Extreme Lagrangian acceleration in confined turbulent flow

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Introduction

Motivation

* Turbulent transport and mixing $\longrightarrow$ Lagrangian point of view.
* Many applications quasi 2D (geophysical flows, plasmas with a strong magnetic field) and 2D first approach $\longrightarrow$ 2D Turbulence.
* Practically all flows bounded $\longrightarrow$ Influence of walls on dynamics (many works focused on Eulerian dynamics).

$\longrightarrow$ **Influence of solid boundaries on Lagrangian dynamics**

Numerical simulation

* Two distinct geometries :

  a **biperiodic** and a circular domain with no-slip boundary conditions.

* Direct numerical simulation using classical pseudo-spectral method.

* $$\frac{\partial \vec{\omega}}{\partial t} + \vec{u} \cdot \nabla \vec{\omega} - \nu \nabla^2 \vec{\omega} = 0$$

  where $\vec{u}$ the velocity, $\omega = \nabla \times \vec{u}$ the vorticity

  and ν the kinematic viscosity.

Numerical simulation

* Two distinct geometries :

  a biperiodic and a **circular domain with no-slip boundary conditions**.

* Direct numerical simulation using classical pseudo-spectral method.

* $$\frac{\partial \vec{\omega}}{\partial t} + \vec{u} \cdot \nabla \vec{\omega} - \nu \nabla^2 \vec{\omega} + \nabla \wedge \left(\frac{1}{\eta}(\chi \vec{u})\right) = 0$$

  where χ is the mask function is 1 outside the flow-domain and 0 inside the flow

  and η the permeability.

* Volume penalization method

  P. Angot & all *Numer. Math.*(1999).

  K. Schneider *Comput. Fluids* (2005).

Parameters

* **freely decaying turbulence**, resolution : 1024^2.
* Semi-implicit time integration $\Delta t = 5.10^{-5}$, permeability $\eta = 10^{-3}$.
* Viscosity $\nu = 10^{-4}$.
* **Initial Reynolds number** $Re \sim 5 \cdot 10^4$.
* Duration : 500000 timesteps.

K. Schneider & M. Farge *Phys. Rev. Lett.* (2005). Similar confined flow

Lagrangian quantities

* Interpolation of the Eulerian quantities

  Integration in time using a second order Runge-Kutta scheme
* **Lagrangian acceleration :** $\vec{a}_L = -\nabla p + \nu \nabla^2 \vec{u}$.
* 1020 Trajectories
* Decaying turbulence $\longrightarrow$ **need to make stationary the statistics** :

  Lagrangian quantities $L(t)$ are divided by their instantaneous standard deviation computed from all particles at each time : $L(t)/\sigma_L(t)$

  P.K. Yeung *Annu. Rev. Fluid Mech.* (2002)

