Turbulent transport of material particles:
Finite size & density effects

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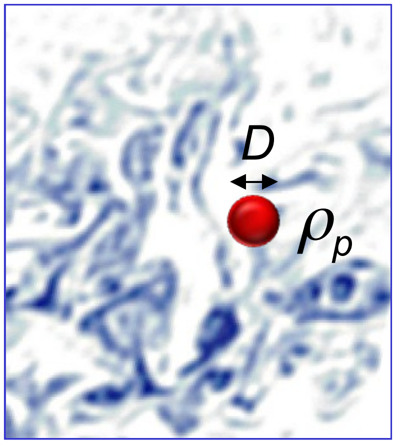
Inertial Particles in turbulence

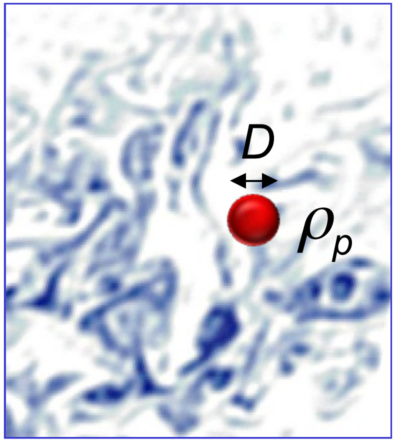


≠ fluid particles

- particles finite size, $D > \eta$?
- particles density, $\rho_p \neq \rho_f$?
- seeding density ?

…

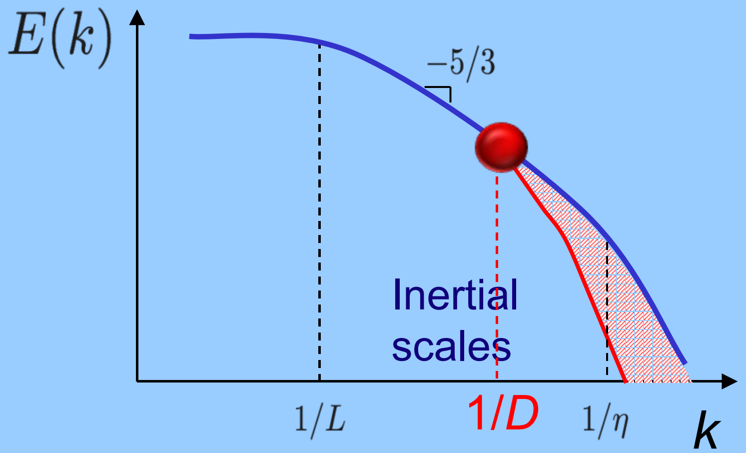
Inertial Particles in turbulence



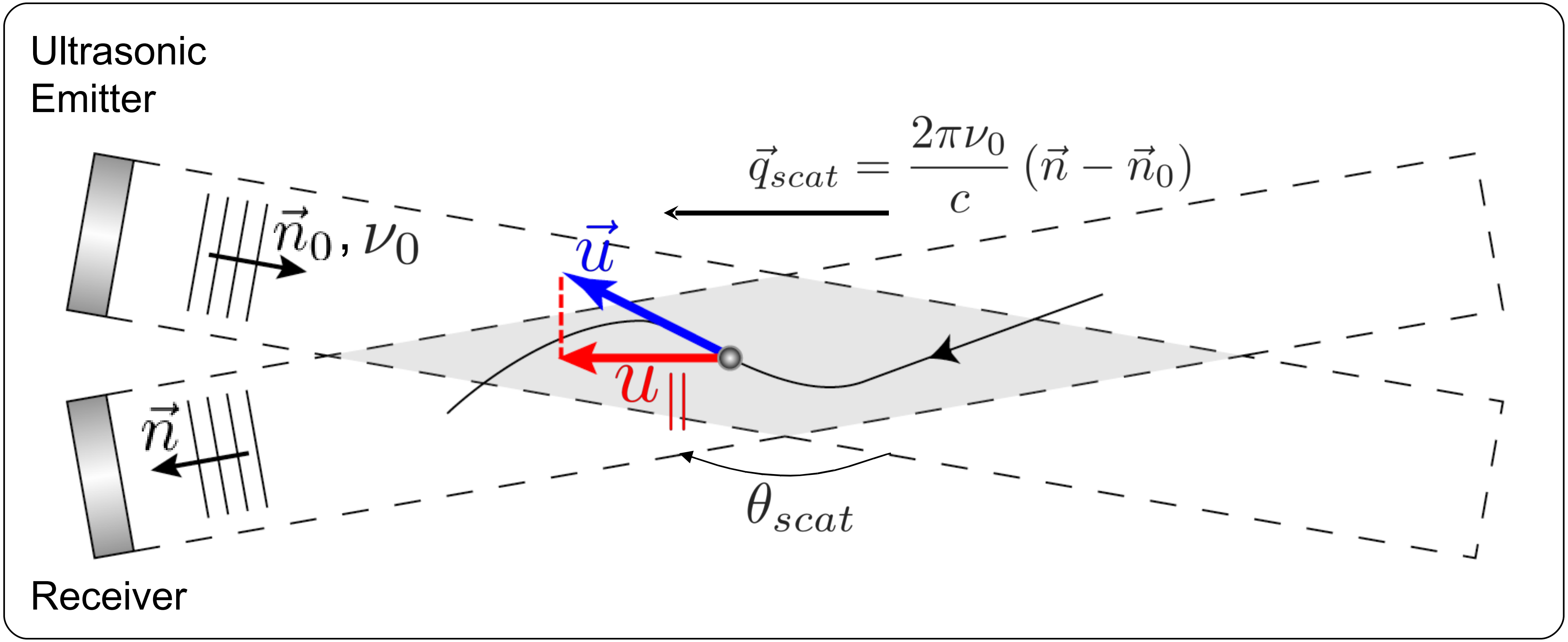
≠ fluid particles

- particles finite size, $D > \eta$?
- particles density, $\rho_p \neq \rho_f$?
- seeding density ?

How is the Lagrangian particles dynamics affected by the filtering of the turbulent structures (in the space domain) due to particles finite size and density ?



Acoustic velocimetry principle



Scattering vector :

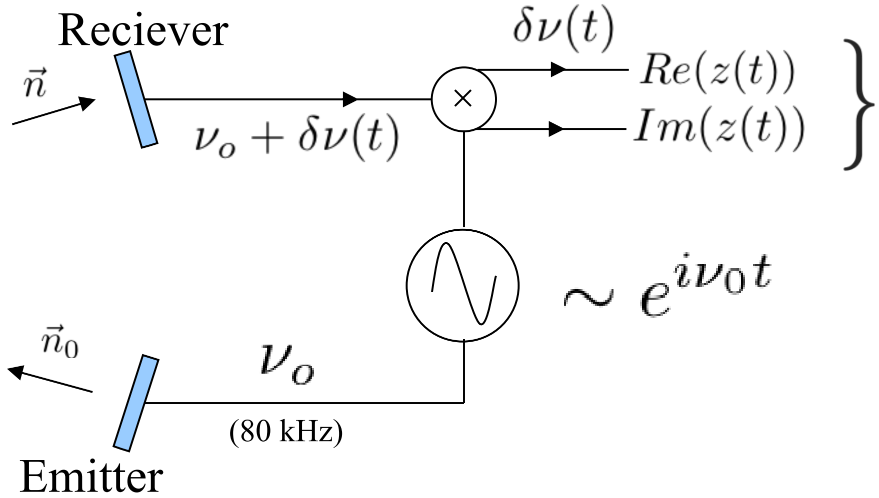
$$\vec{q}_{scat} = \frac{2\pi\nu_0}{c}(\vec{n} - \vec{n}_0)$$

Doppler shift :

$$\delta\nu(t) = \nu(t) - \nu_0 = \frac{1}{2\pi}\vec{q}_{scat} \cdot \vec{u}(t)$$

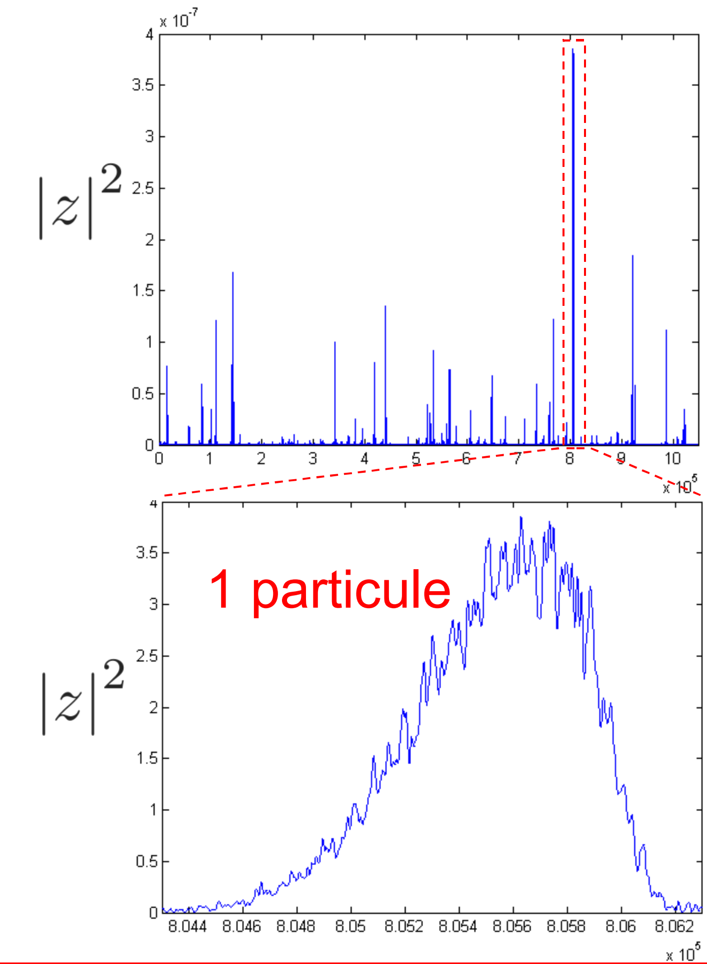
$$u_{//}(t) = \frac{c}{2\nu_0 \sin\left(\frac{\theta_{scat}}{2}\right)}\delta\nu(t)$$

Data Acquisition & Processing

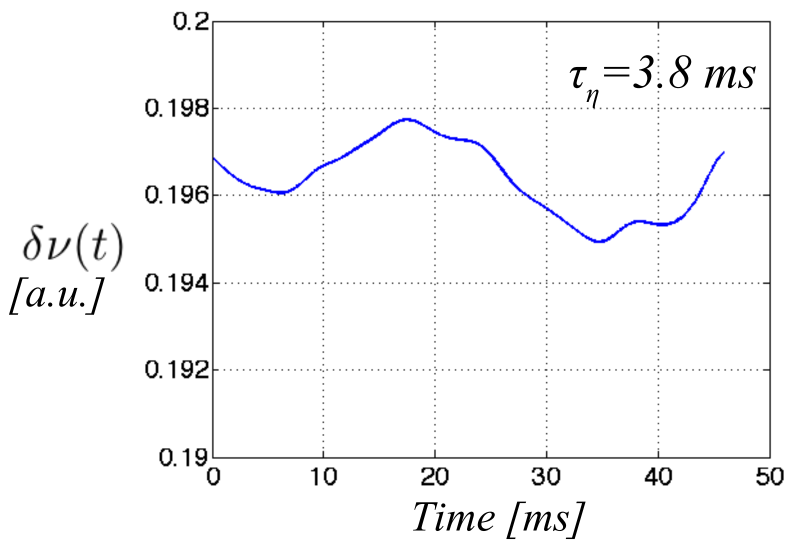


$$z(t) = A(t)e^{i2\pi \int_0^t \delta\nu(t')\mathrm{d}t'}$$

Complex Signal



Maximum de Vraisemblance Approchée
MVA

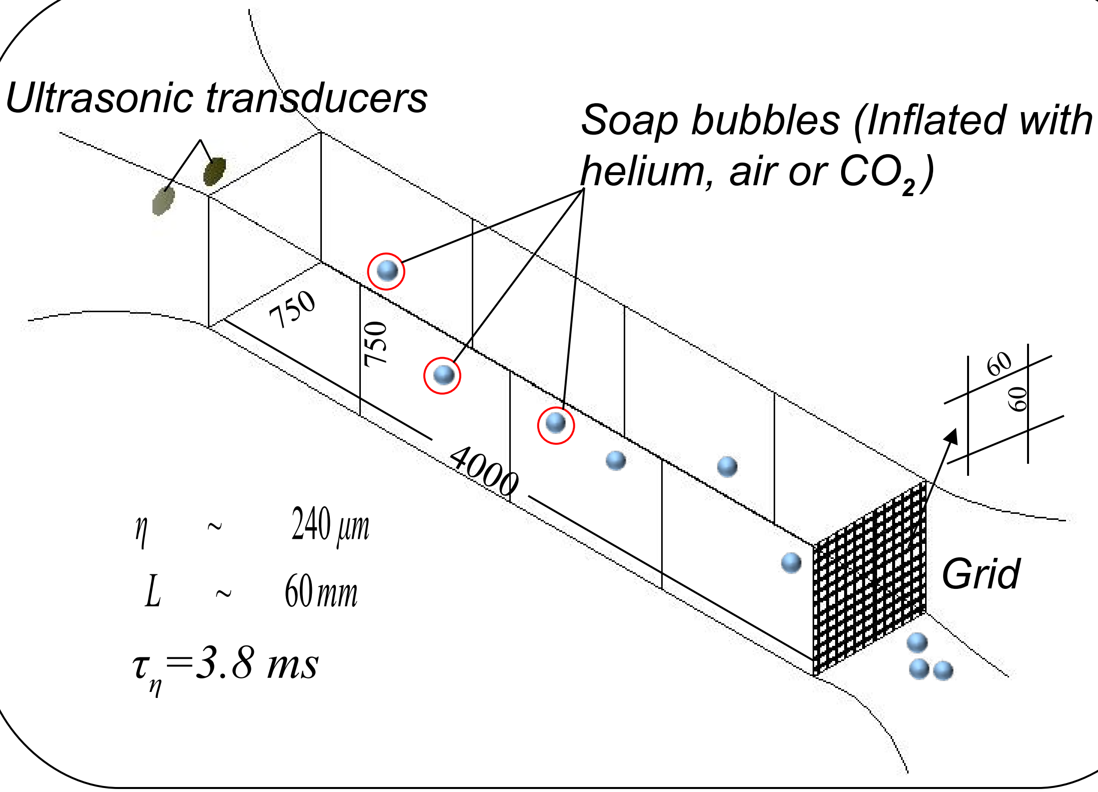


Grid Generated Turbulence in a Wind Tunnel

Isotropic homogeneous turbulent flow

$U \sim 15 \ \ \mathrm{m \cdot s^{-1}}$ $\qquad \dfrac{u_{\mathrm{rms}}}{U} \sim 3\%$

