Galerkin truncation, hyperviscosity and bottlenecks

Uriel Frisch
Observatoire de la Côte d'Azur, Nice

with W. Pauls (Goettingen), A. Wirth (Grenoble), **S. Kurien and J.-Z. Zhu (Los Alamos),**
R. Pandit and S.S. Ray (Bangalore)

Hyperviscous equations

Burgers $\partial_t v + v\partial_x v = -\mu {k_{\mathrm{G}}}^{-2\alpha}(-\partial_x^2)^\alpha v$

$\mu > 0, \quad k_{\mathrm{G}} > 0, \;\; \alpha = \text{dissipativity. Here} \quad \alpha > 1.$

N-S $\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = -\nabla p - \mu {k_{\mathrm{G}}}^{-2\alpha}(-\nabla^2)^\alpha \boldsymbol{v}, \quad \nabla\cdot\boldsymbol{v} = 0$

Dissipation rate $\mu(k/k_{\mathrm{G}})^{2\alpha} \to 0 \text{ or } \infty \text{ when } \alpha \to \infty$

Abstract form $\partial_t v = B(v,v) + L_\alpha v$

Galerkin truncation $\partial_t u = P_{k_{\mathrm{G}}} B(u,u), \qquad u_o = P_{k_{\mathrm{G}}} v_0$

Projector $P_{k_{\mathrm{G}}}$: low-pass filter at wavenumber k_{G}

Large dissipativity limit and thermalization

For $\alpha \to \infty$, and fixed μ and k_G, the solution of the hyperdissipative equations tend to the solution of the Galerkin-truncated equations

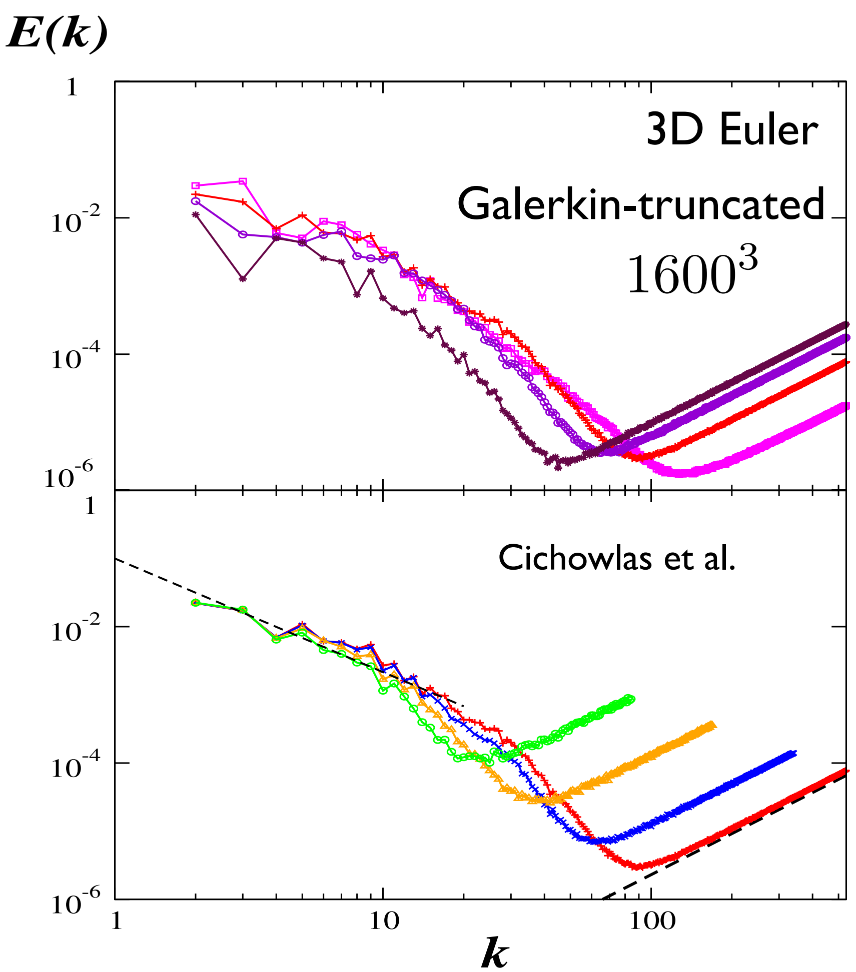
[8] We may regard this as the introduction of infinite damping (infinite resistance) for the degrees of freedom removed.

True for: Burgers, Navier-Stokes, MHD, DIA and EDQNM.
False for: MRCM and resonant wave interaction theory.

