Hyperviscosity and Galerkin truncation for the Burgers equation

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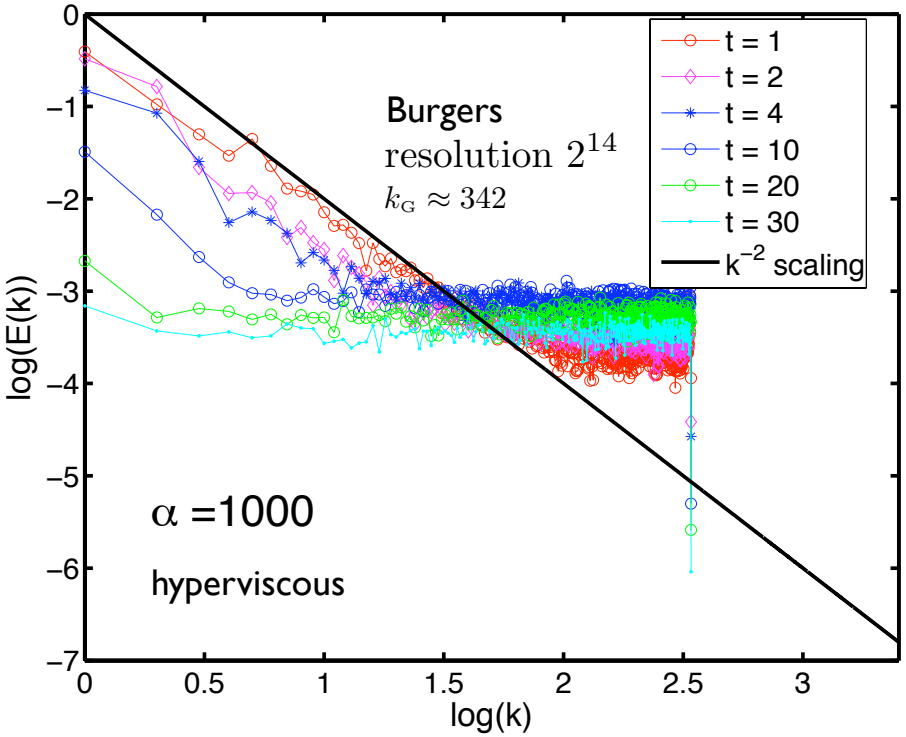
$$\partial_t v + v\partial_x v = -\mu {k_{\rm G}}^{-2\alpha}(-\partial_x^2)^\alpha v$$

$$\mu > 0, \quad k_{\rm G} > 0, \quad \alpha = \text{dissipativity}$$

Hyperviscous and Galerkin-truncated Burgers



Burgers equation with random initial condition

$$u_0(x) = \sin x + \sin(2x + \phi)$$

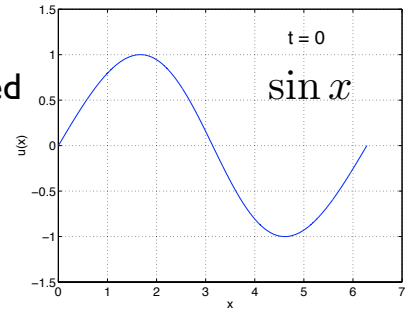
ϕ uniformly distributed in $[-\pi, \pi]$

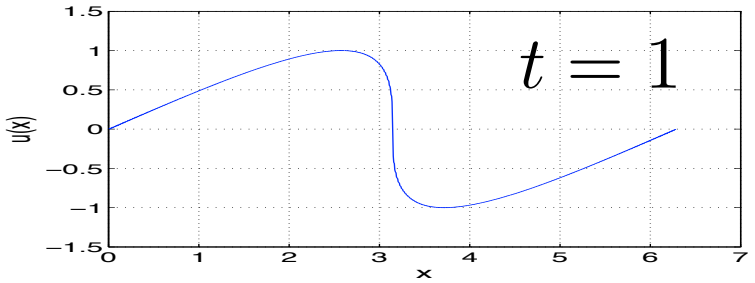
Energy spectrum averaged over 20 realizations

