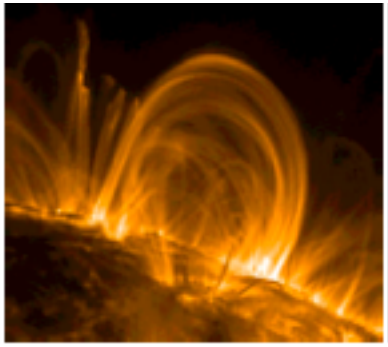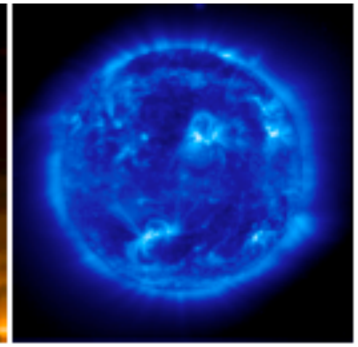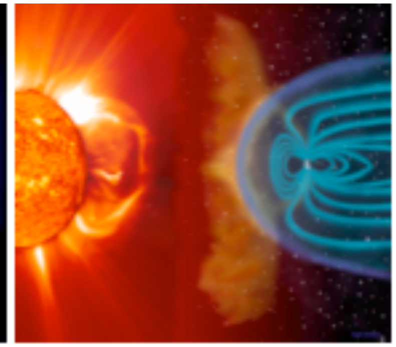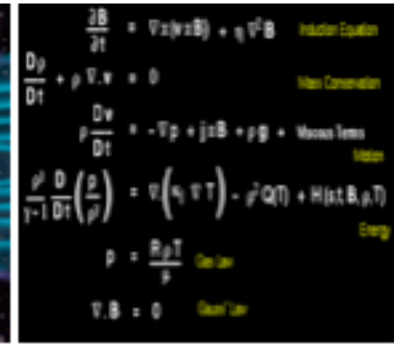

Vent solaire: ondes, turbulence et MHD Hall

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IAS / Université Paris-Sud

GdR Turbulence

Lyon, 1er avril 2008

Le vent solaire

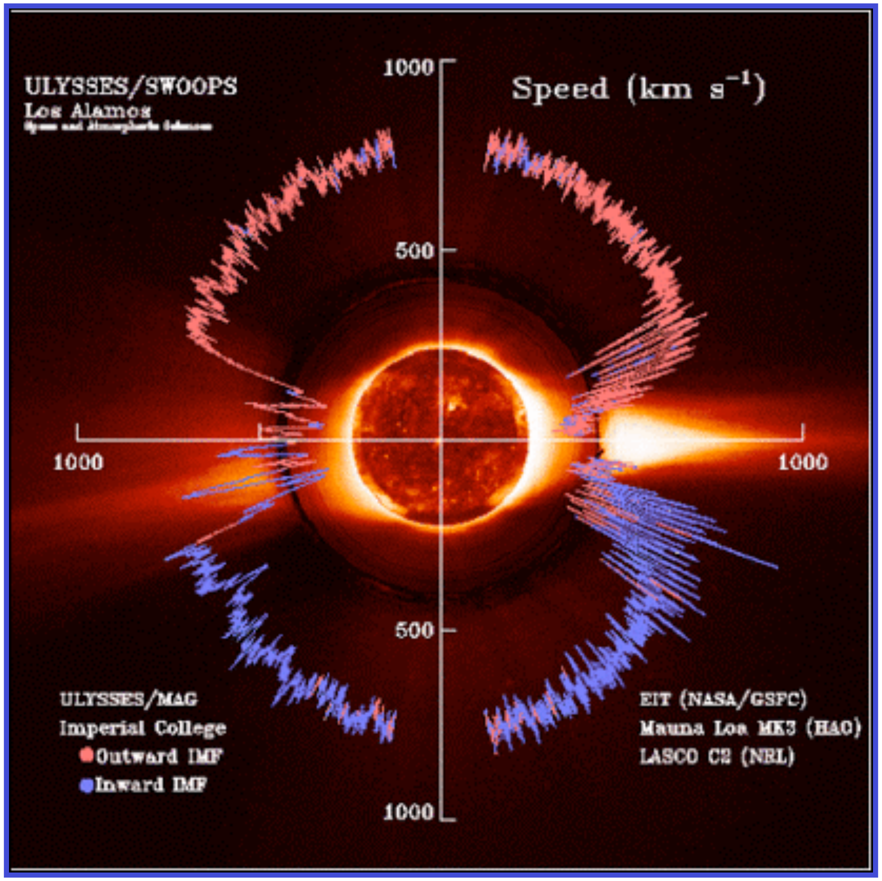
- Vent produit par le Soleil qui inonde toute l'héliosphère (~ 100AU)