Galerkin-truncation ⇒ thermalization (Lee, 1952; Hopf, 1952; Kraichnan, 1958)

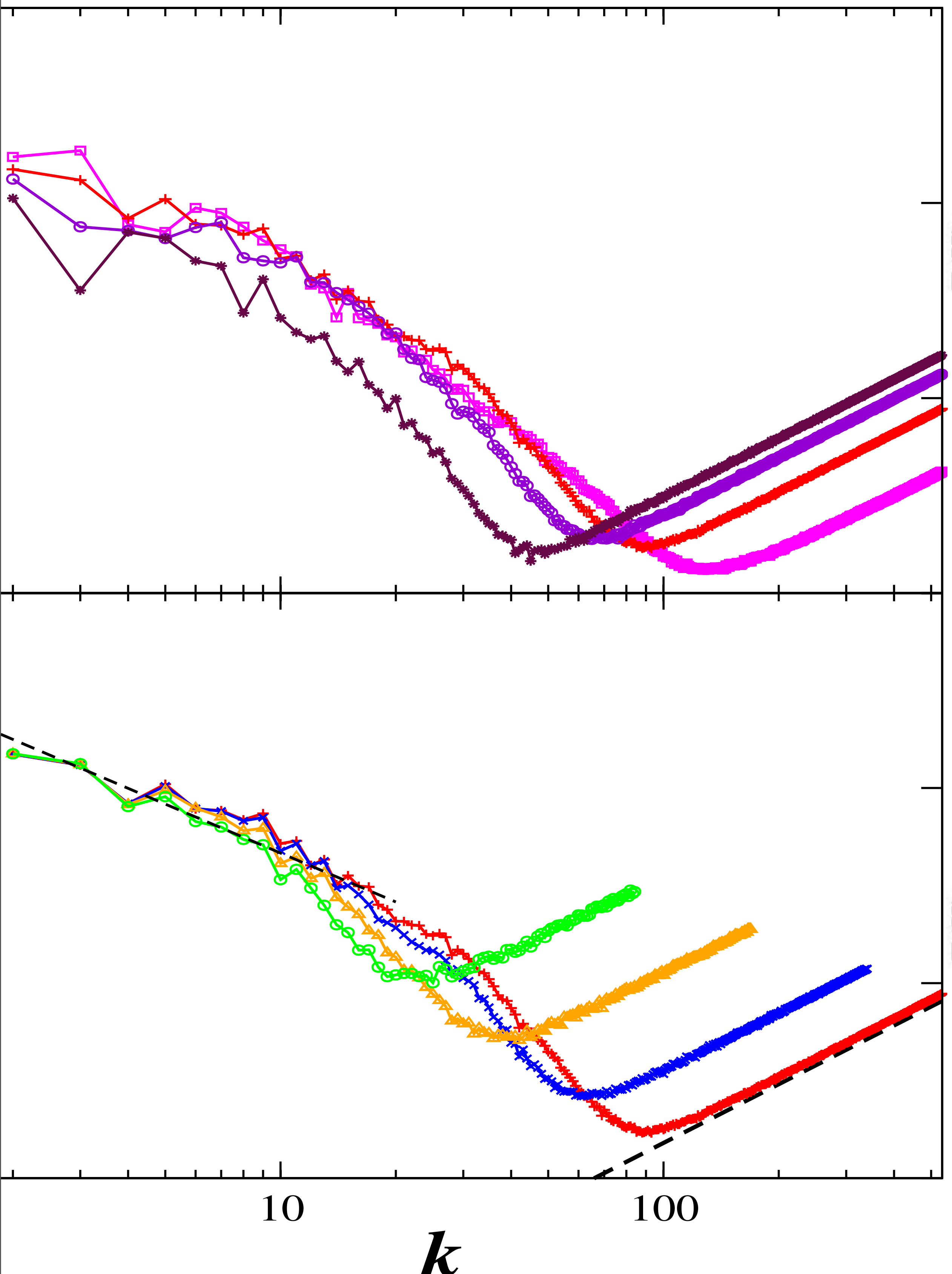
Galerkin-truncated Burgers first studied by Majda and Timofeyev 2000

