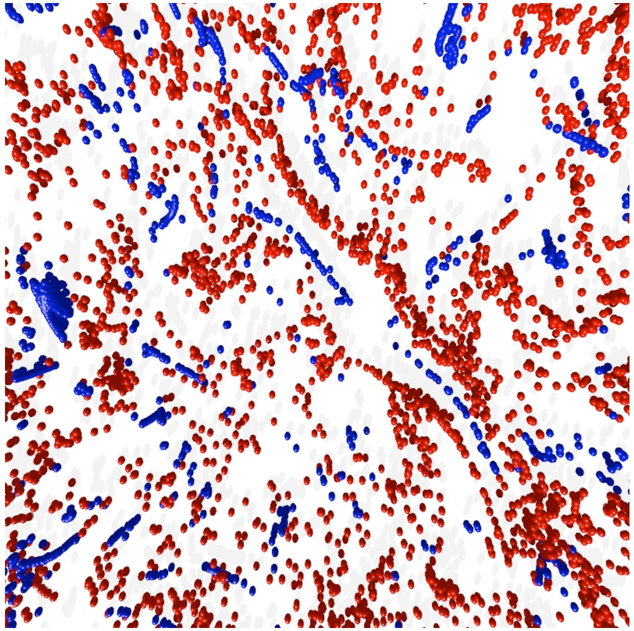
4. Segregation

Probability to be in elliptic regions



Fraction of particles in regions with complex eigenvalues of the strain matrix

$$\hat{\sigma}_{ij} = \partial_i u_j$$

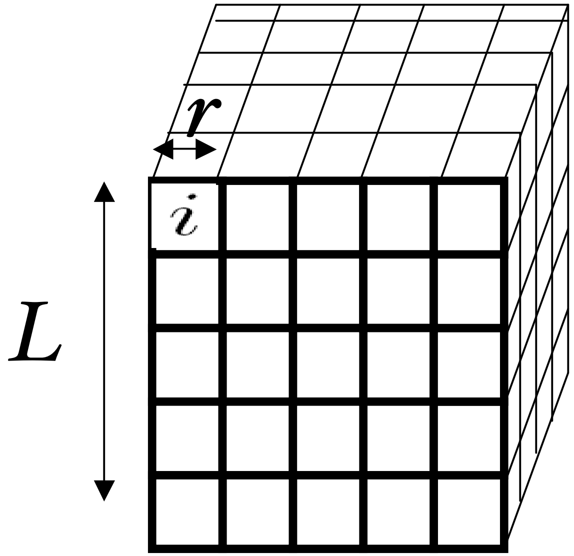


How to quantify the relative segregation of 2 particle species?

Is it possible to define a segregation length?

(see Chong, Perry, Cantwell PF 1990 and Bec et al. JFM 2006)

A possible indicator for segregation



L^3 domain volume

r^3 cublet volume

number of cublets

$\mathcal{M}(r) = (L/r)^3$

Given two species of particles:

$$\alpha_1 = (\beta_1, St_1)$$
$$\alpha_2 = (\beta_2, St_2)$$

$$S_{\alpha_1,\alpha_2}(r) = \frac{1}{N_{\alpha_1} + N_{\alpha_2}} \sum_{i=1}^{\mathcal{M}(r)} |n_i^{\alpha_1} - n_i^{\alpha_2}|$$

N_α : total number of particles of type α

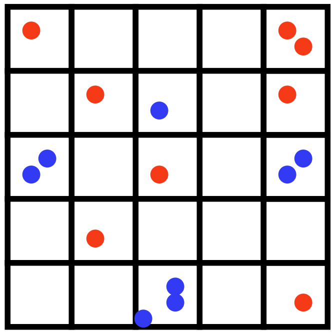
n_i^α : number of particles of type α in box *i*

Properties of $$S_{\alpha_1,\alpha_2}(r) = \frac{1}{N_{\alpha_1}+N_{\alpha_2}} \sum_{i=1}^{\mathcal{M}(r)} |n_i^{\alpha_1} - n_i^{\alpha_2}|$$