(a) Periodic geometry (b) Circular geometry

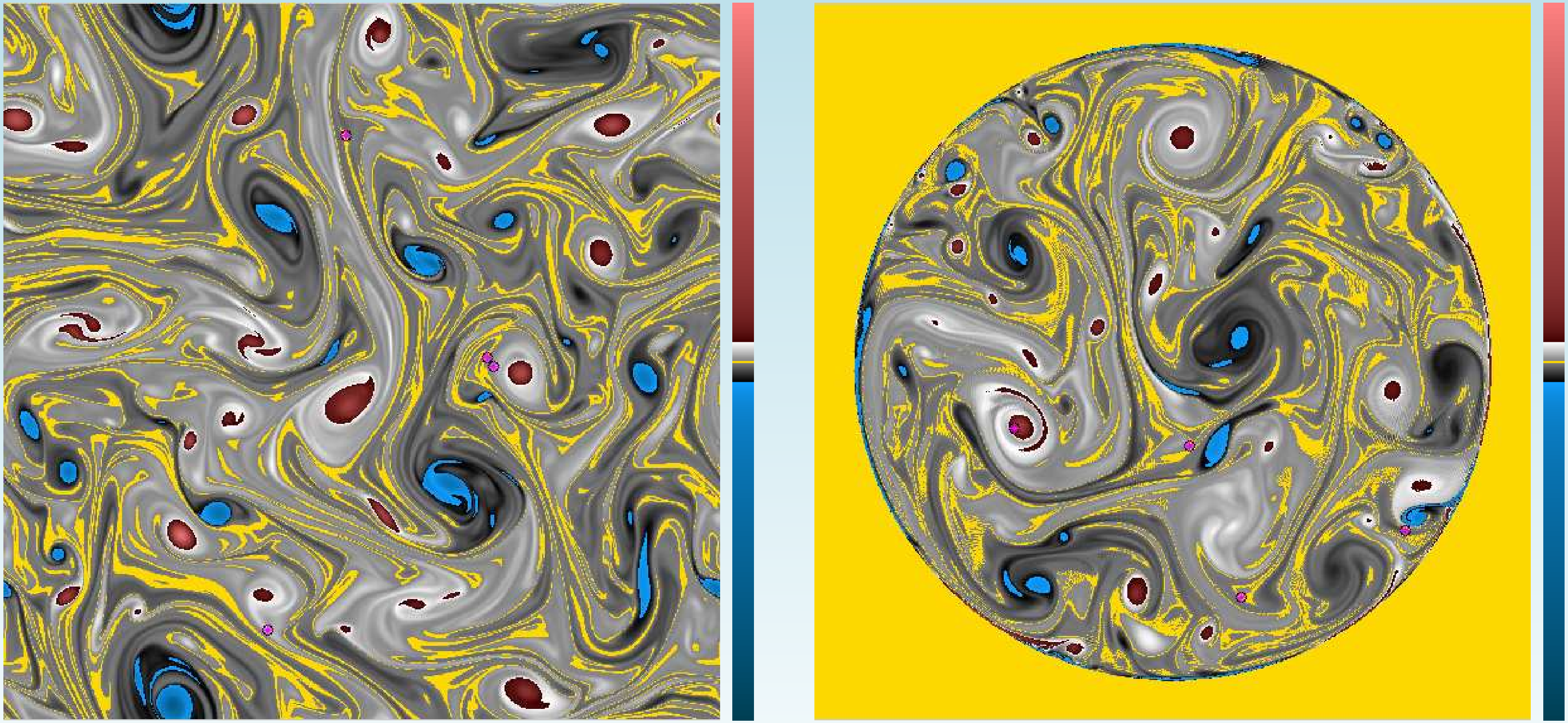

FIG. 1: Snapshots of vorticity fields.