$R_\lambda \sim 160$



$\eta \sim 240\ \mu m$

$L \sim 60\, mm$

$\tau_\eta = 3.8\ ms$

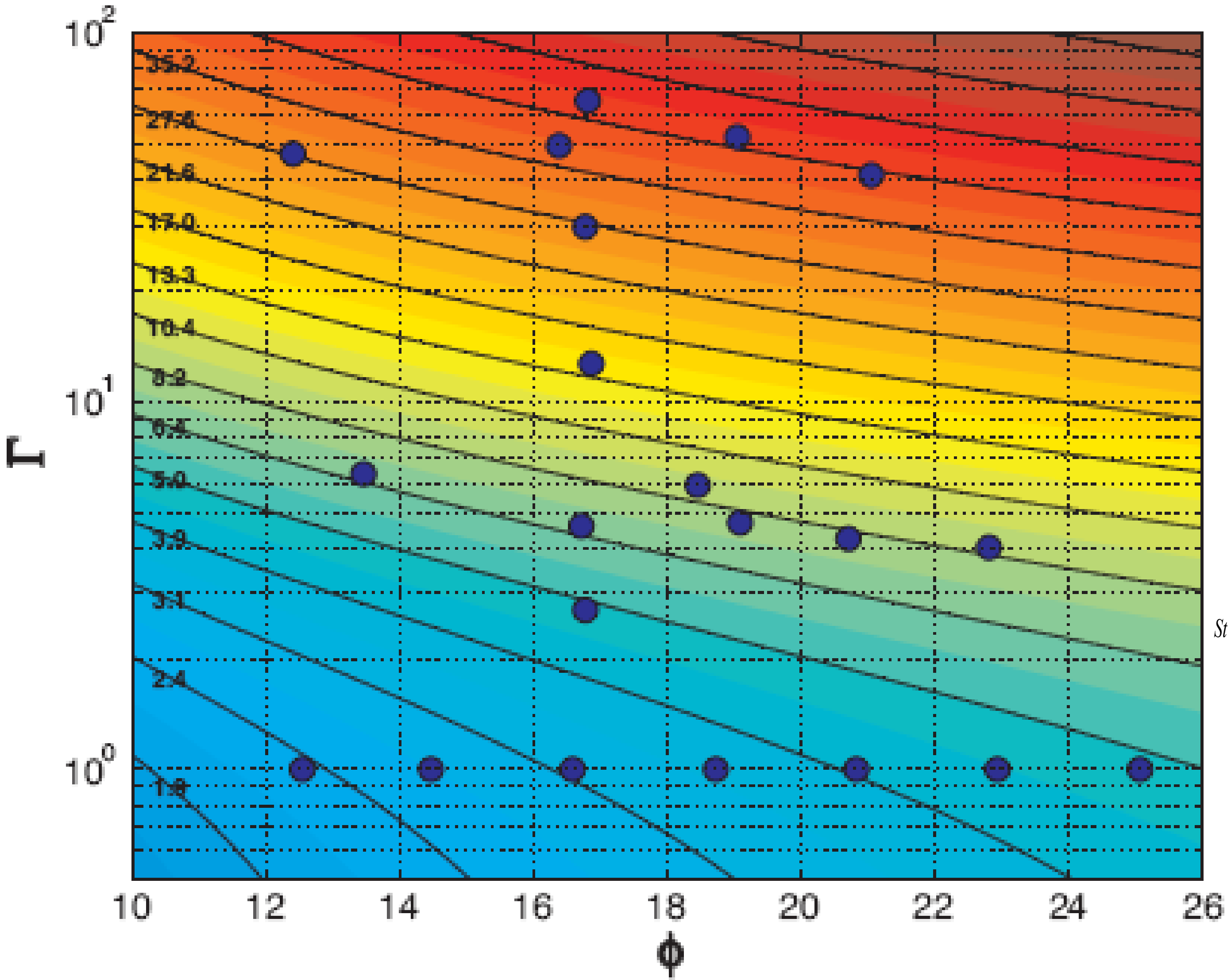
Inertial range sizes

$D = 3 \longrightarrow 6mm$

$12.5\,\eta \longrightarrow 25\,\eta$

$L/20 \longrightarrow L/10$

Studied Particle Sizes, Densities and Stokes Number



$(\varphi = D/\eta)$

$\Gamma = \rho_p / \rho_f$

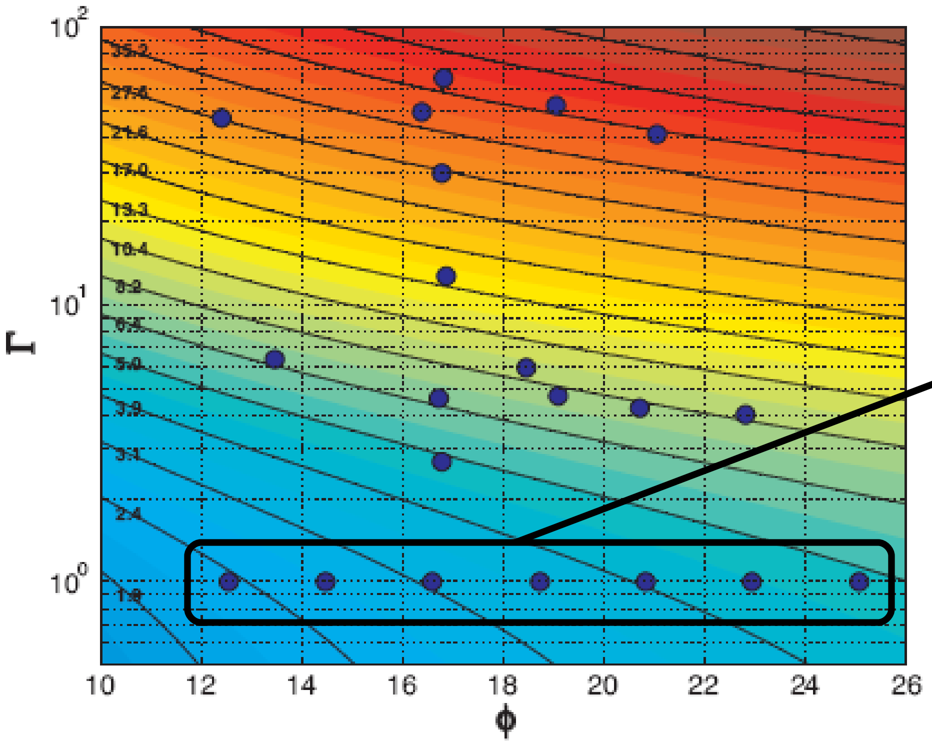
$St = \tau_p / \tau_d$

$Re_p = (\varphi)^{4/3}$

$20 < Re_p < 260$

$$St \equiv \frac{\tau_p}{\tau_d} = \frac{1}{18}\left(\frac{\rho_p}{\rho_f}\right)\frac{Re_p}{1+0.1935\ Re_p^{0.6305}}$$

Finite Size Effects (Neutrally Buoyant)

