Ondes, turbulence, turbulence d' onde

une introduction ...

GDR turbu, ENS-Lyon

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CTRL-L switch

Collaboration et documents

- Equipe ondes et turbulence : Fabien S. Godeferd, Julian F. Scott, Lukas Liechtenstein, Alex Delache, Benjamin Favier
- Documents (hors articles)
  -) http://www.lmfa.ec-lyon.fr/Fabien.Godeferd/perso/ (Summer School in Barcelone)
  -) http://gdr-turbulence.pmmh.espci.fr/Cargese/cargese-turbulence.html
  -) Livre *Homogeneous Turbulence Dynamics*, Sagaut & Cambon, CUP, 2008
- Zakharov et al. 1991, articles Newell et al., Waleffe (1992,1993), + récents Galtier et al.

Résumé

- Rappels de turbulence, cascade, description spectrale
- Ondes, relation de dispersion, interactions faibles
- Dynamique linéaire: mélange de phase (rotation rapide)
- Modèle non-linéaire, triades, triades résonantes, EDQNM et turbulence d’ ondes d’ inertie
- Coexistence de turbulence “faible” et “forte”, stratification stable, MHD

Quantifier la cascade (THI)

- Espace physique

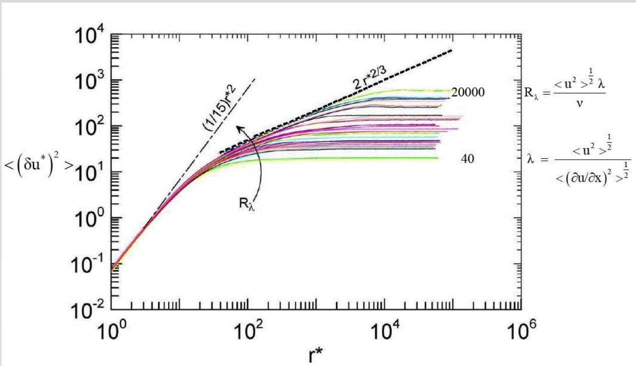
  Equation de Karman-Howarth (1938) $\longrightarrow \underbrace{\langle(\delta u_L)^3\rangle = -\frac{4}{5}\varepsilon r}_{\text{(4/5) Kolmo 41}}$

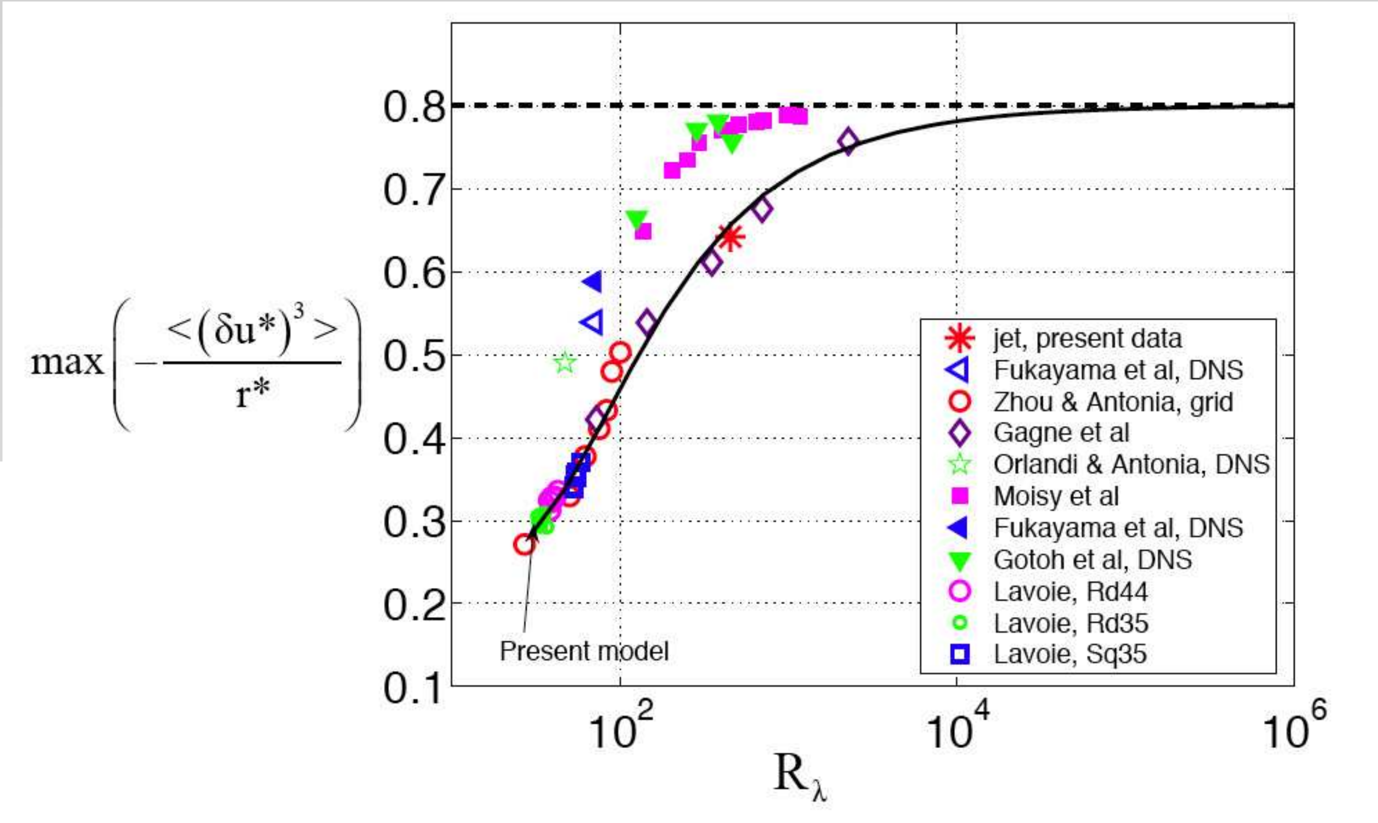
- Espace de Fourier

  Equation de Lin (1949)

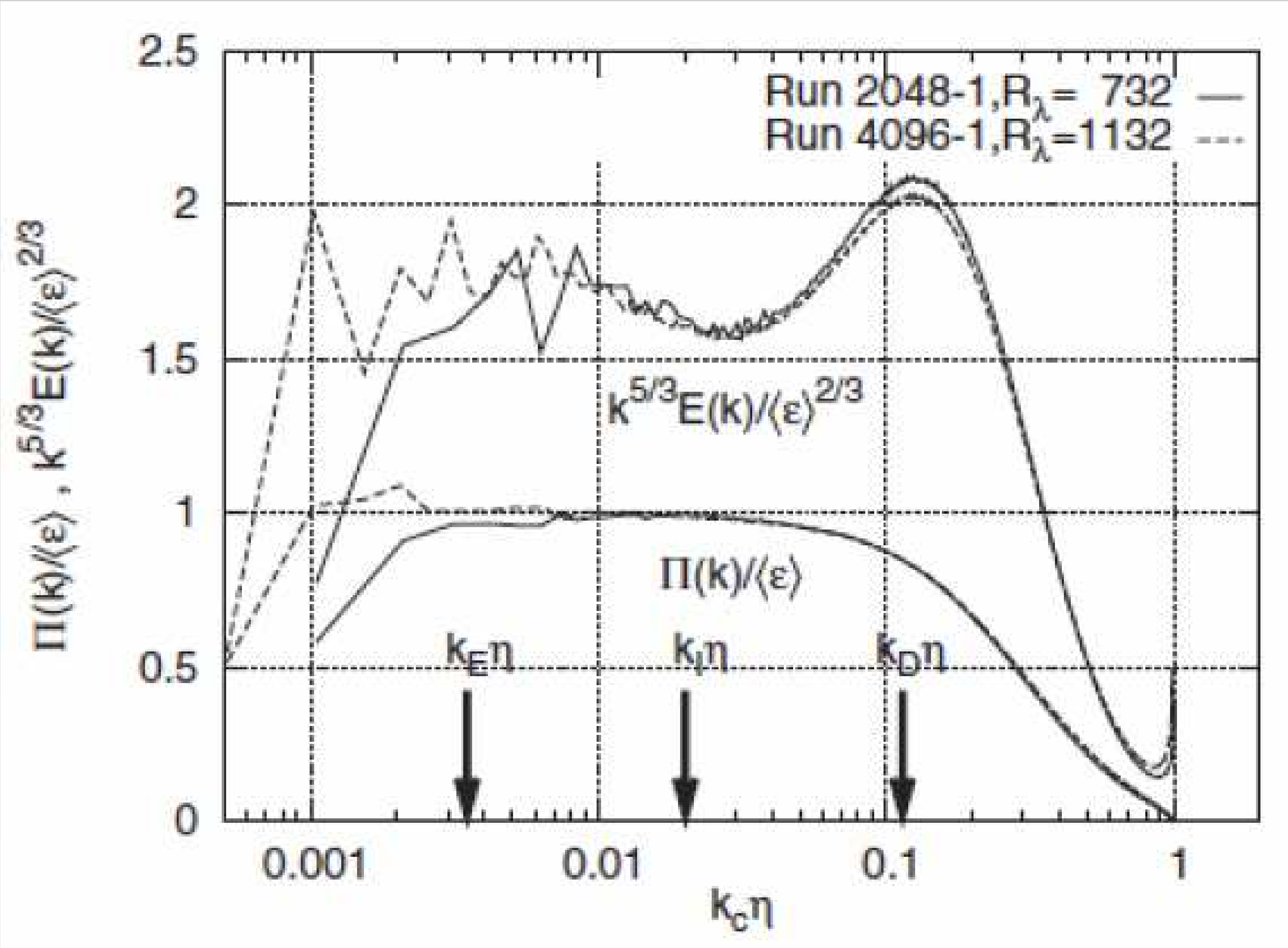
$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) = T(k,t) \qquad \longrightarrow \qquad \int_k^\infty T(p)dp = \varepsilon$$

Utile en turbulence forte et en turbulence d' onde (équations cinétiques)

