Rencontre GDR Turbulence & GDR IFS

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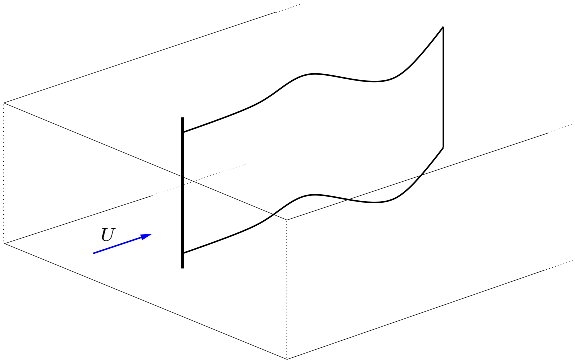
Effet du confinement latéral sur l'instabilité de flottement d'une plaque élastique

Olivier Doaré*, Christophe Eloy+, Martin Sauzade*, David Mano*

* ENSTA-Paristech, Palaiseau, France
+ IRPHE, Marseille, France

INTRODUCTION

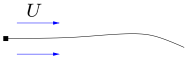
A cantilevered plate in an axial potential flow bounded by two rigid walls



At a critical velocity, plane state of the plate become unstable, flutter appears

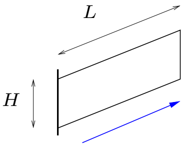
2D problem (∞ spanwise, finite chordwise)

- Kornecki (1976), Huang (1995), Watanabe et al. (2002), Guo and Païdoussis (2000), Michelin et al. (2008,2009) : Theoretical/numerical critical velocity as function of the mass ratio.
- Experimentally, the critical velocity is always higher than that predicted by models.



3D flow around a rectangular plate

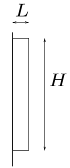
- New parameter : Aspect ratio of the plate $h = H/L$.
- Eloy et al. (2007) : Good correspondence between experiments and theory is found, showing the great importance of three-dimensionality of the flow.



How to approach the 2 dimensional limit ?

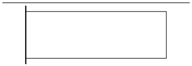
Increase the aspect ratio (short flags)

- When $h \gg 1$, the critical velocity predicted by 3D models is that predicted by 2D models
- Experimentally, 3D deformation of the plate is observed for such high aspect ratios



Force the flow to be 2D by adding walls

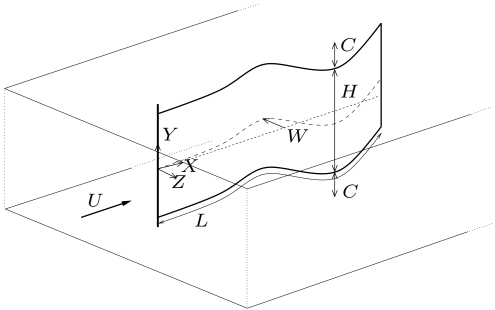
- Few experimental data available, Aurégan and Dépollier (1995), Huang (1995). Even for confined flags, discrepancy is found.
- No theoretical models.



Objectives

- Quantify the phenomenon of confinement with a theoretical model
- Validate the model with experiments

Model



BEAM EQUATION WITH PRESSURE FORCES

$$M \frac{\partial^2 W}{\partial T^2} + D \frac{\partial^4 W}{\partial X^4} = \langle [P] \rangle \qquad (1)$$

- $M \equiv$ surface density of the plate, $D \equiv$ flexural rigidity
- $\langle [P] \rangle$ mean value along the span H of the pressure jump across the plate
- Boundary conditions : Beam clamped at $X = 0$, free at $X = L$

Finite length flags non-dimensional parameters (2D problem)

$$U^* = \sqrt{\frac{M}{D}} LU, \quad M^* = \frac{\rho L}{M}. \tag{2}$$

Geometrical parameters of the 3D problem

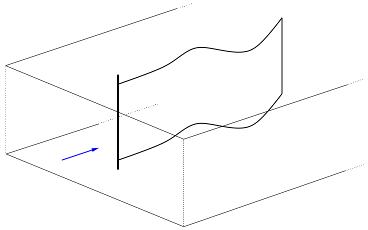
- Aspect ratio :

$$h = H/L \tag{3}$$

- Gap :

$$c = C/L \tag{4}$$

High Reynolds number ⇝ potential flow



Laplace equation for the pressure

$$\Delta p = 0 \tag{5}$$
$$[p_y]_{|y|=h/2+c} = 0$$
$$[p_z]_{z=0} = -(\partial_t + \partial_x)^2 w$$
for $(x, y) \in$ flag