1. Upper/lower bound

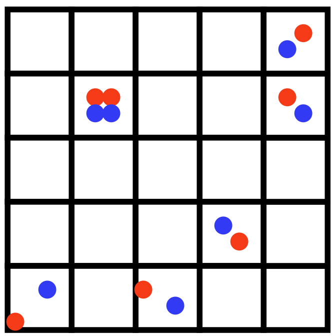
$$S_{\alpha_1,\alpha_2}(r) = 1$$

Always in different boxes



$$S_{\alpha_1,\alpha_2}(r) = 0$$

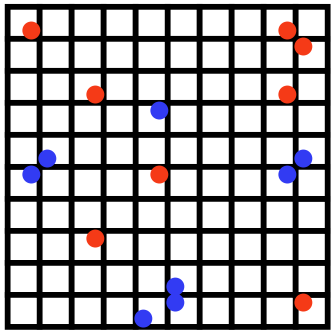
Always in the same boxes



2. Scale dependence (r)

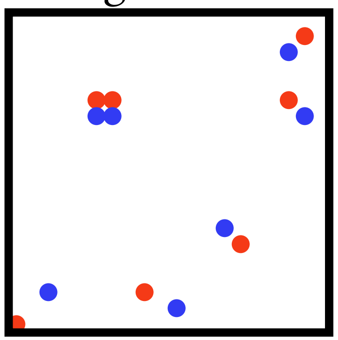
$$\lim_{r\to 0} S_{\alpha_1,\alpha_2} = 1$$

Small scales



$$\lim_{r\to L} S_{\alpha_1,\alpha_2} = 0$$

Large scales

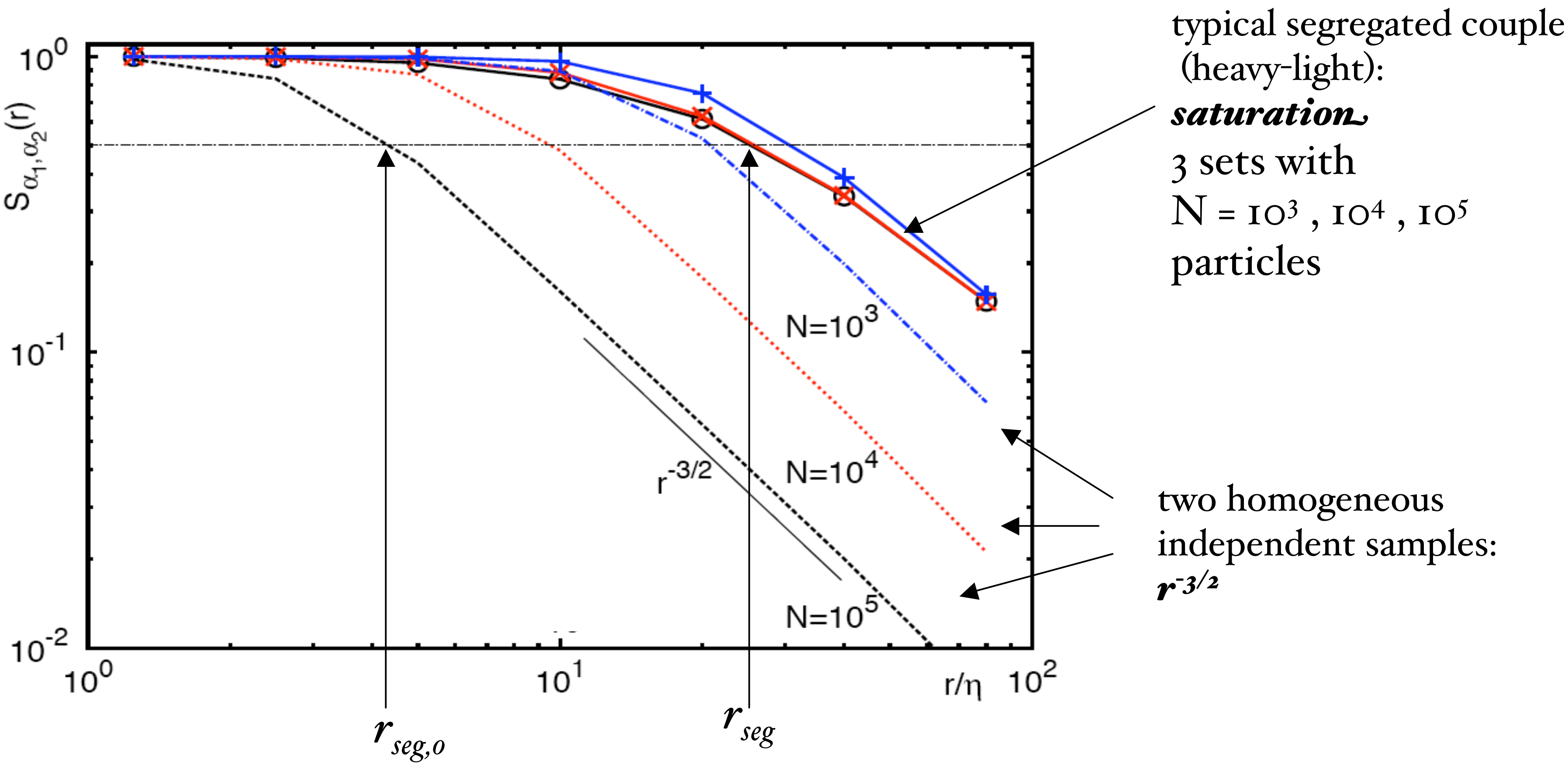


3. Poisson case

$$S(r) \sim N^{1/2}(r/L)^{-3/2}$$

How S(r) behaves for segregated populations of particles?

