Intermittent particle distribution in two-dimensional synthetic compressible turbulence

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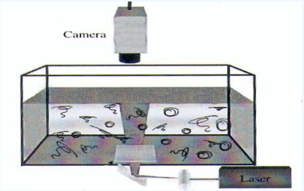
Problem

We consider **passive tracers** in a **2D compressible turbulent** flow

We just integrate : $\frac{\mathrm{d}\mathbf{x}_i(t)}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}_i, t)$
$\mathbf{x}_i(t)$: Position of the ith particle.
$\mathbf{v}(\mathbf{x}, t)$: Turbulent synthetic flow.

Physically : Light particles moving on the surface of a 3D turbulent flow.



$$(\nabla.\mathbf{v})_S = \partial_x v_x + \partial_y v_y = -\partial_z v_z$$

J.R. Cressman, J. Davoudi and al. *New Journ. of Ph.*, **6** (2004).

The compressiblity rate :

$$\mathcal{C} = \frac{\langle (\nabla.\mathbf{v})^2 \rangle}{\langle (\partial_x v_x)^2 \rangle + \langle (\partial_x v_y)^2 \rangle + \langle (\partial_y v_x)^2 \rangle + \langle (\partial_y v_y)^2 \rangle}$$

$\mathcal{C} = 0 \Rightarrow$ incompressible flow
$\mathcal{C} = 1 \Rightarrow$ potential flow (irrotational)

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Interested in the particle distribution $P(n_r)$:

n_r : coarse grained density over scale r.

$\mathcal{C} = 0 \rightarrow$ The particle distribution $P(n_r)$ is Poissonian at all scales.

When $\mathcal{C} \nearrow$, deviation from uniform Poisson distribution $\nearrow$.
We study $P(n_r)$, and its momentum with respect to $\mathcal{C}$.

Numerical method for the flow

Synthetic compressible turbulent flow : $\mathbf{v} = \mathbf{v}_I + \mathbf{v}_C$

$$\left\{ \begin{array}{lcl} \mathbf{v}_I & = & \sum_{n=1}^{m} \mathbf{I}_n \sin(\mathbf{k}_n.\mathbf{x} + \omega_n t + \varphi_n) \\ \mathbf{v}_C & = & \sum_{n=1}^{m} \mathbf{C}_n \sin(\mathbf{k}_n.\mathbf{x} + \omega_n t + \psi_n) \end{array} \right.$$

With, $\mathbf{I}_n = I_n \, (1-\mathcal{C})^{\frac{1}{2}} \, \hat{\mathbf{k}}_{n\perp}$ and $\mathbf{C}_n = C_n \, \mathcal{C}^{\frac{1}{2}} \, \hat{\mathbf{k}}_n$

And,

$$\left\{ \begin{array}{lcr} \hat{\mathbf{k}}_n & = & \cos(\phi_n)\hat{\mathbf{x}} + \sin(\phi_n)\hat{\mathbf{y}} \\ \hat{\mathbf{k}}_{n\perp} & = & -\sin(\phi_n)\hat{\mathbf{x}} + \cos(\phi_n)\hat{\mathbf{y}} \end{array} \right.$$

ϕ_n, φ_n and ψ_n $\Rightarrow$ random variables chosen in $[0, 2\pi]$.

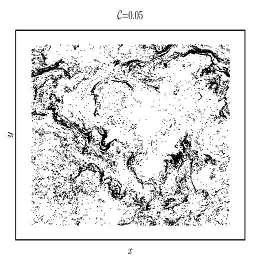
$I_n^2, C_n^2 \sim k_n^{-5/3}$ $\Rightarrow$ Kolmogorov.

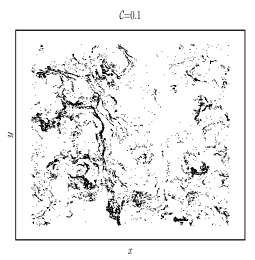
$\omega_n = \lambda\sqrt{k_n^3 E(k_n)}$, $\Rightarrow$ we took $\lambda = 0.5$.

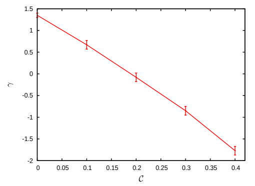
$\eta = 2\pi/k_m$: dissipative scale, $L = 2\pi/k_1$: integral scale

$Re = (L/\eta)^{4/3}$

Example of particle distribution







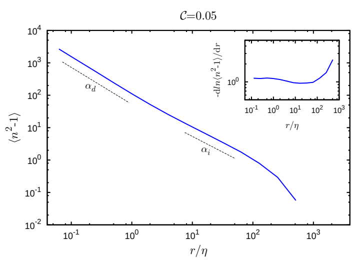
$\Rightarrow \gamma$: largest Lyapunov exponent

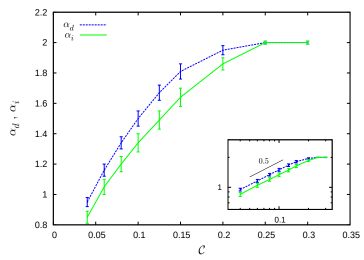
$\Rightarrow \mathcal{C}_c \simeq 0.2$

If $\mathcal{C} > \mathcal{C}_c$, γ becomes negative $\Rightarrow$ particles accumulate on points.