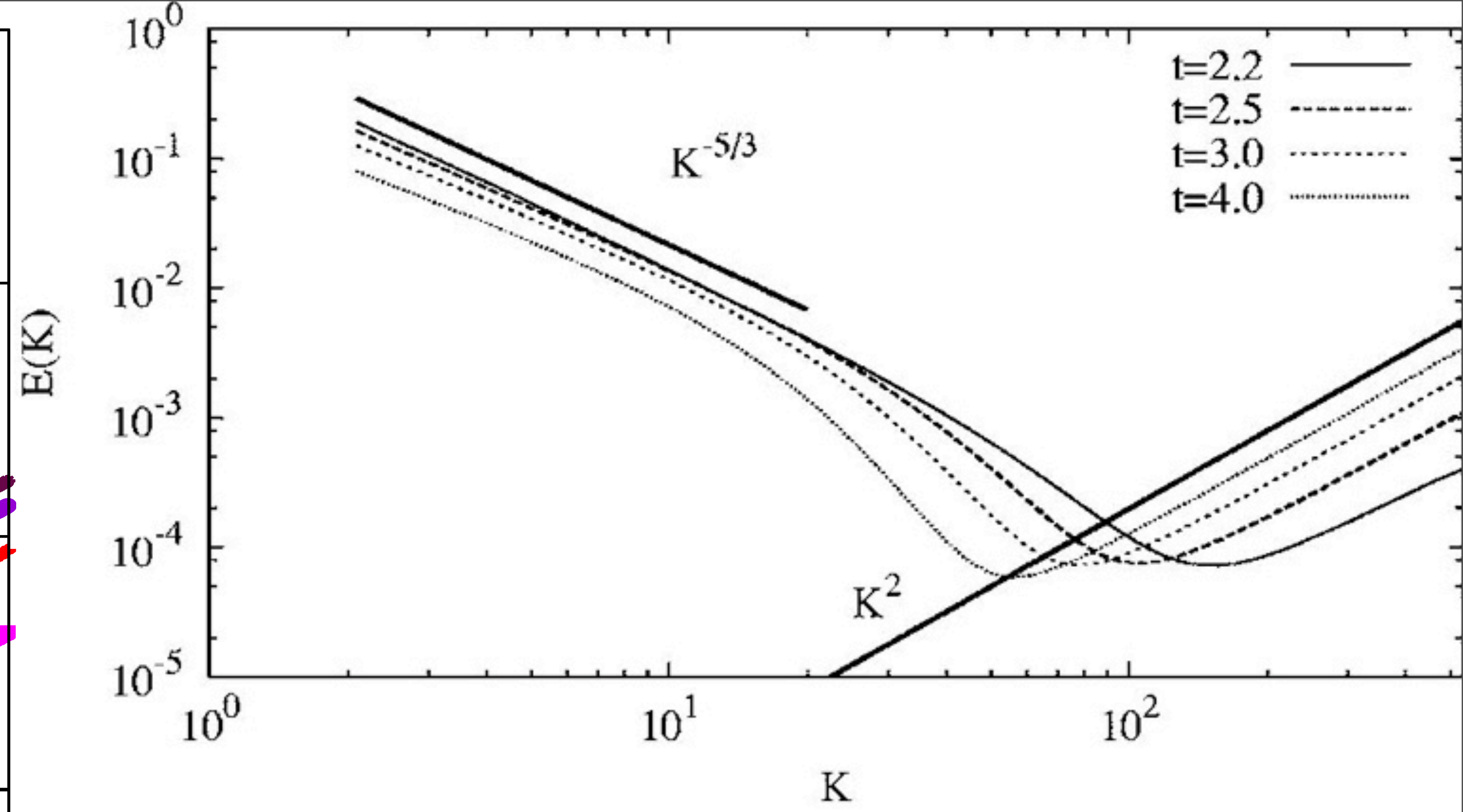
Galerkin-truncated 3D incompressible Euler first studied at high resolution by Cichowlas, Bonaiti, Debbasch and Brachet 2005