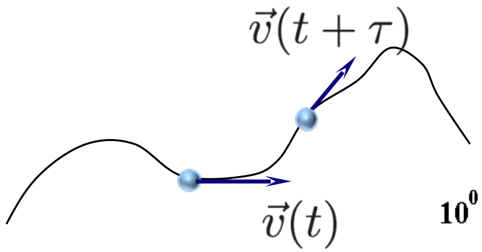


$D \sim 12.5\,\eta \longrightarrow 25\,\eta$

$L/20 \longrightarrow L/10$

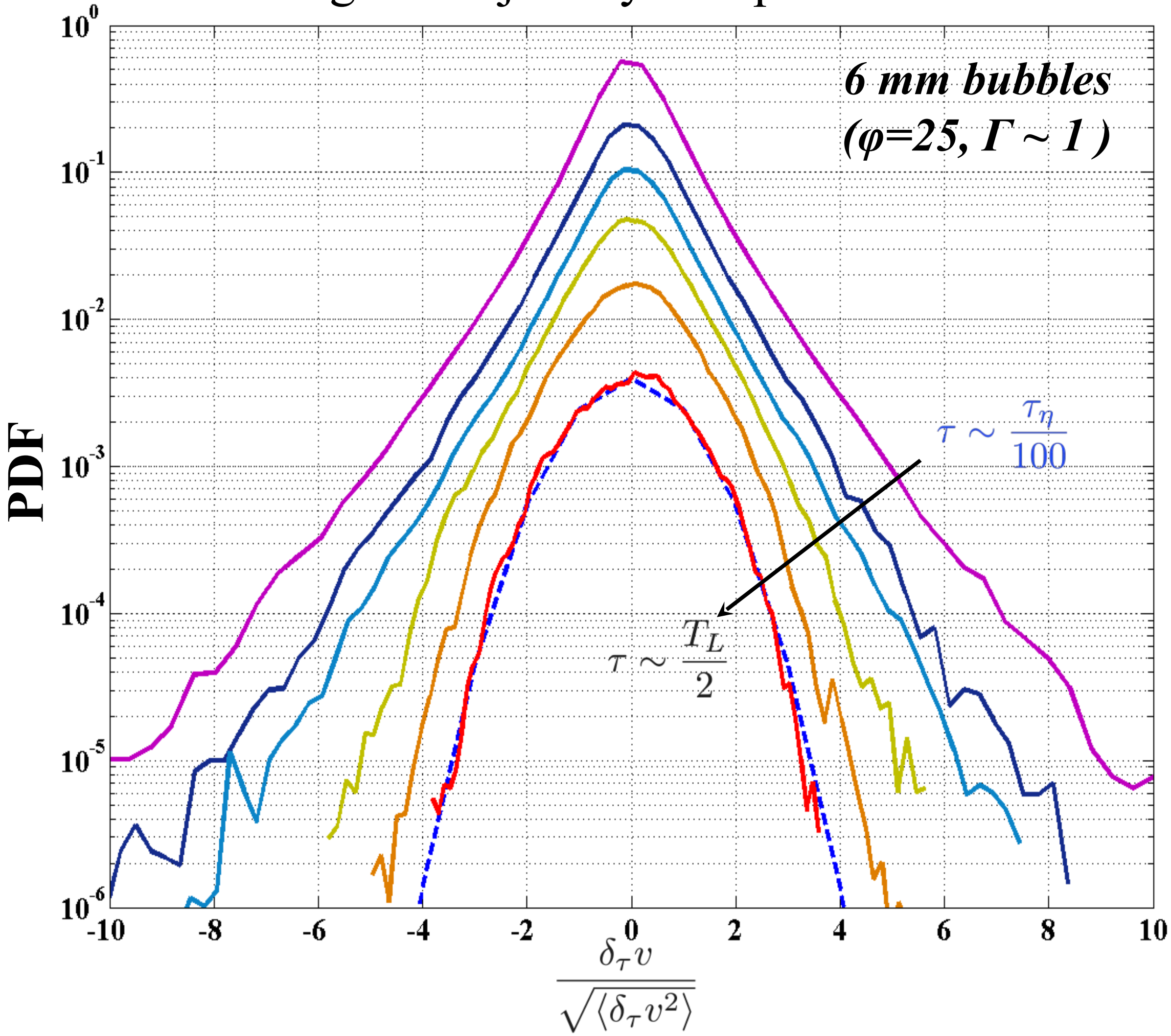
$\Gamma \sim 1$

Lagrangian Velocity Increments

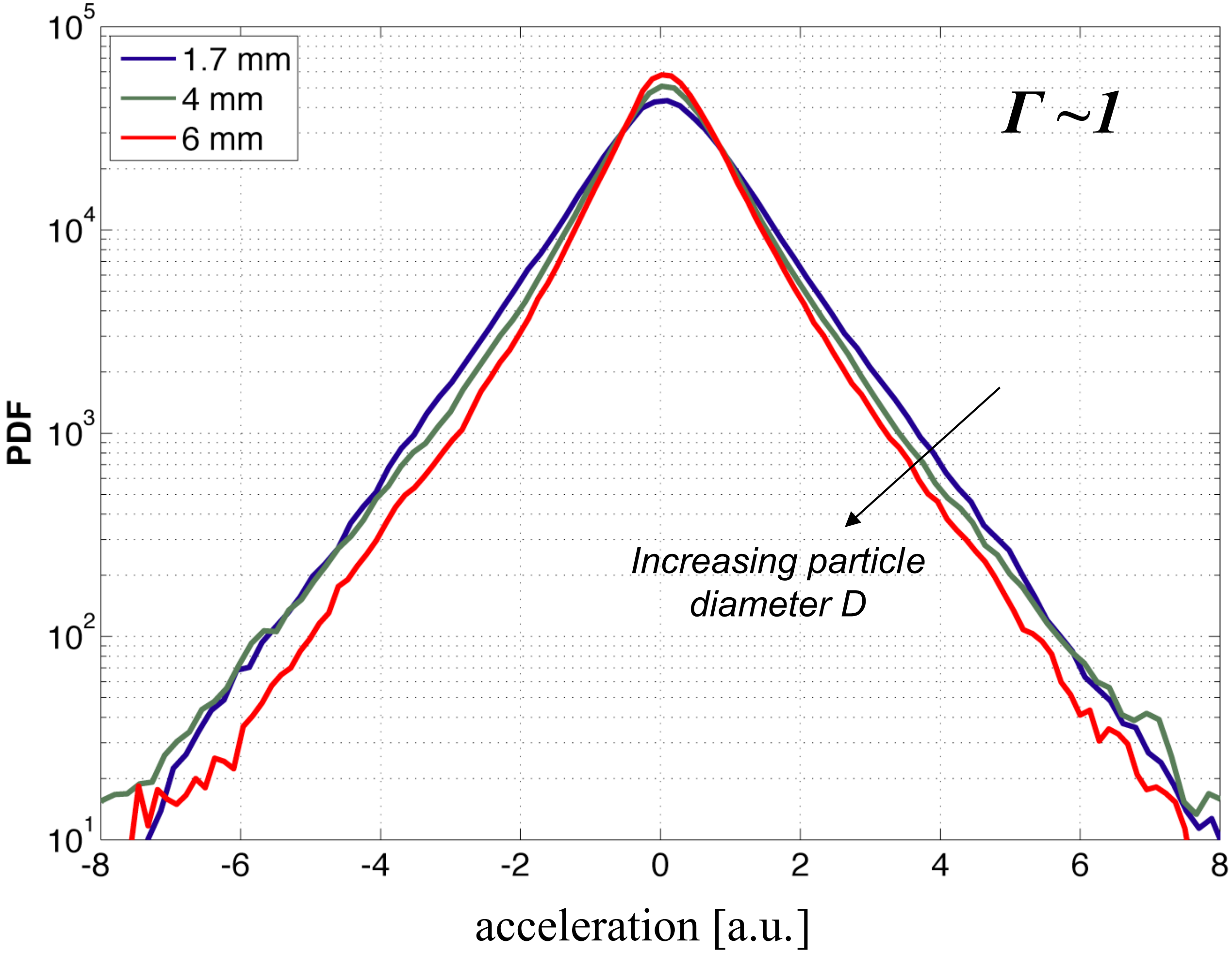


$$\delta_\tau v(t) = v(t+\tau) - v(t)$$

Along the trajectory of a particle

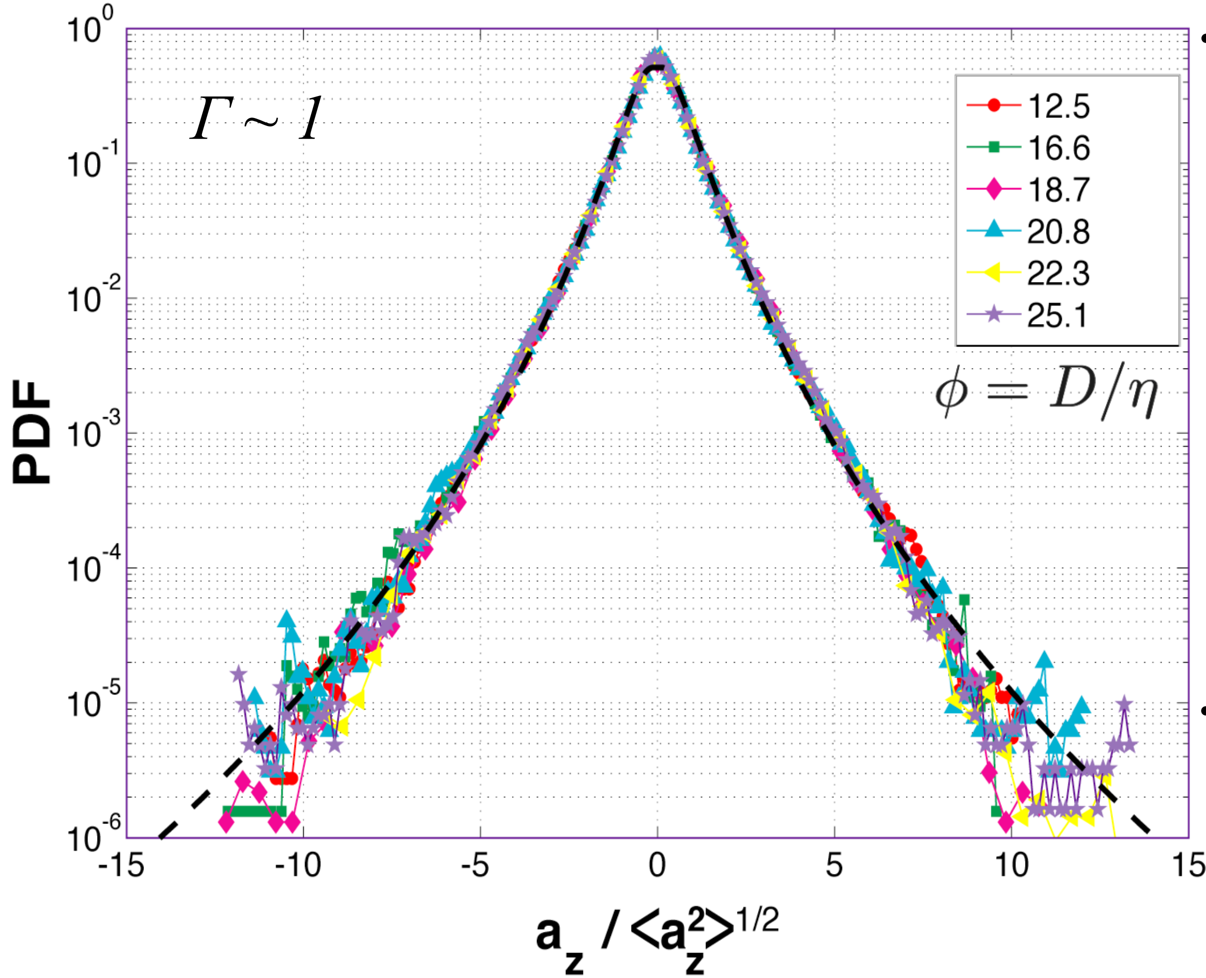


Acceleration PDF (non normalized)



- **PDF** peaks narrows as *D* increases
- Acc. variance decreases with increasing *D*

Acceleration PDF (normalized to variance 1)



- The global shape of the normalized PDF **does not** depend on particle size.
- Correctly described by

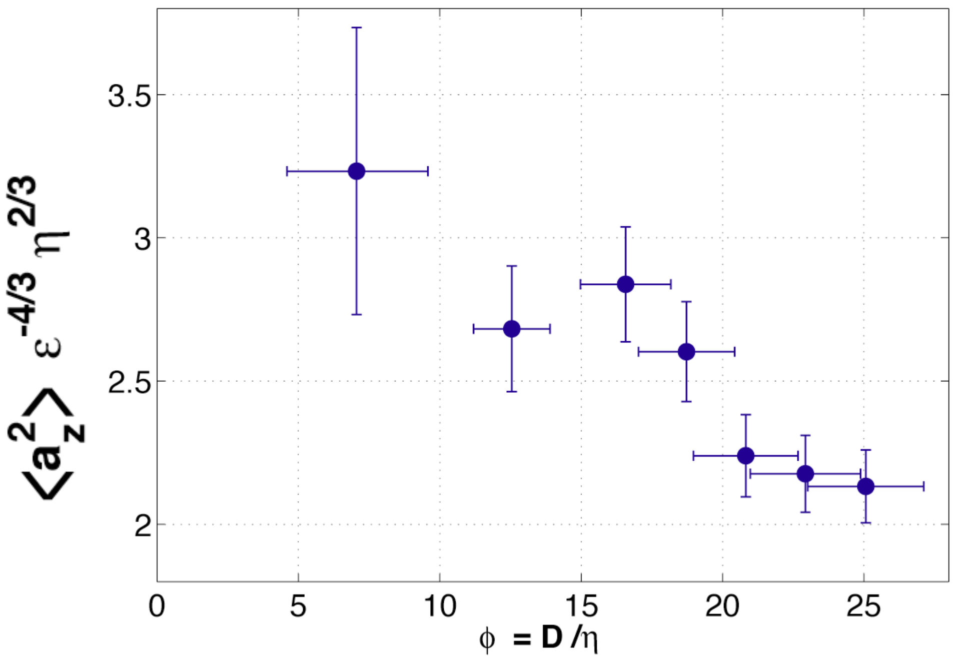
$$\mathcal{P}(x) = \frac{e^{3s^2/2}}{4\sqrt{3}} \left[1 - \mathrm{erf}\left(\frac{\ln\left(\left|x/\sqrt{3}\right|\right) + 2s^2}{\sqrt{2}s} \right) \right]$$

⇨ Lognormal amplitude

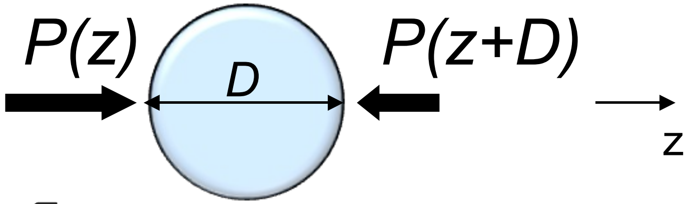
- **Non-gaussian** even for large particle sizes

⇨ **Not trivially related with velocity intermittency**

Acceleration variance (Neutrally Buoyant)



Acceleration = Pressure increments



$$F_z = \frac{\pi}{6} \rho D^3 a_z \propto D^2 \left[P(z+D) - P(z) \right]$$

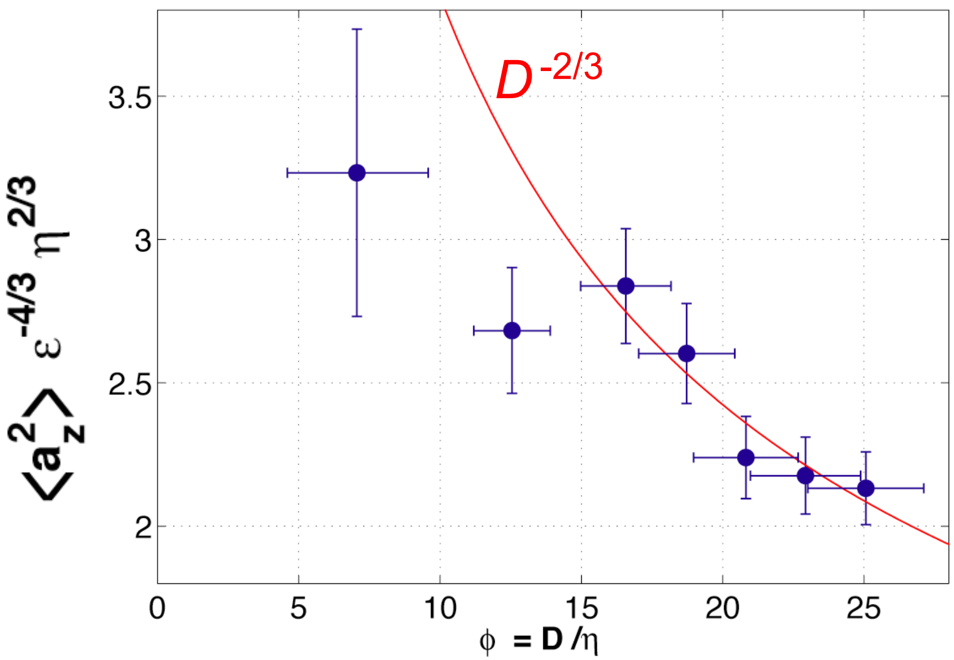
$$\langle a_z^2 \rangle_{\mathrm{part},D} \propto \frac{S_2^P(D)}{D^2}$$

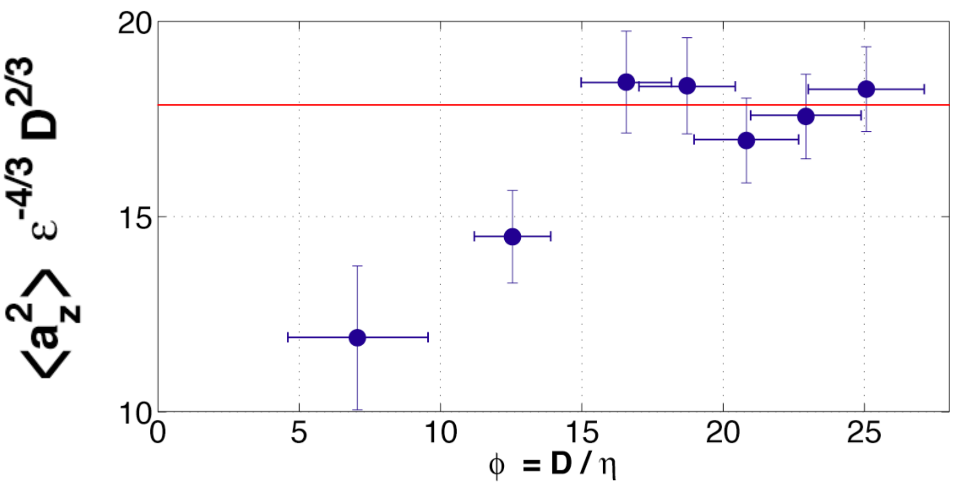
- **K41 inertial scaling**

$$S_2^P(r) = \langle (P(z+r) - P(z))^2 \rangle \propto (\epsilon r)^{4/3}$$

$$\langle a_z^2 \rangle_{\mathrm{part},D} = a_0' \epsilon^{4/3} D^{-2/3}$$

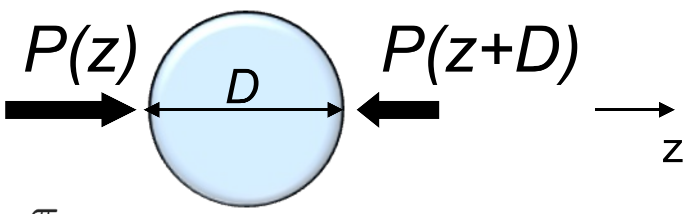
Acceleration variance (Neutrally Buoyant)





a_0'