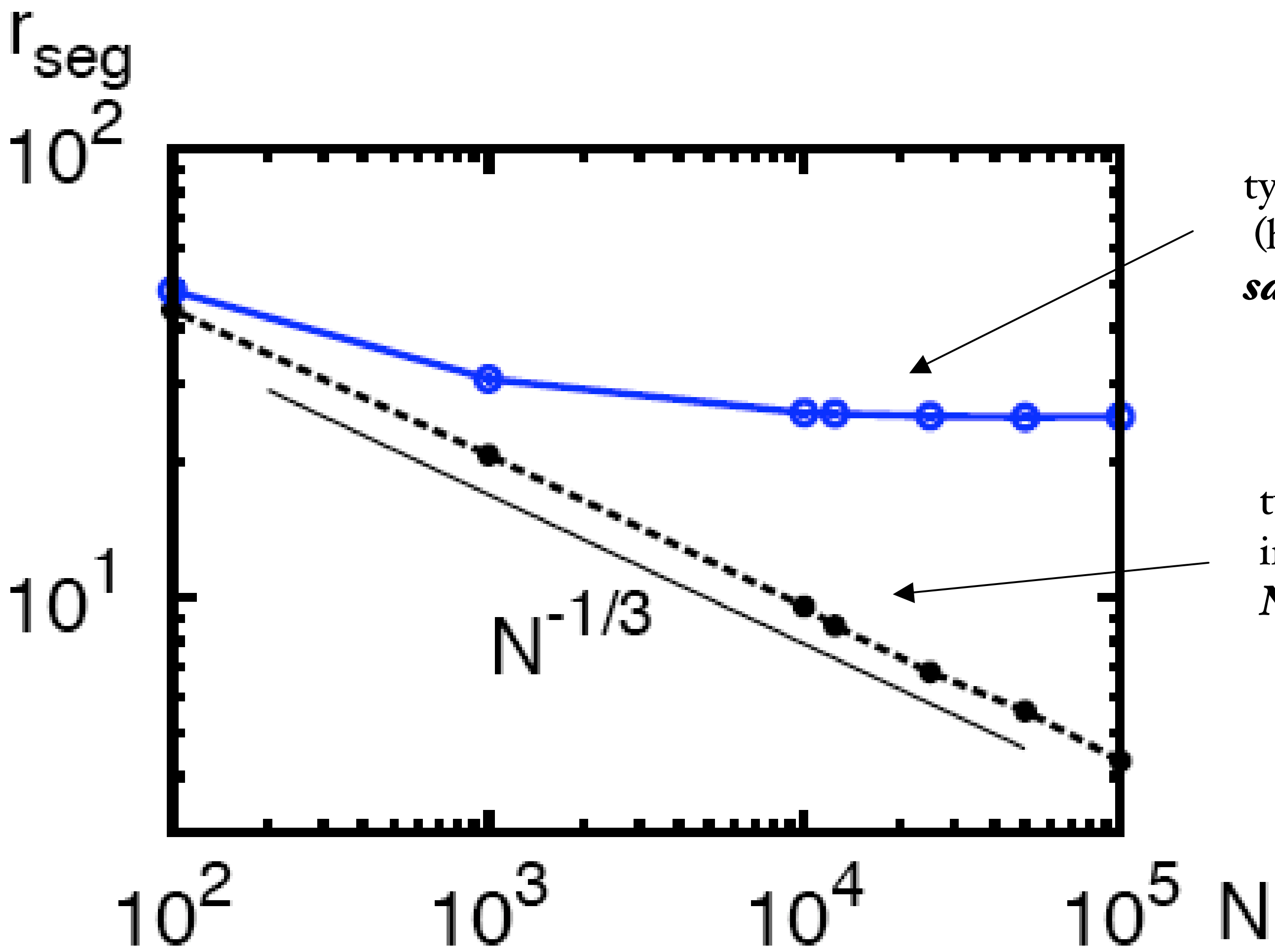
$S_{\alpha^1,\alpha^2}(r)$ vs. the coarsening scale r for segregated families