- Fluctuations **~ 10^{-7} Hz to 10^{2} Hz**

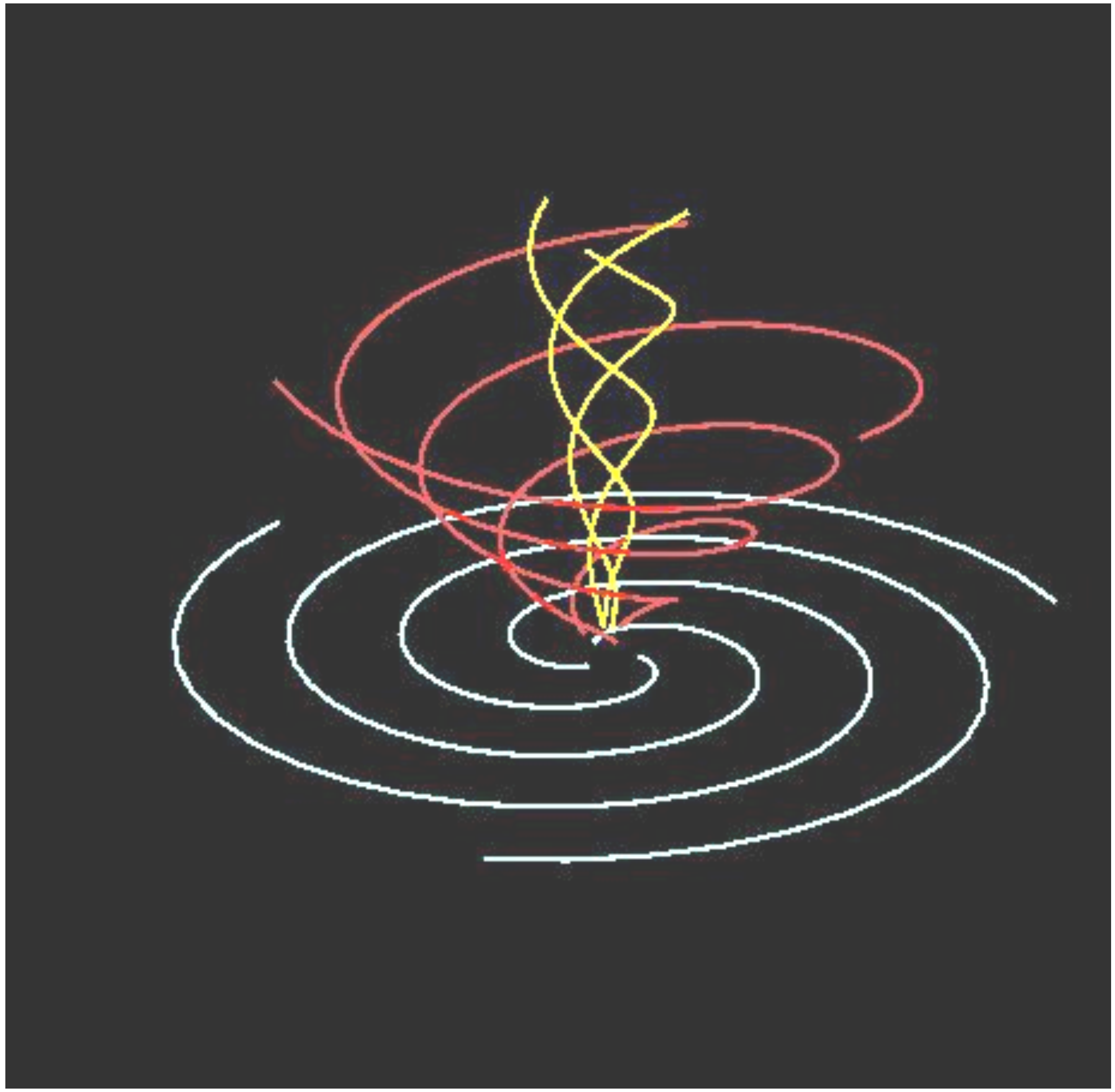
- Vents **rapide** et **lent** (> $20R_{SUN}$)

- Faible variation de densité (qq %)

- Nombre de Reynolds ~ **10^{9}**



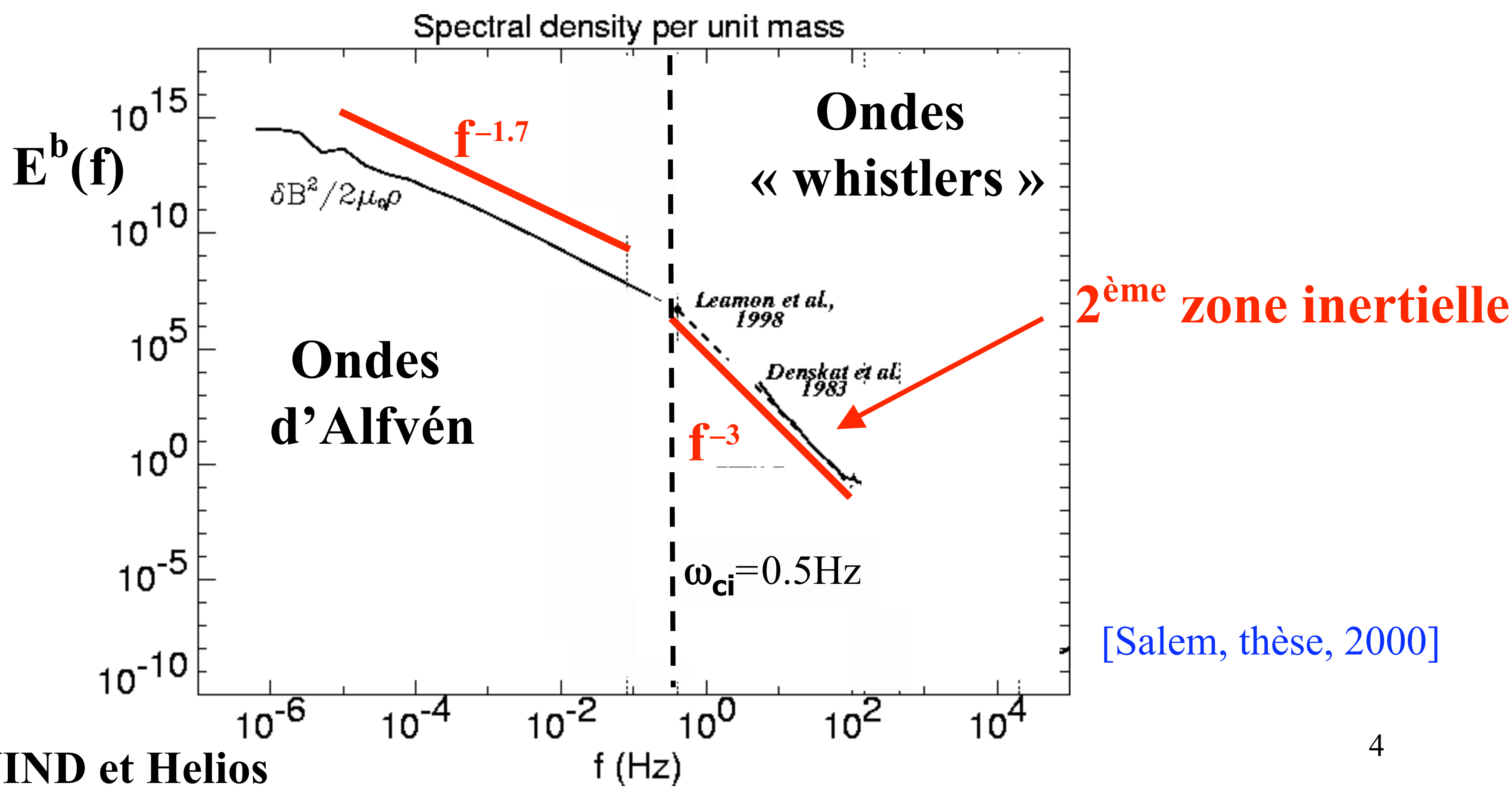
Vent solaire (E. Parker, 1958)