Acceleration = Pressure increments



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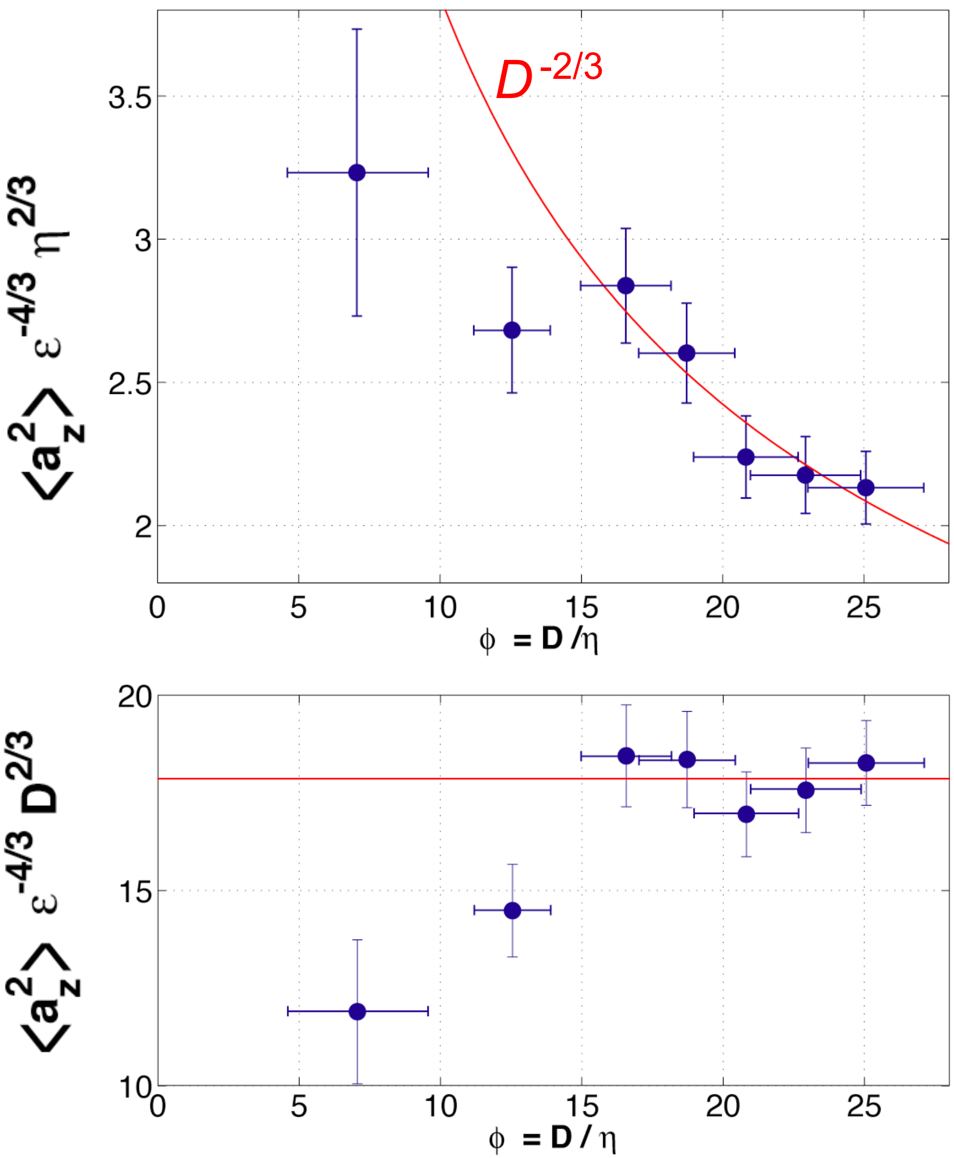

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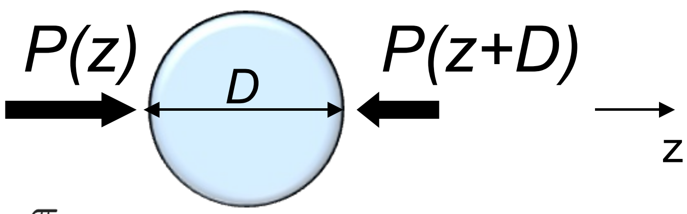

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a_0'

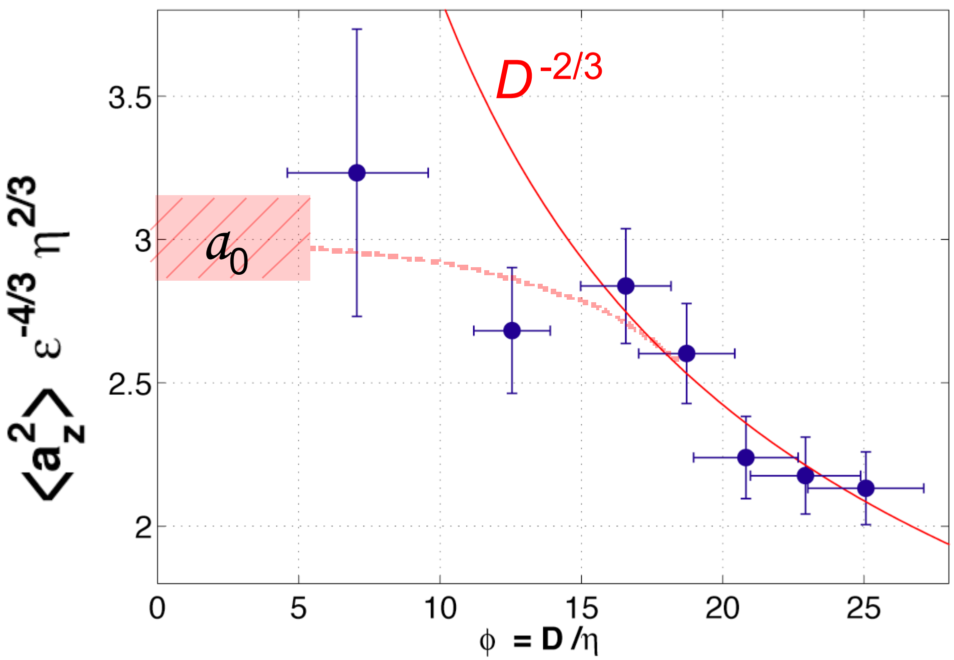
$$\langle a_z^2 \rangle_{\mathrm{part},D} = a_0' \epsilon^{4/3} D^{-2/3}$$

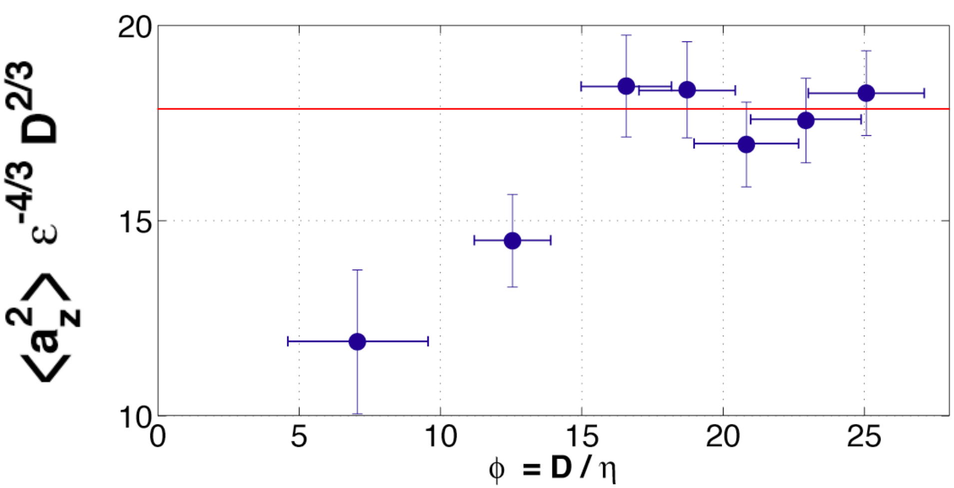
- ***K41 dissipative scaling***

$$\frac{S_2^P(D)}{D^2} \to \left\langle \vec{\nabla} P^2 \right\rangle \propto \epsilon^{4/3} \eta^{-2/3}$$

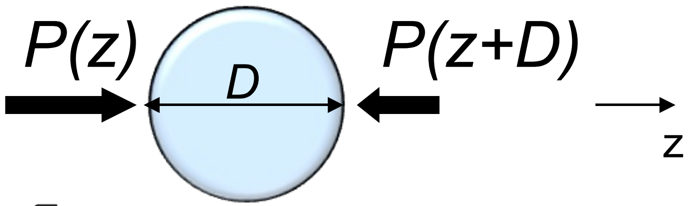
$$\langle a_z^2 \rangle_{\mathrm{fluid}} = a_0 \epsilon^{4/3} \eta^{-2/3}$$

Acceleration variance (Neutrally Buoyant)





Acceleration = Pressure increments



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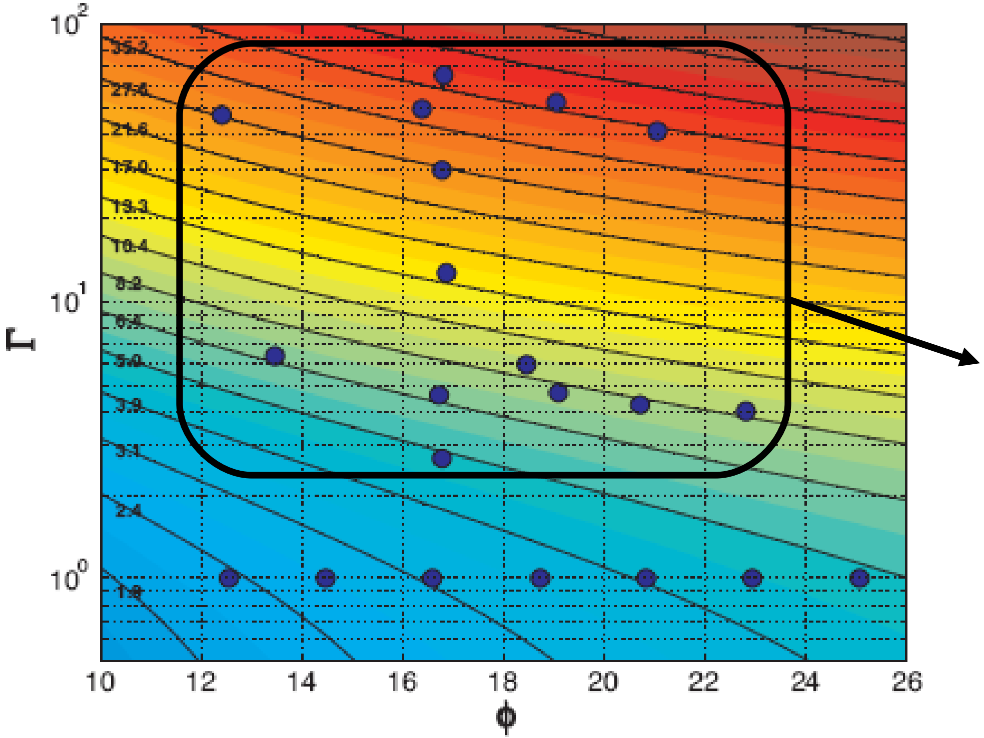
Density Effects