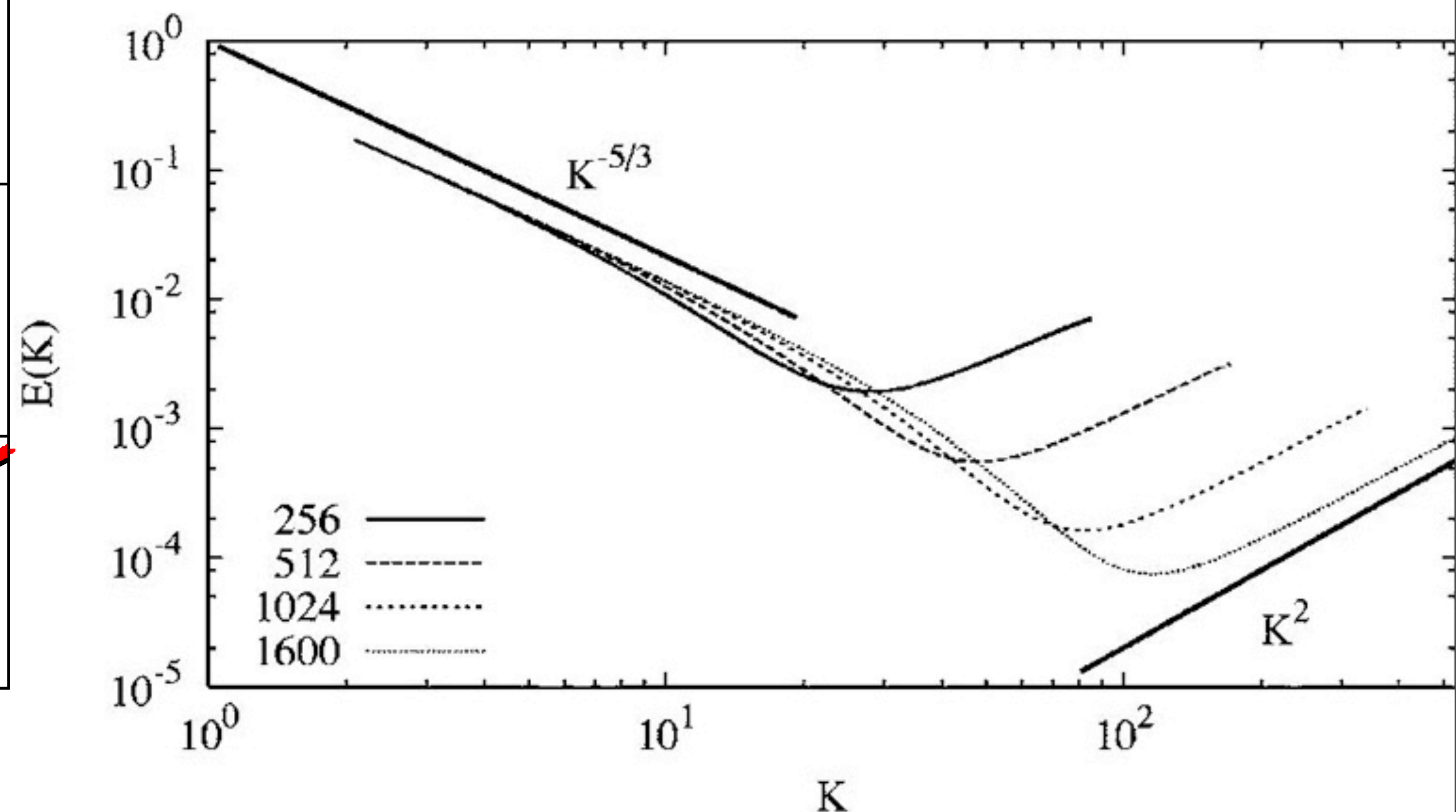


Same resolution; different times

Same time; different resolutions







Cichowlas et al. (2005) “reproduced” by Bos and Bertoglio(2006) with **EDQNM**

Eddy-Damped Quasi-Normal Markovian spectrum

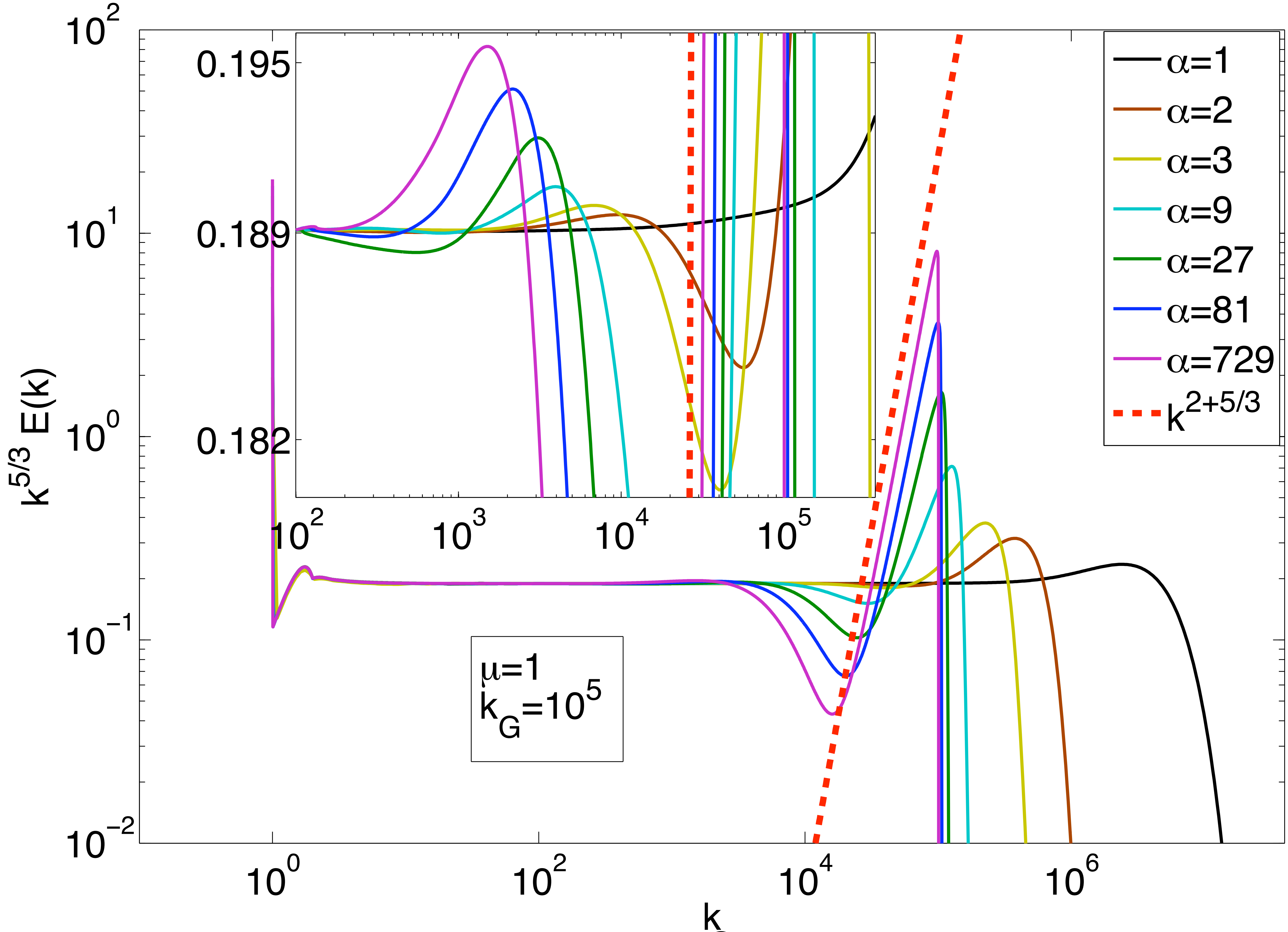
$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) =$$
$$\iint_{\triangle_k} dpdq\theta_{kpq} b(k,p,q) \frac{k}{pq} E(q,t) \left[k^2 E(p,t) - p^2 E(k,t)\right]$$