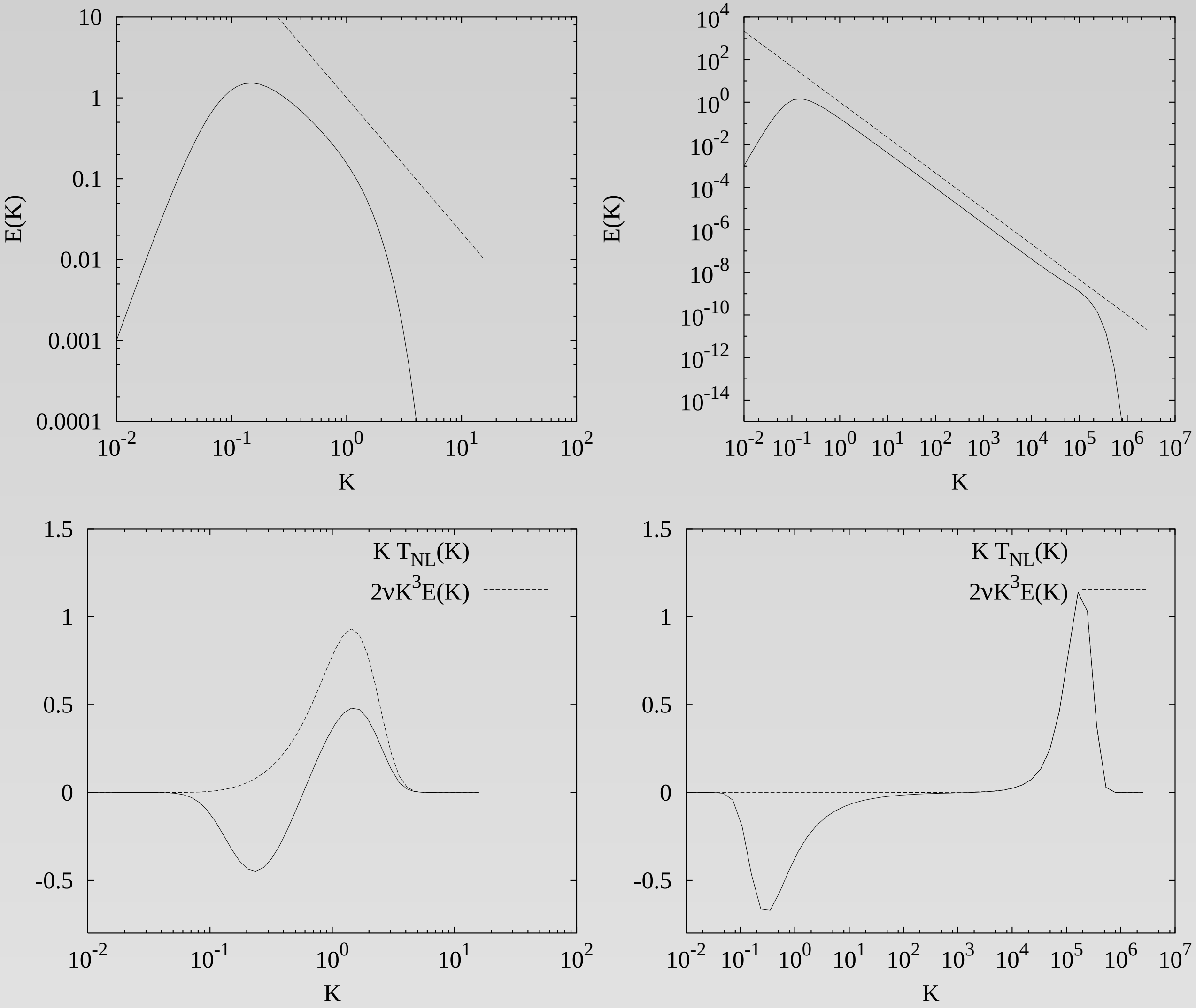




Antonia (Cargèse 2007)

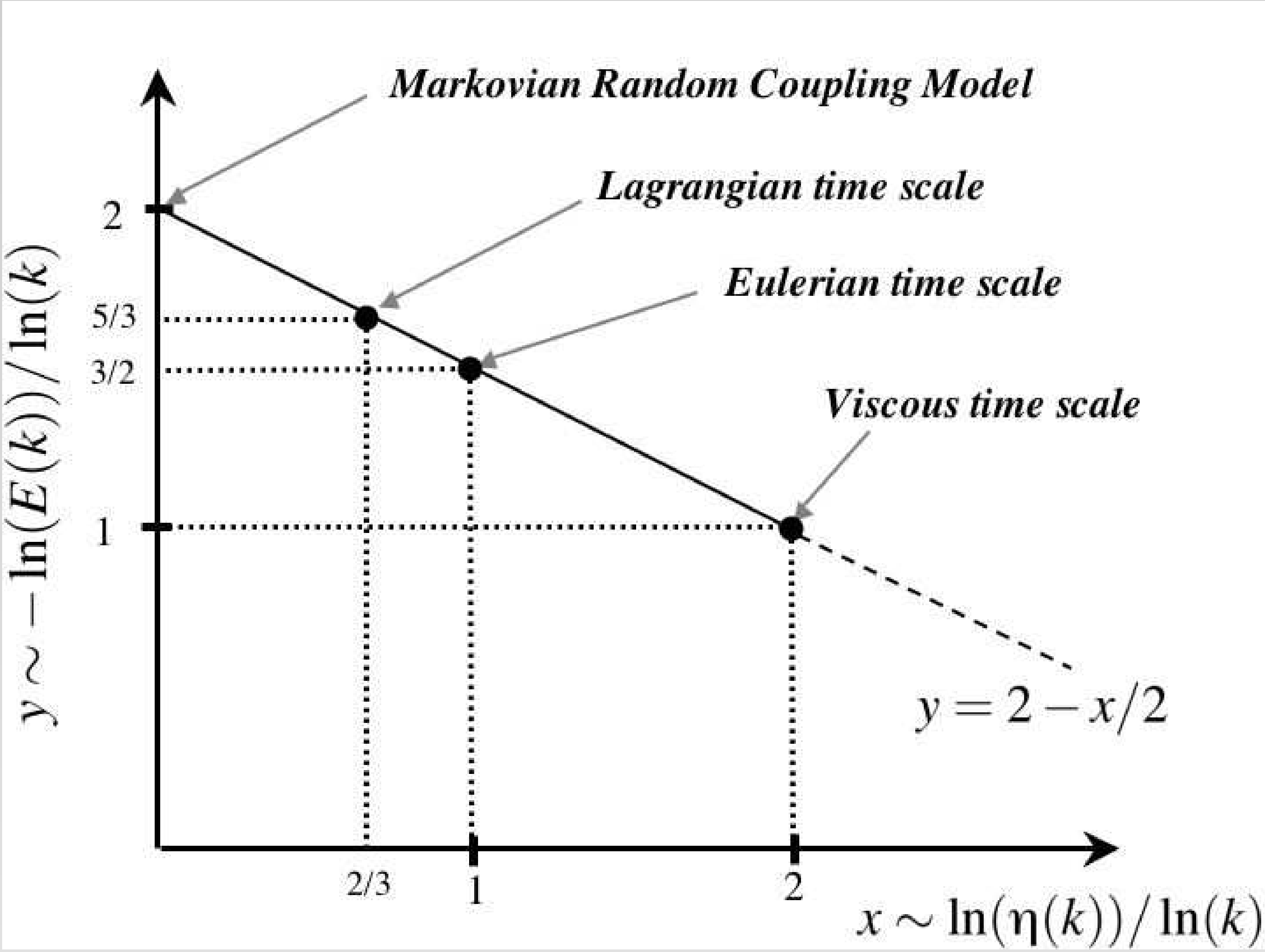


Kaneda (Cargèse)



d' après Wouter Bos,

$R_\lambda = 30, 10^5$, EDQNM (THI)



Temps caractéristique dans les *fermetures spectrales* DIA, EDQNM

$$\eta = \nu k^2, \quad ku', \quad \varepsilon^{1/3}k^{2/3}, \quad Cte$$

Ondes, turbulence d' onde(s)

- Modes de Fourier spatio-temporels $\boldsymbol{u}(\boldsymbol{x},t) = \sum \boldsymbol{A} e^{\imath(\boldsymbol{k}\cdot\boldsymbol{x} - \sigma t)}$
- Loi de dispersion $\sigma = \sigma(\boldsymbol{k})$ (renormalisation nonlinéaire ?) $\boldsymbol{A} = a_{\pm 1}(\boldsymbol{k}, \varepsilon t)$?
- Ondes non dispersives $\sigma = \pm \boldsymbol{k}\cdot\boldsymbol{C}_g$ (cf. convection par une vitesse constante, mais signe PLUS et MOINS)
- Exemples, $\sigma = \pm k c_0$ (acoust.), $\sigma = \pm \boldsymbol{k}\cdot\frac{\boldsymbol{B}_0}{\sqrt{\sigma\mu}} = \pm k_{\parallel}\frac{B_0}{\sqrt{\sigma\mu}}$ (Alfvèn, MHD) $\sigma = f\frac{k_{\parallel}}{k}$ (inertie, rotation), $\sigma = \pm N\frac{k_{\perp}}{k}$ (gravité, stratification stable), $\sigma = \beta\frac{k_x}{k^2}$ (Rossby) ...
- Propriétés: ANISOTROPIE (sauf acoustique, ondes de surface, ...), champs couplés cin-pot (sauf rotation), faible intermittence ? vaste domaine (gas de phonons ...)