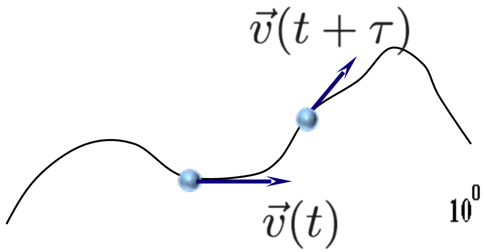


D ~ 13.5 η → 23 η

L / 18 → L / 11

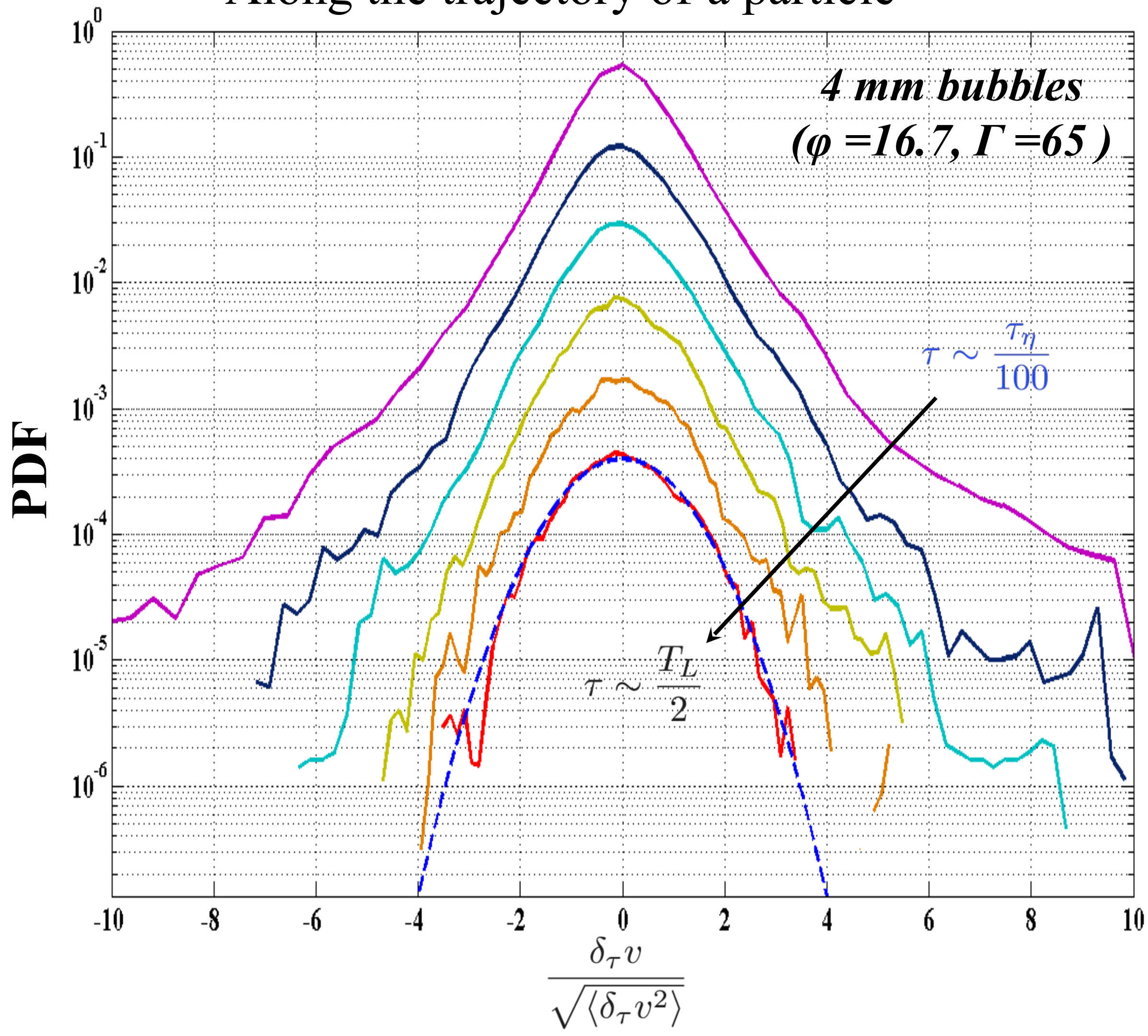
Γ ~ 5 → 65

Lagrangian Velocity Increments

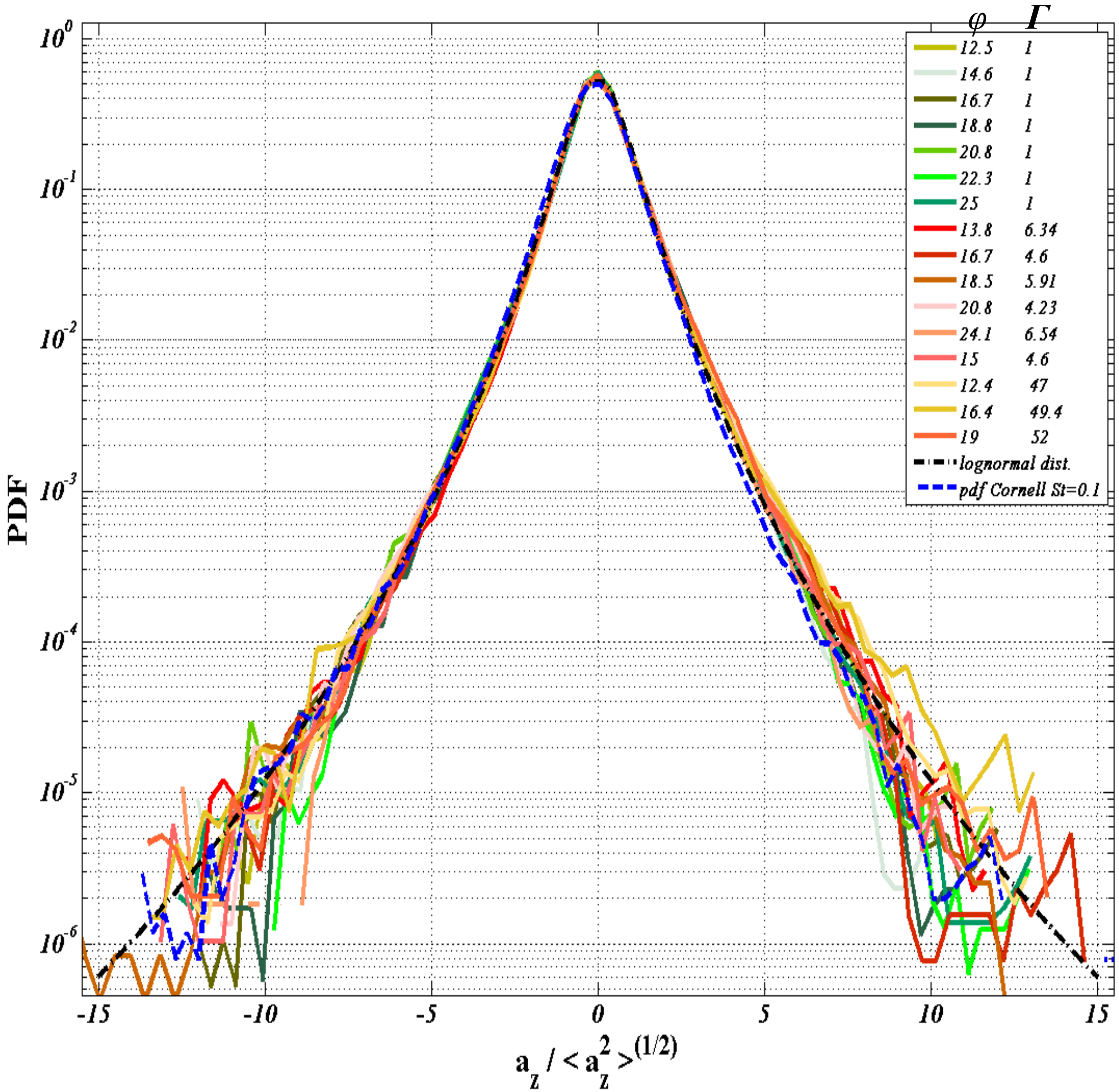


$$\delta_\tau v(t) = v(t+\tau) - v(t)$$

Along the trajectory of a particle



Acceleration PDF (normalized to variance 1)



- **LEGI Experiments**

  $D \sim 12.5\,\eta \longrightarrow 25\,\eta$

  $L / 20 \longrightarrow L / 10$

  $\Gamma \sim 1 \longrightarrow 65$, $St \sim 0.6 \longrightarrow 38$

  $R_\lambda = 160$

- **Experiments Cornell (Warhaft)**

  $D \sim 0.05\,\eta$, $\Gamma \sim 1000$

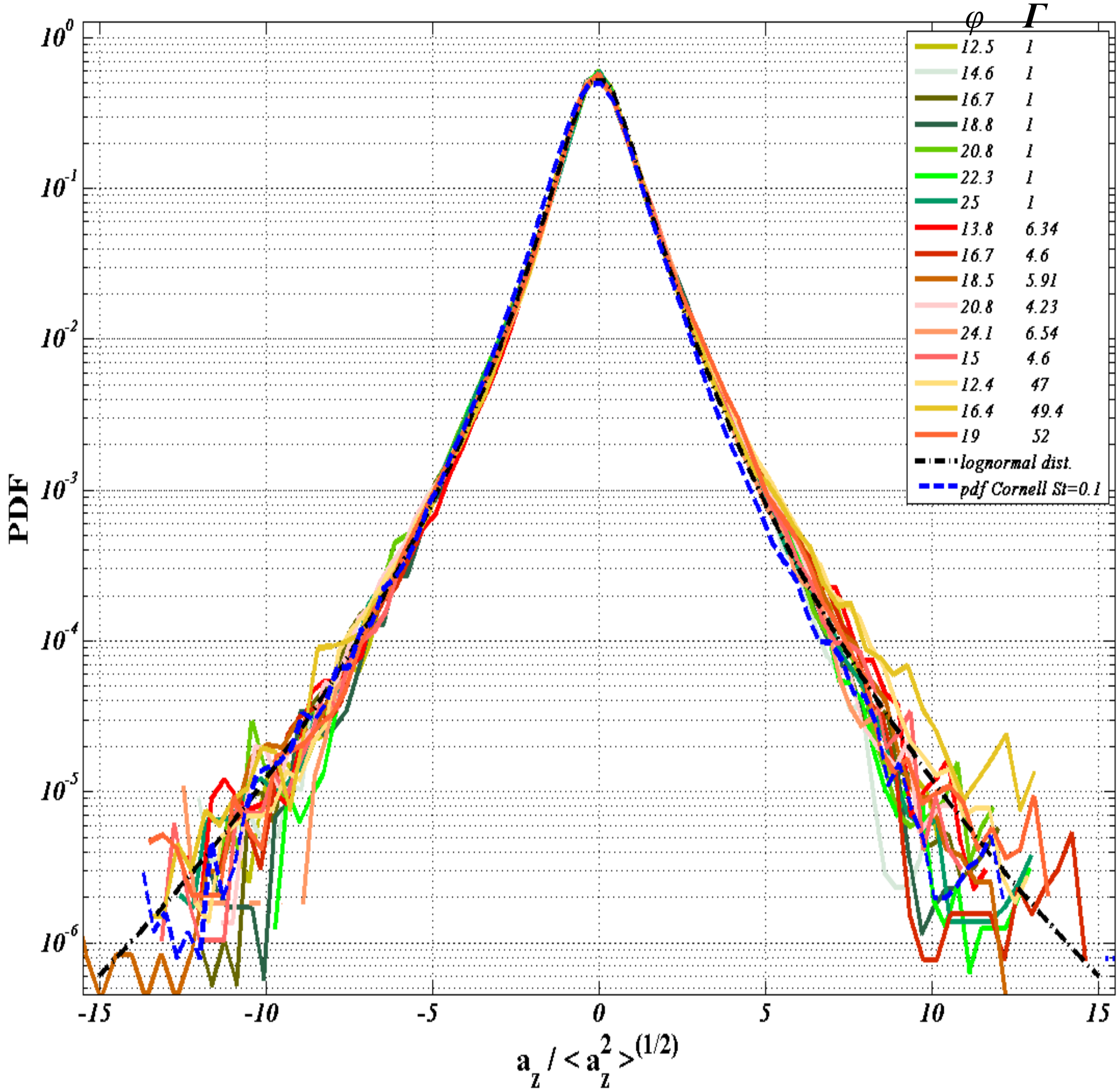
  $R_\lambda = 250$, $St \sim 0.1$

  (water droplets)

- **All normalised PDFs match each other within the statistical error.**

- **Finite size particles acceleration being a physical quantity illustrates an statistical signature for a wide range of sizes and densities**

Acceleration PDF (normalized to variance 1)