Turbulence multiéchelles

Raidissement du spectre des fluctuations magnétiques : $f^{-1.7} \rightarrow f^{-3}$

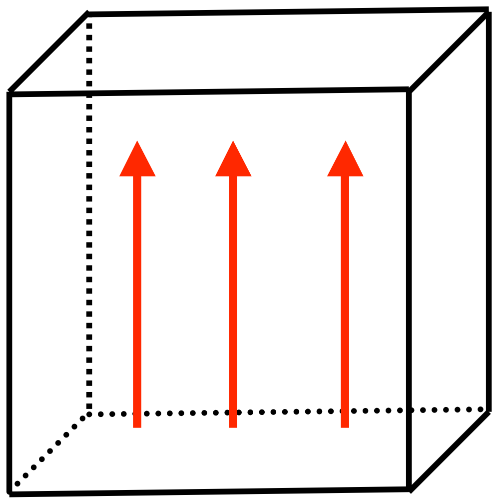
[Coroniti et al., 1982; Denskat et al., 1983; Leamon et al., 1999; Bale et al., 2005]

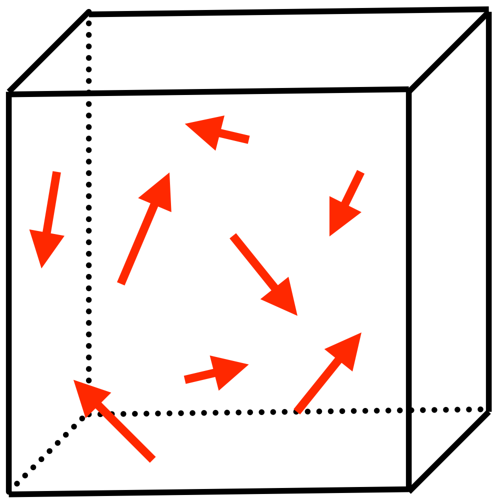


[Salem, thèse, 2000]

WIND et Helios

Approches théoriques





$\mathbf{B}\,(\mathbf{x},t) = B_o\,\mathbf{e}_{//} + \varepsilon\mathbf{b}(\mathbf{x},t)$

Turbulence d’ondes

$\mathbf{B}\,(\mathbf{x},t) = \mathbf{b}(\mathbf{x},t)$

Turbulence forte

MHD Hall incompressible

Equations ($\nu=\eta=0$) inviscides :

$$\nabla . \mathbf{v} = 0$$

$$\partial_t \mathbf{v} + \mathbf{v} . \nabla \mathbf{v} = -\nabla P_* + \mathbf{B} . \nabla \mathbf{B}$$

$$\partial_t \mathbf{B} + \mathbf{v} . \nabla \mathbf{B} = \mathbf{B} . \nabla \mathbf{v} - \mathbf{d_i} \nabla \mathbf{x} [(\nabla \mathbf{x} \mathbf{B}) \mathbf{x} \mathbf{B}]$$

$$\nabla . \mathbf{B} = 0$$

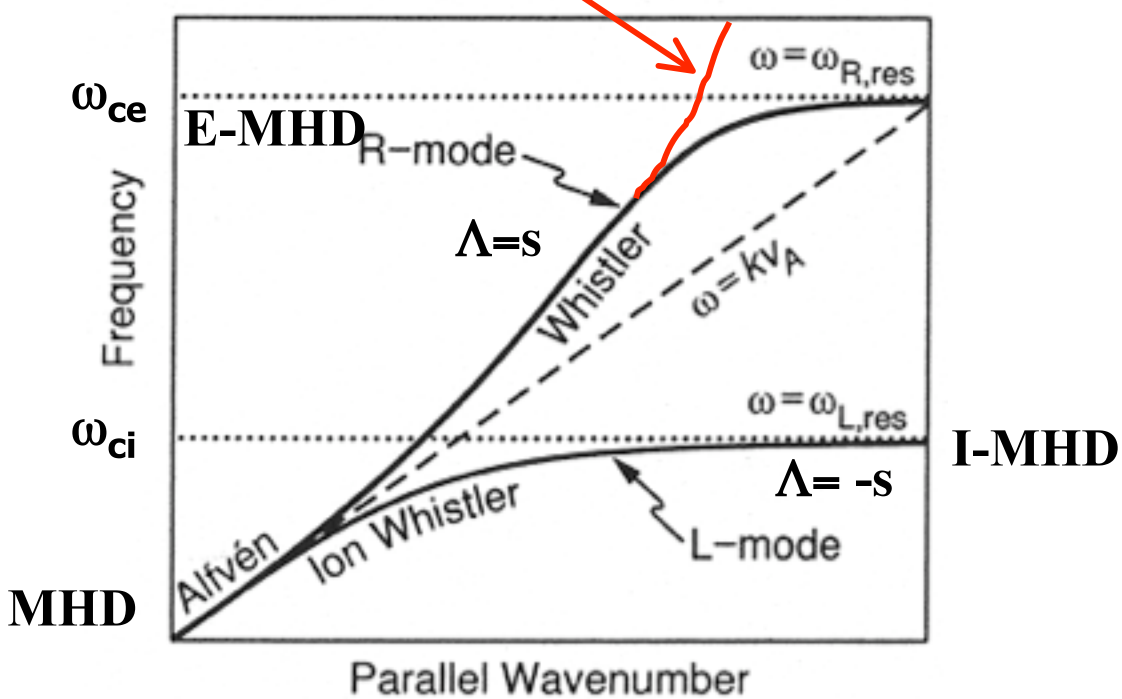
(d_i > 100 km à 1UA)