Results

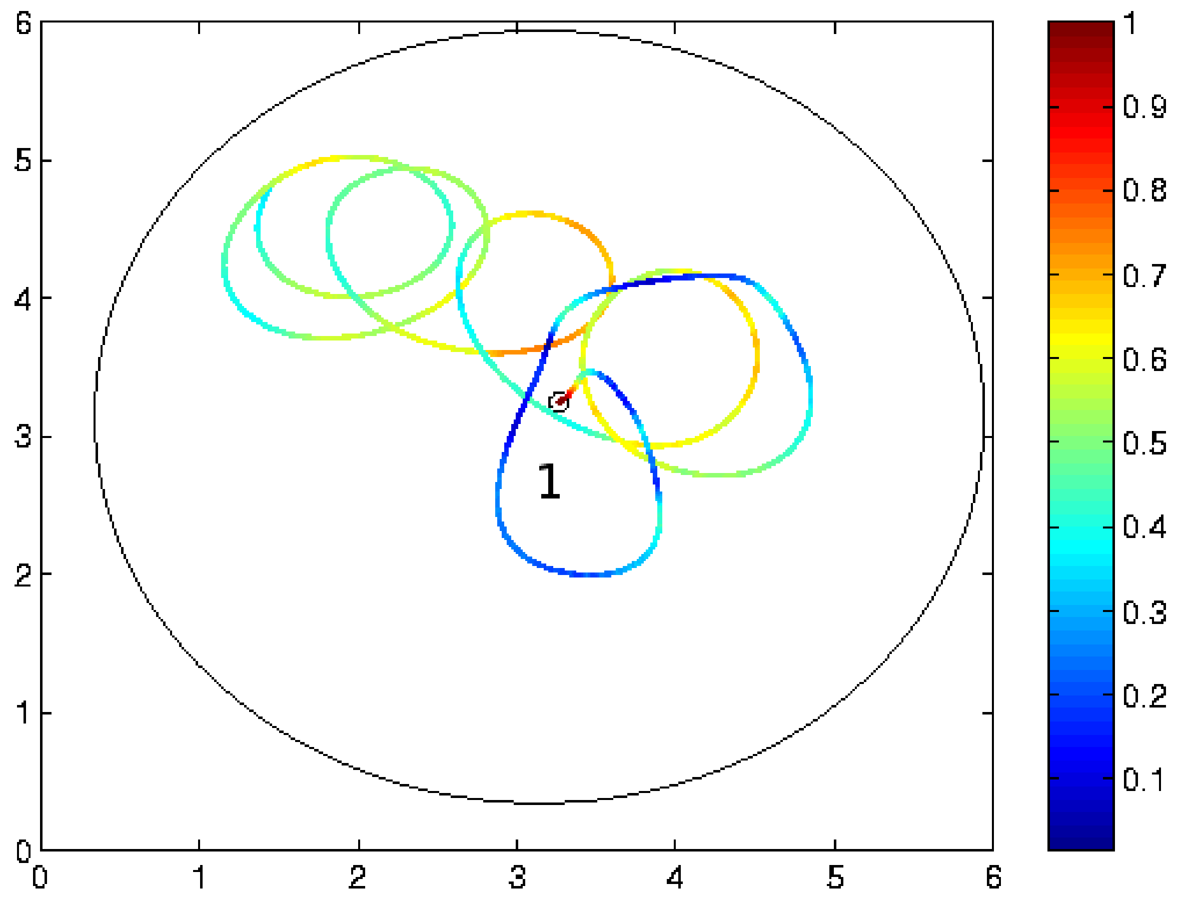

FIG. 2: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

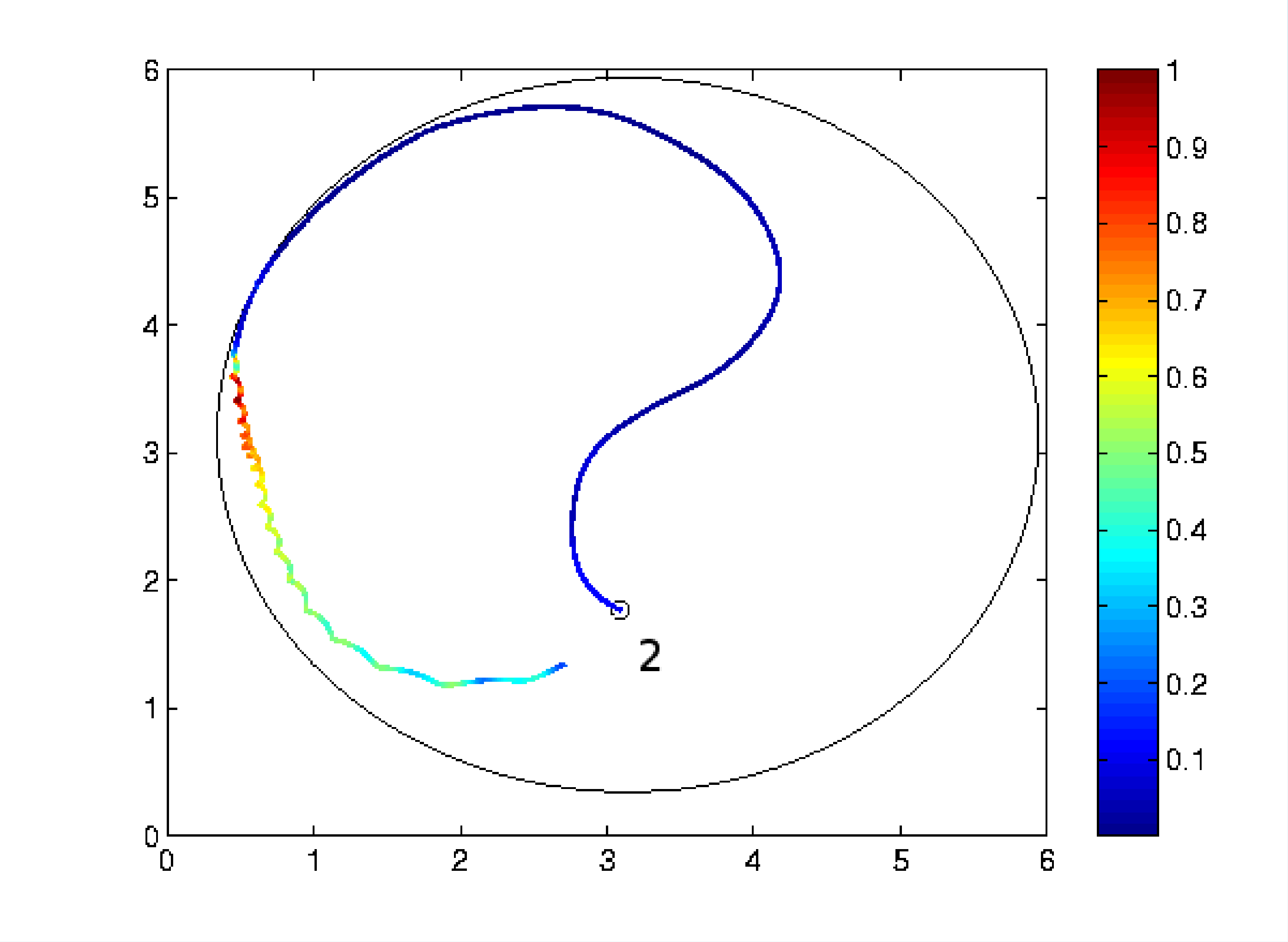

FIG. 3: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