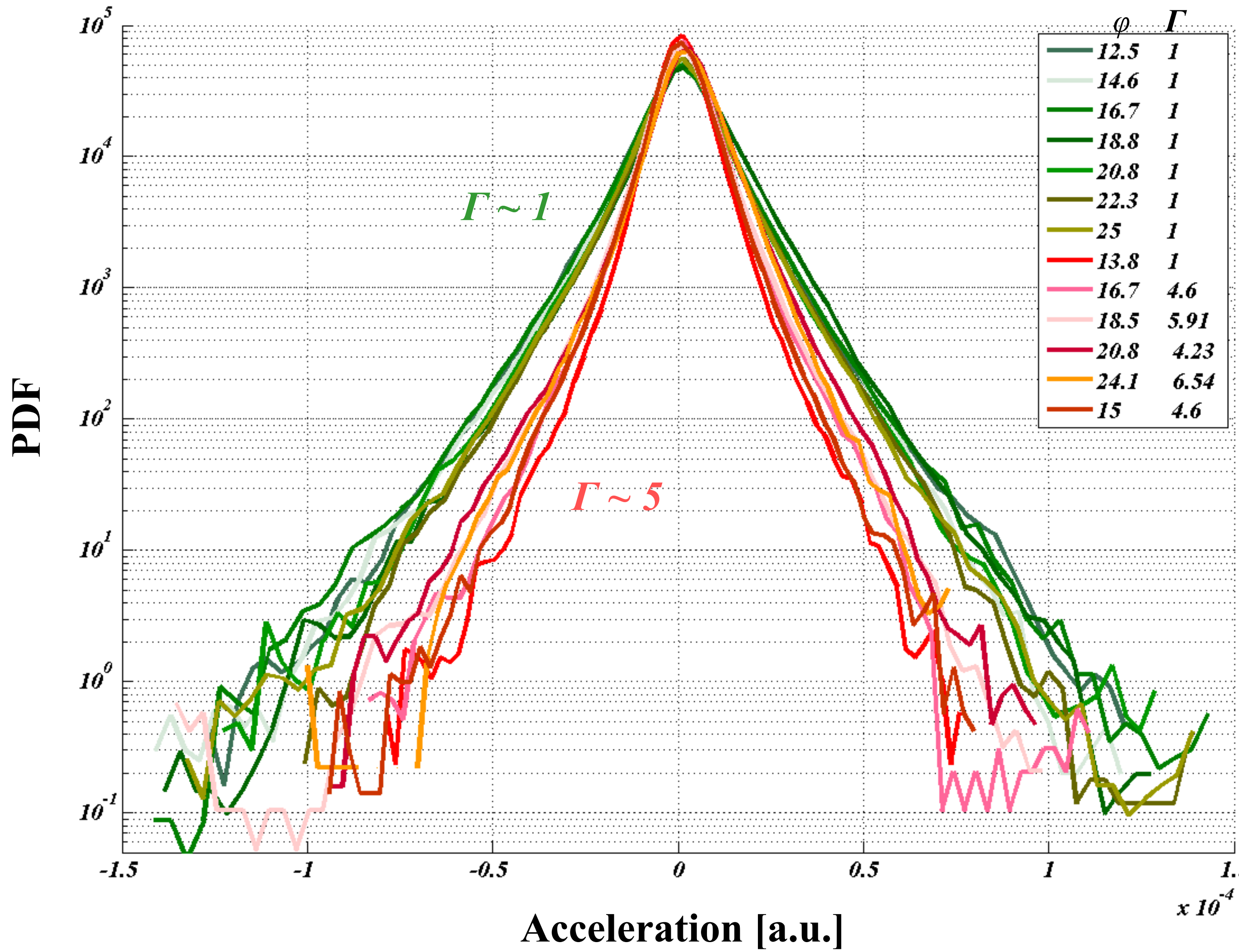


- **Log-normal Distribution**

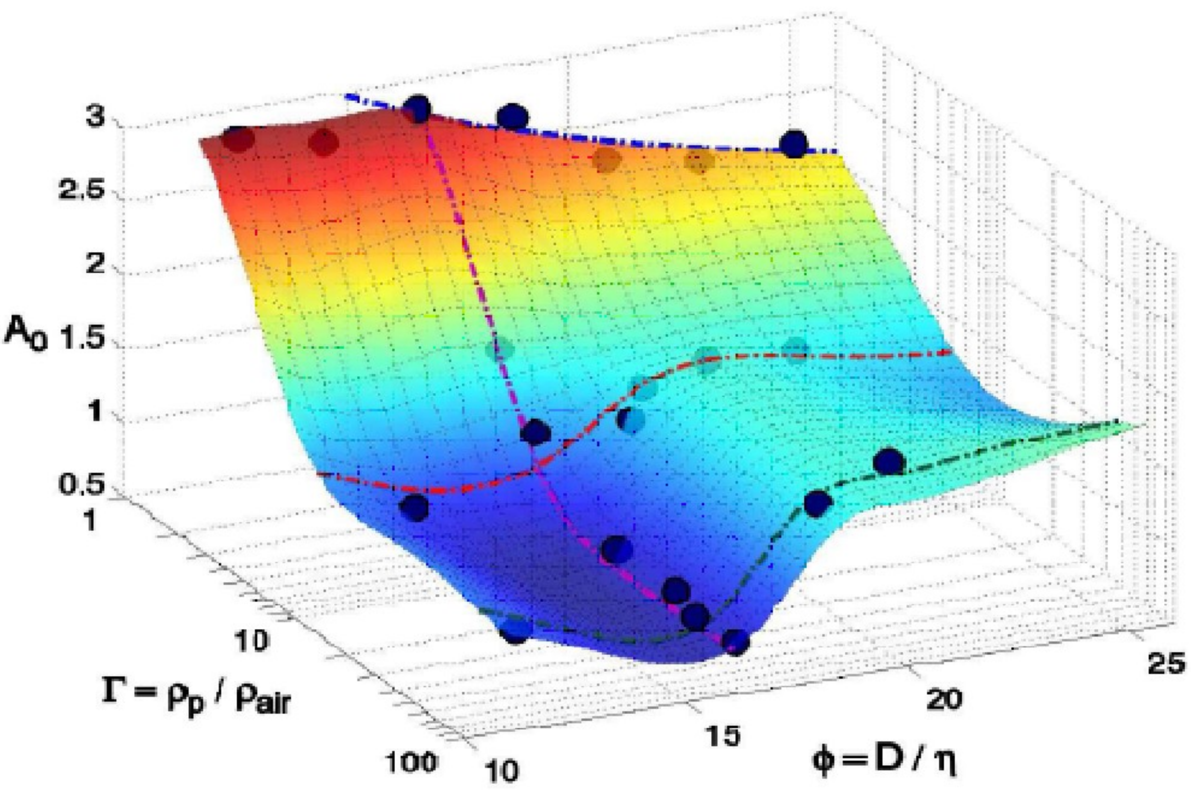
$$\mathcal{P}(x) = \frac{e^{3s^2/2}}{4\sqrt{3}}\left[1 - \mathrm{erf}\left(\frac{\ln\left(\left|x/\sqrt{3}\right|\right) + 2s^2}{\sqrt{2}s}\right)\right]$$

- ***s ~ 0.62 & F ~ 8.4***
- ***F ~ 8.4 << 30***
- ***F ~ 30 fluid particles St < 0.01***
- ***Sudden transition??***

Acceleration PDF (non normalized)



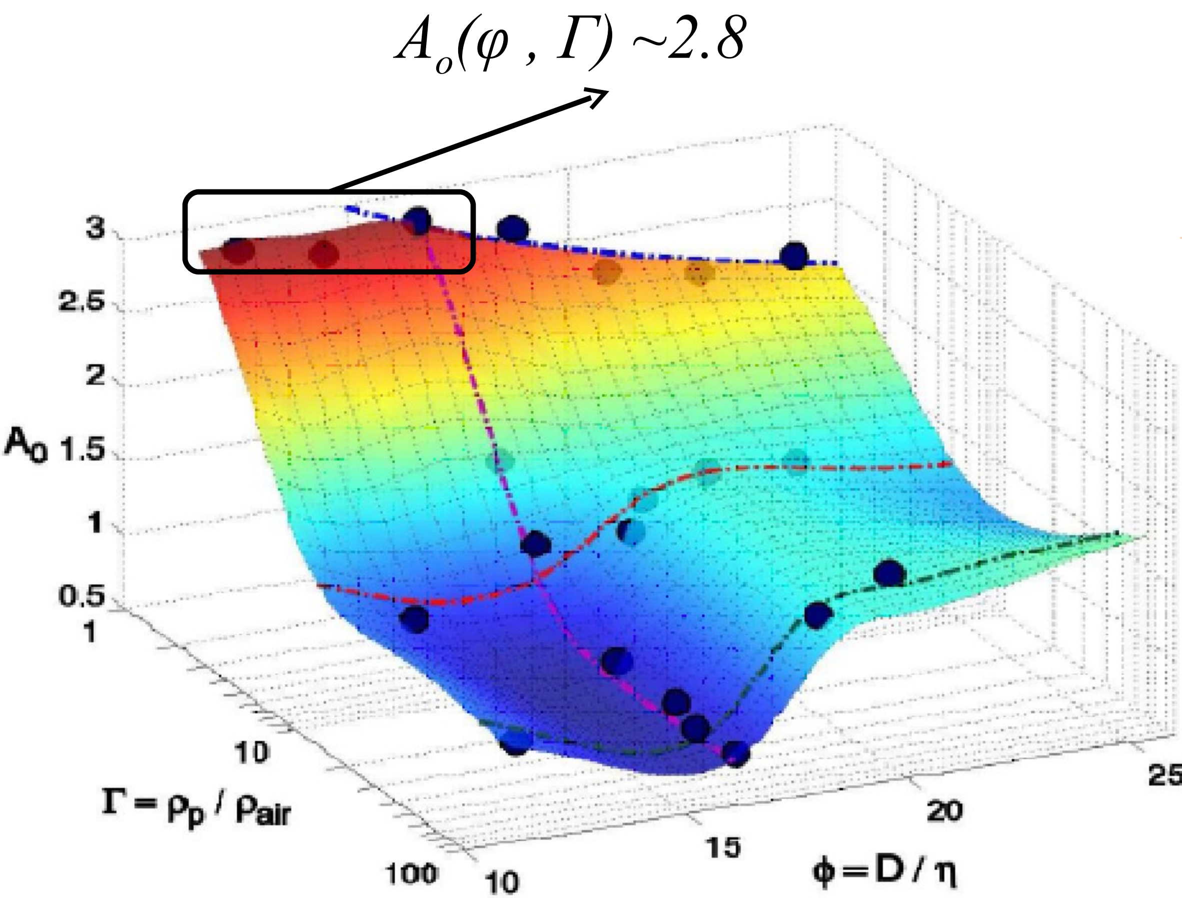
Acceleration Variance Normalised f(φ, Γ)



- **Heisenberg-Yaglom**

$$A_0(\phi, \Gamma) = \langle a_z^2 \rangle \epsilon^{-3/2} \nu^{1/2}$$

Acceleration Variance Normalised f(φ, Γ)



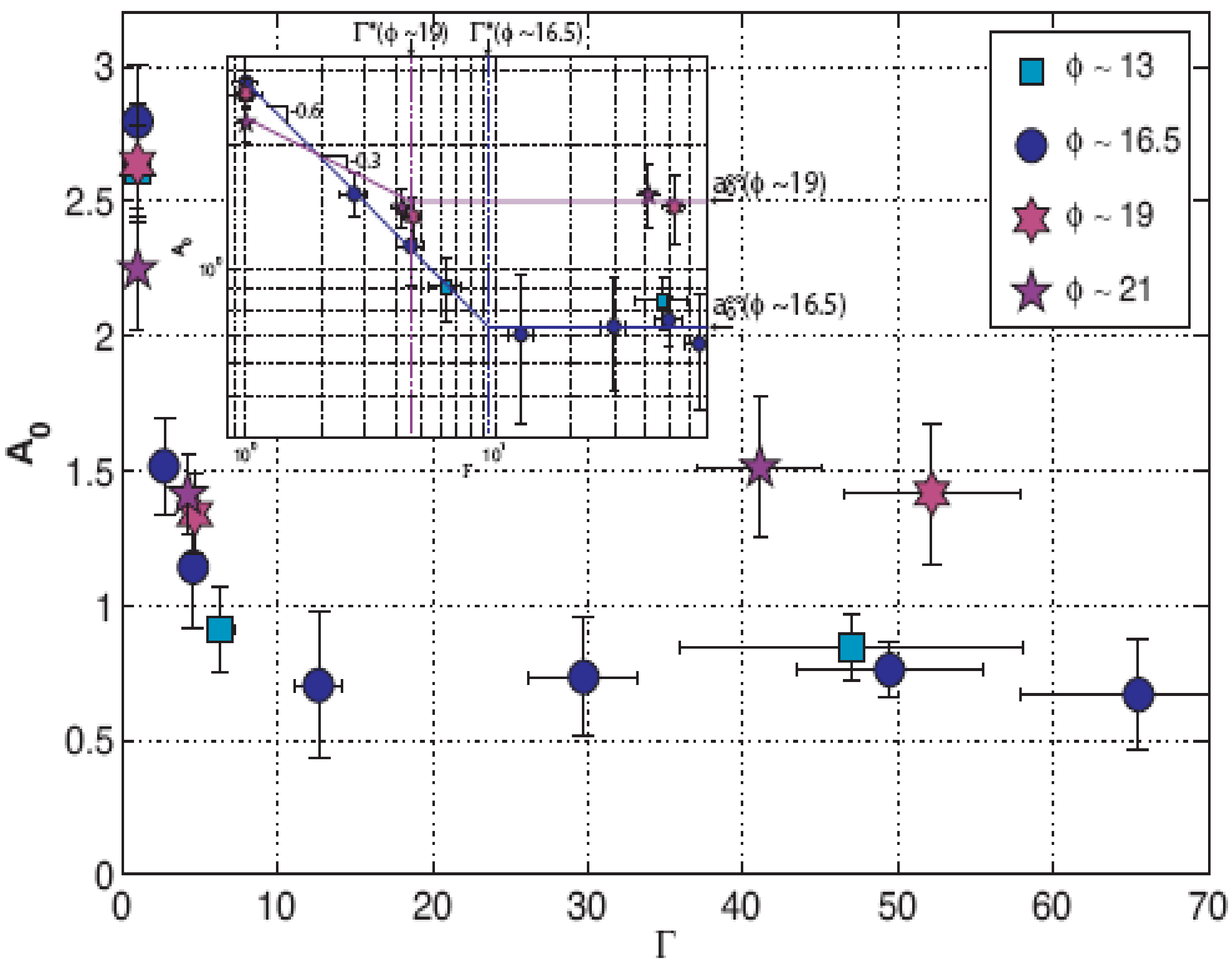
- **Heisenberg-Yaglom**

$$A_0(\phi, \Gamma) = \langle a_z^2 \rangle \, \epsilon^{-3/2} \nu^{1/2}$$

- **$A_o(\varphi, \Gamma) \sim 2.8$**

  Consistent with Voth et al (Cornell)

- **$A_o(\varphi_1, \Gamma)$, $A_0 \downarrow$**



- **Low densities**

$$A_0(\phi,\Gamma) = a_0(\phi)\Gamma^{\alpha}$$

- **Higher densities**

$$A_0 \sim a_0^{\infty}(\varphi)$$

$$a_0^{\infty}(13) \approx a_0^{\infty}(16.5)$$

$$\Gamma^*(13) \approx \Gamma^*(16.5) \geq \Gamma^*(19) \approx \Gamma^*(21)$$

- **Transition at Γ^***
- **Size dependance of α ??**

Conclusion

- Finite sized and heavy particles ($D > \eta$) and ($\Gamma > 1$) still have an ***intermittent dynamics***