- Longueur inertielle des ions : $\mathbf{d_i = B_o / \omega_{ci}}$; $\mathbf{B} = B_o \, \mathbf{e}_{//} + \mathbf{b}$

Ondes en MHD Hall incompressible

En MHD Hall, les électrons n'ont pas de masse

$k = k_{//}$



$(sk_{//} > 0)$ $\qquad \omega_{\Lambda}^{s}(k) = B_o\, s\, d_i\, k^2\, \left(s\Lambda + \sqrt{1 + 4/(d_i k)^2}\right) / 2$

1. Turbulence d'ondes

Turbulence d'ondes en MHD Hall

[Galtier, 2006]

On veut décrire le vent solaire interne (< 1UA)

- On introduit : $\mathbf{B}(\mathbf{x},t) = B_0\, \mathbf{e}_{//} + \varepsilon \mathbf{b}(\mathbf{x},t)$ avec $0 < \varepsilon \ll 1$

  mais $\mathbf{B_0}$ est dans une direction **fixe**

- On développe perturbativement les équations en Fourier

- On calcule asymptotiquement les équations cinétiques d'ondes

$\tau_{nl} \gg \tau_w$ [Zakharov et al., 1992]

Turbulence d'ondes en MHD Hall

• Utilisation d'une **décomposition hélicitaire** complexe (onde hélicitaire):

$$\mathbf{h}^\Lambda(\mathbf{k}) \equiv \mathbf{h}_\mathbf{k}^\Lambda = \hat{\mathbf{e}}_\theta + i\Lambda\hat{\mathbf{e}}_\Phi$$

$$\begin{cases} \hat{\mathbf{e}}_\theta = \hat{\mathbf{e}}_\Phi \times \hat{\mathbf{e}}_k \\ \hat{\mathbf{e}}_\Phi = \dfrac{\hat{\mathbf{e}}_\parallel \times \hat{\mathbf{e}}_k}{|\hat{\mathbf{e}}_\parallel \times \hat{\mathbf{e}}_k|} \end{cases}$$

Λ est la polarisation d'onde ($\Lambda = \pm 1$) ; $\mathbf{k} \cdot \mathbf{h}_\mathbf{k}^\Lambda = 0\,,\ \hat{\mathbf{e}}_k \times \mathbf{h}_\mathbf{k}^\Lambda = -i\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda$

[Craya, 1958; Kraichnan, 1973; Cambon et al., 1989; Turner, 2000; Galtier 2003]

$$\rightarrow \quad \begin{cases} \mathbf{v}_\mathbf{k} = \sum\limits_\Lambda \mathcal{U}_\Lambda(\mathbf{k})\, \mathbf{h}_\mathbf{k}^\Lambda = \sum\limits_\Lambda \mathcal{U}_\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda\,, \\ \mathbf{b}_\mathbf{k} = \sum\limits_\Lambda \mathcal{B}_\Lambda(\mathbf{k})\, \mathbf{h}_\mathbf{k}^\Lambda = \sum\limits_\Lambda \mathcal{B}_\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda\,. \end{cases}$$

Turbulence d'ondes en MHD Hall

- On introduit les variables **d'Elsässer généralisées** :

$$\mathcal{Z}_\Lambda^s \equiv \mathcal{U}_\Lambda + \xi_\Lambda^s \mathcal{B}_\Lambda \,,$$

$$\xi_\Lambda^s(k) = \xi_\Lambda^s = -\frac{s d_i k}{2}\left(s\Lambda + \sqrt{1 + \frac{4}{d_i^2 k^2}}\right) \,.$$

D'où :

$$\begin{cases} \partial_t \mathcal{Z}_\Lambda^s = -i\,\omega_\Lambda^s \mathcal{Z}_\Lambda^s \\ \omega_\Lambda^s(k) = B_o\, s k_{//}\, d_i k \left(s\Lambda + \sqrt{1 + 4/(d_i k)^2}\right) / 2 \end{cases}$$

Turbulence d'ondes en MHD Hall

- On obtient l'équation d'amplitude de l'onde :

$$\partial_t a_\Lambda^s = \frac{\epsilon}{4d_\mathrm{i}} \int \sum_{\substack{\Lambda_\mathrm{p},\Lambda_q \\ s_\mathrm{p},\, s_q}} \xi_\Lambda^{s\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_\mathrm{p}}^{s_p}}{\xi_\Lambda^s - \xi_\Lambda^{-s}} M^{\begin{smallmatrix} \Lambda\Lambda_\mathrm{p}\Lambda_q \\ s\, s_\mathrm{p}\, s_q \end{smallmatrix}}_{-k\,p\,q}\, a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} e^{-i\Omega_{pq,k}t} \delta_{pq,k}\, \mathbf{dp}\, \mathbf{dq},$$

$$\Omega_{pq,k} = \omega_{\Lambda_p}^{s_p} + \omega_{\Lambda_q}^{s_q} - \omega_\Lambda^s$$

$$\partial_t \langle a_\Lambda^s a_{\Lambda'}^{s'} \rangle = \tag{B 1}$$

$$\frac{\epsilon}{4\,d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p,s_q}} \xi_\Lambda^{s\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_\Lambda^s - \xi_\Lambda^{-s}} M^{\begin{smallmatrix} \Lambda\Lambda_p\Lambda_q \\ s\, s_p\, s_q \end{smallmatrix}}_{-k\,p\,q} \langle a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} a_{\Lambda'}^{s'} \rangle e^{-i\Omega_{pq,k}t} \delta_{pq,k}\, \mathbf{dp}\, \mathbf{dq}$$