Solution for the pressure in integral form

$$\frac{1}{2\pi}\int_0^1 \langle[\partial_x p]\rangle(\xi) G(x-\xi, h, c)\,\mathrm{d}\xi = (\partial_t + \partial_x)^2 w, \tag{6}$$

where

$$G(x, h, c) = \int_0^\infty \frac{\sin(kx)}{g(k, h, c)}\mathrm{d}k. \tag{7}$$

2D Helmoltz problem to find g

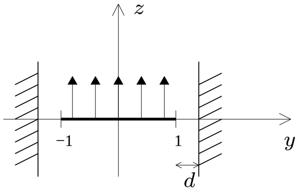
- Helmholtz problem :

$$\begin{aligned} \Delta\varphi &= \kappa^2\varphi, && (8)\\ [\varphi_y]_{|y|=1+d} &= 0, && (9)\\ [\varphi_z]_{z=0} &= 1, \quad \text{for } |y|<1, && (10) \end{aligned}$$

- Function g :

$$g(\kappa,d) = -\frac{\kappa}{2}\langle[\varphi]\rangle = -\kappa\langle\varphi^+\rangle. \qquad (11)$$

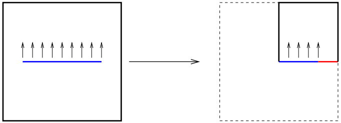
- Rescaling : $\kappa = kh/2$, $d = 2c/h$

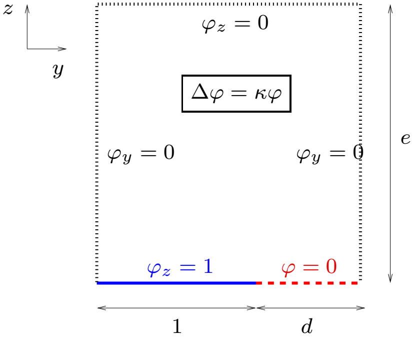


Resolution

Numerical method (good for $\kappa < 1$)

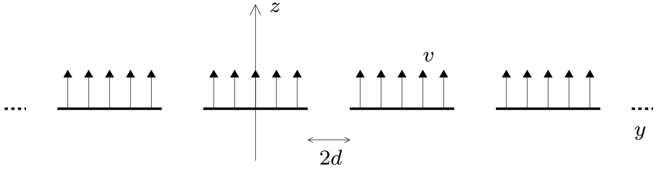
- Finite size problem
  $\rightsquigarrow$ Dimension along the z axis e
- In practice $e = 30$ has been used
- Using symmetry w/r to the z-axis and skew-symmetry w/r to the y-axis
- Finite element software used (COMSOL)





Semi-analytical method (good for $\kappa > 1$)

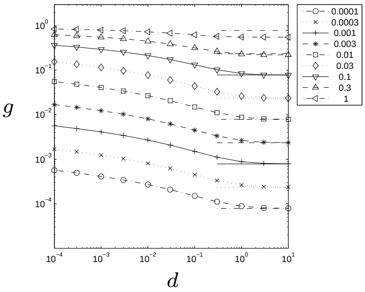
Mirror symmetry



- Green's representation theorem $\rightsquigarrow$ solution for the potential φ in integral form
- Numerical resolution by expanding the solution over Chebychev polynomials of the second kind and projecting the equation on Chebychev polynomial of the first kind $\rightsquigarrow$ linear system

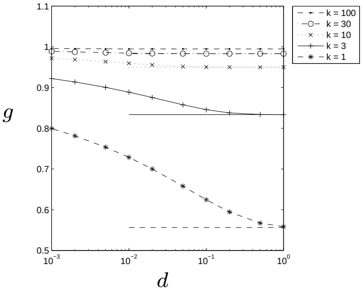
BEHAVIOR OF g

Numerical method



horiz. line $\equiv$ slender body approx

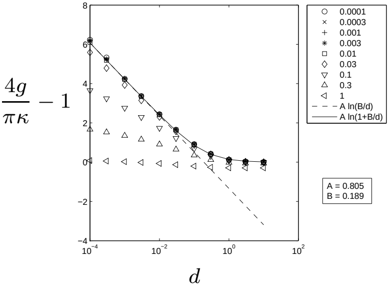
1/2-analytical method



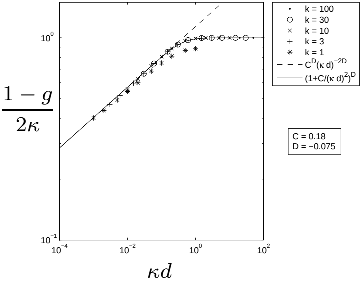
horiz. line $\equiv$ large span approx

BEHAVIOR OF g

Numerical results



Analytical results



- $g_{SB}(\kappa, d) \simeq \frac{\kappa\pi}{4}\left[1 + 0.805 \ln\left(\frac{d+0.189}{d}\right)\right]$, for $\kappa \ll 1$
- $g_{LS}(\kappa, d) \simeq 1 - \frac{1}{2\kappa}\left(1 + \frac{0.18}{(\kappa d)^2}\right)^{-0.075}$, for $\kappa \gg 1$.

