Intermittence multifractale lagrangienne de scalaires passifs

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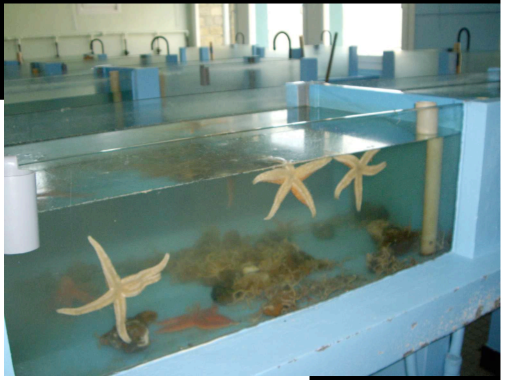
Laboratoire d'Océanologie et de Géosciences

A research laboratory involving:

- *CNRS (National Center for Scientific Research*
- *University of Lille 1*
- *Littoral University*

How marine organisms “see the world”



- Benthic organisms: Eulerian point of view
- Plankton: advected by turbulence; Lagrangian point of view

Eulerian framework: classical Kolmogorov-Richardson cascade picture

Cascade ideas of Richardson (1922)

- Formalized by Kolmogorov (1941)
- **Energy input at large scales; dissipation at small scales: between=intertial range scales; energy cascade**
- Inertial range scales characterized by a –5/3 power law spectrum
- in real space: structure functions

$$< \Delta U_l^{\,q} > \approx l^{\zeta_E(q)}$$

- Exact result: $\zeta_E(3) = 1$
- other property: ζ(q) nonlinear and concave: multifractality

Lagrangian framework: less classical Landau-Novikov Lagrangian cascade

$V(x_0,t)$ velocity of an element of fluid at time t, initially at position $x(0)= x_0$ Denoted V(t) by statistical homogeneity

- Landau and Lipschitz framework: $\Delta V_\tau \approx \varepsilon^{1/2}\tau^{1/2}$
- Later generalized by Novikov (1989) to take into account intermittency: $< \Delta V_\tau{}^q > \approx \tau^{\zeta_L(q)}$
- Exact result: $\zeta_L(2) = 1$
- other property: $\zeta(q)$ nonlinear and concave: multifractality

Eulerian framework: Corrsin-Obukhov cascade

- Obukhov (1949) and Corrsin (1951)
- **Coupling of two cascades: flux of energy and passive scalar variance**

$$\Delta\theta_l \approx \varepsilon_l^{-1/6}\chi_l^{1/3} l^{1/3}$$

- Inertial range scales characterized by a –5/3 power law spectrum
- in real space: structure functions

$$< \Delta\theta_l^{\,q} > \approx l^{\zeta_\theta(q)}$$

- No exact result: $\zeta_\theta(3) \neq 1$
- other property: ζ(q) nonlinear and concave: multifractality