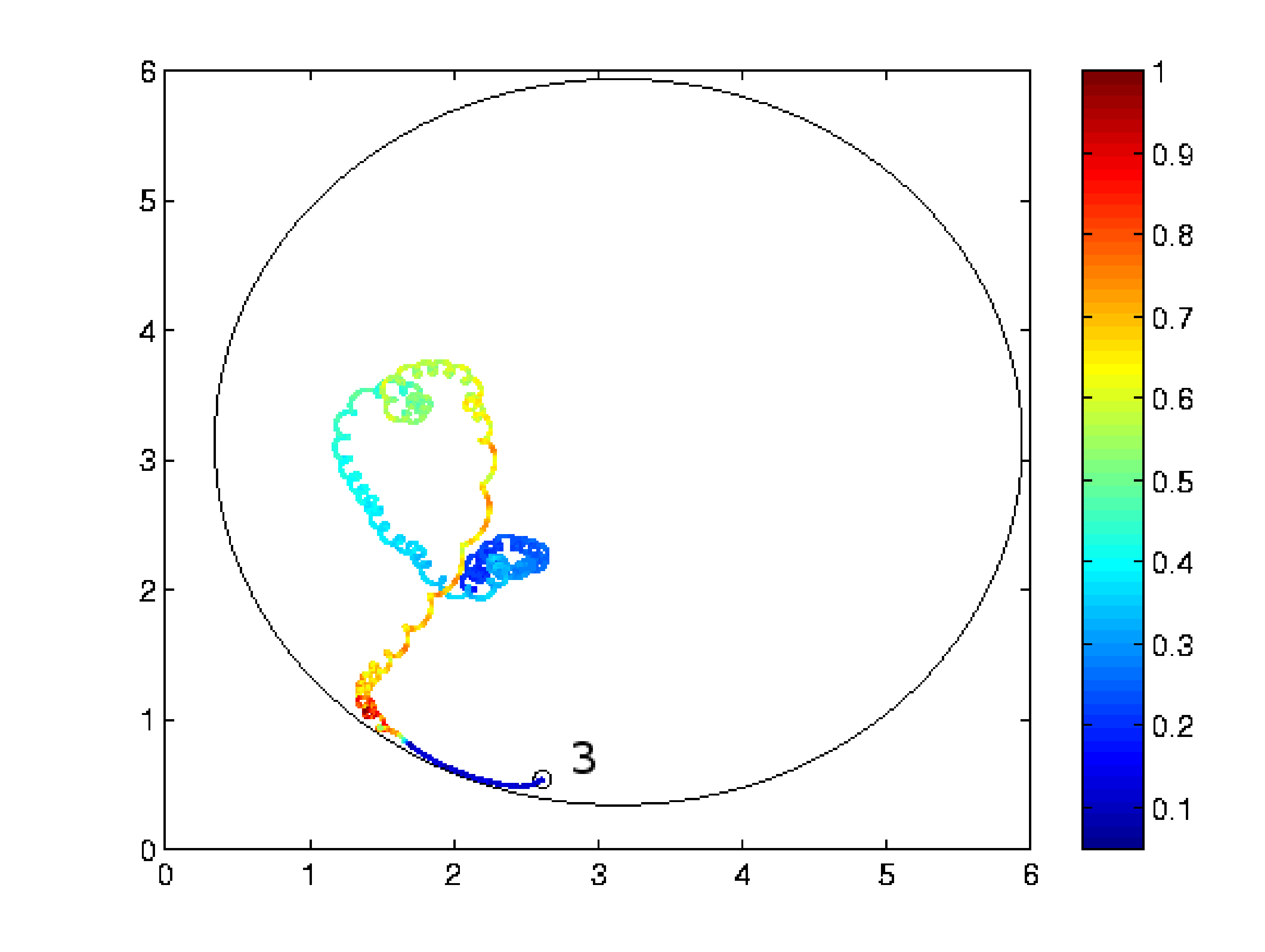

FIG. 4: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

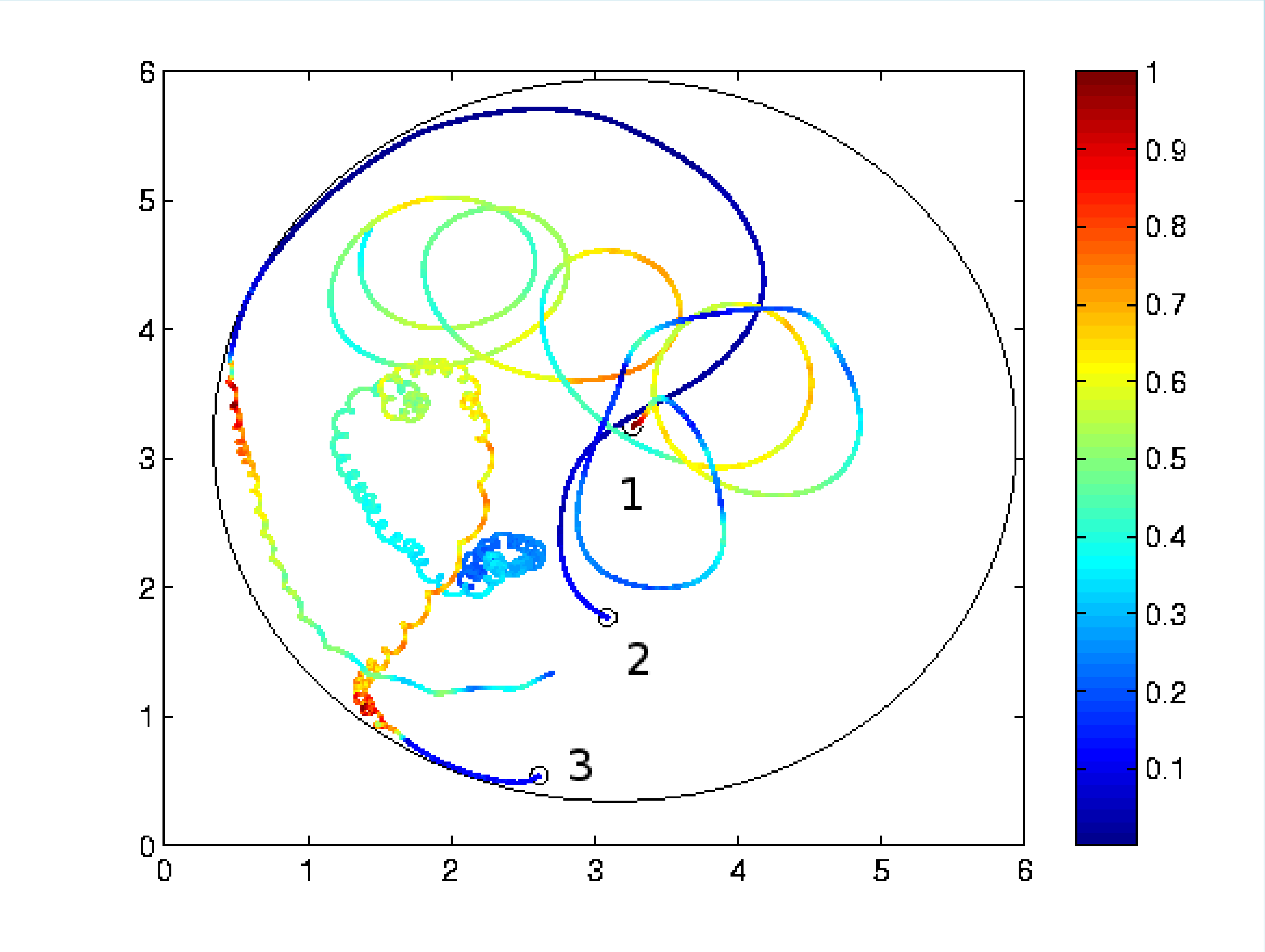

FIG. 5: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

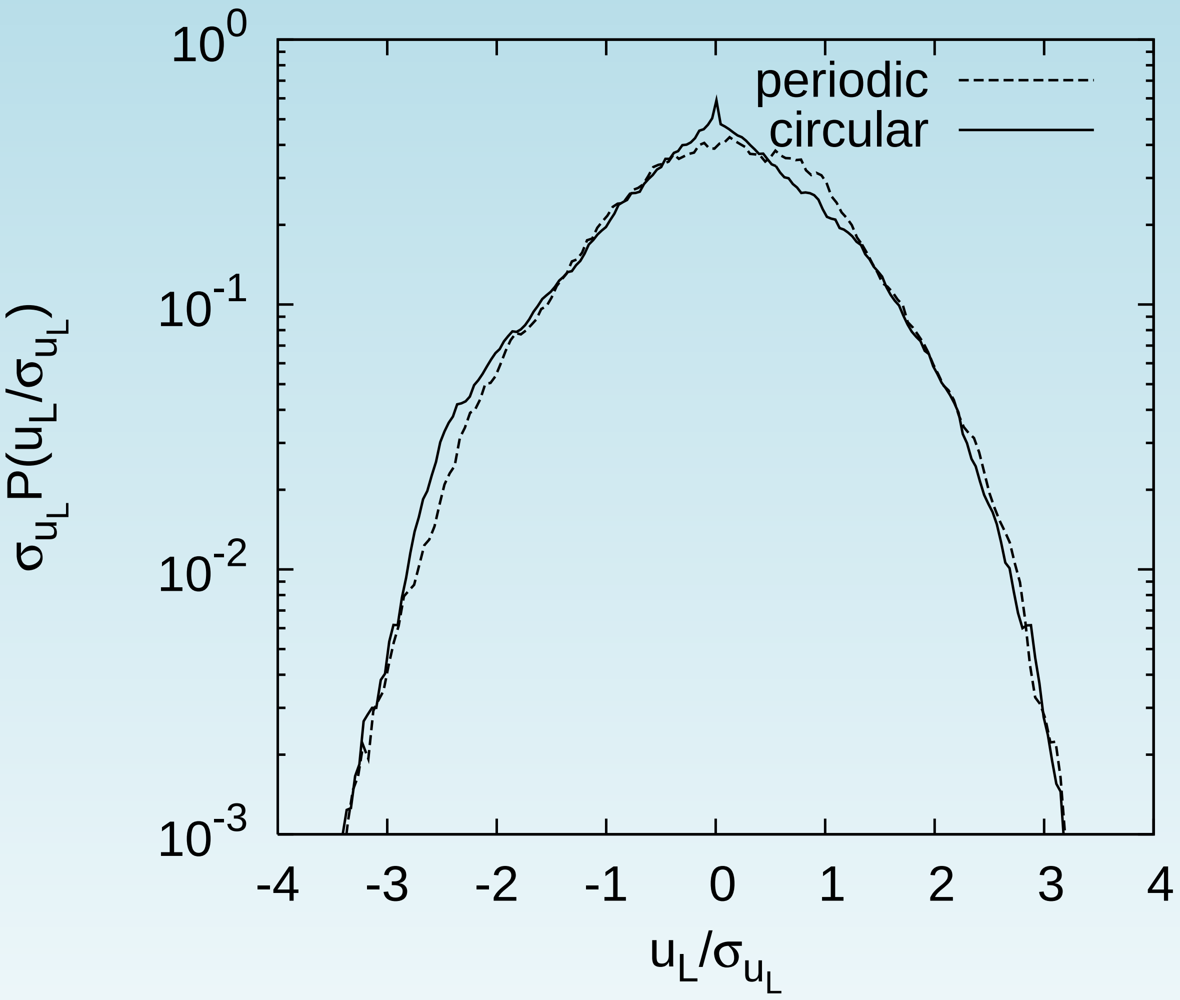

FIG. 6: PDFs of normalized Lagrangian velocities u_L/σ_{u_L} where $\sigma_{u_L} = \langle u_L^2 \rangle^{1/2}$ ($\langle \cdot \rangle$ denotes the ensemble average), for the periodic geometry and for the circular geometry.