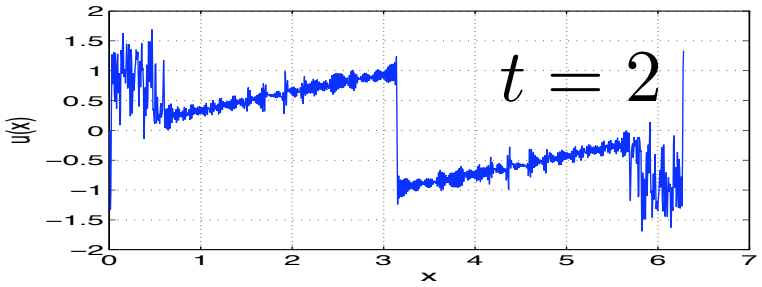
Galerkin-truncated Burgers first studied by Majda and Timofeyev 2000

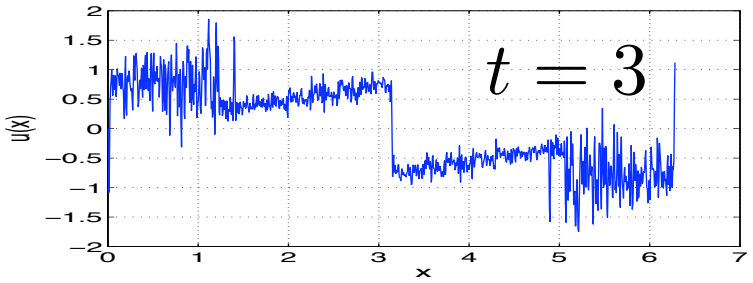
Evolution of Galerkin-truncated

initial condition $\sin x$

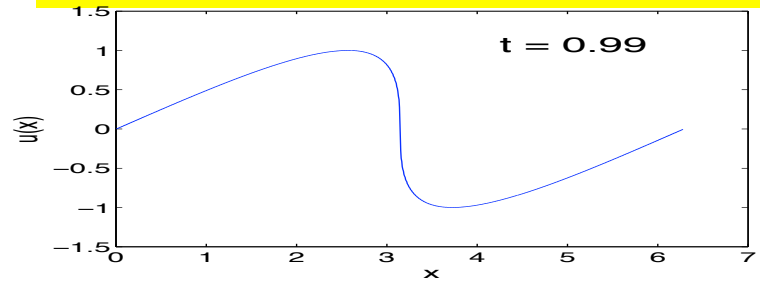


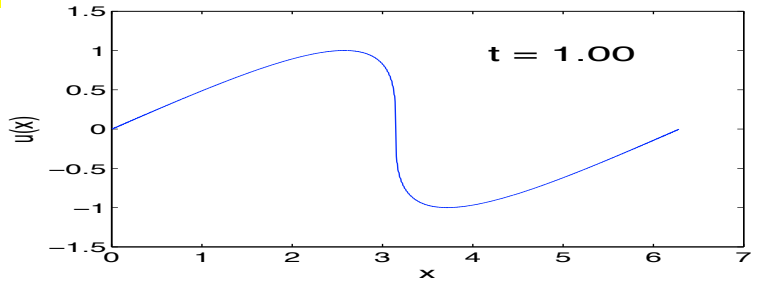


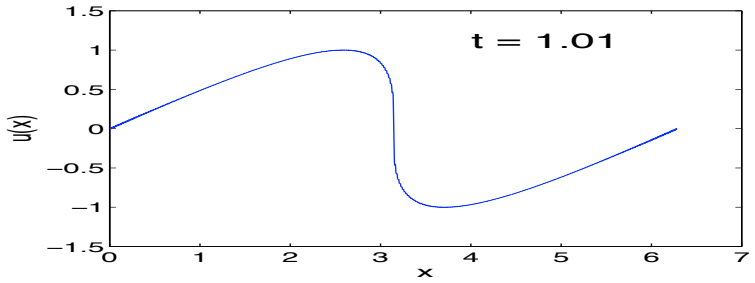


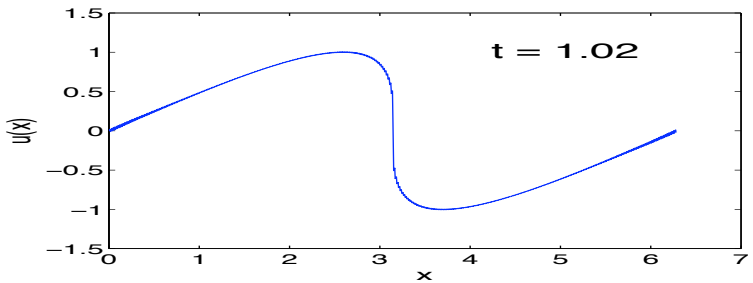


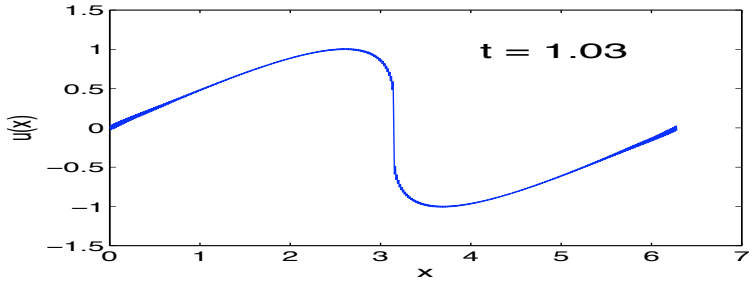
