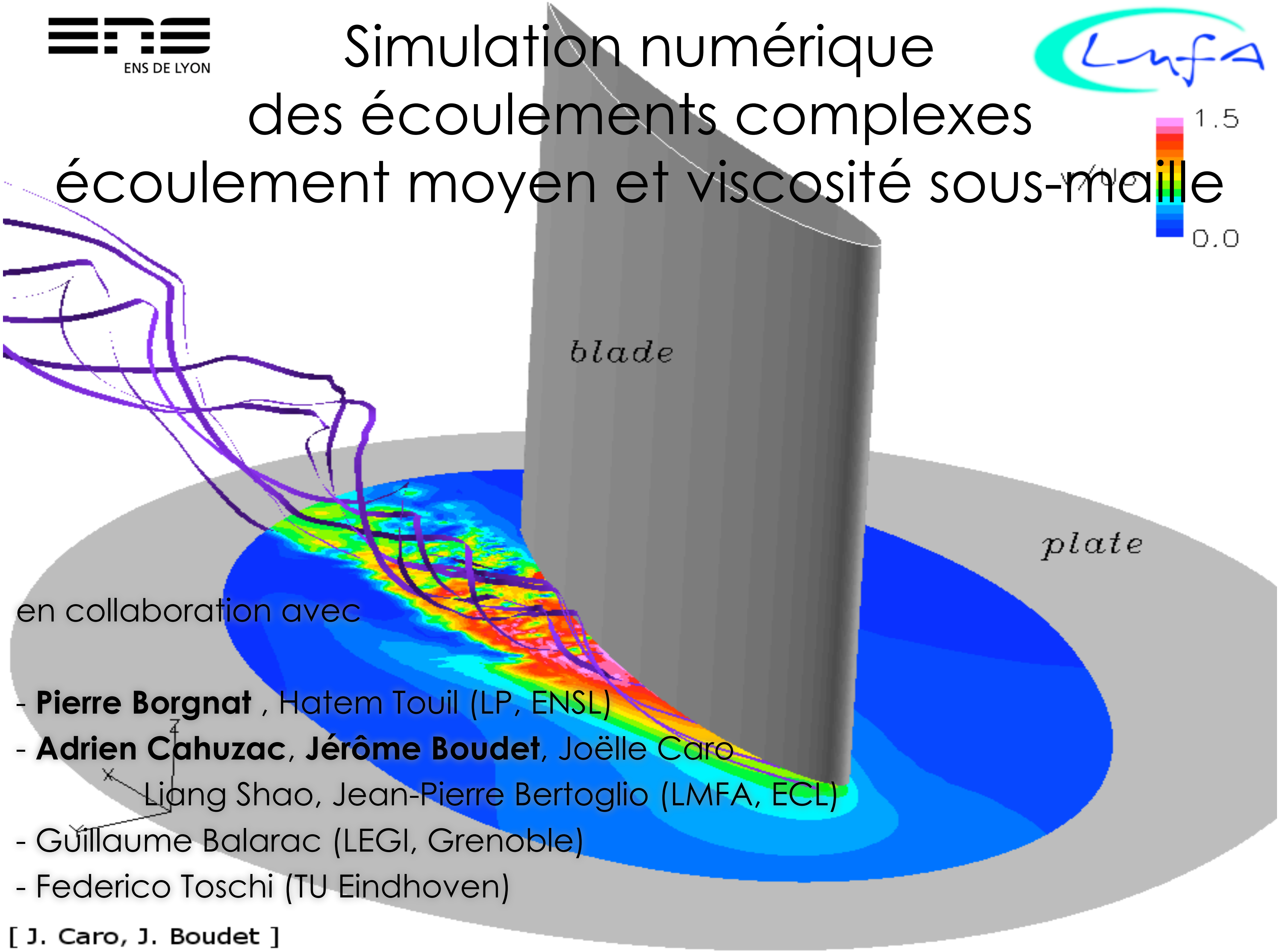
Simulation numérique des écoulements complexes écoulement moyen et viscosité sous-maille

ENS DE LYON



en collaboration avec

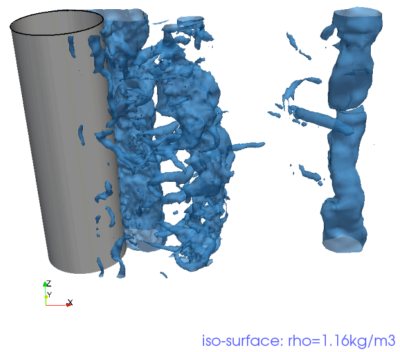
- **Pierre Borgnat** , Hatem Touil (LP, ENSL)
- **Adrien Cahuzac**, **Jérôme Boudet**, Joëlle Caro
  Liang Shao, Jean-Pierre Bertoglio (LMFA, ECL)
- Guillaume Balarac (LEGI, Grenoble)
- Federico Toschi (TU Eindhoven)

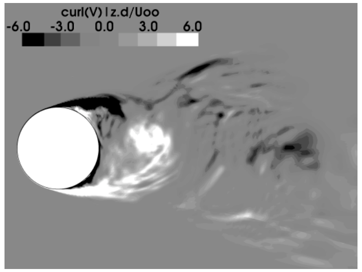
[J. Caro, J. Boudet]