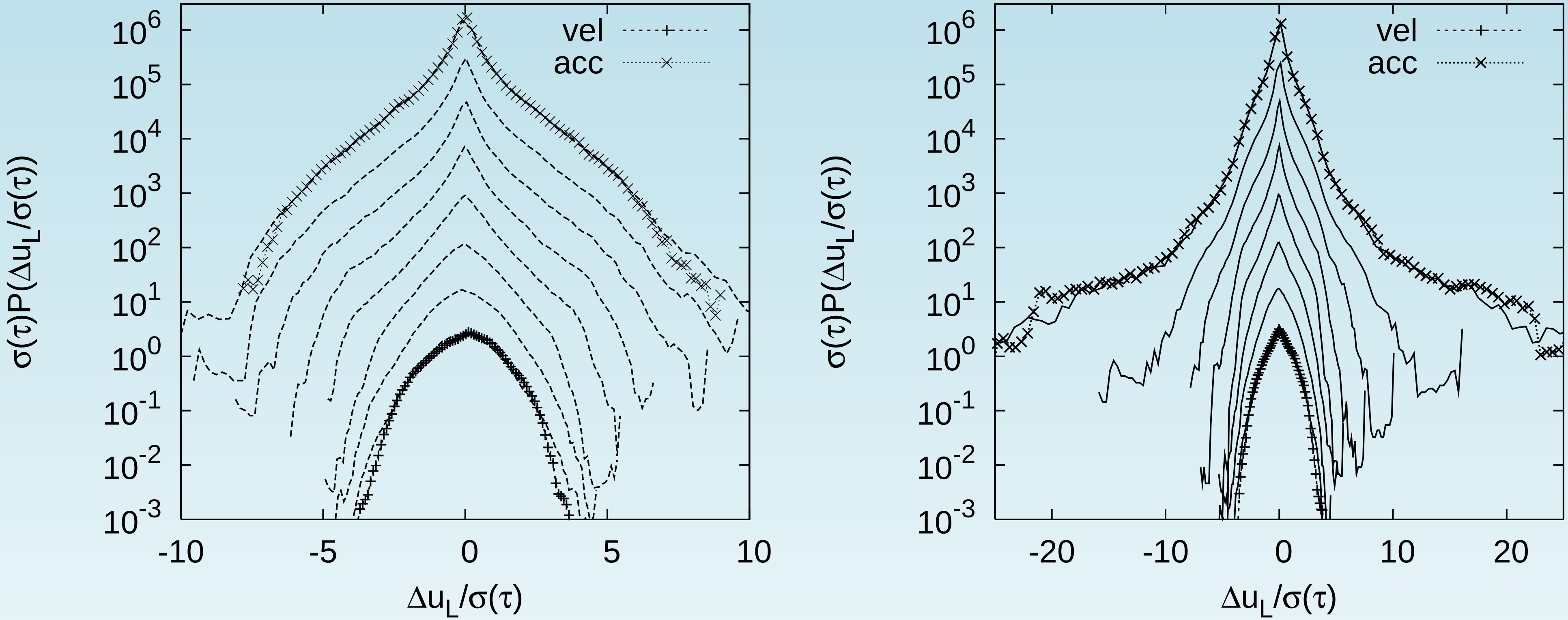

FIG. 7: PDFs of normalized Lagrangian velocity increments $\Delta u_L(\tau)/\sigma(\tau)$ where $\sigma(\tau) = \langle(\Delta u_L(\tau))^2\rangle^{1/2}$, for periodic (left) and circular geometry (right).

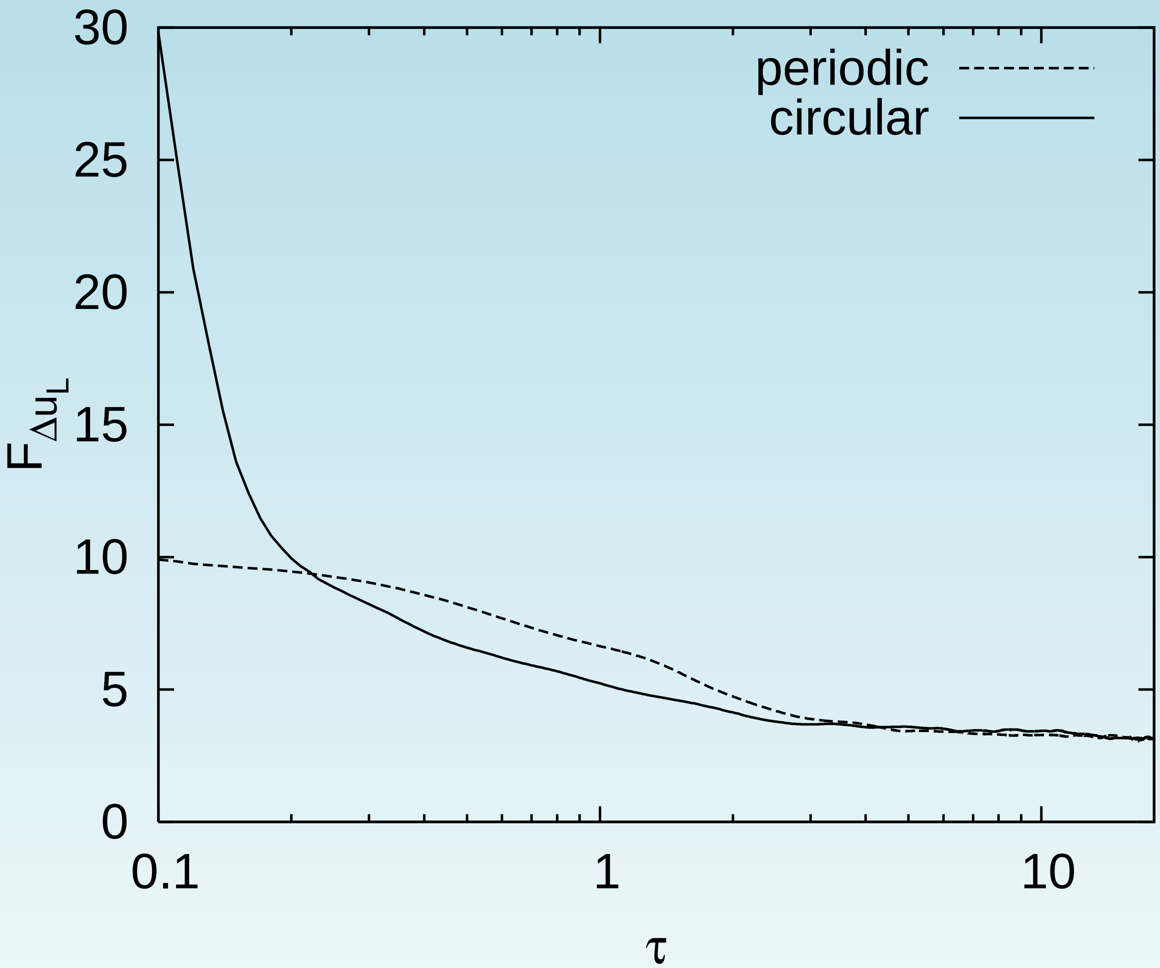

FIG. 8: Flatness of the Lagrangian velocity increments as a function of τ for the periodic and circular geometry.