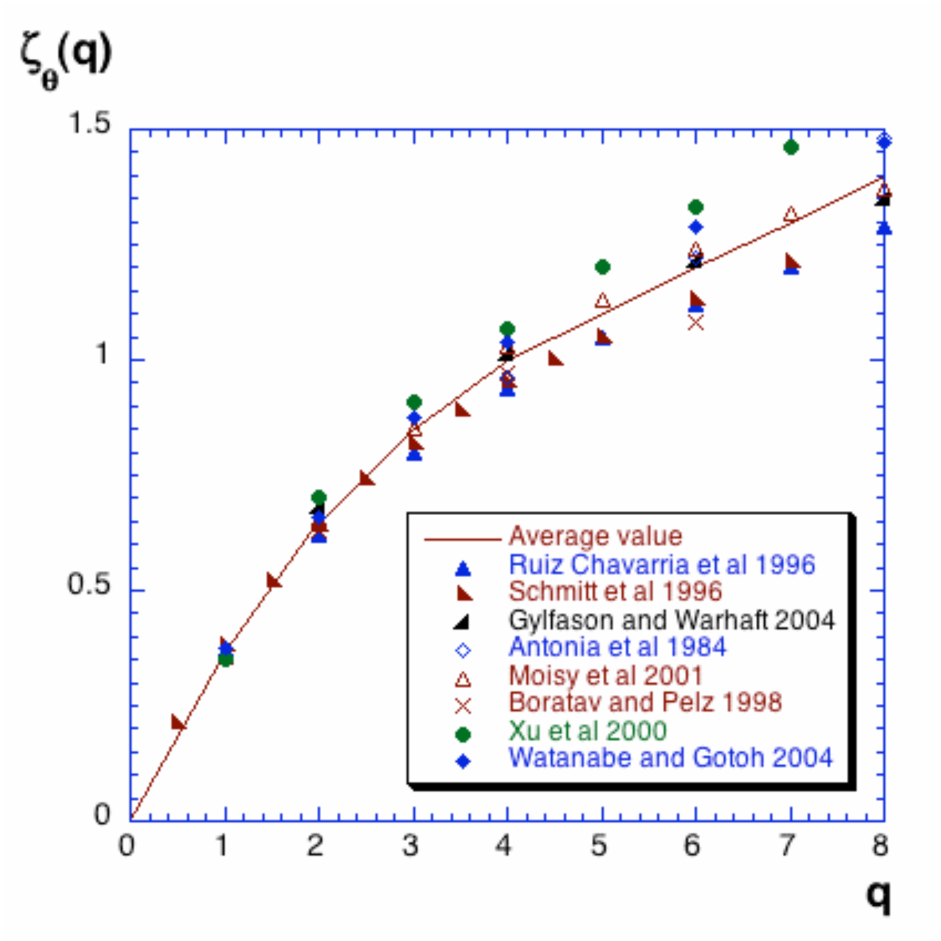
Experimental convergence

Table 1. Average values of $\zeta_\theta(q)$ with error bars, estimated from several published estimates [16–23].

q	$\zeta_\theta(q)$
1	0.365 ± 0.015
2	0.65 ± 0.03
3	0.85 ± 0.05
4	0.99 ± 0.05
5	1.10 ± 0.05
6	1.20 ± 0.08
7	1.30 ± 0.1
8	1.40 ± 0.12

16. R.A. Antonia, E.J. Hopfinger, Y. Gagne, F. Anselmet, Phys. Rev. A **30**, 2704 (1984)
17. G. Ruiz-Chavarria, C. Baudet, S. Ciliberto, Physica D **99**, 369 (1996)
18. F.G. Schmitt, D. Schertzer, S. Lovejoy, Y. Brunet, Europhys. Lett. **34**, 195 (1996)
19. O.N. Boratav, R.B. Pelz, Phys. Fluids **10**, 2122 (1998)
20. G. Xu, R.A. Antonia, S. Rajagopalan, Europhys. Lett. **49**, 452 (2000)
21. F. Moisy, H. Willaime, J.S. Andersen, P. Tabeling, Phys. Rev. Lett. **86**, 4827 (2001)
22. A. Gylfason, Z. Warhaft, Phys. Fluids **16**, 4012 (2004)
23. T. Watanabe, T. Gotoh, New J. Phys. **6**, 40 (2004)

Experimental convergence



16. R.A. Antonia, E.J. Hopfinger, Y. Gagne, F. Anselmet, Phys. Rev. A **30**, 2704 (1984)
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Mixed structure functions

- Yaglom (1949)'s relation $<\Delta\theta_l^{\,2}\Delta U_l>=-\dfrac{4}{3}\chi l$
- **Generalized for other order of moments:**

$$<\left(\Delta\theta_l^2\Delta U_l\right)^{q/3}>\approx l^{\zeta_m(q)}$$

- Exact result: $\zeta_m(3)=1$

- Other property: $\zeta_m(q)$ nonlinear and concave; close to velocity structure functions

Mixed structure functions

Table 2. Some recent experimental estimations for $\zeta_m(q)$. A: Large Re: atmospheric data [18]; B: DNS data, $Pr = 1$ and $R_\lambda = 141$ [19]; C: Experimental data, coaxial rotating disks, $Pr = 0.7$ and $R_\lambda = 367$ [28]; D: Experimental data, downstream of a cylinder, $Pr = 0.7$ and $R_\lambda = 300$ [29]; E: DNS data, $Pr = 1$ and $R_\lambda = 427$ [23]; F: Experimental data, round jet, $Pr = 0.7$ and $R_\lambda = 550$ [20]; G: Experimental data, grid turbulence, $Pr = 0.7$ and $R_\lambda = 582$ [31].

q	A	B	C	D	E	F	G
0.5	0.21		0.19				
1	0.39		0.37	0.39	0.38		
1.5	0.56		0.54				
2	0.72	0.72	0.70	0.73	0.70		
2.5	0.87		0.85				
3	1	1	1	1	0.96	1.06	1
3.5	1.12		1.14				
4	1.24	1.25	1.27	1.23	1.19		
4.5	1.35		1.39				
5	1.45		1.50	1.43			
6	1.65	1.83		1.61	1.55	1.83	1.52
7	1.83			1.80			
8	2.00			1.95	1.86		

$$< \left(\Delta\theta_l^2 \Delta U_l \right)^{q/3} > \approx l^{\zeta_m(q)}$$

A F.G. Schmitt, D. Schertzer, S. Lovejoy, Y. Brunet, Europhys. Lett. **34**, 195 (1996)