- ***Normalized Acceleration PDF does not depend on size and density; remains non gaussian***

Finite size and density effects are not trivially related to velocity intermittency

- ***Acceleration*** ⟺ ***Eulerian Pressure Increments*** at scale D (for $\Gamma = 1$)

$a^2_{rms} \sim D^{-2/3}$ $S_2(r) \sim r^{4/3}$ (K41)

- ***LES*** models for turbulent transport of finite sized particles

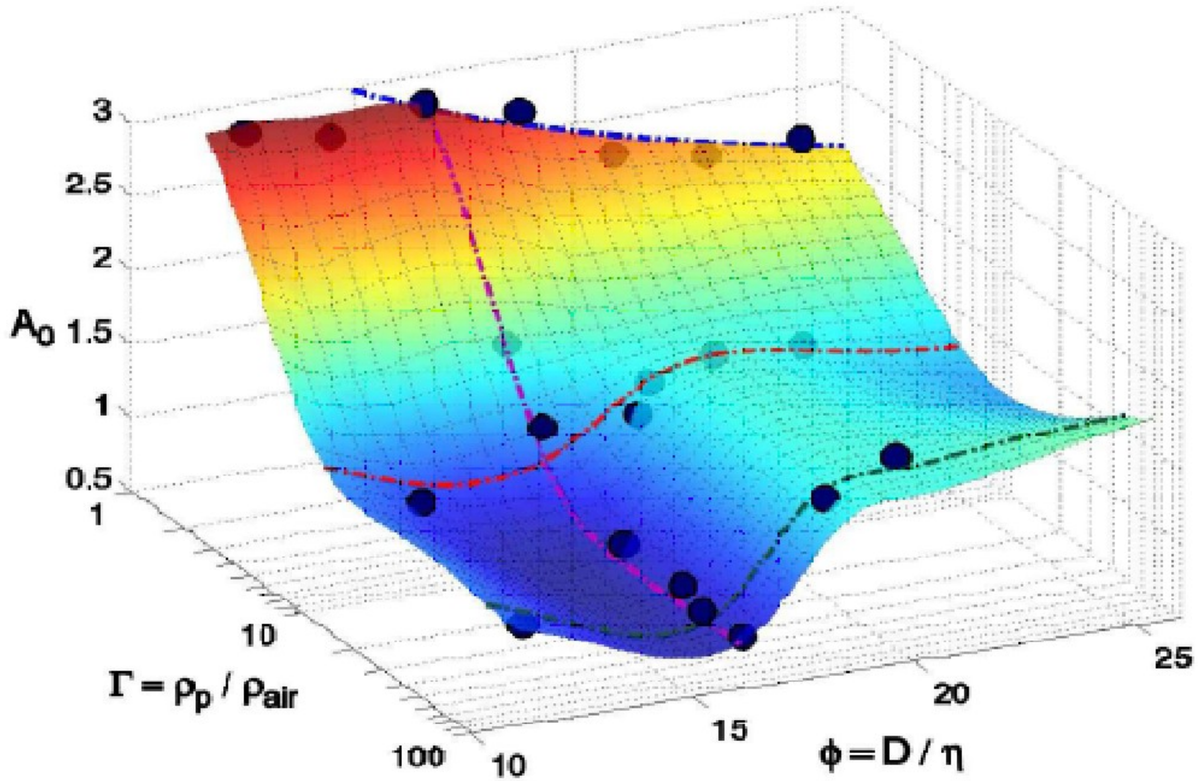
Perspectives

- Relation Lagrangian - Eulerian via the pressure field ?

- Density effects ?

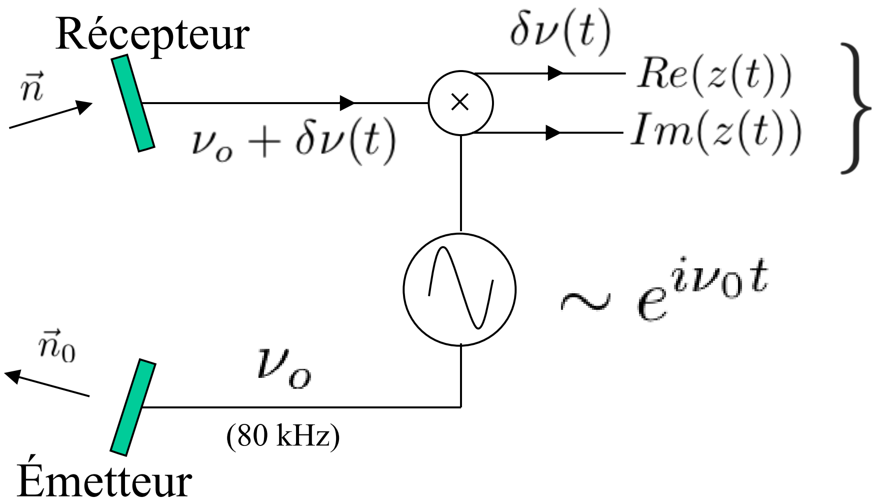
- Collective effects (seeding density) ?

N. Qureshi et al., arXiv:physics/07063042



3
2.5
2
A_0
1.5
1
0.5
1
10
100
$\Gamma = \rho_p / \rho_{air}$
10
15
20
25
$\phi = D / \eta$