"QN" --- Chou(1940), Millionshtchikov(1941): realizability problem

"N" --- Lee (1952), Hopf(1952): statistics of absolute equilibria of truncated Euler

DIA (Kraichnan): tractability problem

"ED", "M" --- Orszag(1970, 1977)



Hyperviscous EDQNM:

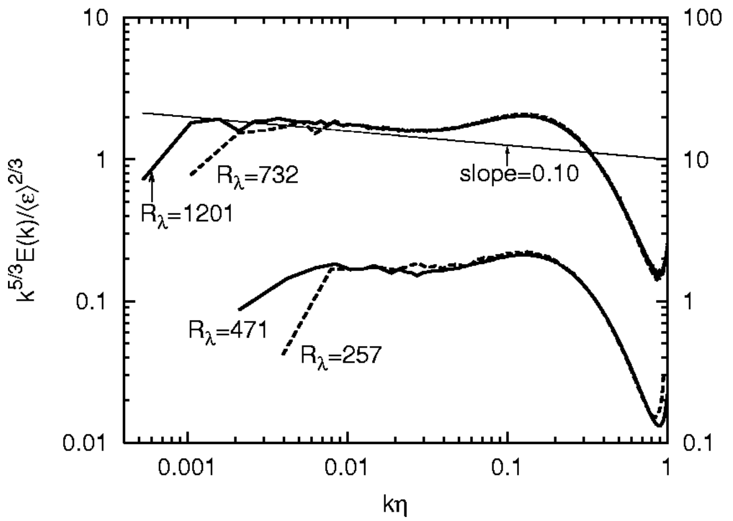
convergence to Galerkin truncation and secondary bottleneck ...

Bottleneck, thermalization, depletion of intermittency, etc

- Large α produces a huge thermalized bottleneck

- The standard $\alpha = 1$ bottleneck may be viewed as an *aborted thermalization*

Kaneda et al. 2003 (Earth Simulator). Compensated energy spectrum



- Thermalization is accompanied by Gaussianization and isotropization

- Spurious effects are expected: depletion of intermittency and isotropization

Hyperviscosity and Galerkin truncation for the Burgers equation

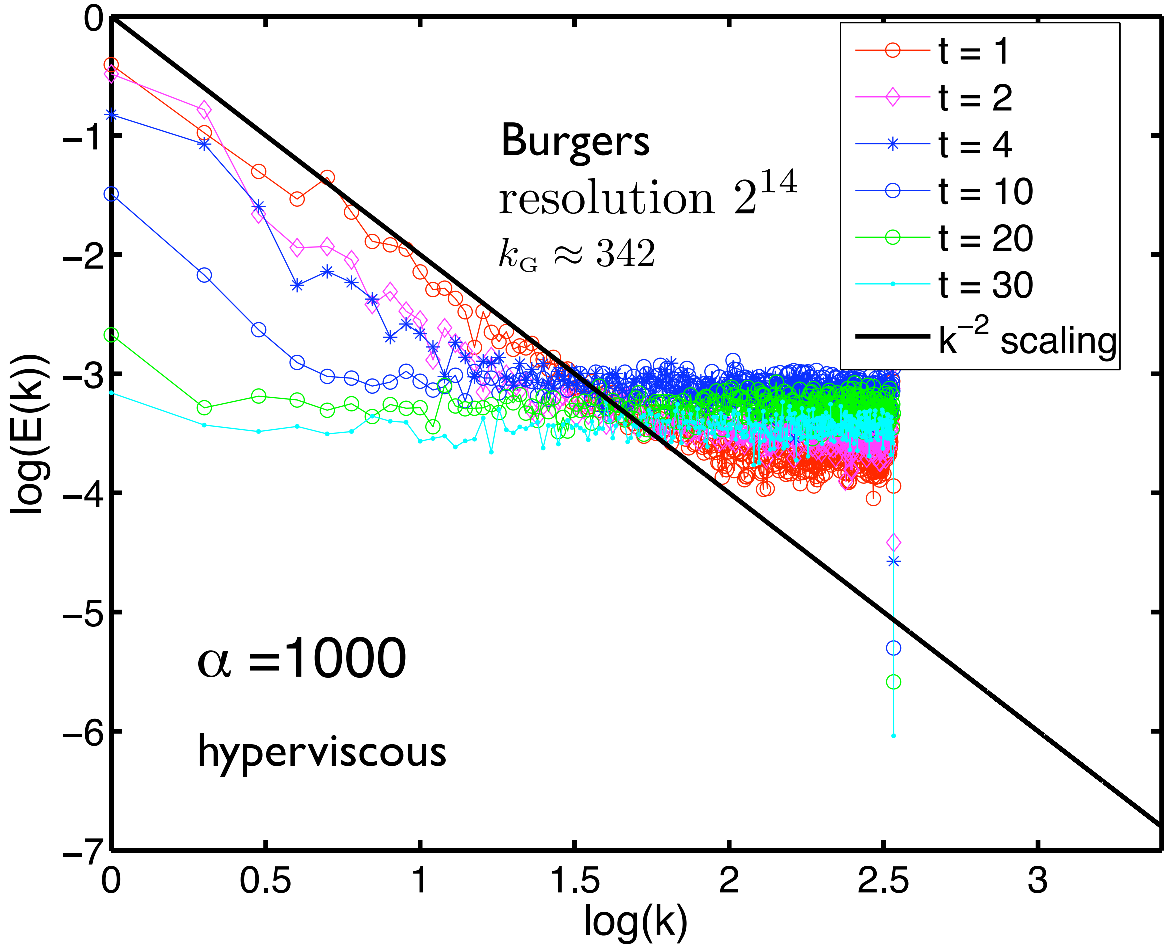
Samriddhi Sankar Ray
Indian Institute of Science, Bangalore