The shock acts as a black hole

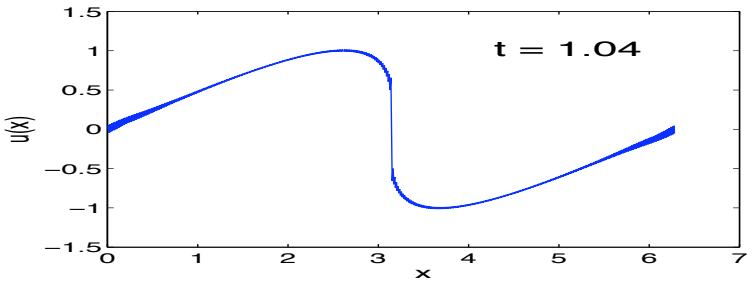


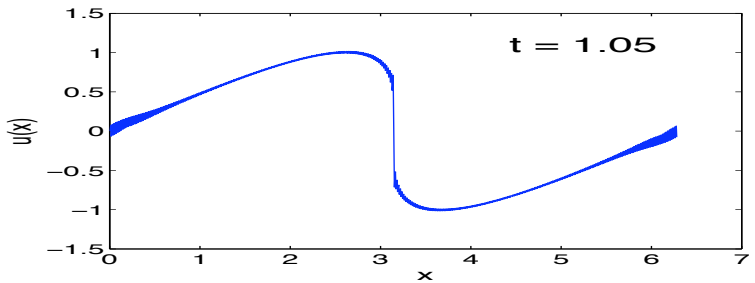


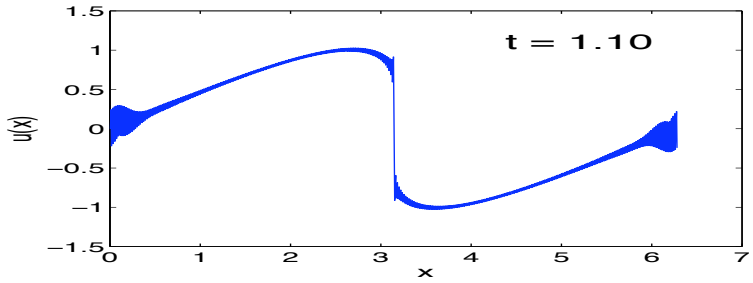












resolution 2^{10}

$k_{\mathrm{G}} = 342$

Are these genuine shocks? Mathematical question:
do the solutions of the inviscid truncated Burgers eq.
converge to the "entropy solution" when $k_{\mathrm{G}} \to \infty$?