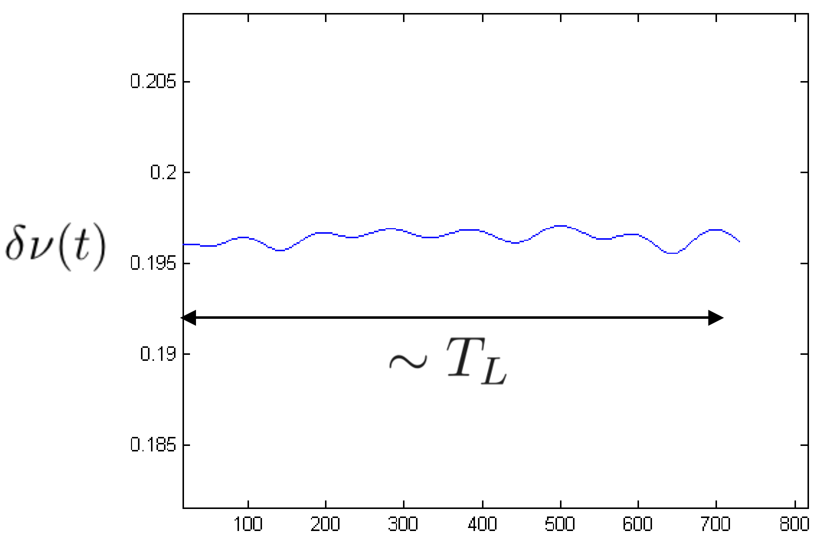
Data Acquisition & Processing

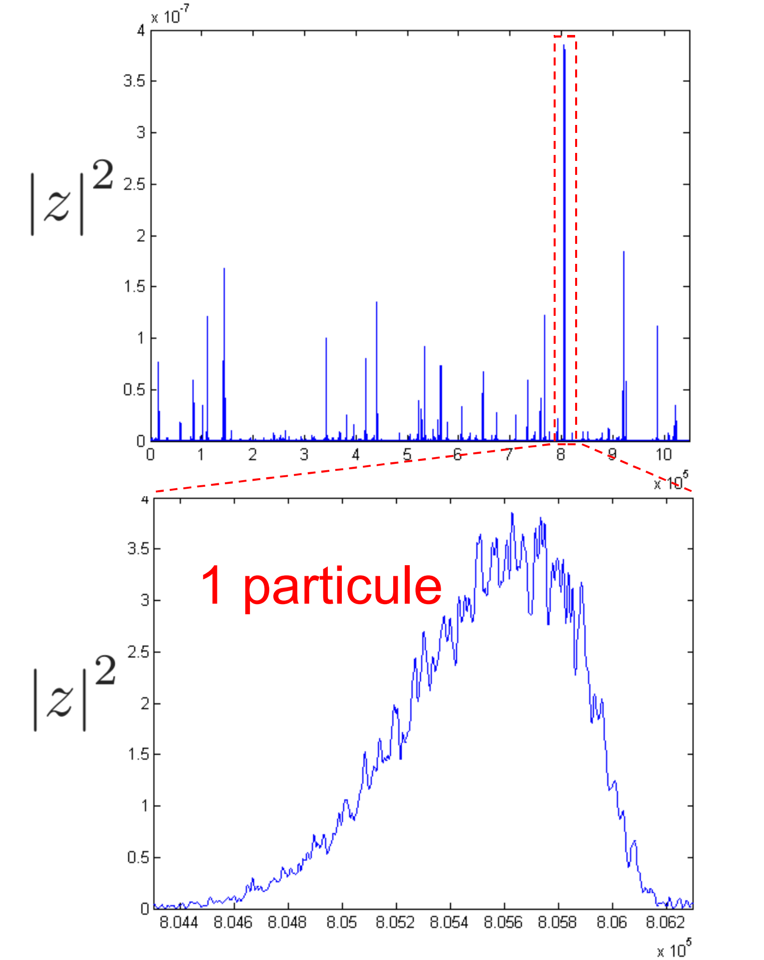


$$z(t) = A(t)e^{i2\pi \int_0^t \delta\nu(t')\mathrm{d}t'}$$

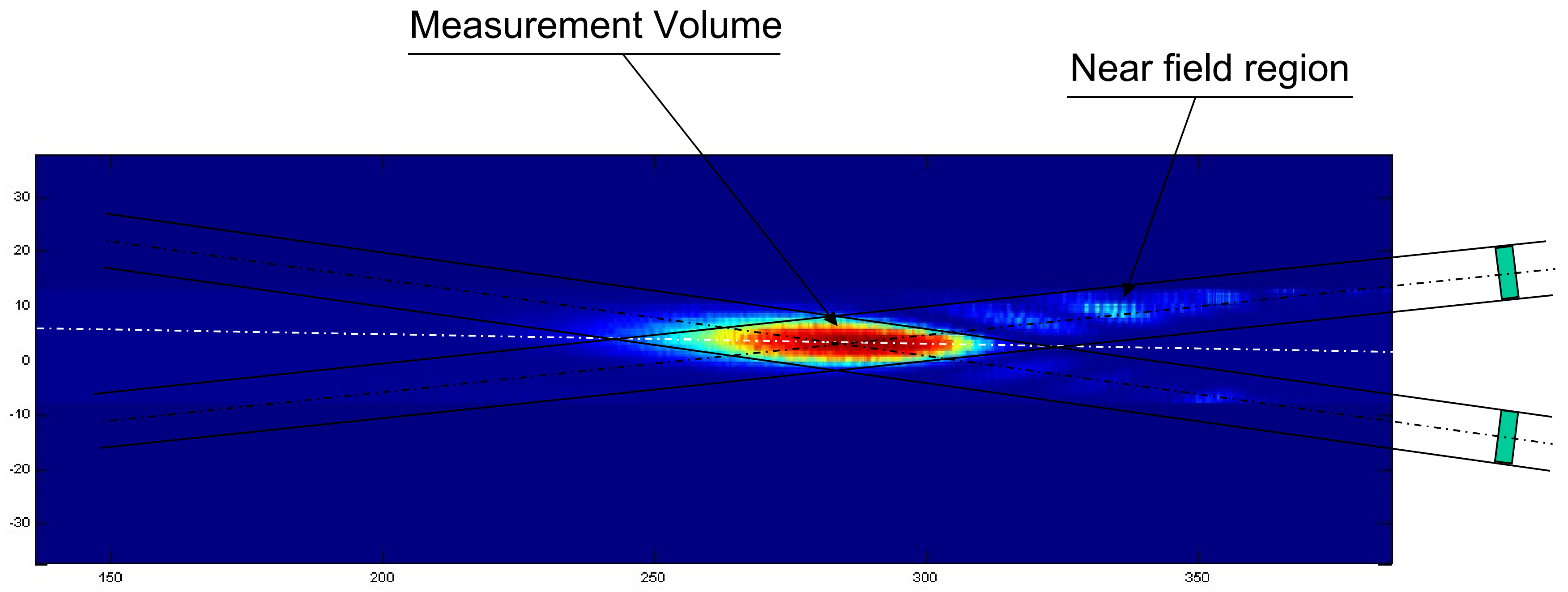
Signal Complex

Maximum de Vraisemblance Approchée
MVA

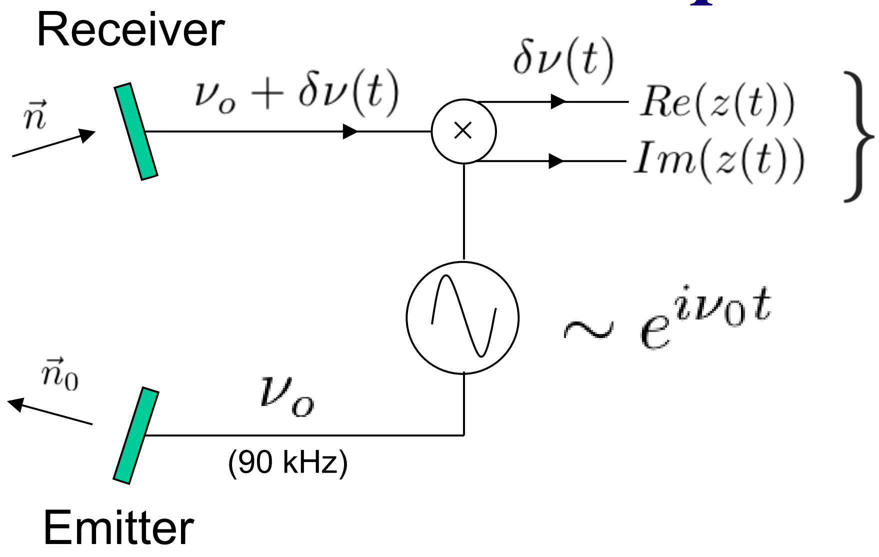




Measurement Volume

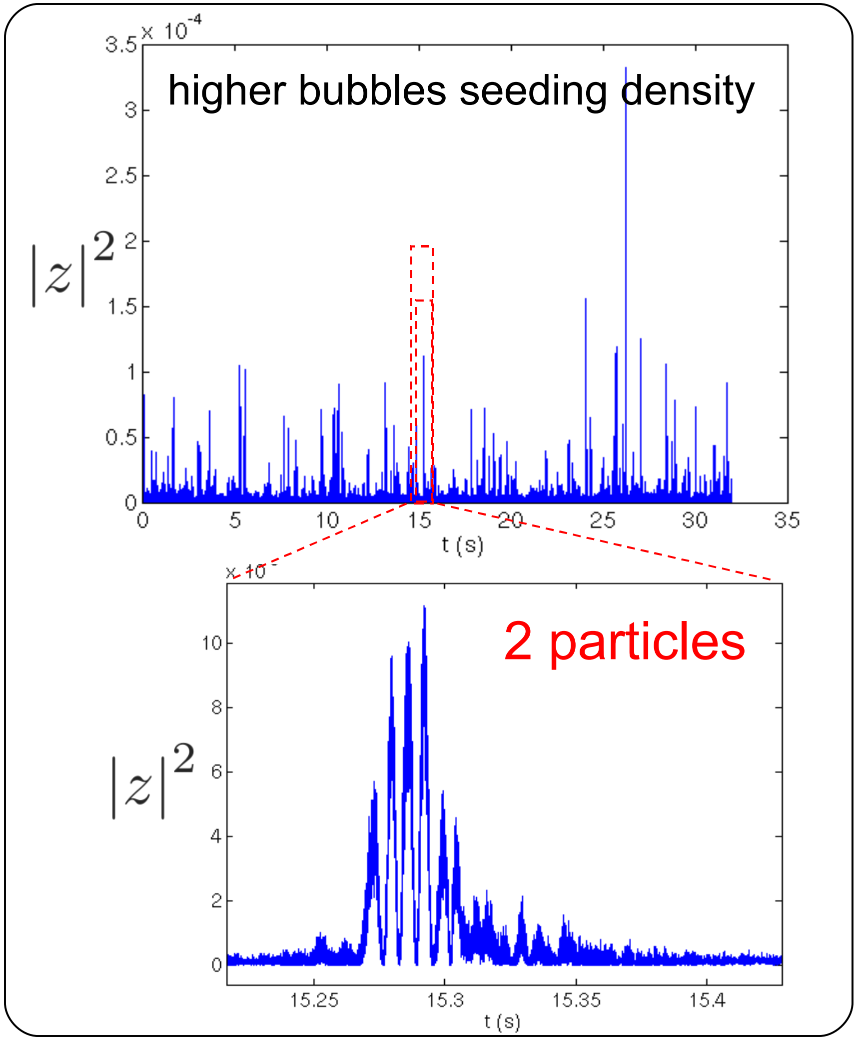


Data Acquisition - Processing



$$z(t) = A(t)e^{i2\pi \int_0^t \delta\nu(t')\mathrm{d}t'}$$

Complex downmixed signal

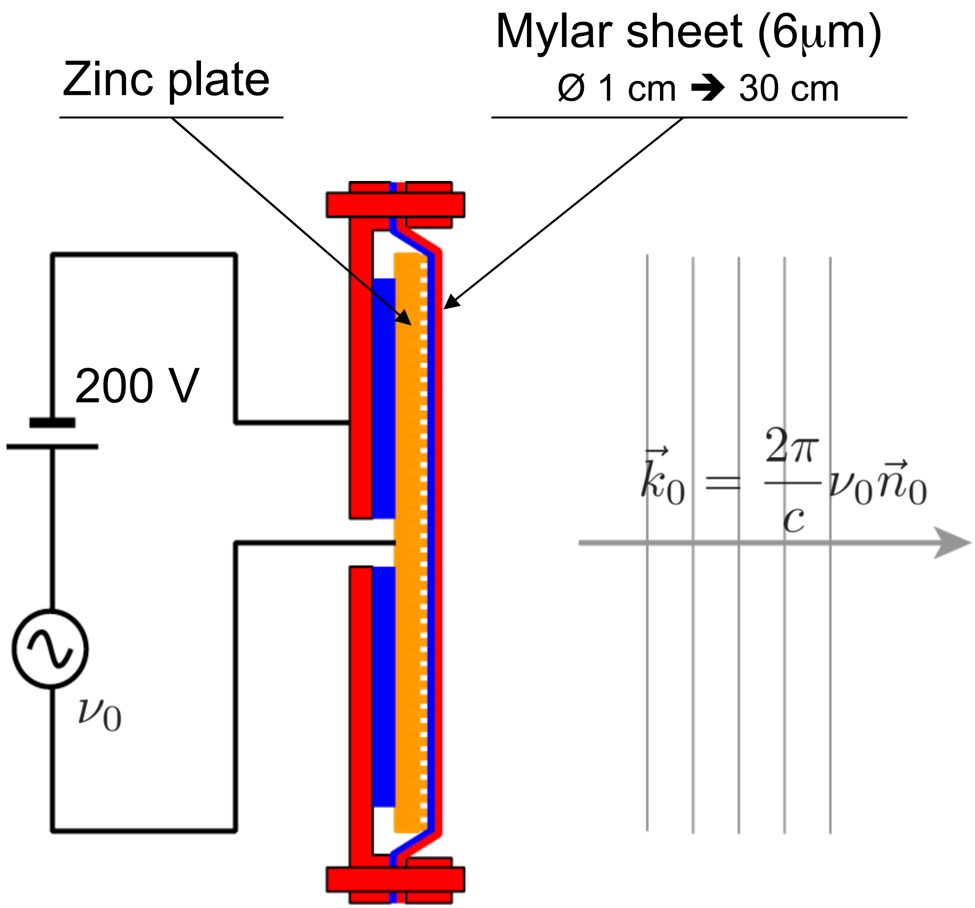


$$z(t) = A_1(t)e^{i2\pi \int_0^t \delta\nu_1(t')\mathrm{d}t'} + A_2(t)e^{i2\pi \int_0^t \delta\nu_2(t')\mathrm{d}t'}$$

$$|z|^2 = |A_1|^2 + |A_2|^2 + 2A_1A_2\cos\left(i2\pi\int_0^t \delta\nu_1(t') - \delta\nu_2(t')\mathrm{d}t'\right)$$

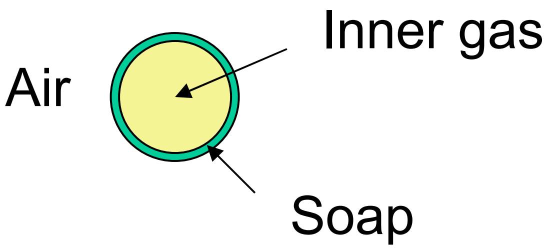
Ultrasonic transducers

Sell-type transducers (*electro-acoustical circular piston*)

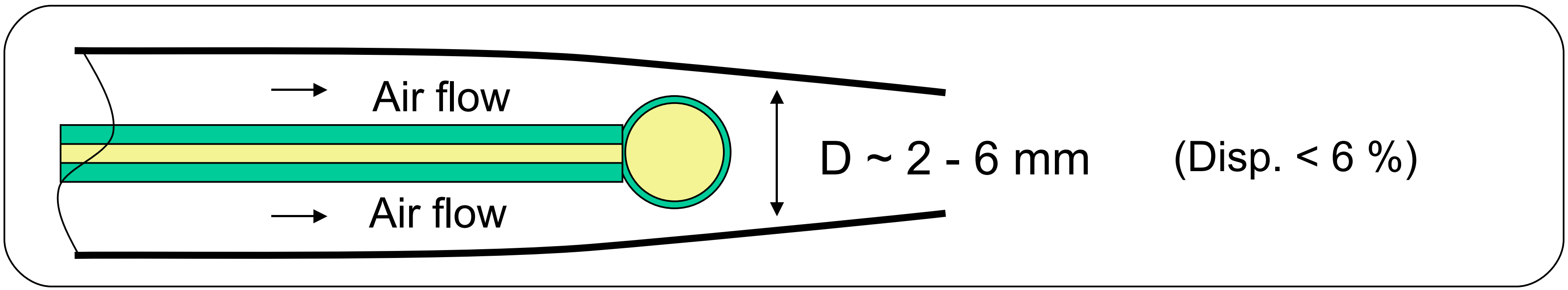


- Reciprocal
- Linear
- Large spectral band width
  (20kHz ➔ 150 kHz)
- Directional
- Home made

Particles : Gas filled soap bubbles

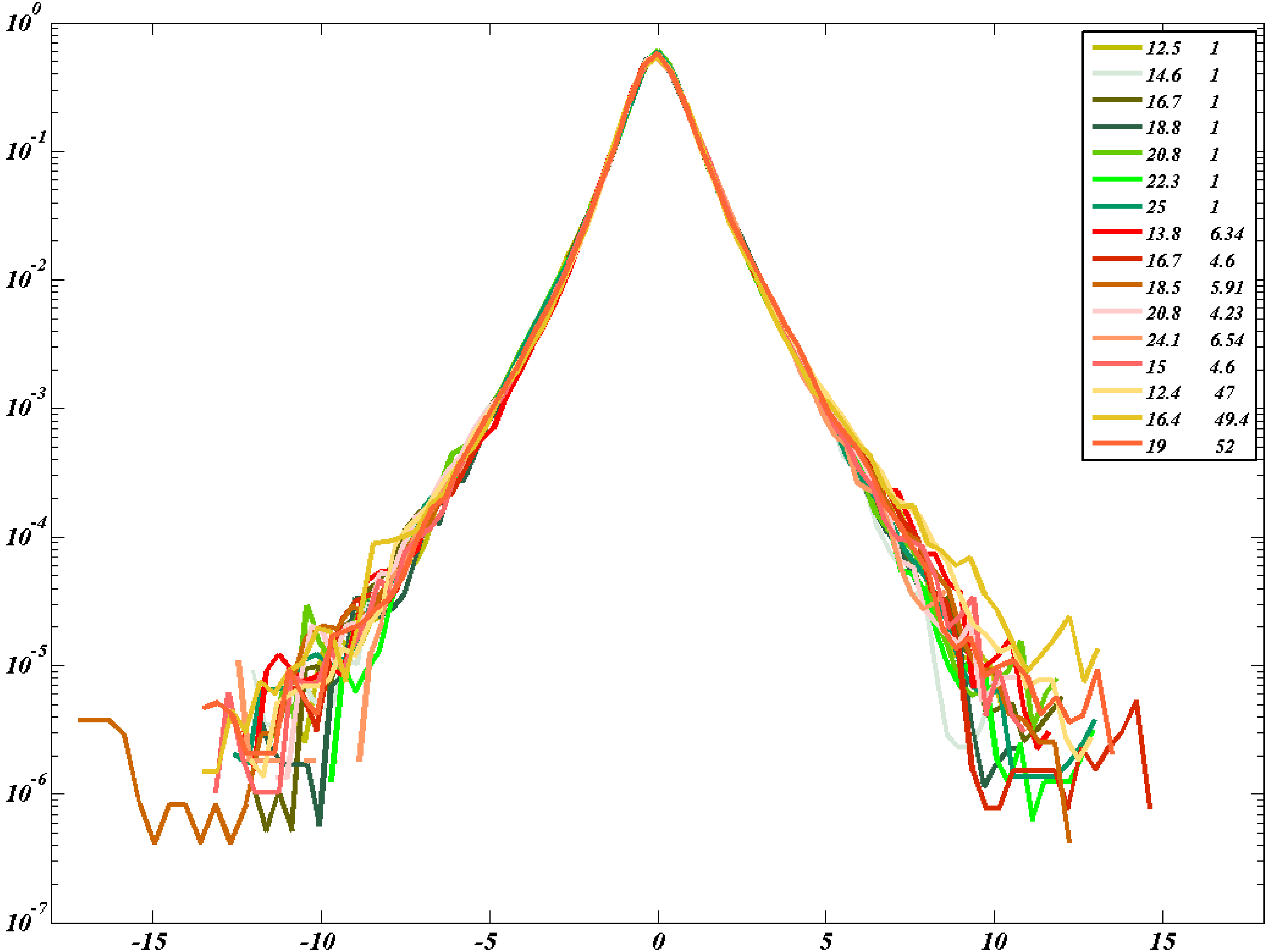


Using Helium as inner gas,
we can compensate the weight of soap
➡ Neutrally buoyant particles



- Adjustable parameters :
  - soap, gas and air flow rates
  - inner gas type

➡ Bubbles density, size and production rate adjustable
Stokes number effects *: Lagrangian tracers ➔ inertial particles*



10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
-15
-10
-5
0
5
10
15
12.5 1
14.6 1
16.7 1
18.8 1
20.8 1
22.3 1
25 1
13.8 6.34
16.7 4.6
18.5 5.91
20.8 4.23
24.1 6.54
15 4.6
12.4 47
16.4 49.4
19 52

- *Our Experiments:*
  - ***D ~ 12.5 η → 30 η or L / 20 → L / 8***
  - ***Γ ~ 1 → 65***
- *Warhaft Cornell:*
  - ***D ~ 0.05 η***
  - ***Γ ~ 1000***