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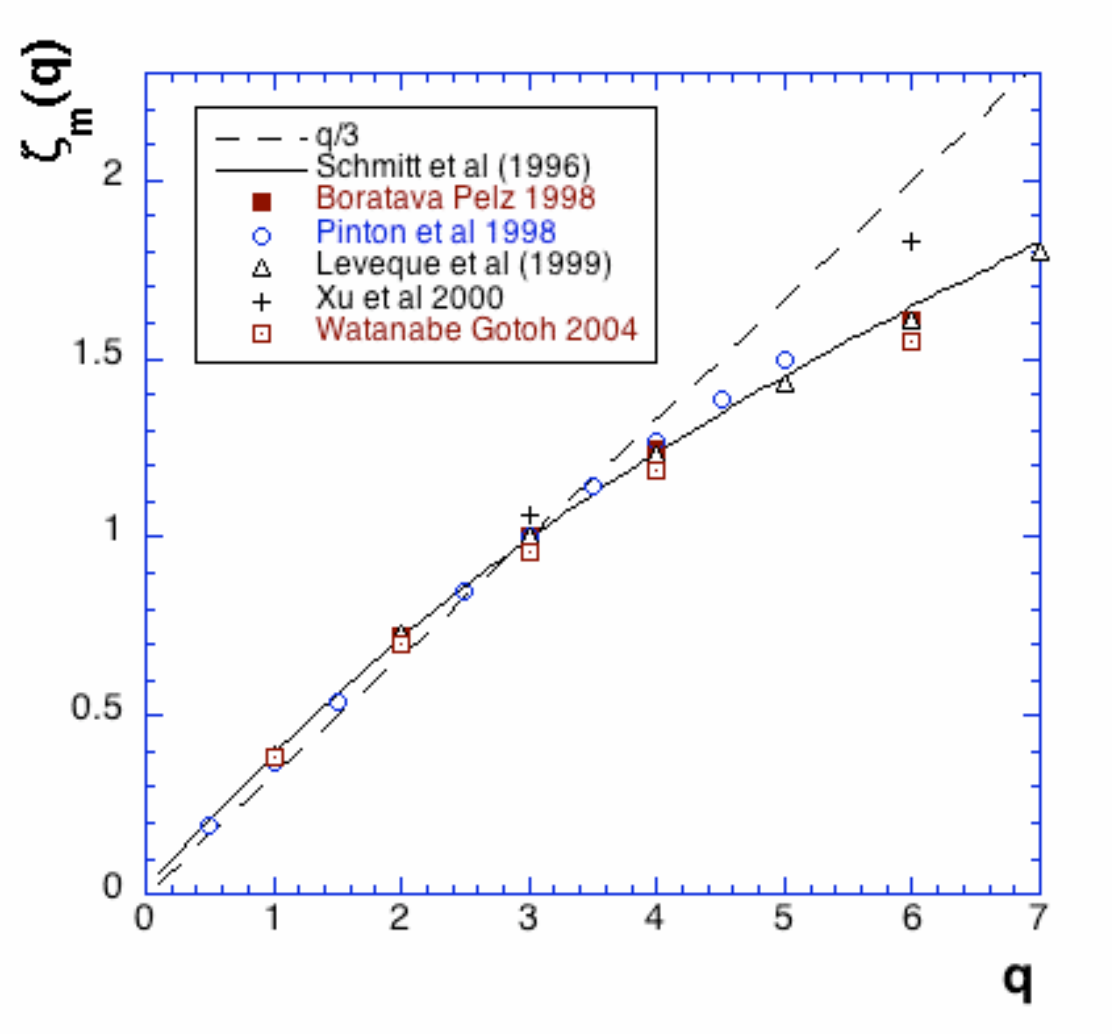
G L. Mydlarski, J. Fluid Mech. **475**, 173 (2003)

Mixed structure functions



$$< \left(\Delta\theta_l^2 \Delta U_l \right)^{q/3} > \approx l^{\zeta_m(q)}$$

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Lagrangian framework: Inoue-Novikov Lagrangian intermittency

$\Theta(x_0,t)$ velocity of an element of fluid at time t, initially at position $x(0)= x_0$ Denoted $\Theta(t)$ by statistical homogeneity

- Inoue (1952)'s framework: $$\Delta\Theta_\tau \approx \varphi^{1/2}\tau^{1/2}$$
- Later generalized by Novikov (1989) to take into account intermittency: $$< \Delta\Theta_\tau^{\ q} > \approx \tau^{\xi_\theta(q)}$$
- Exact result: $\xi_\theta(2) = 1$
- other property: $\xi(q)$ nonlinear and concave: multifractality

→ No experimental results up to now

How to relate Eulerian and Lagrangian scaling exponents?

$$< \Delta\theta_l^{\,q} > \approx l^{\zeta_\theta(q)} \qquad < \left(\Delta\theta_l^2 \Delta U_l\right)^{q/3} > \approx l^{\zeta_m(q)} \qquad < \Delta\Theta_\tau^{\,q} > \approx \tau^{\xi_\theta(q)}$$

$\xi_\theta(q)$ vs. $\zeta_\theta(q)$, $\zeta_m(q)$

2 sets of hypotheses

1. A statistical relation linking space and time

2. A statistical relation between Eulerian and Lagrangian variables

2 sets of hypotheses: 4 possible relations

1. A statistical relation linking space and time

Direct scaling relation by dimensional analysis:

$$l^2 \approx \tau^3$$

α

Other law taking into account the intermittency of velocity fluctuations (Boffetta et al, 2002)

$$\Delta U_l \approx l^{h_u}$$

$$\tau \approx l^{1-h_u}$$

β

2. A statistical relation between Eulerian and Lagrangian variables

Ergodicity arguments for the dissipation (Borgas, 1993)

$$<\chi_l^{\,q}> = <\varphi_l^{\,q}>$$

A

Decorrelation of eddies argument (Boffetta et al, 2002)

$$\Delta\theta_l \approx \Delta\Theta_\tau$$

B

Results for the links between ξ_θ and ζ_θ, ζ_m

1. Deterministic time and eddies argument

α

B

$$\xi_\theta(q) = \frac{3}{2}\zeta_\theta(q); \qquad \text{case I}$$

→ $\xi_\theta(2) = \frac{3}{2}\zeta_\theta(2) \approx 1$ since $\zeta_\theta(2) \approx 0.65$