Interactions faibles ? sans *fermeture*, triades

Ondes créées par mécanisme extérieur (gradient, forces massiques, champ couplés), non-linéarité quadratique, base des modes propres $\dot{a}_s + \imath s \sigma_k a_s \propto \widehat{uu}$

$$\dot{\boldsymbol{a}}_s(\boldsymbol{k}, t) = \sum_{s', s''=0,\pm 1} \int_{k+p+q=0} \underbrace{e^{\imath(s\sigma_k + s'\sigma_p + s''\sigma_q)t}}_{\text{effet de phases “rapide”}} \underbrace{M_{ss's''} a_s(\boldsymbol{p}, t) a_s(\boldsymbol{q}, t)}_{\text{non-linéarité}} d^3\boldsymbol{p}$$

$s = 0, \pm 1$, triade $\boldsymbol{k} + \boldsymbol{p} + \boldsymbol{q} = 0$, non-linéarité réduite si $s\sigma_k + s'\sigma_p + s''\sigma_q$ grand ?

Ondes ou pas ondes ? ondes NON dispersives ? pseudo-ondes (vitesse convectante) ?

annulation possible de la phase (reson.) ou non

Dynamique *statistique*, $< aa >$, $< aaa >$, pourquoi ?

Turbulence *faible* (avec) et *forte* (sans) rotation

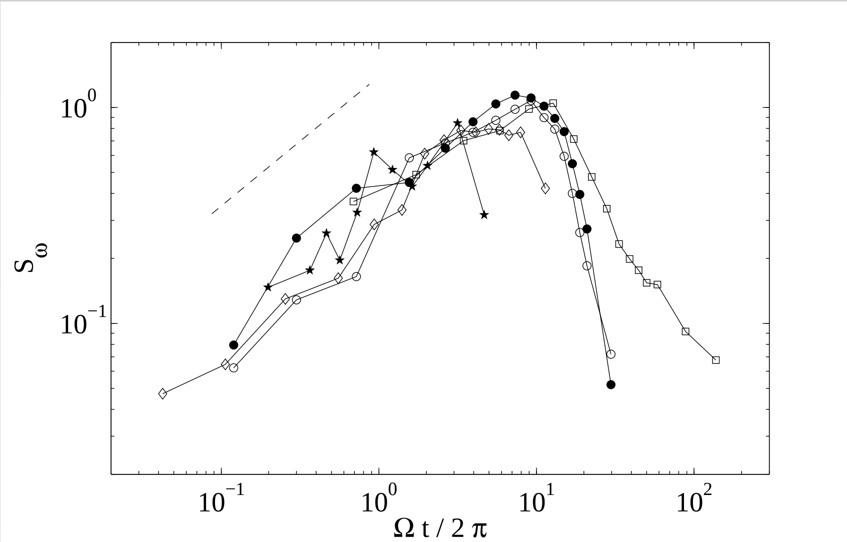
- cas 3D sans frontières, la résonance triple existe (pas vrai en eaux peu profondes ...): non-linéarité $\sim o(Ro)$, $\Omega t \sim Ro^{-2}$
- possibilité de justifier une hypothèse QN de cumulants (4) nuls (Benney & Newell, 1969, "Random Phase Approx")
- On passe de l' EDQNM(3) à la théorie de turbulence d' onde par la limite de ED tendant vers 0 (Bellet et al. 2006)...
  $\frac{1}{\mu - \imath X} \rightarrow 2\pi\delta(X) + \imath\mathcal{P}\left(\frac{1}{X}\right)$ ...
- ... en utilisant les modes propres des ondes d' inertie, qui sont aussi les modes propres du rotationnel AVEC et SANS rotation (helical modes, ondes d' hélicité).
- Mais il n' y a pas que la limite asymptotique de turbulence d' onde avec non-linéarité restreinte aux triades résonantes ! Presque tous les transitoires dûs au mélange de phase sont intéressants (cf polémique avec Peter Davidson).

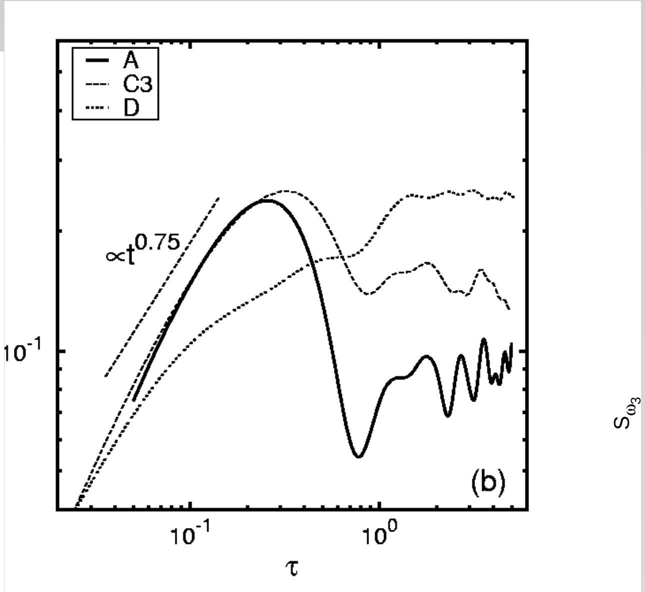
Rotation, mélange de phase: linéaire et/ou non-linéaire

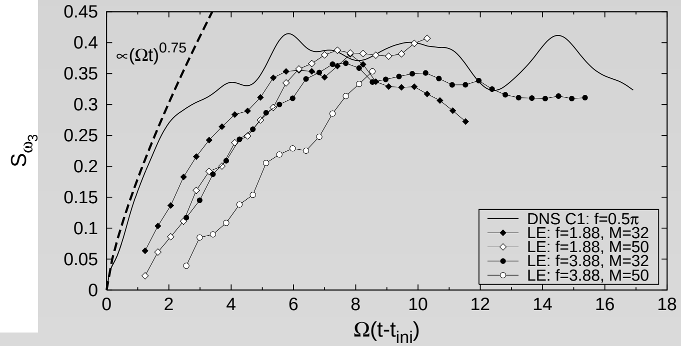
- Relevance of linear solution depends on the *order and type* of statistical correlations
  -) doubles: 2 point 1 time: $e^{\imath\sigma_k t}$, $e^{-\imath\sigma_k t}$
  -) doubles: 2 point 2 time: $e^{\imath\sigma_k(t\pm t')}$
  -) triples: 3 point 1 time: $e^{\imath(\pm\sigma_k\pm\sigma_p\pm\sigma_q)t} \longrightarrow$ nonlinear $\cdots\cdot$

$$\langle \omega_3^3 \rangle = \sum \int \exp[\imath f t(\cos\theta_k + s'\cos\theta_p + s''\cos\theta_q] S(\boldsymbol{k}, \boldsymbol{p}, \epsilon t) d^3\boldsymbol{p} d^3\boldsymbol{k}$$

($\cos\theta_k = k_3/k$) Need for initial triple correlations at THREE point. Many other correlations.







Cyclonic / anticyclonic asymmetry: (64^3 LES ?) Bartello et al. (1994), Morize et al. (2005), Gence & Frick (2001), Staplehurst et al. (2008), van Bokhoven et al. (2008). No need for centrifugal inst.

$$\frac{d}{dt}\langle\omega_3^3\rangle = 6\Omega\langle\frac{\partial u_3}{\partial x_3}\omega_3^2\rangle + \text{4th-order, viscous}$$

Equations exactes pour la théorie non-linéaire

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) e(k, \cos\theta, t) = T^{(e)}(k, \cos\theta, t)$$

Poincaré transformation $\hat{u}_i(\boldsymbol{k}, t) = \sum_{s=\pm 1} N_i(s\boldsymbol{k}) e^{\imath s f t \cos\theta} a_s(\boldsymbol{k}, \epsilon t)$,

$$T^{(e)} = \int_{t_0}^{t} \sum_{s,s',s''=\pm 1} \int_{p+q=k} e^{\imath f (t-t')(s\cos\theta_k + s'\cos\theta_p + s''\cos\theta_q)}$$

$$S_{ss's''}(\boldsymbol{k}, \boldsymbol{p}, t, \epsilon t') d^3\boldsymbol{p} dt'$$

Système dynamique de triade isolée

Fiortjoft, Kraichnan (2D), Waleffe (3D) : pas de rotation

$$\dot{a}_s(\boldsymbol{k}) = (s'p - s''q)G_{kpq}a^*_{s'}(\boldsymbol{p})a^*_{s''}(\boldsymbol{q}),$$

$$\dot{a}'_s(\boldsymbol{p}) = (s''q - sk)G_{kpq}a^*_{s''}(\boldsymbol{q})a^*_s(\boldsymbol{k})$$

$$\dot{a}''_s(\boldsymbol{q}) = (sk - s'p)G_{kpq}a^*_s(\boldsymbol{k})a^*_{s'}(\boldsymbol{p})$$

Avec rotation, $G_{kpq} \rightarrow G_{kpq}e^{\imath X}$, triades résonantes,
$X = f(s\cos\theta_k + s'\cos\theta_p + s''\cos\theta_q) = 0$, avec
$k\cos\theta_k + p\cos\theta_p + q\cos\theta_q = 0$:

$$\frac{cos\theta_k}{s'q - s''p} = \frac{cos\theta_p}{s''k - sq} = \frac{cos\theta_q}{sp - s'k}$$

Analogie avec le “solide d’ Euler”