with U. Frisch (Nice) and R. Pandit (Bangalore)

$$\partial_t v + v\partial_x v = -\mu {k_{\mathrm{G}}}^{-2\alpha}(-\partial_x^2)^{\alpha} v$$

$$\mu > 0, \quad k_{\mathrm{G}} > 0, \quad \alpha = \text{dissipativity}$$

Hyperviscous and Galerkin-truncated Burgers



Burgers equation with random initial condition

$$u_0(x) = \sin x + \sin(2x + \phi)$$

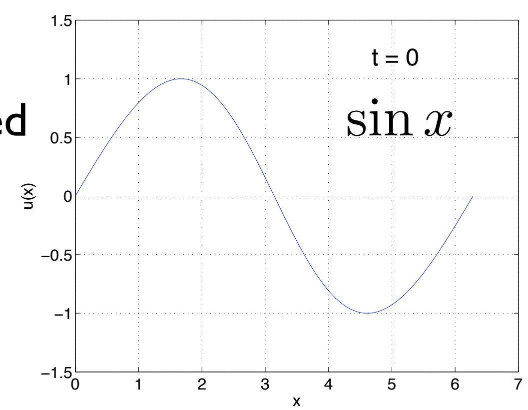
ϕ uniformly distributed in $[-\pi,\ \pi]$

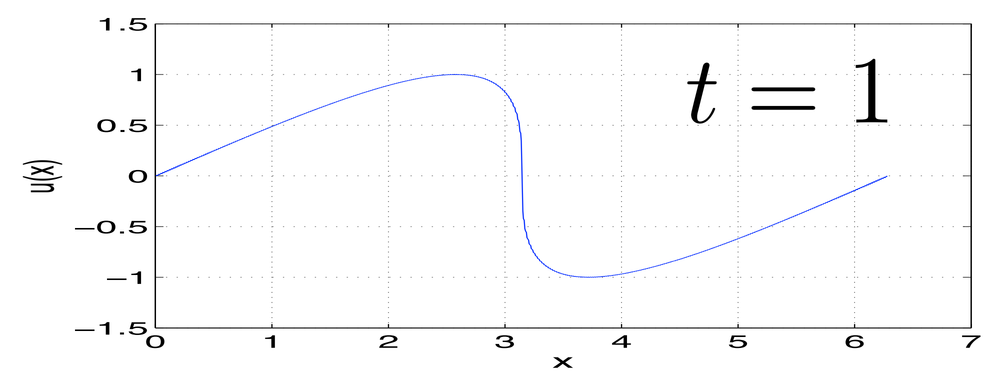
Energy spectrum averaged over 20 realizations

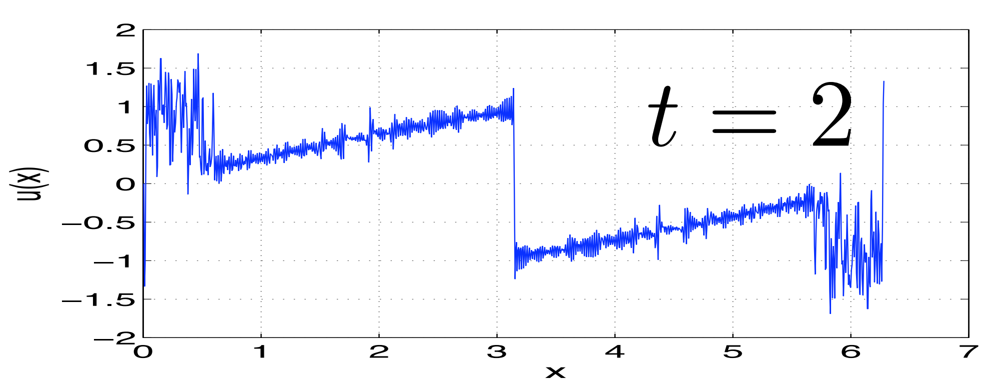
Galerkin-truncated Burgers first studied by Majda and Timofeyev 2000