Results for the links between ξ_θ and ζ_θ, ζ_m

2. Deterministic time and ergodicity argument

α

A

$$\xi_\theta(q) = \frac{3}{2}\zeta_m\left(\frac{3q}{2}\right) - \frac{q}{4}; \qquad \text{case II}$$

→ $\xi_\theta(2) = 1$ since $\zeta_m(3) = 1$

Results for the links between ξ_θ and $\zeta_{\theta,}$ ζ_m

3. Intermittent time and eddies argument

β

A

$$\langle \chi_\ell^q \rangle \sim \frac{\langle \Delta\theta_\ell^{2q} \Delta U_\ell^q \rangle}{\ell^q}$$

$$\sim \int_{(h_\theta,h_u)} \ell^{2qh_\theta+qh_u+c(h_\theta,h_u)-q} d(h_\theta,h_u)$$

$$\sim \int_{(h_\theta,h_u)} \tau^{\frac{2qh_\theta+qh_u+c(h_\theta,h_u)-q}{1-h_u}} d(h_\theta,h_u)$$

$$-K_\varphi(q) = \min_{(h_\theta,h_u)} \left(\frac{2qh_\theta + qh_u + c(h_\theta,h_u) - q}{1-h_u} \right)$$

$$\begin{cases} \zeta_m(3q) = 2qh_u + c_1(h_u) \\ q = -\frac{1}{2}c_1'(h_u). \end{cases}$$

$$c_1(h_u) = c(h_u/2, h_u)$$

$$h_\theta = h_u/2$$

$$-K_\varphi(q) = \min_{h_u} \left\{ \frac{2qh_u + c_1(h_u) - q}{1-h_u} \right\}$$

Results for the links between ξ_θ and ζ_θ, ζ_m

3. Intermittent time and eddies argument

β

A

$$-K_\varphi(q) = \min_{h_u}\left\{\frac{2qh_u + c_1(h_u) - q}{1-h_u}\right\}$$

$$H_q(h_u) = \frac{2qh_u + c_1(h_u) - q}{1-h_u}$$

Minimal for h_0

$$H_q{}'(h_0) = 0$$

gives h_0 q_0

$$\begin{cases} \xi_\theta(q) = \zeta_m\left(q_0\right) \\ \dfrac{q}{2} = \dfrac{2q_0}{3} - \zeta_m\left(q_0\right) \end{cases}$$

case III

$\zeta_m(3) = 1$ gives $\xi_\theta(2) = 1$

Results for the links between ξ_θ and $\zeta_{\theta,}$ ζ_m

4. Intermittent time and ergodicity argument

{ β
B

$$\tau^{\xi_\Theta(q)} \sim \langle \Delta\Theta_\tau^q \rangle \sim \langle \Delta\theta_\ell^q \rangle$$
$$\sim \int \ell^{qh_\theta + c(h_\theta)} dp(h_\theta)$$
$$\sim \int \tau^{\frac{qh_\theta + c(h_\theta)}{1-h_u}} dp(h_\theta).$$

$$\xi_\Theta(q) = \min_{(h_\theta, h_u)} \left(\frac{qh_\theta + c(h_\theta)}{1 - h_u} \right)$$

Here an additional hypothesis is needed:

if $h_u = 1/3$ $\quad \xi_\theta(q) = \frac{3}{2}\zeta_\theta(q);$ case I

if h_u, h_θ independent

$$\xi_\Theta(q) = \frac{\min_{h_\theta}\{qh_\theta + c(h_\theta\}}{\max_{h_u}\{1 - h_u\}} = \frac{\zeta_\theta(q)}{1 - h_{\min}}$$

Results for the links between ξ_θ and $\zeta_{\theta,}$ ζ_m

4. Intermittent time and ergodicity argument

β

B

$$\xi_\Theta(q) = \min_{(h_\theta, h_u)} \left(\frac{q h_\theta + c(h_\theta)}{1 - h_u} \right)$$ if $h_\theta = h_u/2$ $$\xi_\Theta(q) = \min_{h_\theta} \left(\frac{q h_\theta + c(h_\theta)}{1 - 2h_\theta} \right)$$