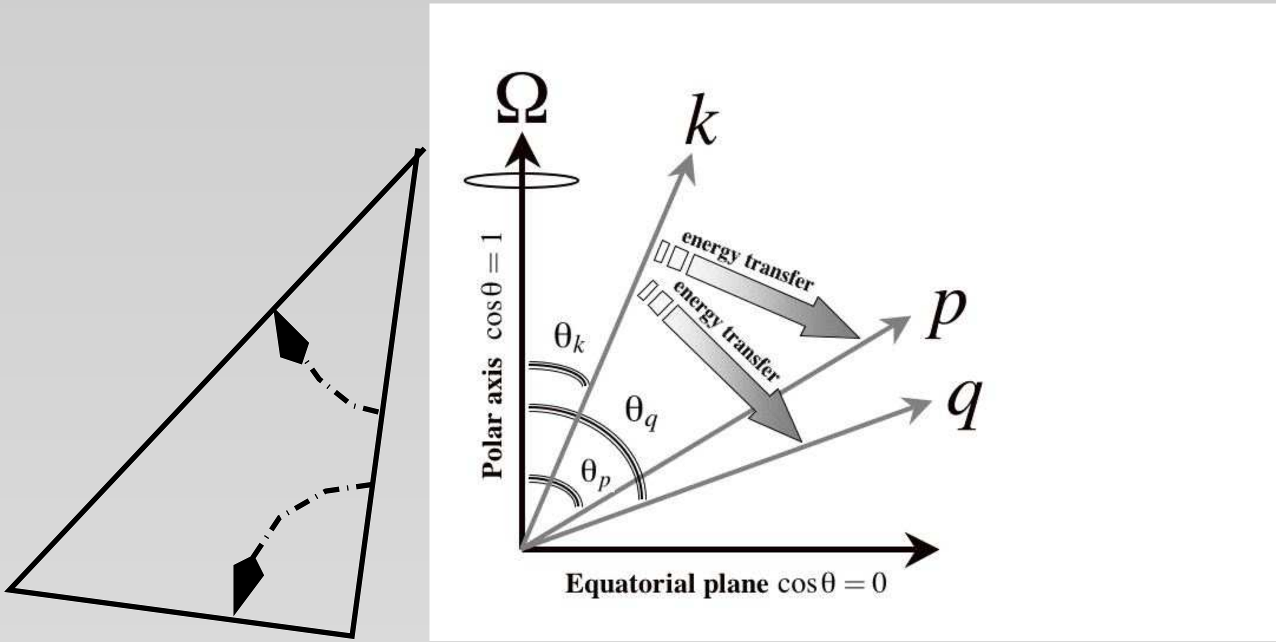
- Moment angulaire dans les axes principaux d’ inertie

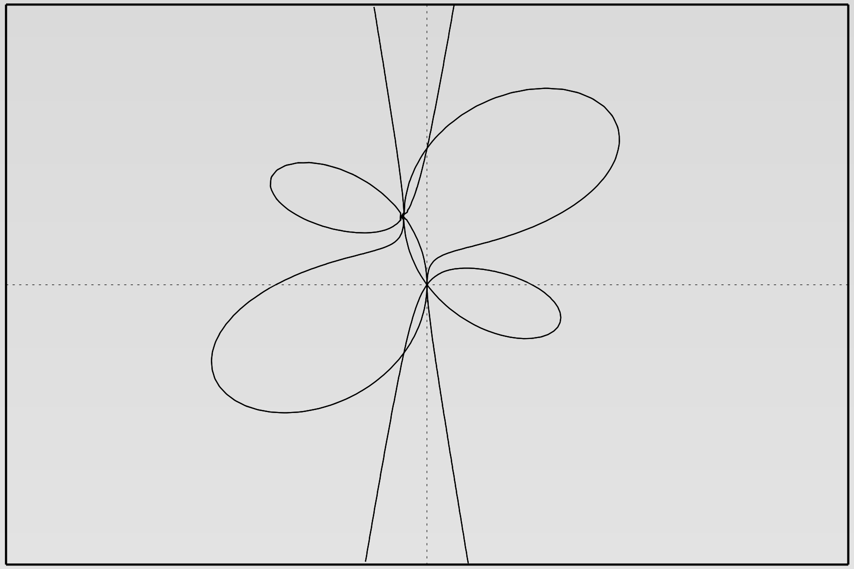
$$I_1\dot{\Omega}_1 = (I_2 - I_3)\Omega_2\Omega_3, \quad (1)$$

$$I_2\dot{\Omega}_2 = (I_3 - I_1)\Omega_3\Omega_1 \quad (2)$$

$$I_3\dot{\Omega}_3 = (I_1 - I_2)\Omega_1\Omega_2, \quad (3)$$

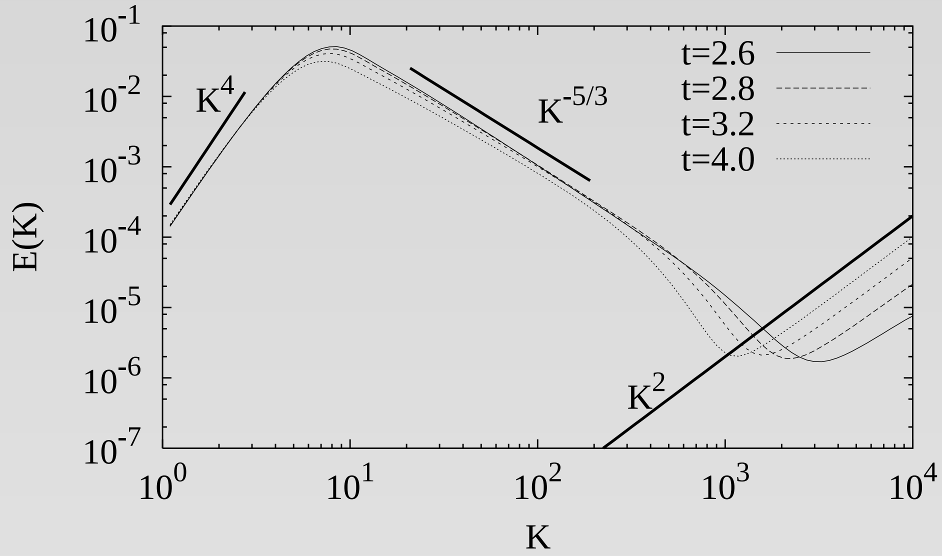
- Lois de conservation: énergie rot. $(I_1\Omega_1^2 + ...) \longrightarrow$ énergie cin. (triade), norme du moment angulaire $(I_1\Omega_1)^2 + ... \longrightarrow$ helicité (triade)
- Signes (s, s', s''), $I_1 \longrightarrow sk$, instabilité par rapport au moment *intermédiaire*

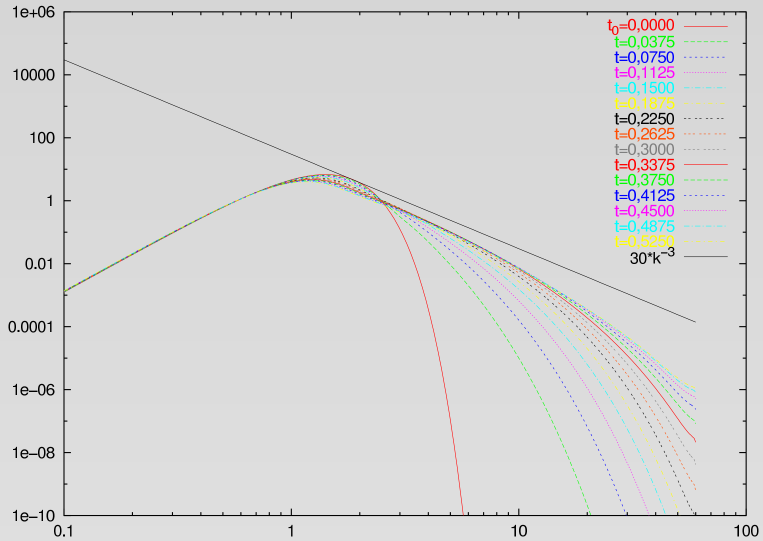

Ω
k
energy transfer
energy transfer
p
q
θ_k
θ_q
θ_p
Polar axis cosθ = 1
Equatorial plane cosθ = 0



Results. NONLINEAR statistical theory

- From classical EDQNM (isotropic, no rotation, Orszag 1970, Bos & Bertoglio, 2006) ...

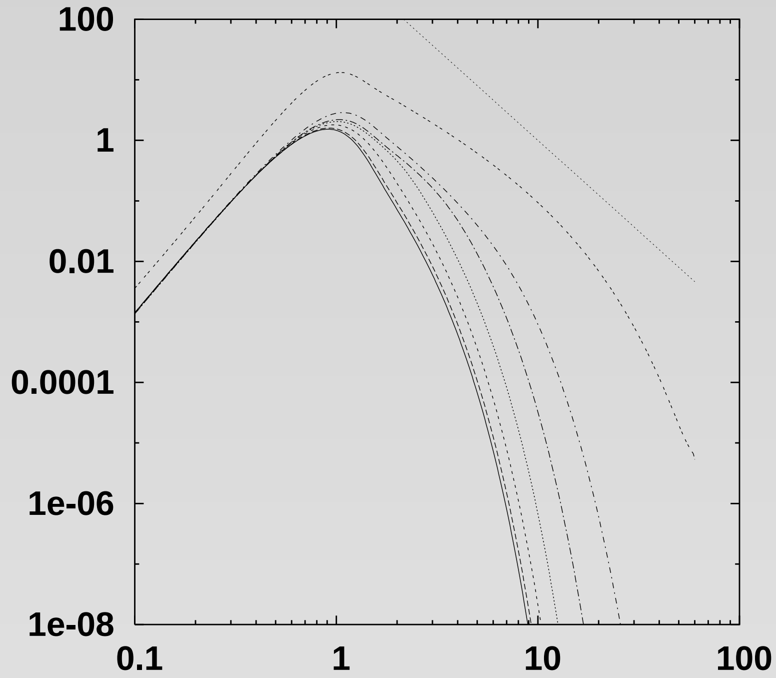




- ... to EDQNM3 $\longrightarrow$ (A) QNM energy equation (Bellet *et al.*, JFM, 2006)