$$+$$

$$\frac{\epsilon}{4\,d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p,s_q}} \xi_{\Lambda'}^{s'\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_{\Lambda'}^{s'} - \xi_{\Lambda'}^{-s'}} M^{\begin{smallmatrix} \Lambda'\Lambda_p\Lambda_q \\ s'\, s_p\, s_q \end{smallmatrix}}_{-k'\,p\,q} \langle a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} a_{\Lambda}^{s} \rangle e^{-i\Omega_{pq,k'}t} \delta_{pq,k'}\, \mathbf{dp}\, \mathbf{dq}\,.$$

and

$$\partial_t \langle a_\Lambda^s a_{\Lambda'}^{s'} a_{\Lambda''}^{s''} \rangle = \tag{B 2}$$

$$\frac{\epsilon}{4\,d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p,s_q}} \xi_\Lambda^{s\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_\Lambda^s - \xi_\Lambda^{-s}} M^{\begin{smallmatrix} \Lambda\Lambda_p\Lambda_q \\ s\, s_p\, s_q \end{smallmatrix}}_{-k\,p\,q} \langle a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} a_{\Lambda'}^{s'} a_{\Lambda''}^{s''} \rangle e^{-i\Omega_{pq,k}t} \delta_{pq,k}\, \mathbf{dp}\, \mathbf{dq}$$

$$+$$

$$\frac{\epsilon}{4\,d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p,s_q}} \xi_{\Lambda'}^{s'\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_{\Lambda'}^{s'} - \xi_{\Lambda'}^{-s'}} M^{\begin{smallmatrix} \Lambda'\Lambda_p\Lambda_q \\ s'\, s_p\, s_q \end{smallmatrix}}_{-k'\,p\,q} \langle a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} a_{\Lambda}^{s} a_{\Lambda''}^{s''} \rangle e^{-i\Omega_{pq,k'}t} \delta_{pq,k'}\, \mathbf{dp}\, \mathbf{dq}$$

$$+$$

$$\frac{\epsilon}{4\,d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p,s_q}} \xi_{\Lambda''}^{s''\,2} \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_{\Lambda''}^{s''} - \xi_{\Lambda''}^{-s''}} M^{\begin{smallmatrix} \Lambda''\Lambda_p\Lambda_q \\ s''\, s_p\, s_q \end{smallmatrix}}_{-k''\,p\,q} \langle a_{\Lambda_p}^{s_p} a_{\Lambda_q}^{s_q} a_{\Lambda}^{s} a_{\Lambda'}^{s'} \rangle e^{-i\Omega_{pq,k''}t} \delta_{pq,k''}\, \mathbf{dp}\, \mathbf{dq}\,.$$

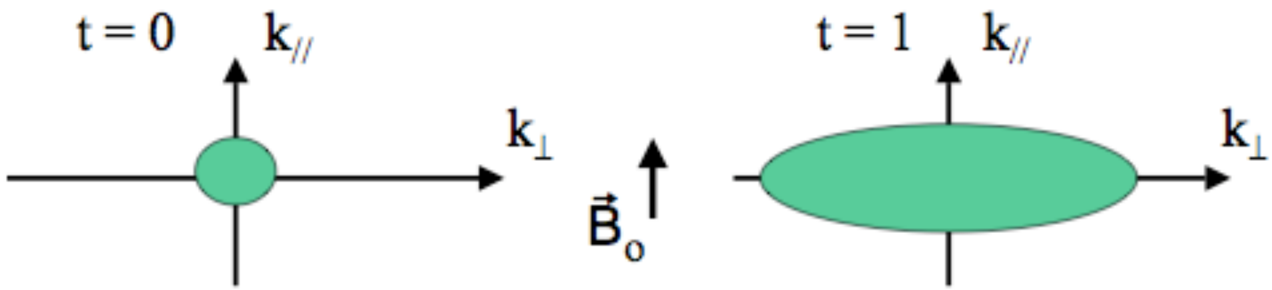
Technique de turbulence d'ondes

Aux temps longs (**$\tau \sim \tau_{tr} >> \tau_A$**), seuls les termes résonants survivent

Fermeture asymptotique des équations cinétiques !

Turbulence d'ondes en MHD Hall

- Tendance globale (∀ l'échelle) vers l'**anisotropie** spectrale :



- Les équations maîtresses sont :

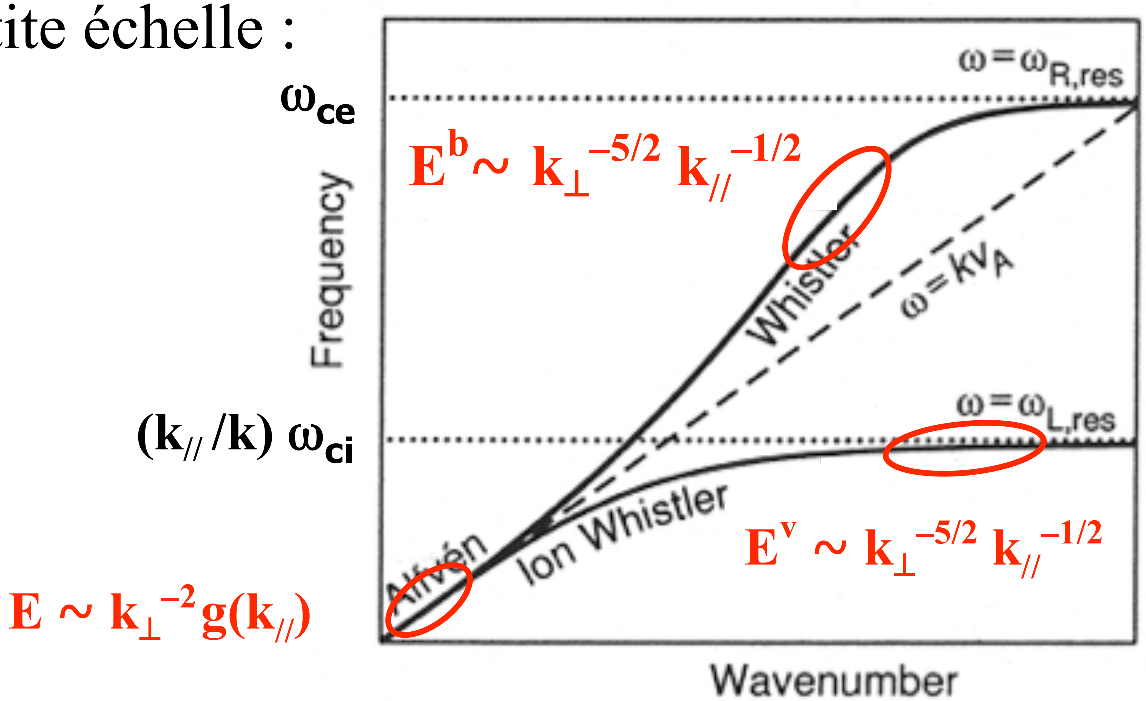
$$\partial_t \begin{Bmatrix} E^V(\mathbf{k}) \\ E^B(\mathbf{k}) \end{Bmatrix} = \frac{\pi \epsilon^2}{8\, d_i^2 B_0^2} \int \sum_{\substack{\Lambda,\Lambda_p,\Lambda_q \\ s,s_p,s_q}} \left(\frac{\sin\psi_k}{k}\right)^2 \frac{(\Lambda k + \Lambda_p p + \Lambda_q q)^2 \left(1 - \xi_\Lambda^{-s^2} \xi_{\Lambda_p}^{-s_p{}^2} \xi_{\Lambda_q}^{-s_q{}^2}\right)^2}{(1+\xi_\Lambda^{-s^2})(1+\xi_{\Lambda_p}^{-s_p{}^2})(1+\xi_{\Lambda_q}^{-s_q{}^2})}$$