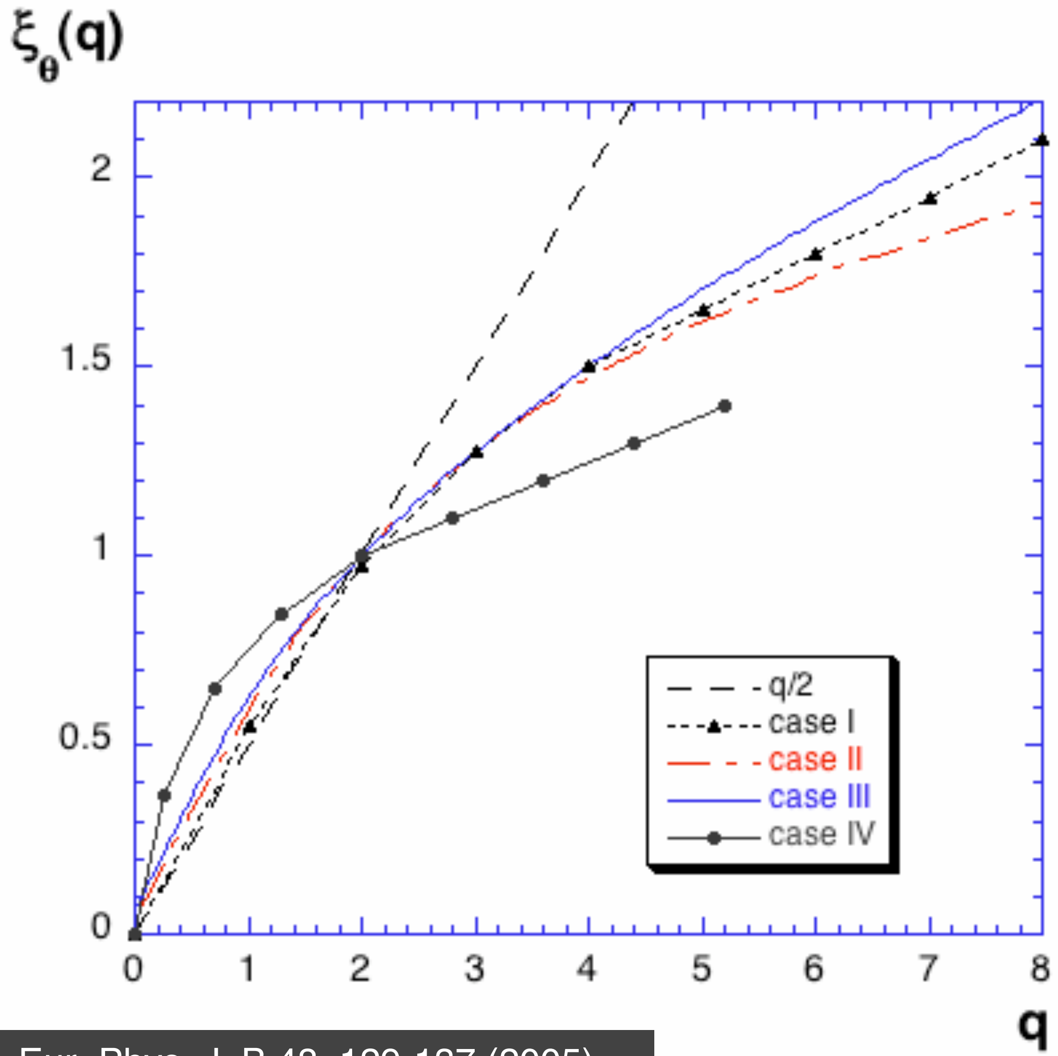
Gives using several Legendre transforms:

$$\begin{cases} \xi_\theta(q) = \zeta_\theta(q_0) \\ q = q_0 - 2\zeta_\theta(q_0) \end{cases}$$

case IV

To obtain $\xi_\theta(2) = 1$ one needs $\zeta_\theta(4) = 1$ which is experimentally approximatively verified