Angle-dependent spectrum

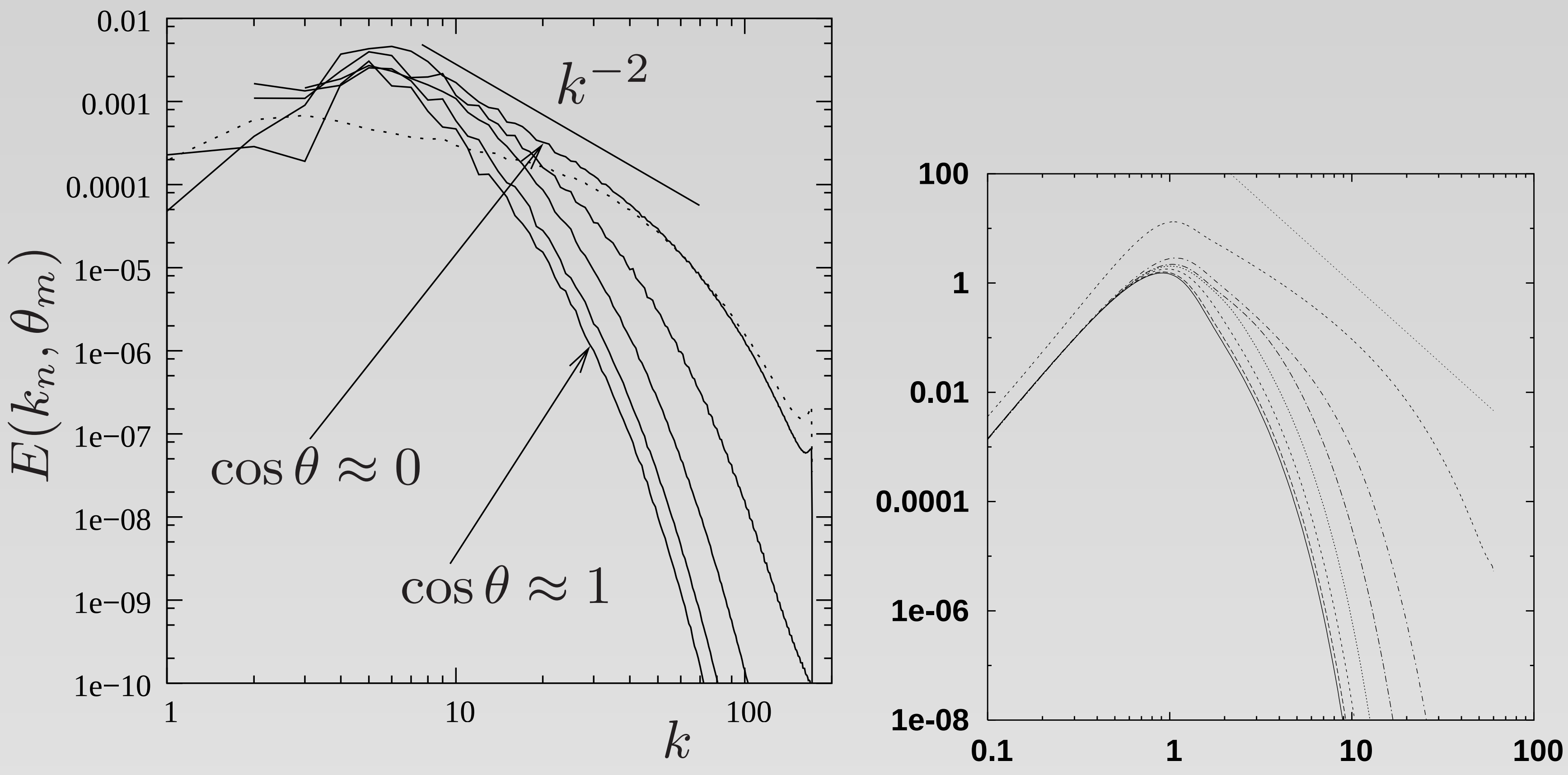
- Isotropy breaking by spectral transfer $T^{(e)}(\boldsymbol{k})$: directional anisotropy:



$$4\pi k^2 e(\boldsymbol{k}, t_f) = 4\pi k^2 e(k, \underbrace{\cos\theta}_{k_{||}/k}, t_f)$$

- Spherical averaging $\longrightarrow E(k, t_f)$, prefactor $E \sim \frac{\Omega}{t} k^{-3}$, not 2D !

AQNM and DNS



512^3 DNS by Liechtenstein *et al.*, JOT, 2005

Inertial wave-turbulence, resonant interactions, 2D or not 2D

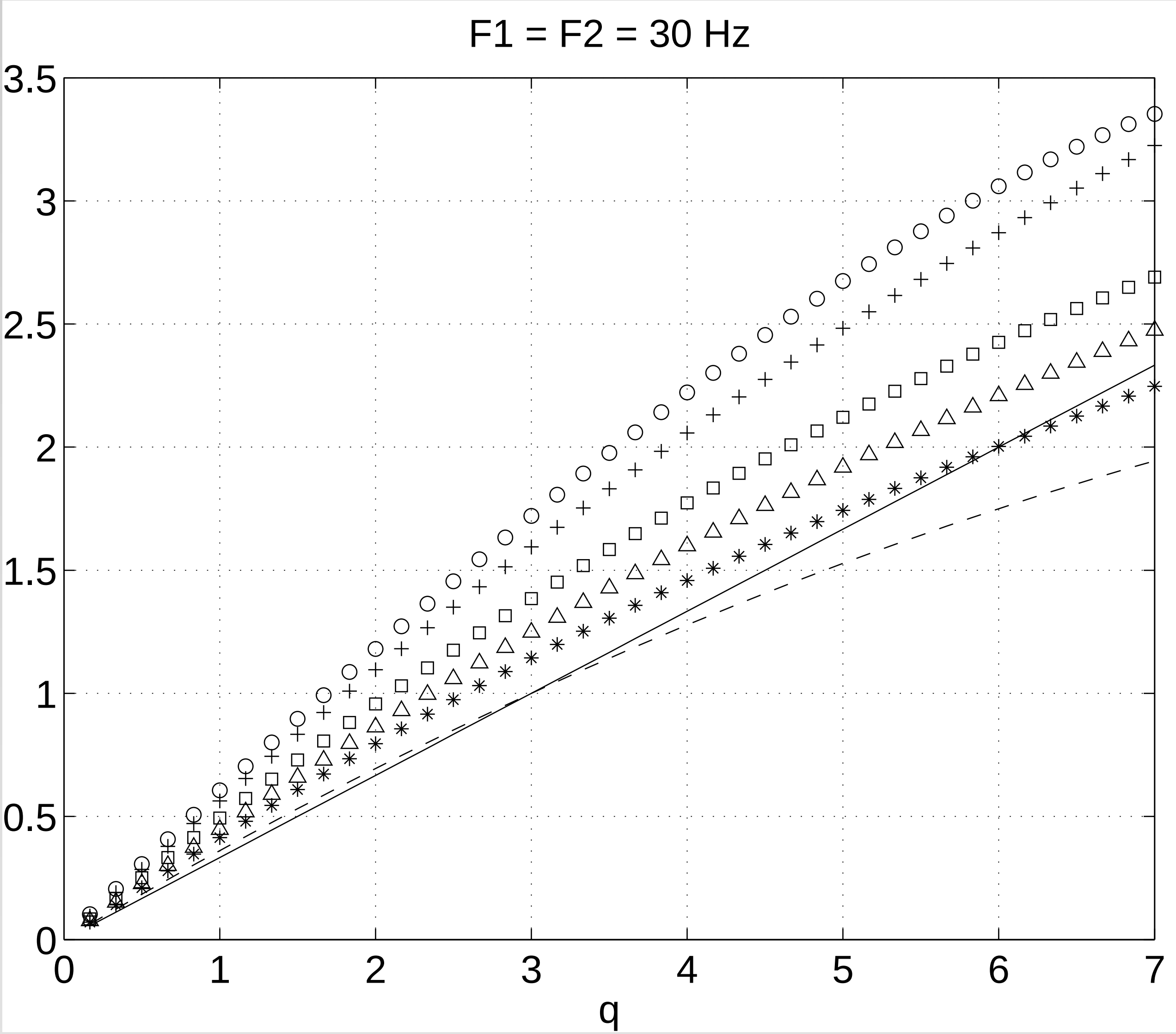
- *Low dimension* of active manifolds : overestimated in forced ? DNS/LES ?
  'TRUE' 2D embedded in 3D : a DIRAC singularity

$$E(k) \sim f^2 k^{-3}, \qquad e(k_\perp, k_\parallel) = \underbrace{\frac{E(k_\perp)}{2\pi k_\perp}}_{\sim f^2 k_\perp^{-4}} \delta(k_\parallel)$$

- integral singularity from theoretical wave-turbulence

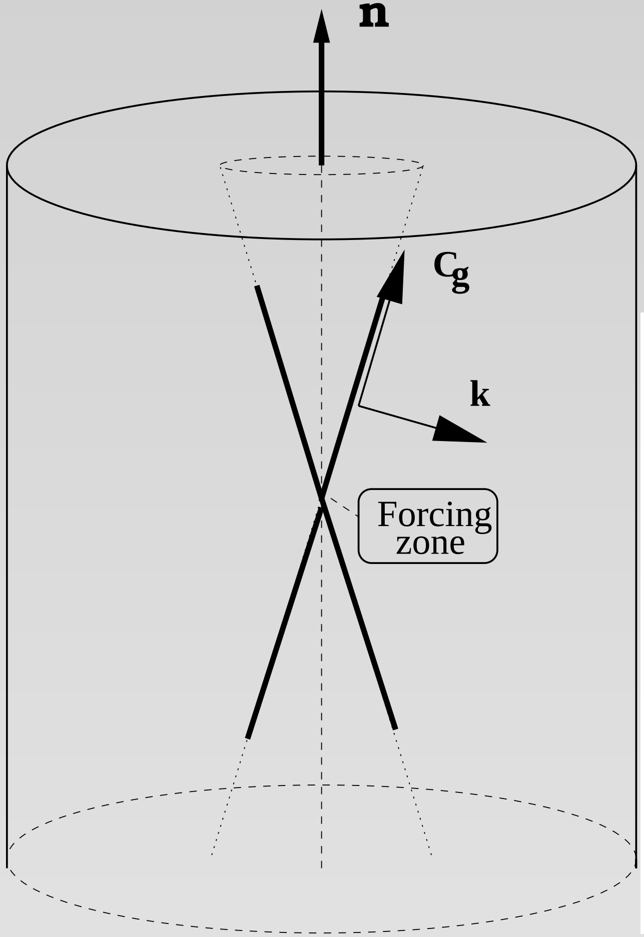
$$e(k_\perp k_\parallel) \sim k_\parallel^{-1/2} k_\perp^{-7/2} \quad (k_\parallel \ll k_\perp) \quad = k^{-4} x^{-1/2}$$ Galtier 2003