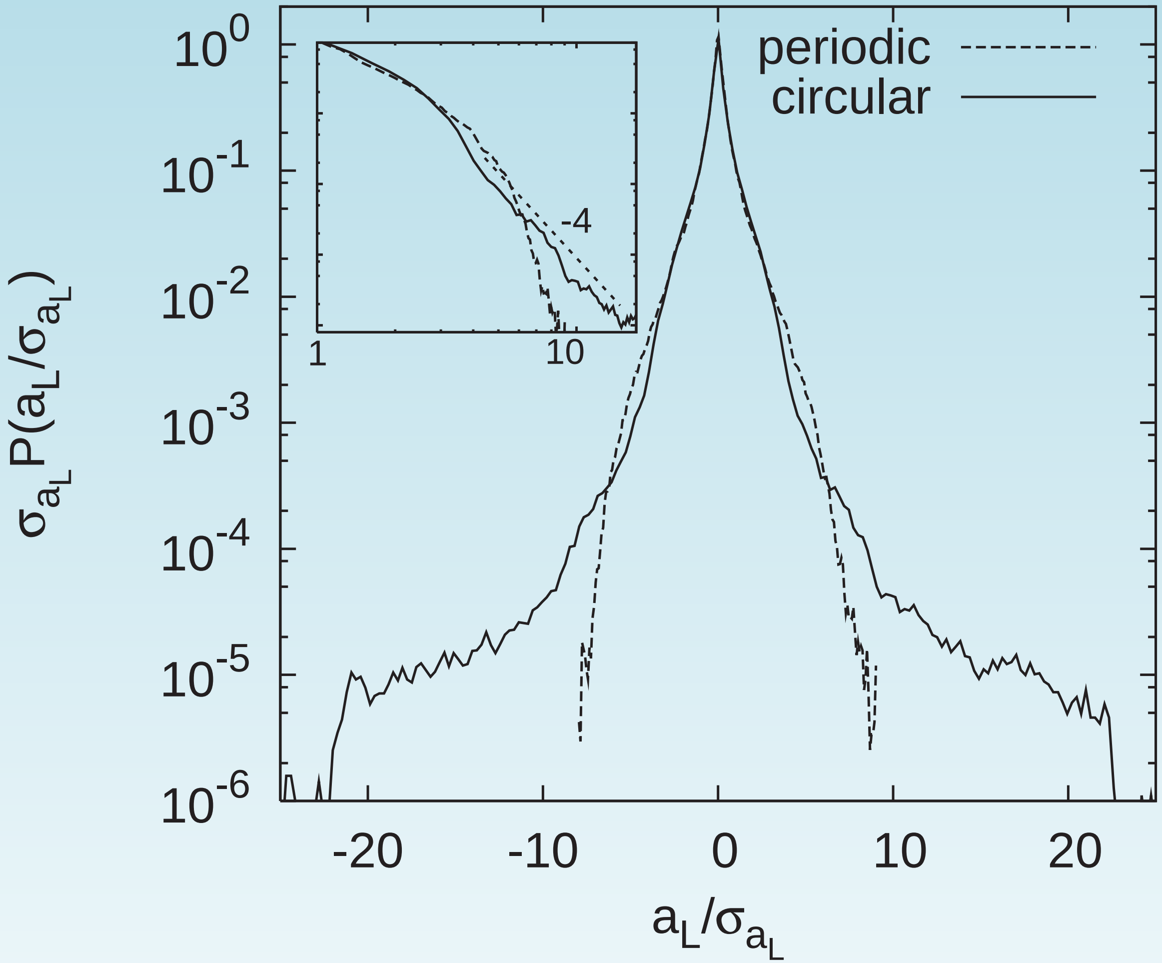

FIG. 9: PDFs of the normalized Lagrangian acceleration a_L/σ_{a_L} where $\sigma_{a_L} = \langle a_L^2 \rangle^{1/2}$ for both cases. Inset : PDFs of the normalized Lagrangian acceleration in double logarithmic scale