Comparing predictions

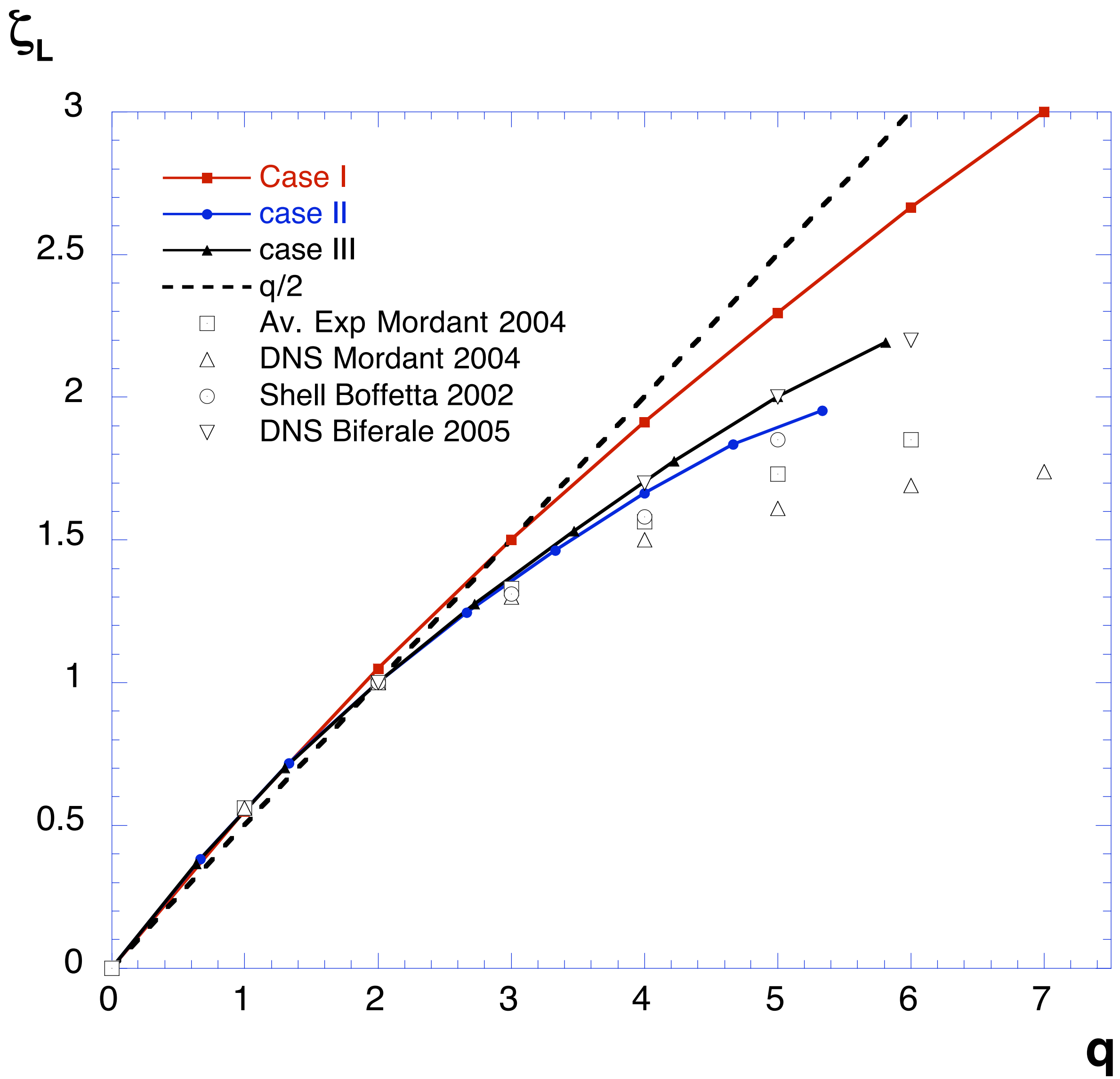


Using experimental averages for $\zeta_{\theta}, \zeta_{m}$

Cases I and II are not realistic
Cases III and IV are more realistic
Case IV's curve is quite different from the others

Schmitt, Eur. Phys. J. B 48, 129-137 (2005)

The same approach for velocity: 3 predictions



Lagrangian experimental and numerical values

Schmitt, Phys. Lett. A 342, 448-458 (2005)
Schmitt, Physica A 368, 377-396 (2006)

Approche expérimentale

Flotteur lagrangien océanique de petite taille

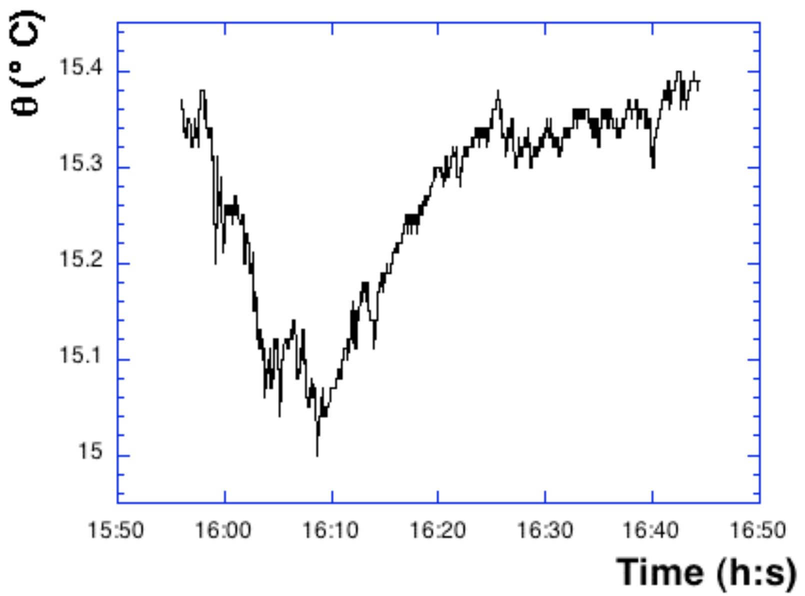
8 cm de long
50g
Alec Electronics MDS MkV/T