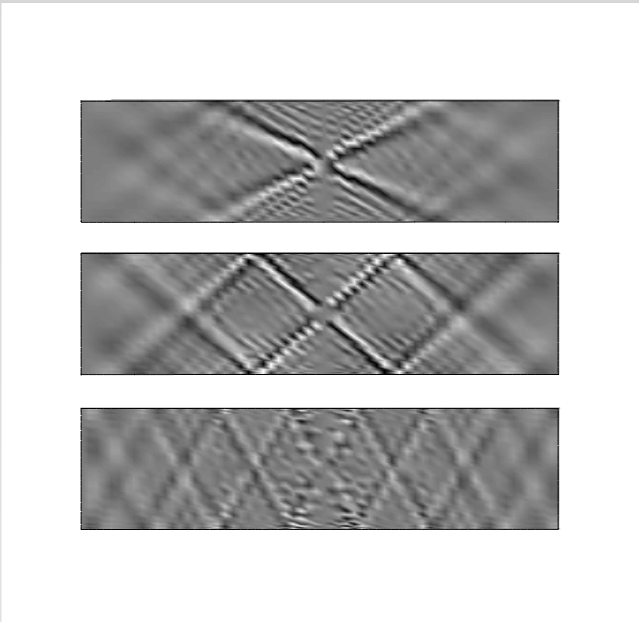
$$E(k) \sim \frac{f}{t} k^{-3}, \quad e(k, x \sim 0) \sim k^{-4}$$ Bellet et al. 2006



Catherine Simand, voisinage d' un vortex intense, (Baroud et al., Mueller & Thiele, Morize et al. ??)

Wave aspects

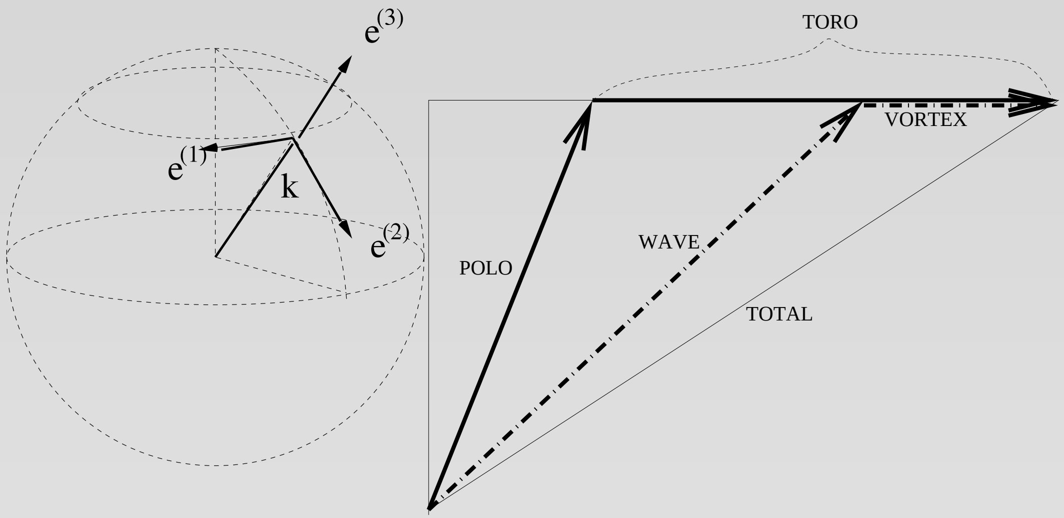




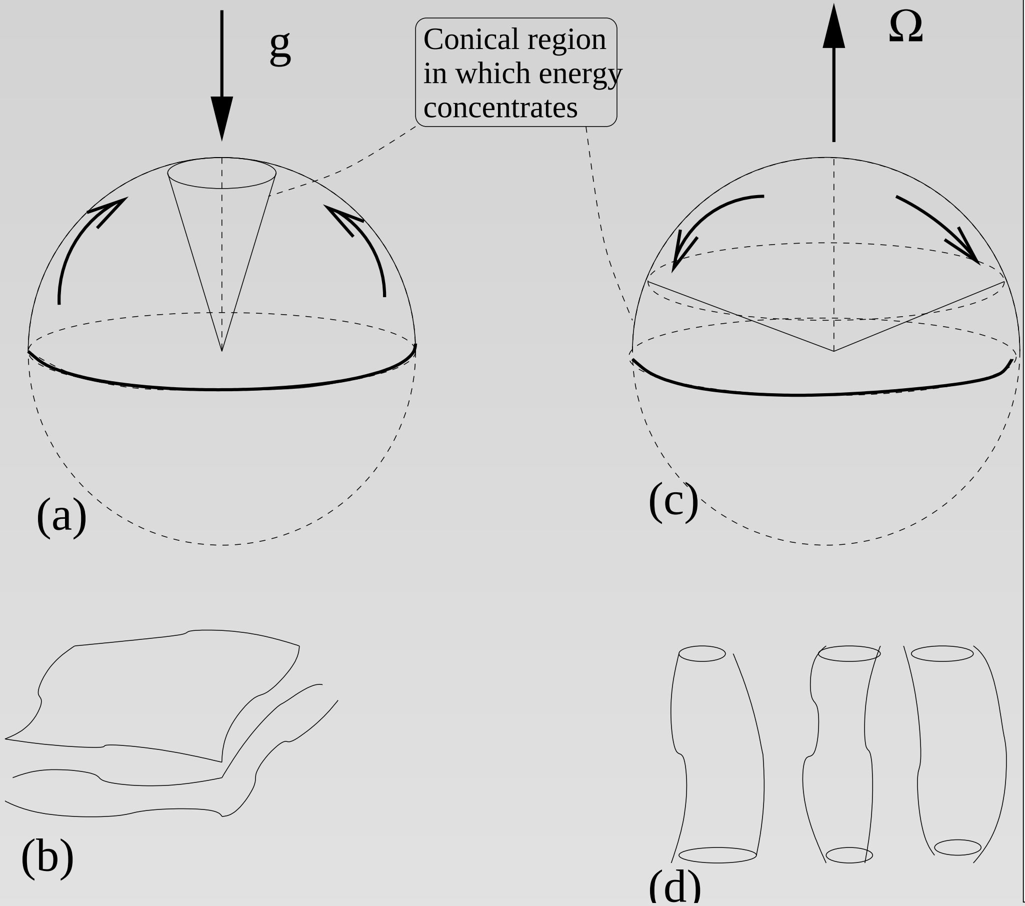
Mc Ewan (1967), Mowbray & Rarity (1967), Godeferd & Lollini, JFM (1999)

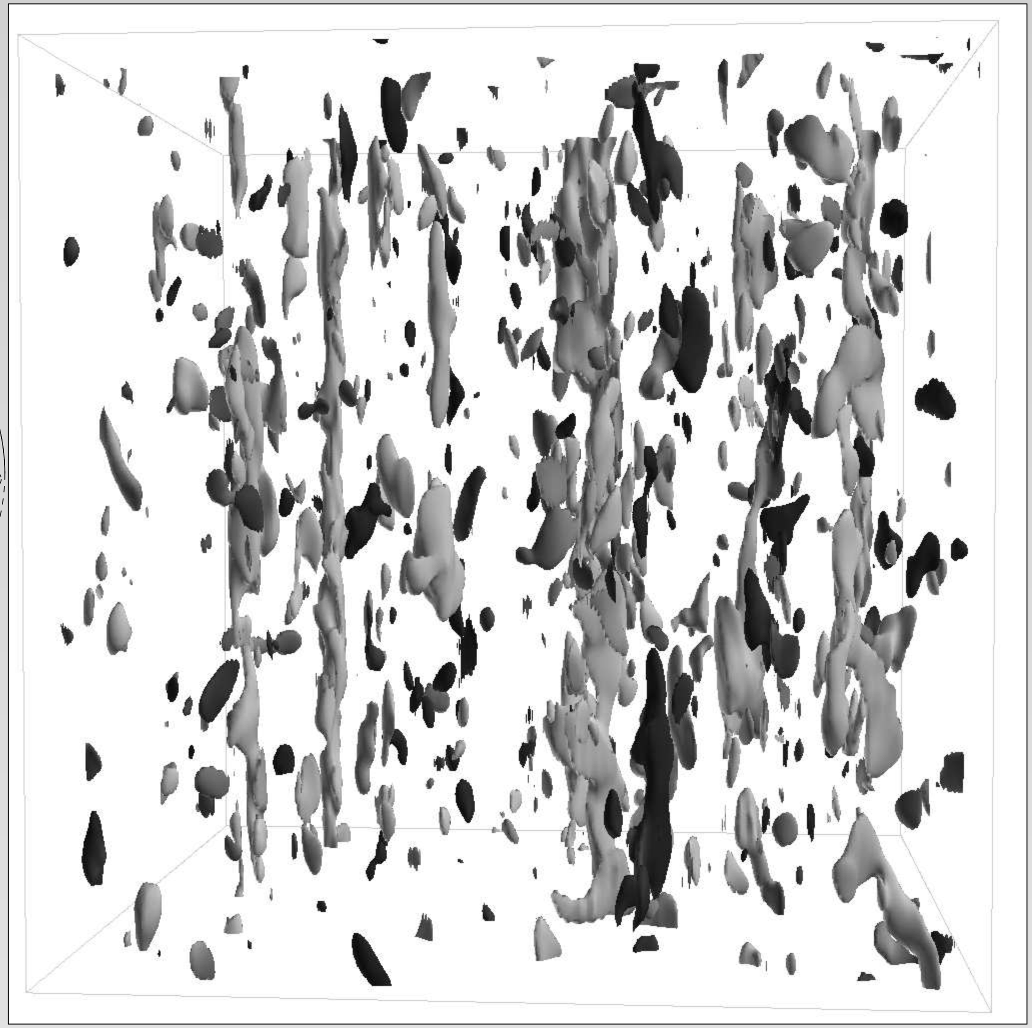
Stratification stable : ANTI 2D et cascade toroidale