Conditional statistics

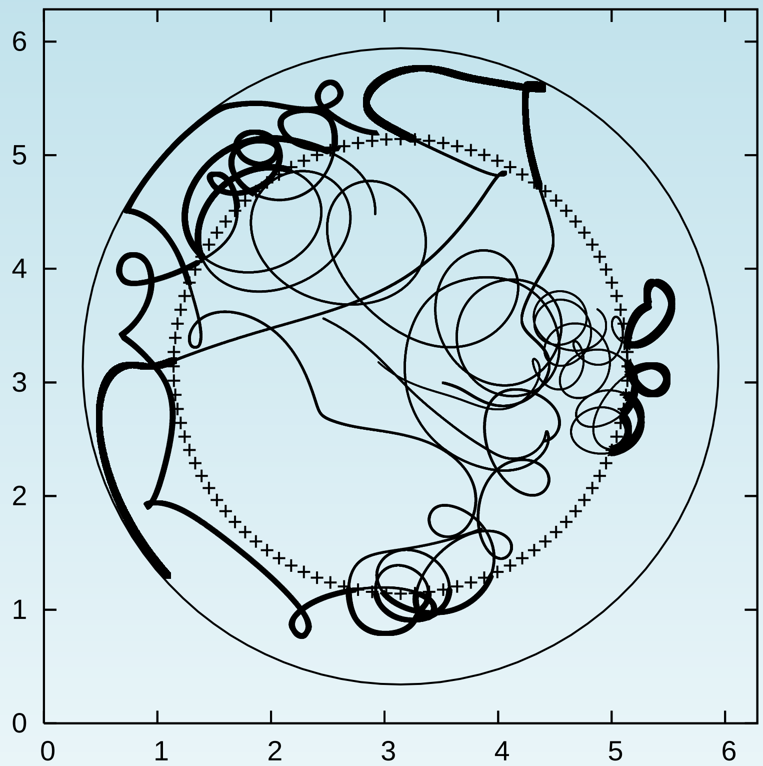

FIG. 10: Trajectories in the circular geometry. The trajectories are divided into particles inside and outside the disk defined by the radius r_0 (circle in dotted line).

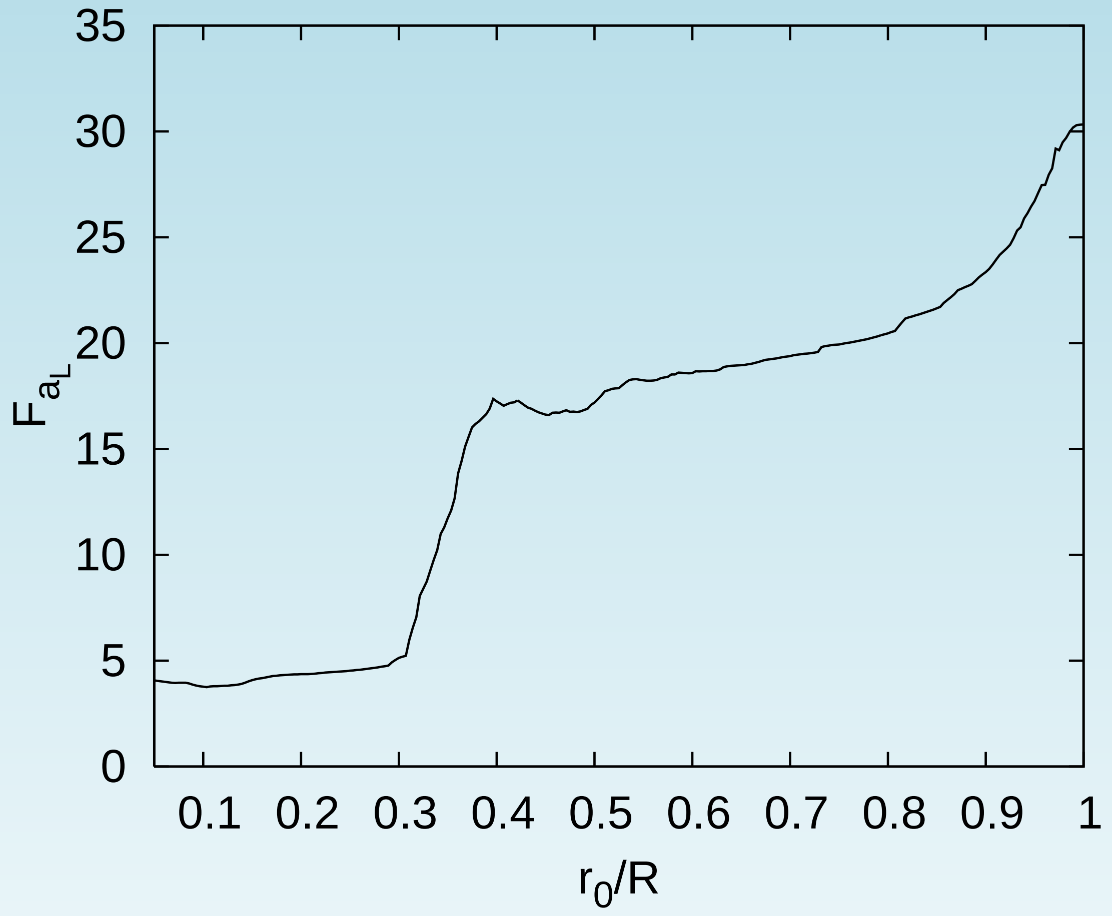

FIG. 11: Conditional flatness of the Lagrangian acceleration as a function of radius r_0/R.

$\longrightarrow$ *Lagrangian boundary layer thickness* δ_L defined by a **critical radius** $r_0/R = 0.3$.

Conclusions

* Lagrangian acceleration in periodic case in 2D similar to 3D.
* Influence of no-slip boundaries on Lagrangian velocity and acceleration :
  - **no significant influence on Lagrangian velocity** except the small cusp around zero in the PDF.
  - **heavy tails in the Lagrangian acceleration PDF** $\longrightarrow$ extreme values .
* Conditional statistics :
  - presence of a ***Lagrangian boundary layer thickness***.
  - Influence of the wall in approximatively 90% of the domain surface.
* B. Kadoch ; W.J.T. Bos ; K. Schneider *Phys. Rev. Lett.* (2008) accepted

Future work

– Influence of Reynolds number

– Comparison with Eulerian quantities

Eulerian acceleration Vs Lagrangian acceleration

– 3D $\longrightarrow$ careful reassessment in experimental results

influence of the wall with conditional statistics in experience ?