Slender body model ($\kappa \ll 1$)

$$g_{SB}(\kappa, d) \simeq \frac{\kappa\pi}{4}\left[1 + 0.805 \ln\left(\frac{d + 0.189}{d}\right)\right] \tag{12}$$

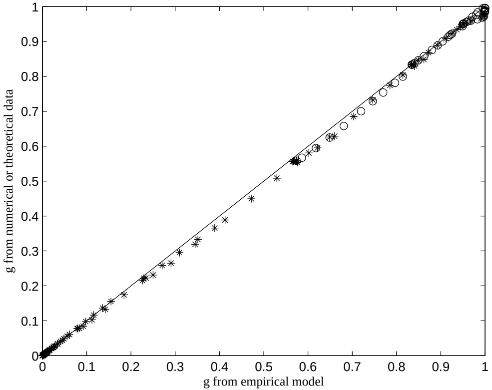
Large-span model ($\kappa \gg 1$)

$$g_{LS}(\kappa, d) \simeq 1 - \frac{1}{2\kappa}\left(1 + \frac{0.18}{(\kappa d)^2}\right)^{-0.075} \tag{13}$$

Composite extension $\rightsquigarrow$ Empirical model for g

$$g_e(\kappa, d) = 1 - \left[\frac{1}{1 - g_{LS}} + \exp\left(g_{SB} - \frac{1}{1 - g_{LS}}\right)\right]^{-1}, \tag{14}$$

Validity of the empirical model



Stability analysis

Methodology [same as in Eloy et al. (2007,2008)]

- Inverse transform of $g \rightsquigarrow G$.
- Decomposition of the plate deformation on Galerkin modes (beam modes)
- Hypothesis of harmonic time dependence of the deformation at a complex frequency ω
- Pressure distribution associated with a given Galerkin mode sought in the form of a combination of Chebychev polynomials of the first kind.
- Integral equation for the pressure projected on Chebychev polynomials of the second kind $\rightsquigarrow$ solution for the pressure corresponding to each Galerkin mode.
- Projection of the beam equation forced by pressure in the flow on Galerkin modes $\rightsquigarrow$ linear eigenvalue problem.
- One eigenfrequency with a negative imaginary part $\rightsquigarrow$ instability.

Stability analysis results

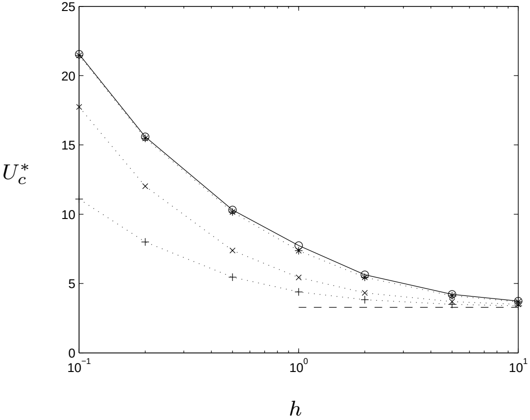


FIGURE: $M^* = 0.5$ $\quad c = \infty$ (circle), $c = 1$ (star), $c = 10^{-2}$ (crosse), $c = 10^{-4}$ (plus); $h = \infty$ (dashed line)

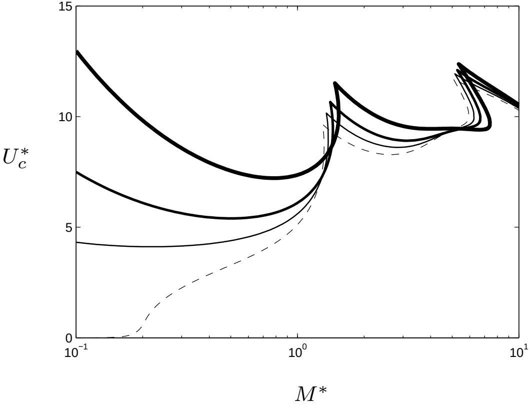
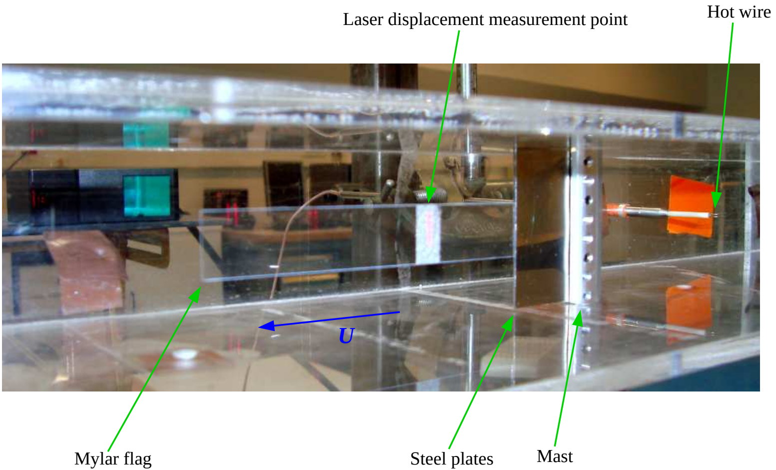