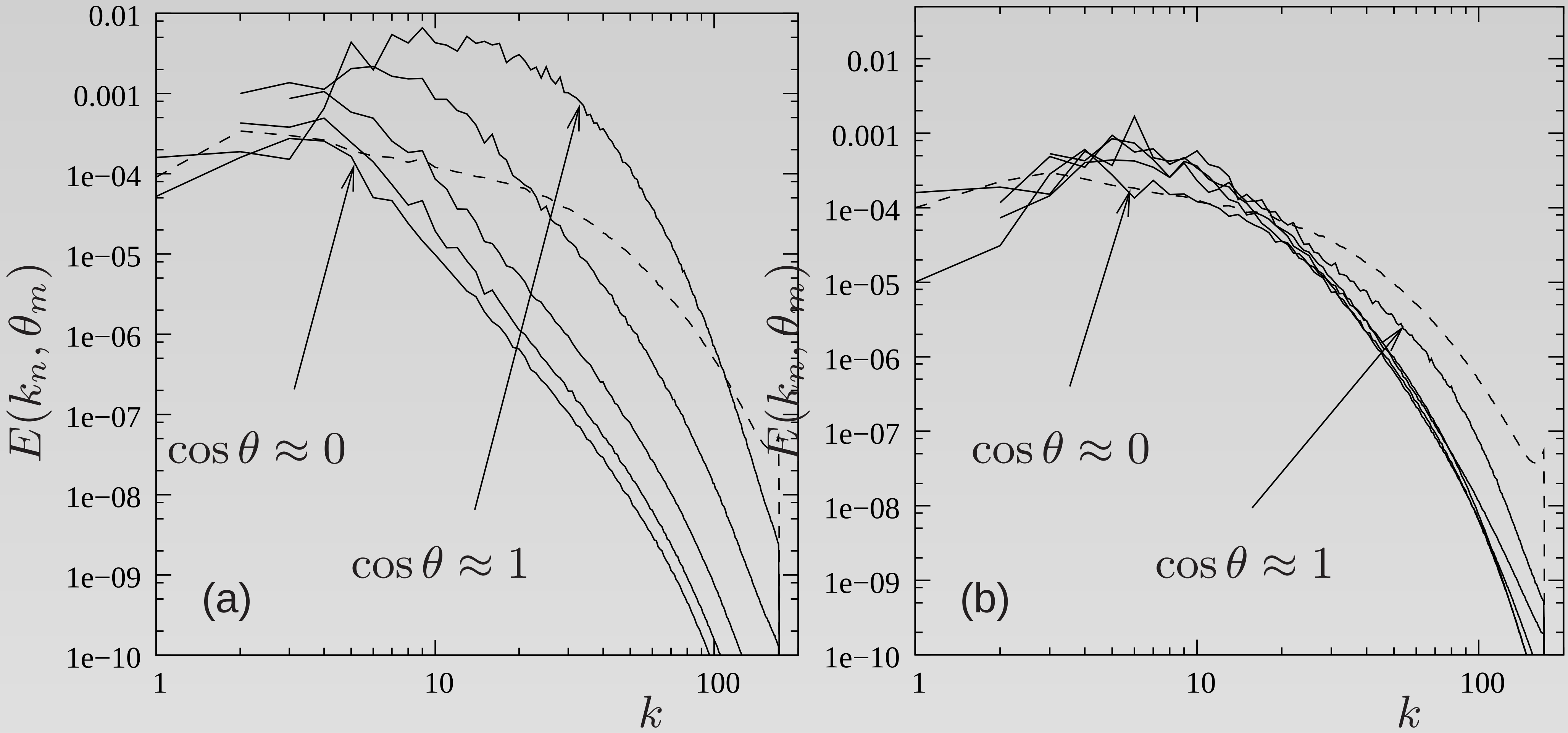
Mode non-propagatif: toroidal et QG



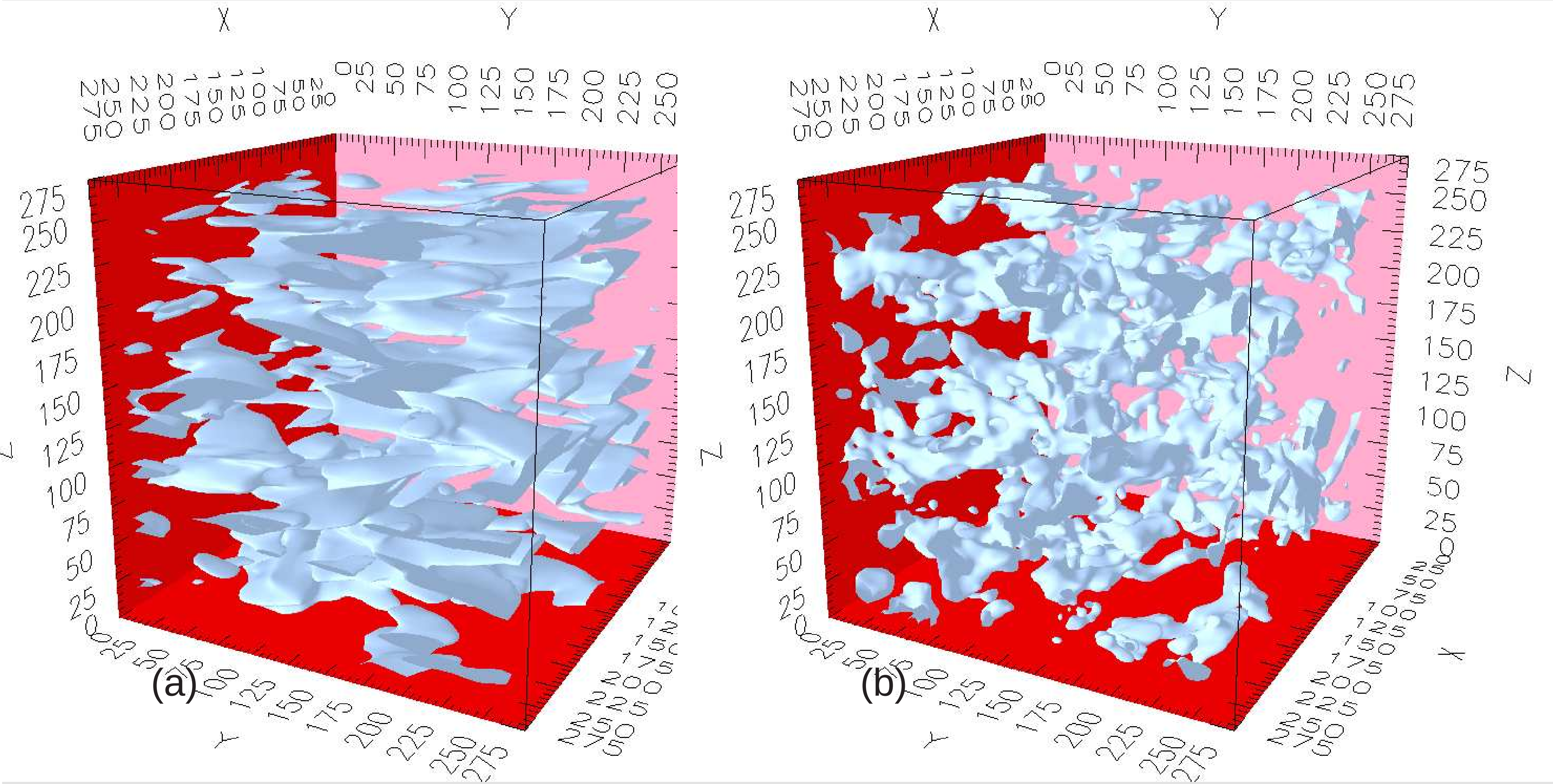
Toroidal/poloidal (Craya-Herring, standard) and “Vortex/wave” ($f/N-$ depending)


g
Conical region
in which energy
concentrates
Ω
(a)
(c)
(b)
(d)





Angle-dependent toroidal and poloidal modes (Liechtenstein, 2006)


X
Y
Z
0
25
50
75
100
125
150
175
200
225
250
275
(a)
(b)

Pure stratification: Generalized Lin equations

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) e^{(tor)} = T^{(tor)} \tag{4}$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) e^{(w)} = T^{(w)} \tag{5}$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2 + 2\imath N \frac{k_\perp}{k}\right) Z' = T^{(z')} \tag{6}$$

Energy spectra $e^{(tor),(pol),(pot)}(k_\perp, k_{||})$, imbalance deviator Z', $\Re Z' = e^{(pol)} - e^{(pot)}$, $e^{(w)} = e^{(pol)} + e^{(pot)}$.

A lot of information can be generated, vs. second and third-order structure functions.

The toroidal cascade. Why not 2D ?