$$\left(\frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{k_\parallel}\right)^2 \begin{Bmatrix} \xi_\Lambda^{-s^2} \\ 1 \end{Bmatrix} \frac{\omega_\Lambda^s \, \omega_{\Lambda_p}^{s_p}}{\xi_\Lambda^{-s^2} + 1} \left(\frac{\xi_{\Lambda_q}^{-s_q{}^2} E^V(\mathbf{q}) - E^B(\mathbf{q})}{\xi_{\Lambda_q}^{-s_q{}^2} - 1}\right)$$

$$\left[\left(\frac{\xi_{\Lambda_p}^{-s_p{}^2} E^V(\mathbf{p}) - E^B(\mathbf{p})}{\xi_{\Lambda_p}^{-s_p{}^2} - 1}\right) - \left(\frac{\xi_\Lambda^{-s^2} E^V(\mathbf{k}) - E^B(\mathbf{k})}{\xi_\Lambda^{-s^2} - 1}\right)\right] \delta(\Omega_{k,pq})\, \delta_{k,pq}\, d\mathbf{p}\, d\mathbf{q}\,.$$

Turbulence d’ondes en MHD Hall

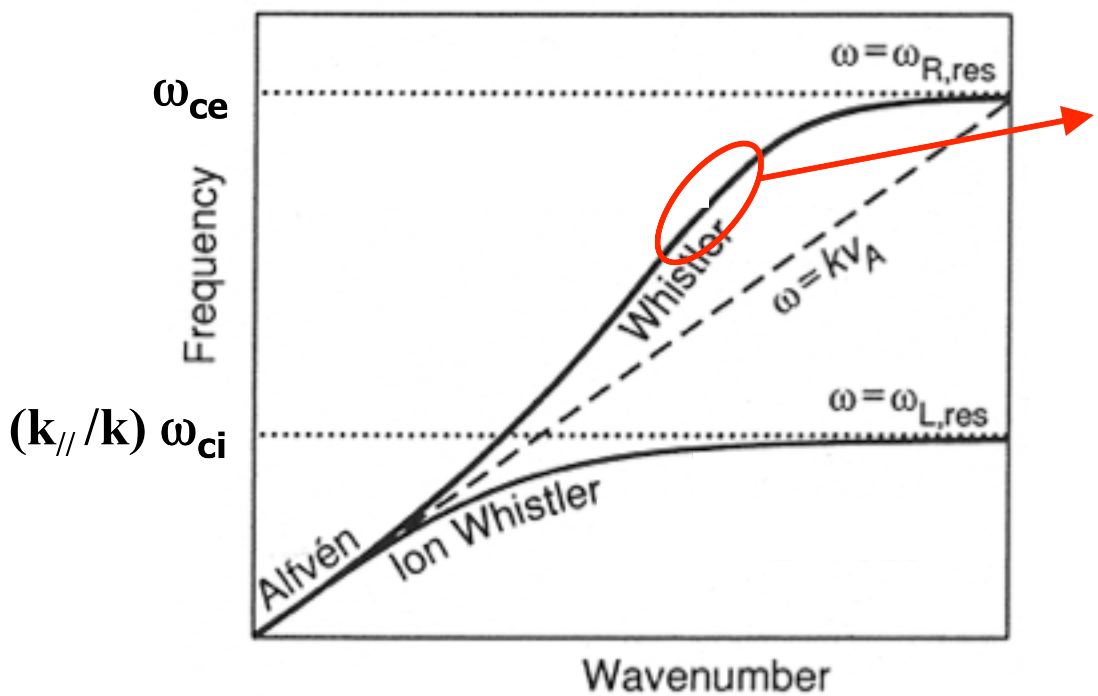
• Les solutions exactes en loi de puissance montrent un **raidissement** à petite échelle :



Kraichnan Anisotrope : $E(k_\perp, k_\parallel) \sim \sqrt{\Pi B_0}\, k_\perp^{-2} k_\parallel^{-1/2} (1 + k_\perp^2 d_i^2)^{-1/4}$

(d_i ~ 100 km)

Turbulence d'ondes en MHD Hall



Comme Navier Stokes en rotation rapide !

2. Turbulence forte

Turbulence (forte) en MHD Hall

- Que sait-on de la turbulence (forte) MHD Hall ? *Peu de choses…*

→ DNS sont restreintes aux (très) bas nombres de Reynolds

→ Difficultés d'obtenir un comportement multiéchelles

[*eg*. Ghosh et al., 1996; Mininni et al., 2003-2006]

Attendre quelques décennies ou changer de stratégie !