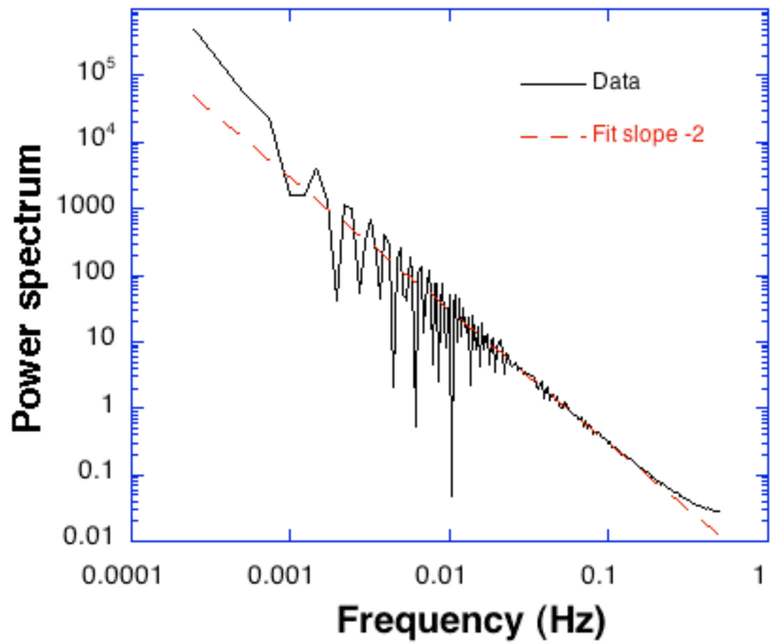
Capteur de température miniature
Fréquence d'échantillonnage: 1 Hz

Etude préliminaire: 2 séries temporelles de 80 et 50 min, enregistrées à partir d'un zodiac près du flotteur lagrangien (contact visuel)