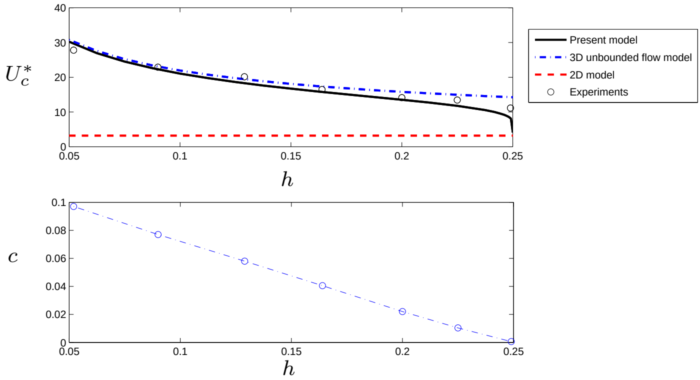
FIGURE: $h = 1$ $c = \infty$ (thick), $c = 10^{-2}$ (medium), $c = 10^{-4}$ (thin), $h = \infty$ (dashed)

Experiments

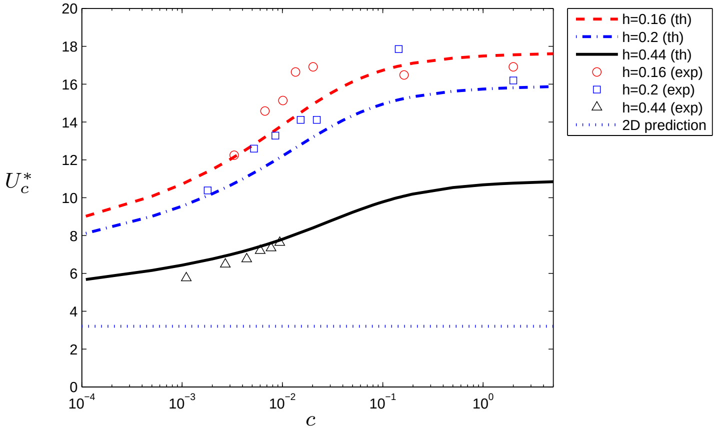
Experiments



Experiment 1 : Fixed channel, fixed length plate, varying height



Experiment 2 : Fixed size plate, varying channel height



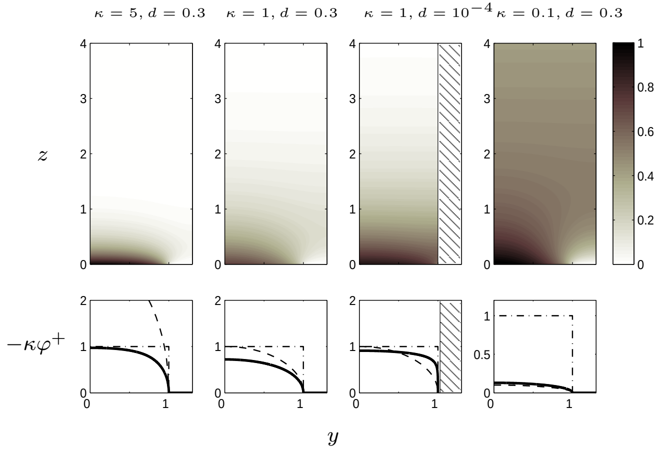
CONCLUSIONS

- 3D model for the pressure distribution in a flow bounded by two walls in the spanwise direction
- Stability analysis using this model
- Model validated by experiments
- Main result : the 2D limit is very difficult (impossible?) to achieve experimentally by bounding the flow in the spanwise direction

IMPROVEMENTS

- Small gaps, confined flow $\rightsquigarrow$ viscosity effects ?
- Effect of the confinement in the z-direction ?

NUMERICAL RESULTS



— — —, Slender-body model $-\kappa\phi^+ = \kappa\sqrt{1-y^2}$

— . — . — 2D model, $-\kappa\phi^+ = 1$