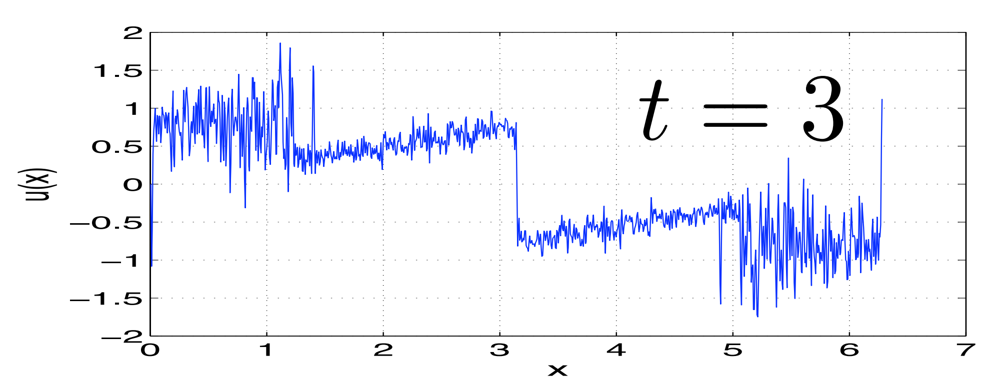
Evolution of Galerkin-truncated

initial condition $\sin x$

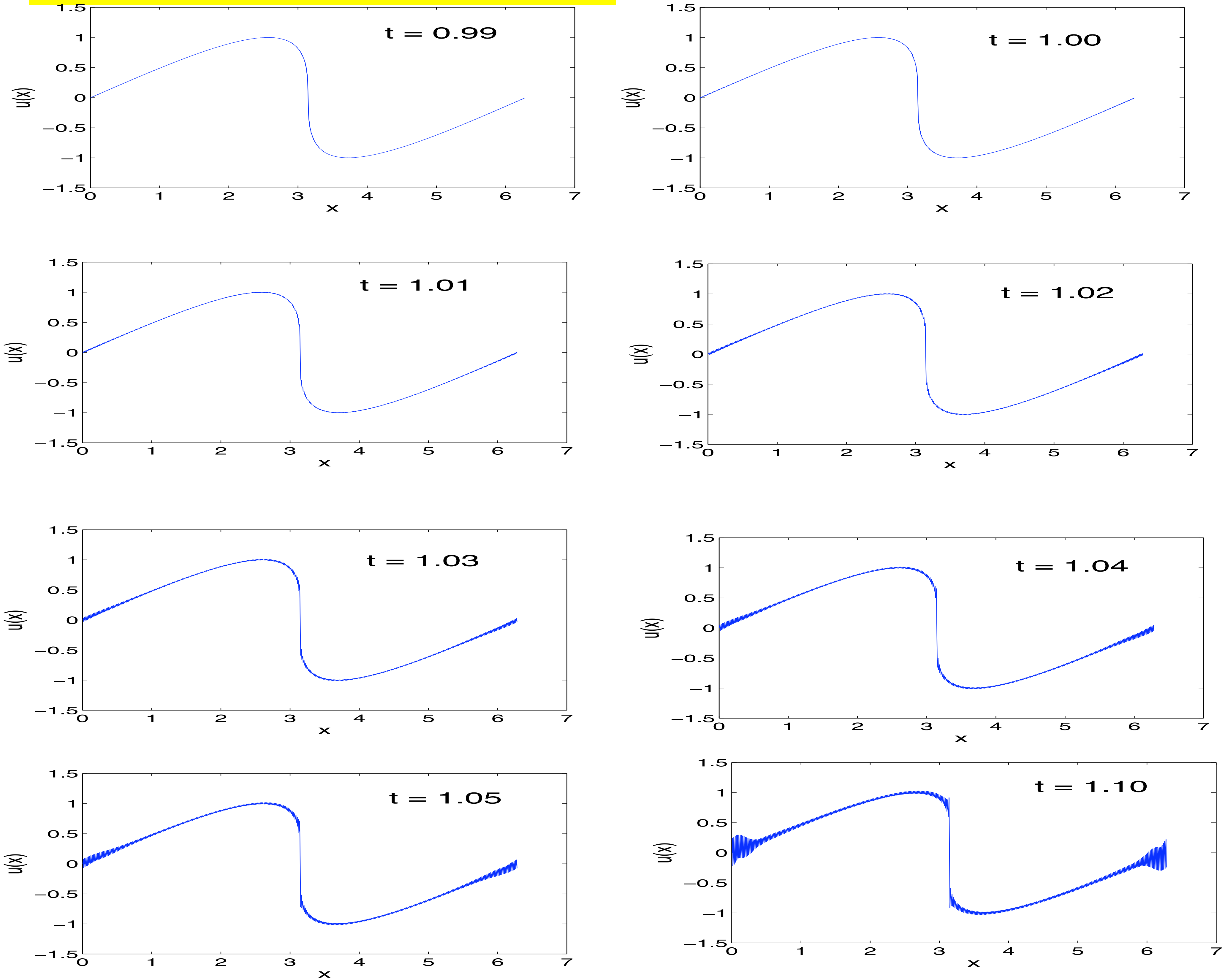








The shock acts as a black hole

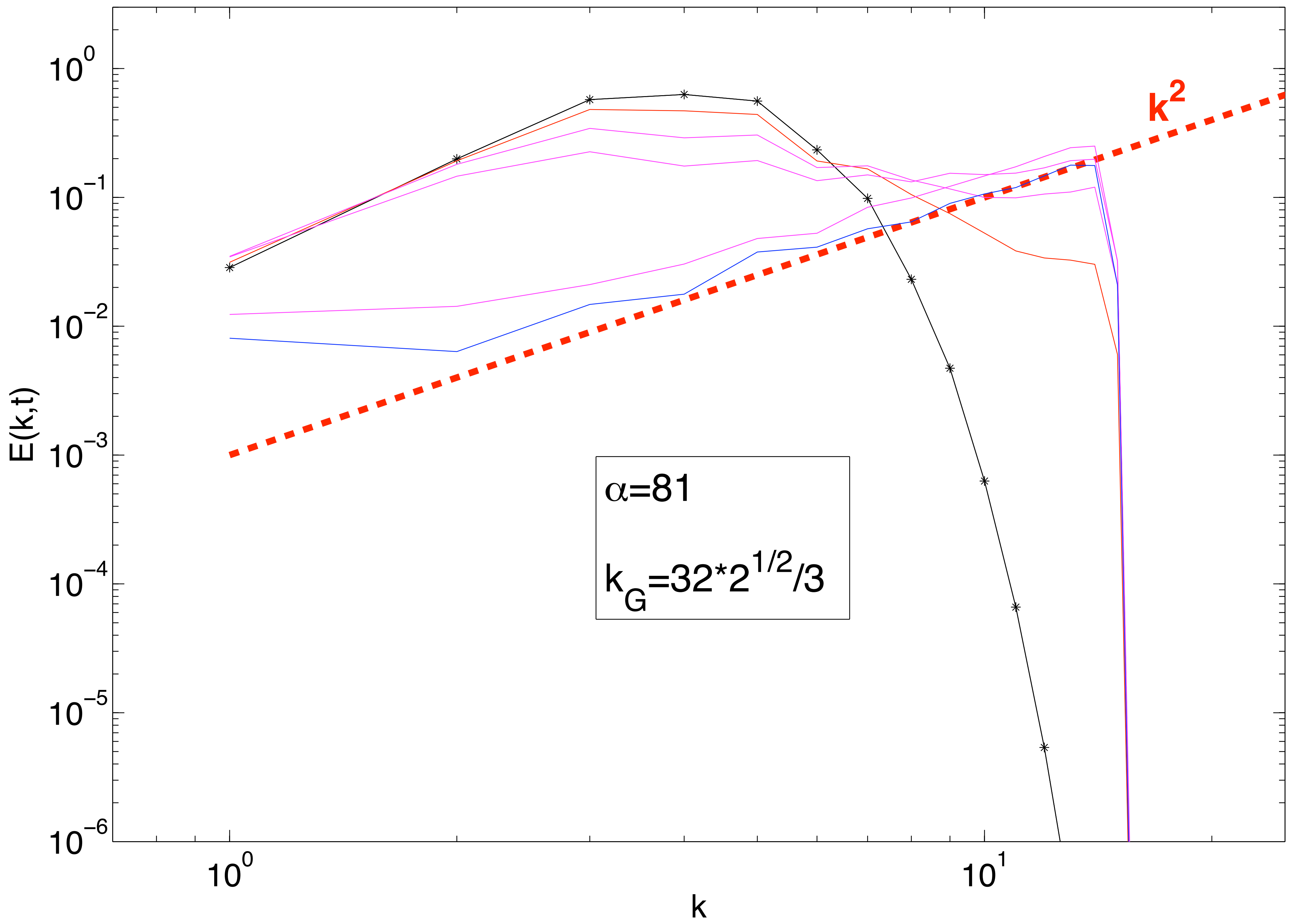


resolution 2^{10}

$k_{\mathrm{G}} = 342$

Are these genuine shocks? Mathematical question:
do the solutions of the inviscid truncated Burgers eq.
converge to the "entropy solution" when $k_{\mathrm{G}} \to \infty$?

Hyperviscous Navier-Stokes



64^3 DNS