Approche expérimentale

Flotteur lagrangien océanique de petite taille



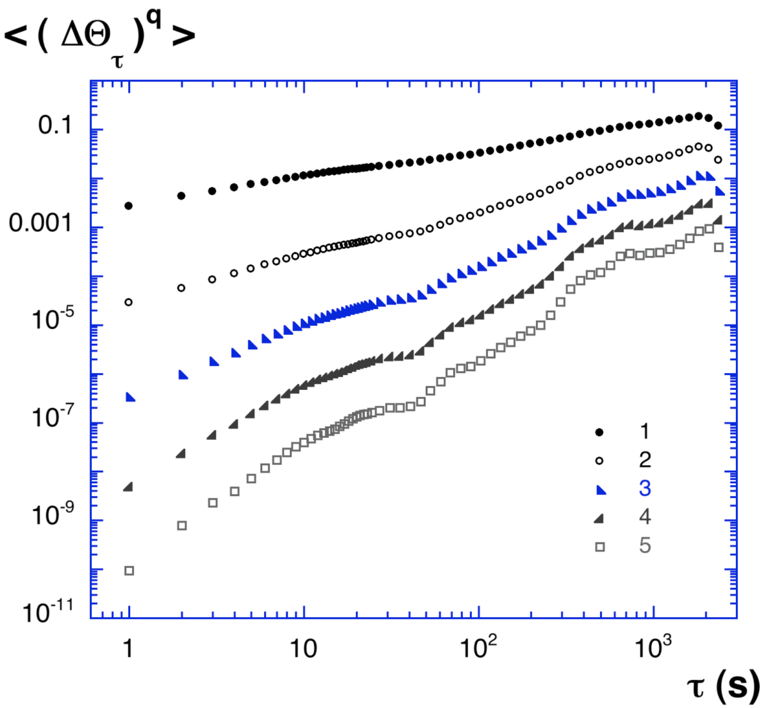


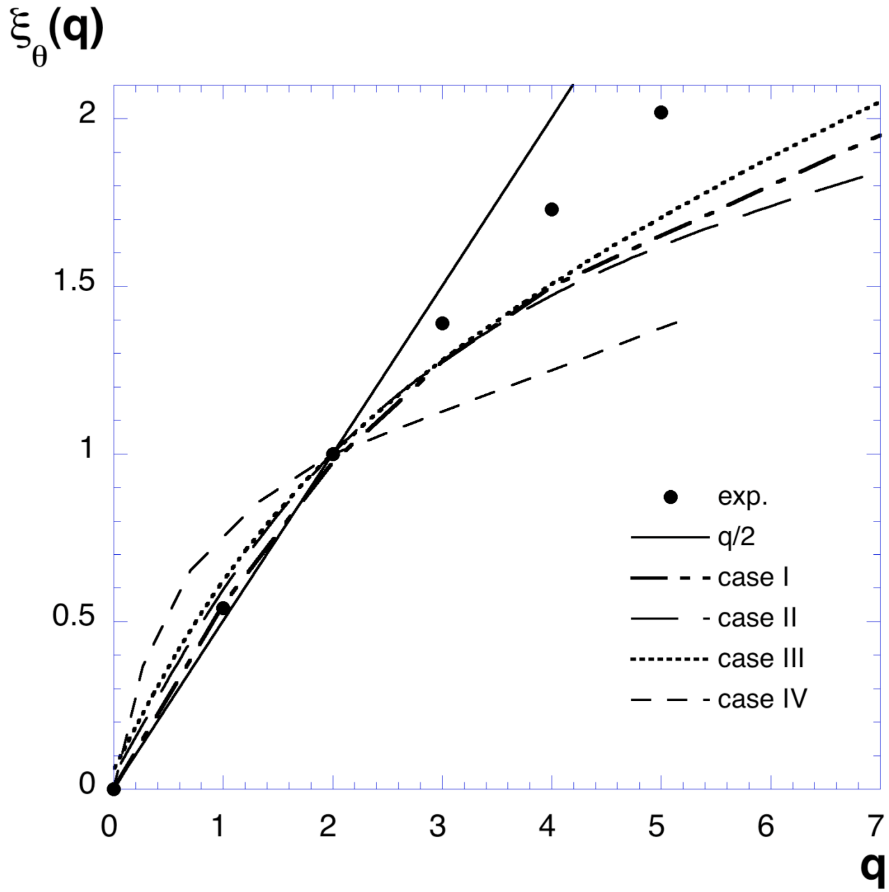
Spectre en ω^{-2}

Intermittence malgré des problèmes de précision des mesures (discrétisation)

Approche expérimentale

Flotteur lagrangien océanique de petite taille





Multifractalité des fluctuations de scalaire passif lagrangien

mais les relations proposées ne semblent pas vérifiées

Conclusion

• Multifractalité des fluctuations de scalaire passif lagrangien mais les relations proposées ne semblent pas vérifiées

• Plus de données? Nouvelle campagne prévue, avec capteur plus précis, et durée plus longue

Perspectives

• Capteur de plus petite taille?

• Données DNS?

Ecole thématique du CNRS

Barcelone, sept 2008

International Summer School on

Turbulence, Plankton and Marine Snow

Barcelona, Spain

1 - 5 September 2008

A one-week summer school is planned in the field of (geophysical) turbulence and its role on the swimming behaviour and contact rates of plankton, patchiness of plankton, bloom formation, marine snow formation. Attention will also be paid to the theory and modeling of geophysical turbulence, laboratory experiments, and in situ observation on plankton in turbulence.

The aim of the meeting is to provide an intense course on current advances in the field with six invited didactical lectures by top specialists in the subject. The main emphasis will be put on **the fluid-mechanical aspects of ocean flows, dispersion of species in small-scale turbulence** and the available numerical and experimental techniques to analyze those. Additionally, it is aimed to promote the **interactions between marine biology and fluid mechanics** by bringing together senior and junior scientists from both disciplines, and to stimulate discussions and interaction between colleagues from the two disciplines. Besides the keynote lectures there will be room for several lectures by senior researchers on specialized topics.

Location: Moli de Mar, Po Ermita de St. Cristofor (Far), Vilanova i la Geltrú, 08800 Barcelona, Spain
Full fee: 200 euro. The course fee includes lodging, lecture notes, coffee breaks, and conference diner. Payment must be received by August 1st, 2008.

The following keynote lectures will be given:

- turbulence in the ocean: an introduction to its physics (Steve Thorpe, Bangor, UK)
- experiments and observation of plankton in turbulence (Francesc Peters, Barcelona, Spain)
- planktonic population dynamics in turbulent boundary layers (David Lewis, Liverpool, UK)
- mesoscale turbulence and plankton patchiness (Marina Levy, Paris, France)
- small-scale hydrodynamics and plankton (Andy Visser, Copenhagen, Denmark)
- the formation and fate of marine snow: the role of hydrodynamics (Thomas Kioerboe, Copenhagen, Denmark)

Besides short lectures on special topics will be given, covering:

- turbulent exchange in the benthic boundary layer (Luca van Duren, Delft, The Netherlands)
- planktonic contact and capture rates in turbulent environments (Hans Pecseli, Oslo, Norway)
- using CFD to investigate the copepod hydrodynamics (Houshuo Jiang, Woods Hole, USA)
- animal and robot strategies for tracking odors underwater (Frank Grasso, New York, USA)
- physical gradients and biological responses across the edge of the continental shelf (Jonathan Sharples, Liverpool, UK)

Finally, also the junior participants will be invited to present their work in separate oral sessions.

The Summer School is organized by:

- **Herman Clercx**, Fluid Dynamics Laboratory, Eindhoven University of Technology, The Netherlands (chairman)
- **Tim Pedley**, DAMTP, University of Cambridge, UK
- **Jaume Piera**, Marine Technology Unit, CMIMA-CSIC, Barcelona, Spain
- **François Schmitt**, CNRS, Director of Laboratory of Oceanology and Geosciences, France
- **Anton van Steenhoven**, Eindhoven University of Technology, The Netherlands

A scientific advisory committee consists of: José Redondo (Barcelona), Cèlia Marrasé (Barcelona), Thomas Kioerboe (Copenhagen), Antonello Provenzale (Torino), David Lewis (Liverpool).

Registration: Send an email to Herman Clercx (h.j.h.clercx@tue.nl) before June 1st, 2008. It should include a brief CV (2 pages maximum), a letter specifying why he/she should attend the Summer School and providing a brief 10-line description of their PhD or PD project, a motivation for some limited financial support to cover travel costs, a letter of recommendation of their supervisor, and administrative details (including billing address). Selection will be made by the organising committee and the scientific advisory committee.[1] Notification of admittance will be sent by email before July 1st, 2008.

[1] Maximum number of participants is 25.