- Why the toroidal component only ? $\boldsymbol{e}^{(1)}\cdot\ldots\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\omega}\times\boldsymbol{u} + \boldsymbol{\nabla}\left(p + \frac{u^2}{2}\right) - b\boldsymbol{n}$,
  $\dot{u}^{(1)} + \boldsymbol{e}^{(1)}\cdot\sum_{p+q=k}(\hat{\boldsymbol{\omega}}(\boldsymbol{p})\times\hat{\boldsymbol{u}}(\boldsymbol{q})) = 0$,
  $\hat{\boldsymbol{u}} = u^{(1)}\boldsymbol{e}^{(1)} + u^{(2)}\boldsymbol{e}^{(2)}$, $\hat{\boldsymbol{\omega}} = \imath k\left(u^{(1)}\boldsymbol{e}^{(2)} - u^{(2)}\boldsymbol{e}^{(1)}\right)$
- Following Kraichnan and Waleffe (1992, 1993): stability of a single triad

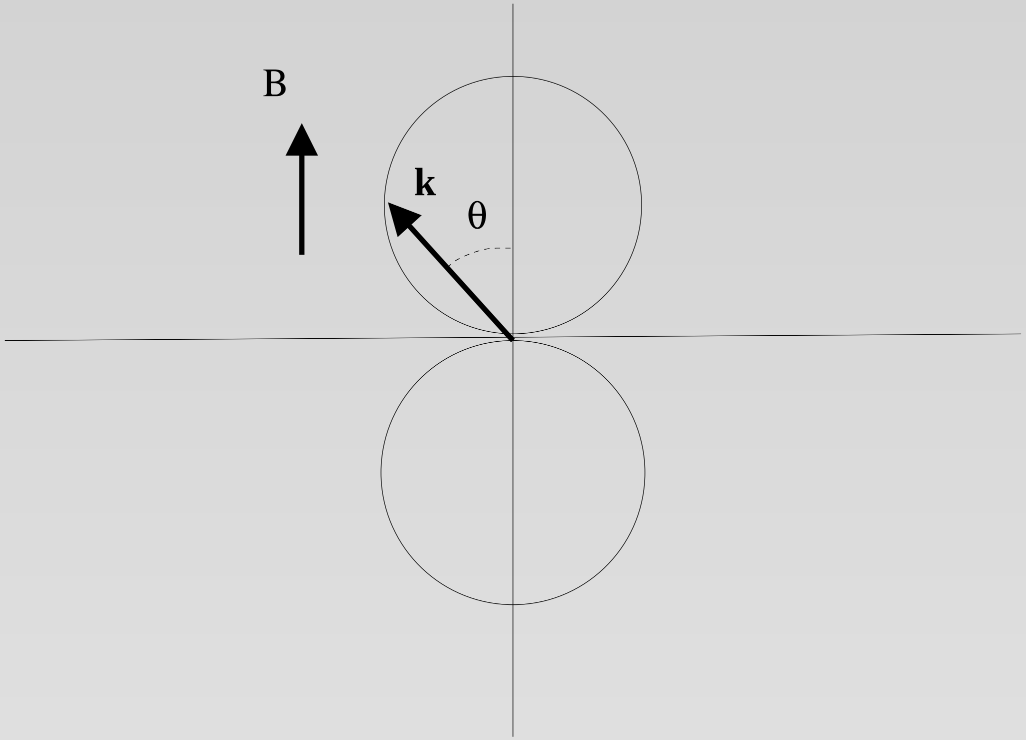
$$\dot{u}_k^{(1)} = (p_\perp^2 - q_\perp^2)Gu_p^{(1)*}u_q^{(1)*}, \qquad (7)$$

$$\dot{u}_p^{(1)} = (q_\perp^2 - k_\perp^2)Gu_q^{(1)*}u_k^{(1)*}, \qquad (8)$$

$$\dot{u}_q^{(1)} = (k_\perp^2 - p_\perp^2)Gu_k^{(1)*}u_p^{(1)*}, \qquad (9)$$

- quasi 2D or not, reverse or direct cascade ? cylinder to cylinder, shell to shell, angle to angle : very rich and various morphology ...

Coexistence of weak and strong turbulence: MHD



Alfven-wave turbulence competing with strong turbulence with additional Joule dissipation effect (Moffat 1967, T. Alboussière, etc)

Remarques finales. Problèmes ouverts

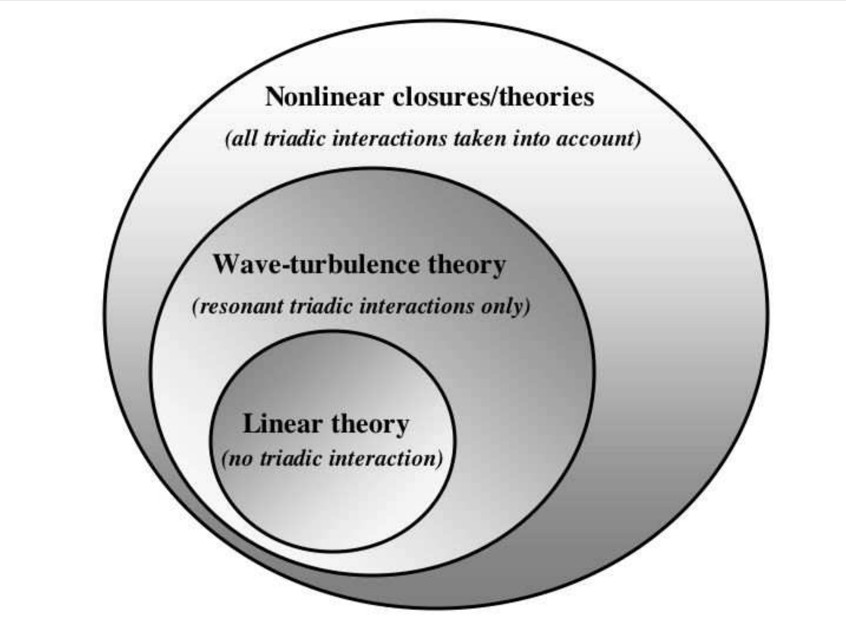
1. Intérêt de l' espace de Fourier 3D, $\boldsymbol{k} \cdot \hat{\boldsymbol{u}} = 0$, modes (propres) *solénoidaux*

2. Cascade mieux décrite (et comprise ?) $\langle (\delta u_{\parallel})^3 \rangle$ contre $T^i(k_{\parallel}, k_{\perp})$?
   -) Equation(s) de Lin *exactes* pour les modes d' énergie pertinents et les termes de déséquilibre
   -) Une façon systématique de construire les corrélations triples en accord avec la *conservation détaillée* (triade)
   -) MAIS problème de renormalisation non-linéaire (ED) en turbulence "forte" ? Renormaliser la fréquence σ ?

3. Turbulence forte et faible face à face ? D' abord comprendre la cascade induite par le mode non-propagatif. Cas MHD ? (pas une simple fréquence de coupure visqueuse)

- Introduire un term kV_0 dans une version isotropisée pour tenir compte d' ondes (non-dispersives) ?? MHD, ANISOTROPIE, PLUS et MOINS ...
- Intérêt de modèles purement linéaires de mélange de phase: effets (croissance !)transitoires, corrélations en DEUX temps et statistique lagrangienne (compétition ondes-tourbillons, stratifié-tournant, plasmas)
- Formalisme hamiltonien, en linéaire ($H = \boldsymbol{k} \cdot \boldsymbol{U} + \sigma(\boldsymbol{k})$) et non-linéaire ? (IMPASSE !)
- Pas d' interactions résonantes pour les triades ? (ex. $\sigma = N, f/N = 1$) $\rightarrow$ quartets (IMPASSE !)

Towards a theory of axisymmetric weak+strong HAT turbulence

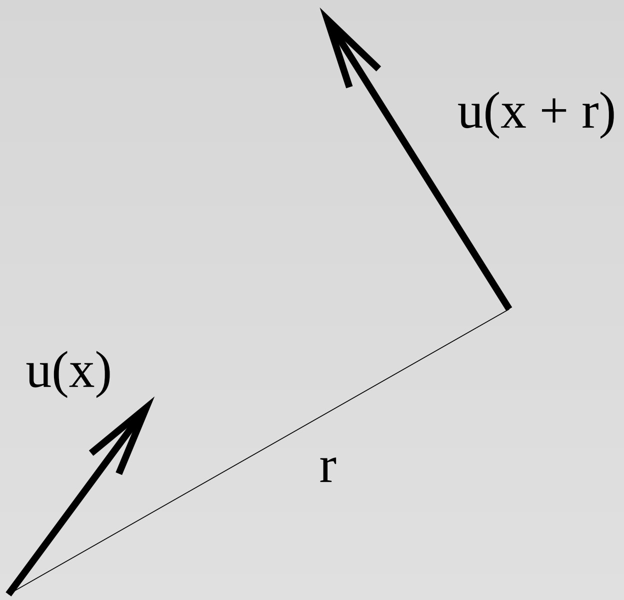


- Work remain to be done for toroidal, QG, + main waves interactions (catalytic ?)
- Toro-polo with Elsasser variables, MHD, plasmas, dumbell ? ALFIN. Weakly compressible turbulence.
- Not to forget DNS/LES with dedicated post-processing (Marenostrum project, etc) et effets de frontières. Eviter la *schizophénie* ! ANISO

Strong anisotropy

Anisotropic description

- ANISOTROPY/ inhomogeneity/ Intermitency
- structure functions or correlations, two-point : $R_{ij}(\boldsymbol{r}) = \langle u_i(\boldsymbol{x}) u_j(\boldsymbol{x}+\boldsymbol{r})\rangle$



-) Single-point: componentality only

-) Two-point: directional anisotropy

- Low dimension parameterization, SO(3) symmetry group (Arad *et al.*,PRE,1999)

Anisotropic description. 3D Fourier space

- Anisotropic scalar (e. g. spherical harmonics) for both ‘physical’ and ‘spectral’

$$\frac{1}{2}R_{ii}(\boldsymbol{r}) \quad \rightarrow \quad \frac{1}{2}\hat{R}_{ii}(\boldsymbol{k}) = e(\boldsymbol{k})$$

$$\sum r_n^m(r) Y_n^m(\theta_r, \phi_r) \quad \rightarrow \quad \sum \varphi_n^m(k) Y_n^m(\theta_k, \phi_k)$$

Avoiding a ‘schizophrenic’ viewpoint ! (Cambon & Teissèdre 1985, CRAS Paris)

- A trace-deviator decomposition restricted to solenoidal space

$$\hat{R}_{ij} = \underbrace{\underbrace{U(k)P_{ij}}_{\text{isotropic}} + \underbrace{\mathcal{E}(\boldsymbol{k})P_{ij}}_{\text{directional}}}_{eP} + \underbrace{\Re\left(Z(\boldsymbol{k})N_iN_j\right)}_{\text{polarization}}.$$

(Cambon & Jacquin, JFM, 1989), $P_{ij} = \delta_{ij} - \frac{k_i k_i}{k^2}$, $\boldsymbol{N}$ ‘helical mode’. Helicity ?

Rotating turbulence, MHD simplified case, $\boldsymbol{k} \perp \hat{\boldsymbol{u}}$