Turbulence (forte) en MHD Hall

[Galtier & Buchlin, 2007]

Etude numérique à l'aide d'un modèle **« 3D »** de **cascade turbulente** :

$$\frac{\partial V_n}{\partial t} + \nu_2 k_n^4 V_n =$$

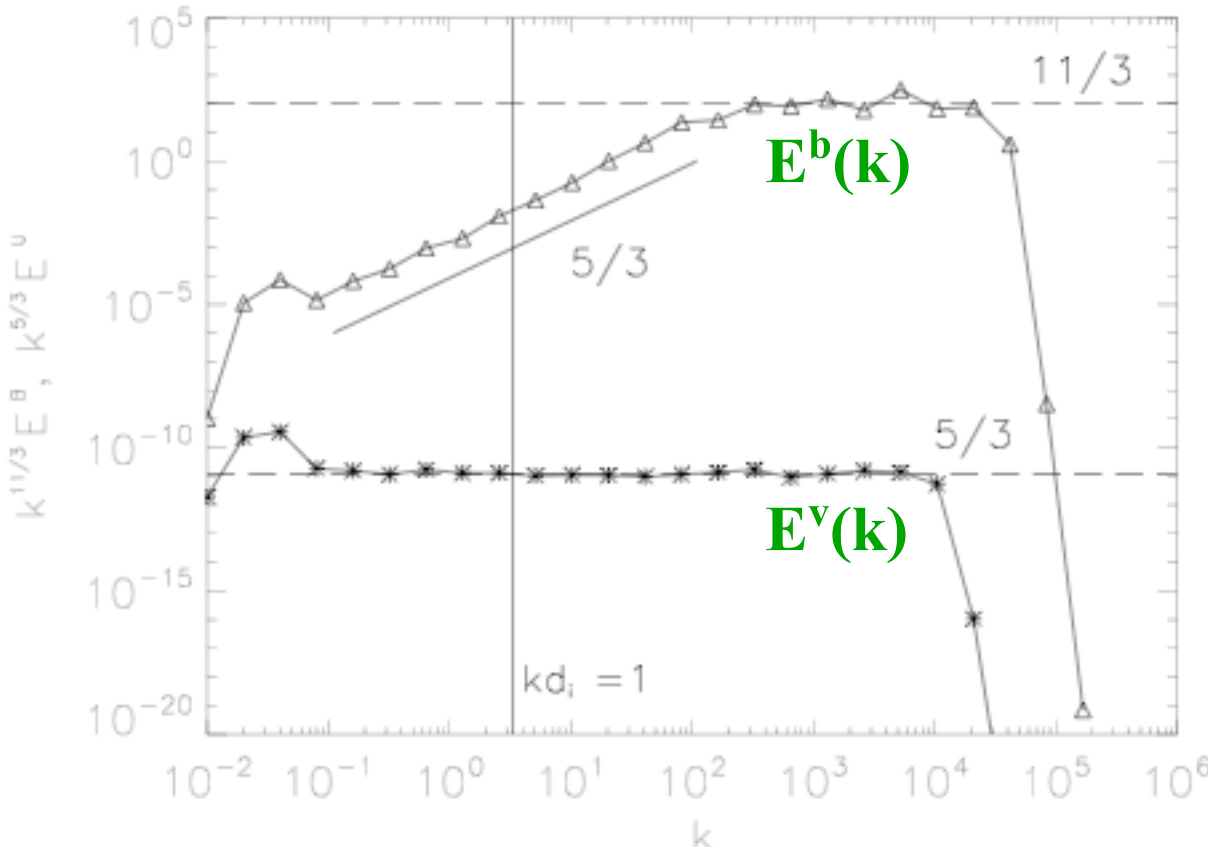
$$ik_n \left[(V_{n+1}V_{n+2} - B_{n+1}B_{n+2}) - \frac{1}{4}(V_{n-1}V_{n+1} - B_{n-1}B_{n+1}) - \frac{1}{8}(V_{n-2}V_{n-1} - B_{n-2}B_{n-1}) \right]^*$$

$$\frac{\partial B_n}{\partial t} + \eta_2 k_n^4 B_n =$$

$$\frac{ik_n}{6} [(V_{n+1}B_{n+2} - B_{n+1}V_{n+2}) + (V_{n-1}B_{n+1} - B_{n-1}V_{n+1}) + (V_{n-2}B_{n-1} - B_{n-2}V_{n-1})]^*$$

$$+(-1)^n i d_i k_n^2 \left[B_{n+1}B_{n+2} - \frac{1}{4}B_{n-1}B_{n+1} - \frac{1}{8}B_{n-2}B_{n-1} \right]^* ,$$