Measurements

We measured the variance of the PDF $P(n_r)$ at different scales in the inertial and the dissipative range.



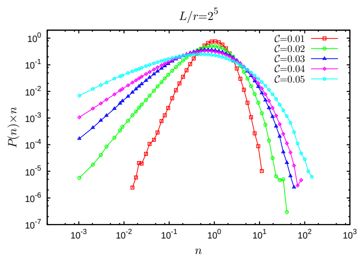


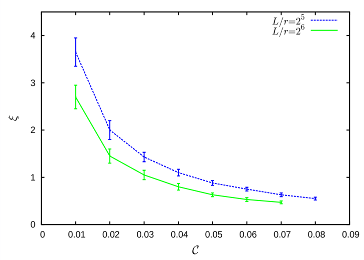
We observe : $\langle n_r^2 - 1 \rangle \sim r^{-\alpha}$, with two different exponents α_d (dissipative ra.) and α_i (inertial ra.) ($\alpha_i < \alpha_d$). $0 < \alpha < 2$.

$\Rightarrow$ The inhomogeneities grow faster in the dissipative range than in the inertial range.

No significant dependance of $\alpha_{d,i}$ with L/η has been found.

Shape of the PDF $P(n_r)$ in the inertial range :





Algebraic tails are observed at small concentrations :

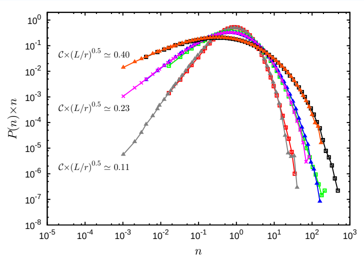
- $P(n_r) \times n_r \sim n_r^{\xi}$ for $n_r \ll 1$
- $\xi \searrow$ with $\mathcal{C}$ and $1/r$
  $\Rightarrow$ Probability of almost empty regions $\nearrow$.

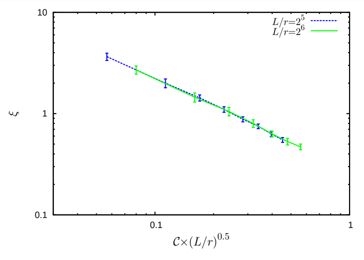
$\Rightarrow$ Behavior observed for inertial particles.

Bec and al., *PRL* **98**, 084502 (2007).

Superposition of the PDF in the inertial range.

We suppose : $P(n_r) = f(\mathcal{C}^\beta \times (L/r)^\kappa)$.





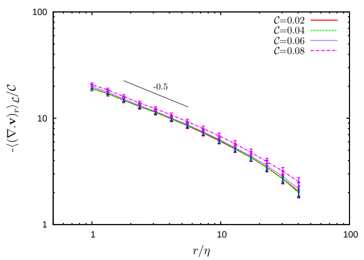
We found : $\beta \simeq 1$ and $\kappa \simeq 0.5$.

ξ seems to behave as : $(\mathcal{C} \times (L/r)^{0.5})^{0.9}$

$P(n_r)$ should depends only on the contraction rate $\rightarrow \langle(\nabla.\mathbf{v})_r\rangle_\mathcal{L}$.

$$\langle(\nabla.\mathbf{v})_r(\mathbf{x_0})\rangle_\mathcal{L} = \langle \int \mathrm{d}^2\mathbf{x}\, \mathcal{G}_r(\mathbf{x},\mathbf{x_0})\ (\nabla.\mathbf{v})_r(\mathbf{x})\rangle_\mathcal{L}$$

Where $\mathcal{G}_r(\mathbf{x},\mathbf{x_0}) = 1/(2\pi r^2)\ \exp(-(\mathbf{x}-\mathbf{x_0})/2r^2)$: Low pass Gaussian filter.



$\langle(\nabla.\mathbf{v})_r\rangle_\mathcal{L} \sim \mathcal{C} \times (L/r)^{0.5}$ in a part of the inertial range.

$\Rightarrow P(n_r) = f(\langle(\nabla.\mathbf{v})_r\rangle_\mathcal{L})$

Conclusion

Properties of $P(n_r) \Rightarrow$ similarites with the inertial particles problem.

- Scaling behavior of $\langle n_r^2 - 1 \rangle$:
  - $\langle n_r^2 - 1 \rangle \sim r^{-\alpha_{d,i}}$ with $\alpha_i < \alpha_d$.
  - Inertial particles $\Rightarrow$ behavior only observed in the dissipative range.
- $P(n_r)$ in the inertial range :
  - Algebraic tails at small concentrations.
    Probability of finding empty regions increase with $\mathcal{C}$ and $1/r$.
  - $P(n_r)$ depends only on $\langle (\nabla.\mathbf{v})_r \rangle_{\mathcal{L}}$
- No significant effect of L/η.