Definition of a segregation length r_{seg} :

$$S_{\alpha_1,\alpha_2}(r_{seg}) = 1/2$$

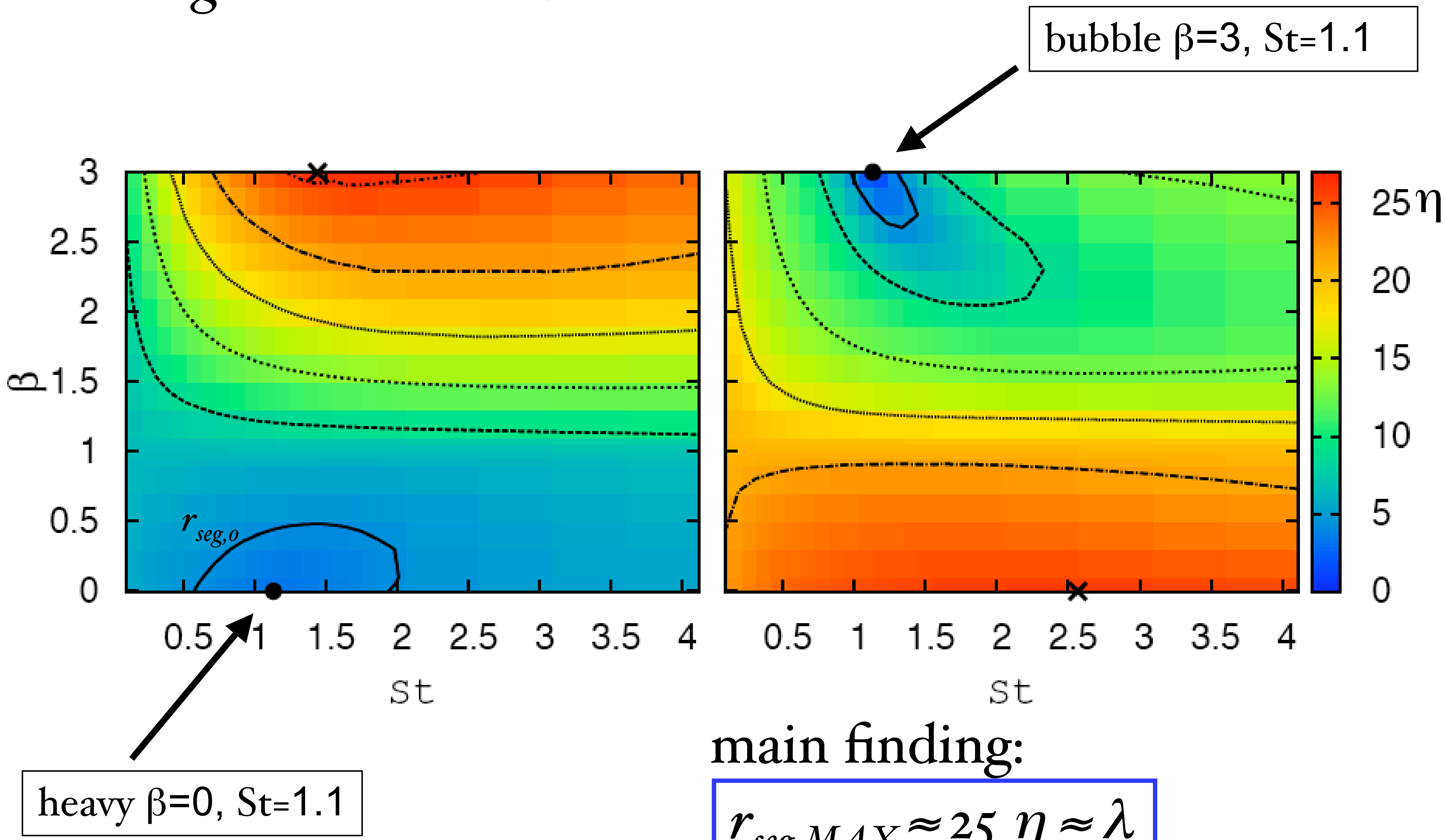
r_{seg} behavior *vs.* particle number N



typical segregated couple (heavy-light): ***saturation***

two homogeneous independent samples: $\boldsymbol{N^{-1/3}}$

r_{seg} for heavy particle and bubble



main finding:

$r_{seg,MAX} \approx 25\ \eta \approx \lambda$

Preprints on spatial clustering:

- *Dimensionality and morphology of particle and bubble clusters in turbulent flow,*
  E. C. , M. Kerscher, D. Lohse and F. Toschi,
  ArXiv:/nlin.CD/0710.1705

- *Quantifying turbulence induced segregation of inertial particles,*
  E. C., M. Cencini, D. Lohse, F. Toschi,
  ArXiv:/nlin.CD/0802.0607

- *Quantifying microbubble clustering in turbulent flow from single-point measurements,*
  E. C. , T. H. van den Berg, F. Toschi and D. Lohse,
  ArXiv: physics/0607255 (Phys Fluids *in press*)

iCFDdatabase

Lagrangian data $(x(t), v(t), a(t), u(t), \partial_i u_j(t))$ available at *http://cfd.cineca.it/*

Acknowledgments: sara Amsterdam The Netherlands CINECA Consorzio Interuniversitario Bologna Italy CASPUR Roma Italy