Turbulence (forte) en MHD Hall



Raidissement non trivial

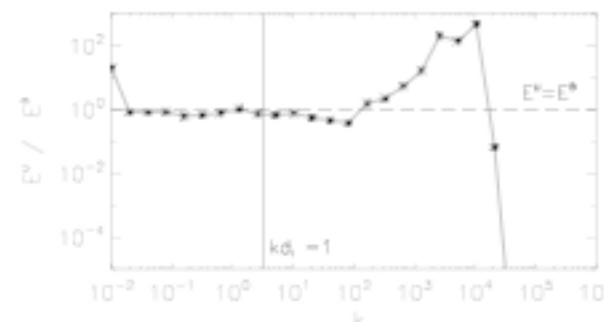
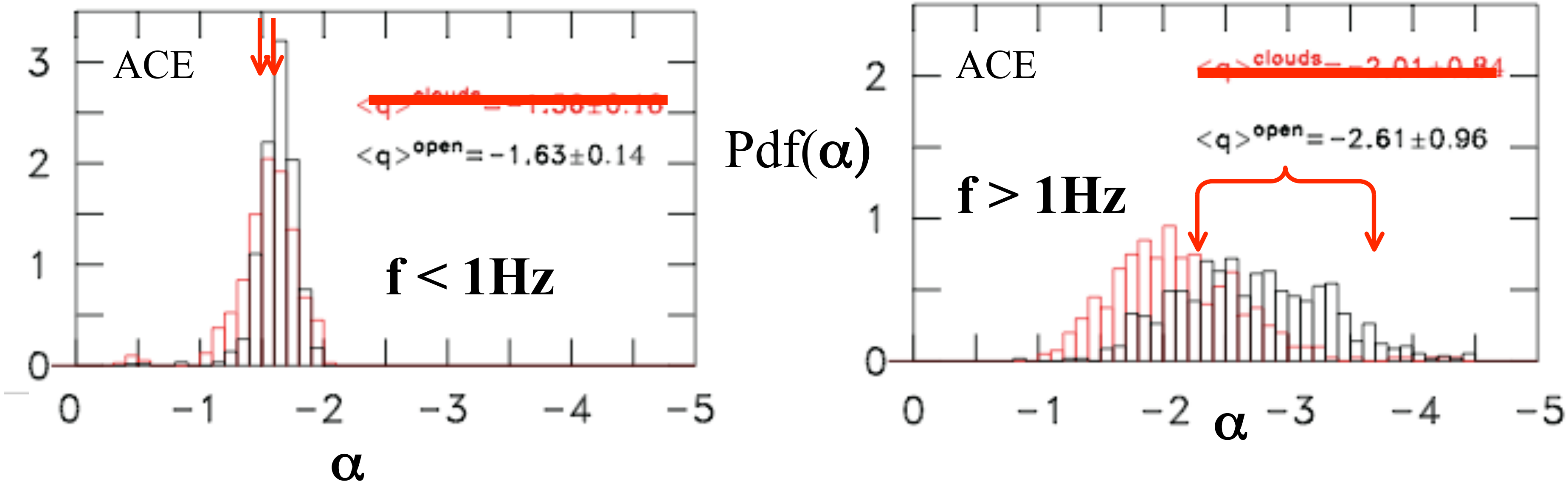


Fig. 2.— Compensated magnetic (triangles) and kinetic (stars; for clarity, they are shifted to lower values) energy spectra in Hall-MHD. The vertical solid lines indicate the critical value $kd_i = 1$.

Spectre des fluctuations magnétiques

[Smith et al., 2006]



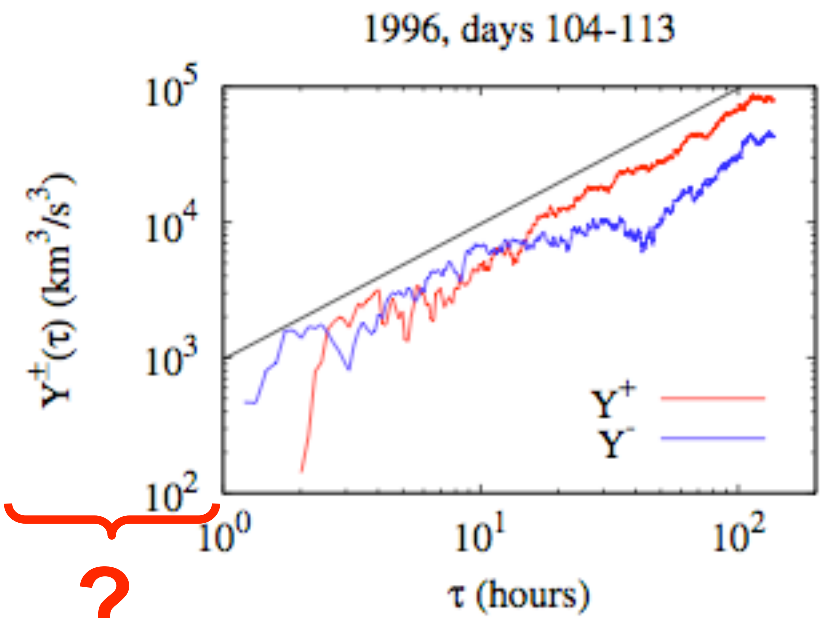
- Turbulence MHD Hall : $E^b(k) \sim k^{-\alpha}$, où $\alpha =$ **7/3** → **11/3**

→ Peut dépendre de l'efficacité de **l'absorption cyclotron**

PRL 99, 115001 (2007) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2007

Observation of Inertial Energy Cascade in Interplanetary Space Plasma

L. Sorriso-Valvo,[1] R. Marino,[2] V. Carbone,[2] A. Noullez,[3] F. Lepreti,[2] P. Veltri,[2]
R. Bruno,[4] B. Bavassano,[4] and E. Pietropaolo[5]



?

Prédiction exacte pour la MHD :

[Politano & Pouquet, 1998]

$$\boxed{Y^{\pm}(r) = -\tfrac{4}{3}\epsilon^{\pm} r} \qquad Y^{\pm}(r) = \langle |\Delta z^{\pm}|^2 \Delta z^{\mp}_{\parallel} \rangle$$

Prédiction exacte pour la MHD Hall :

$$\boxed{-\frac{4}{3}\varepsilon^T r = B^{vvv}_{\parallel ii} + B^{vbb}_{\parallel ii} - 2B^{bvb}_{\parallel ii} - 4d_I (S^4_{\parallel ii} - S^4_{i\parallel i})}$$

[Galtier, 2008]

$$B^{\alpha\beta\gamma}_{ijk} = \langle (\alpha'_i - \alpha_i)(\beta'_j - \beta_j)(\gamma'_k - \gamma_k) \rangle \qquad S^4_{ijk}(\mathbf{r}) = \langle J_i(\mathbf{x}) b_j(\mathbf{x}) b_k(\mathbf{x}') \rangle$$

Conclusions

- Présence de **deux** zones inertielles dans le vent solaire

- La turbulence **MHD Hall** est un modèle pertinent

  → Turbulence d'ondes/forte mènent à un raidissement

  → Turbulence compressible aussi…

- Rôle d'un flux **asymétrique**, hypothèse de Taylor…?

- Intermittence…

- Données actuelles **ne sont pas** suffisantes (f → <u>vecteur d'onde</u> **k**)