Inhomogénéité - phénoménologie

$u = \langle u \rangle + u'$: décomposition de Reynolds

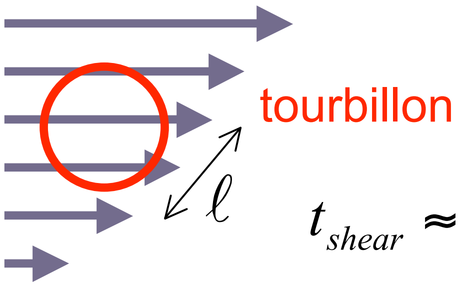
$\Sigma = \sqrt{\left|\langle \nabla u \rangle\right|^2}$: cisaillement



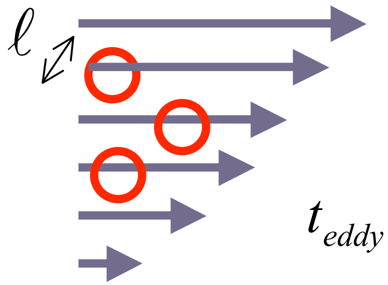


écoulement autour d'un cylindre à Re_D=47000 (régime turbulent sous-critique)

écoulement moyen + fluctuations



$$t_{shear} \approx \frac{1}{\Sigma}$$



$$t_{eddy} \approx \ell / \delta u(\ell) \approx \varepsilon^{-1/3} \ell^{2/3}$$

Kolmogorov's theory

$$\varepsilon \approx u'^2 \cdot \Sigma$$

taux de dissipation d'énergie

$$L_\Sigma \approx \frac{u'}{\Sigma}$$

turbulence cisaillée | **turbulence homogène et isotrope**

Simulation numérique des écoulements turbulents

goal= get a physically-sound **numerical representation of the flow**

-**D**irect **N**umerical **S**imulation (all scales of motion) is usually prohibitive

- **alternatives = partial representation of the flow :**
  - **R**eynolds-**A**veraged **N**avier-**S**tokes : mean flow
  - **L**arge-**E**ddy **S**imulation : large-scale dynamics of the flow

→**Does it make sense (from a physical viewpoint) ?**

→ **What are the governing equations?**

$$\partial_t \widetilde{U}_{grid} = \mathrm{NS}(\widetilde{U}_{grid}) + \mathrm{div}\,\tau_{subgrid}$$

$$\tau_{subgrid} - \mathrm{tr}(\tau_{subgrid})\mathrm{I} = -2\nu_{subgrid}\,\widetilde{S}$$

SGS viscosity = property of the flow, not of the fluid… (very) bad news !

$$\nu_{subgrid}(\Delta, \widetilde{U}_{grid}, \nabla\widetilde{U}_{grid}, \text{etc.})?$$

Effet du cisaillement en équations

$$u_i = u'_i + \frac{\partial \langle u \rangle_i}{\partial x_j} \cdot x_j : \quad \text{locally uniform shear}$$

- after some (exact) calculations from the Navier-Stokes equations…

$$\mathrm{T}_{sgs}(l) = \frac{1}{4\pi l^2} \oint_{\partial B_l} \left(\left\langle \left| \delta u'(x,l) \right|^2 \cdot \delta u'_i(x,l) \right\rangle + \frac{\partial \langle u \rangle_i}{\partial x_j} \cdot l_j \left\langle \left| \delta u'(x,l) \right|^2 \right\rangle \right) \mathrm{dS_i}$$

- in the context of LES :

$$-\left\langle \tau_{sgs} \cdot \overline{S} \right\rangle = 2 \left\langle \nu_{sgs}(\Delta) \cdot \left| \overline{S} \right|^2 \right\rangle \approx \mathrm{T}_{sgs}(\Delta)/\Delta \qquad \text{with } \Delta = \text{grid spacing}$$

$$\delta u'(\Delta) \approx \left| \overline{S}' \right| \cdot \Delta \qquad \text{and} \qquad \delta U(\Delta) \approx \left| \left\langle \overline{S} \right\rangle \right| \cdot \Delta \Rightarrow \nu_{sgs}(\Delta) = (C_s \Delta)^2 \cdot \left(\left| \overline{S} \right| - \left| \left\langle \overline{S} \right\rangle \right| \right)$$

Shear-Improved Smagorinsky Model
Lévêque et al., *J. Fluid Mech.* (2007)

$$\nu_{sgs}(\Delta) = (C_s \Delta)^2 \cdot \left(\left| \bar{S} \right| - \left| \langle \bar{S} \rangle \right| \right)$$

If $\left| \tilde{S}' \right| >> \left| \langle \tilde{S} \rangle \right|$: Smagorinsky model for HI turbulence (Cs =0.17)

$$\nu_{sgs} \approx (C_s \Delta)^2 \cdot \frac{\delta u'(\Delta)}{\Delta} \approx (C_s \Delta)^2 \cdot \left| \tilde{S}' \right|$$

If $\left| \langle \tilde{S} \rangle \right| >> \left| \tilde{S}' \right|$: Shear-dominated SGS viscosity

$$\nu_{sgs} \approx (C_s \Delta)^2 \cdot \frac{\left| \tilde{S}' \right|^2}{\left| \langle \tilde{S} \rangle \right|}$$

PLUS :

→ physically-sound, no ad-hoc parameter
→ low computational cost
→ easy to parallelize
→ convenient for boundary conditions

MINUS :

→ requires **estimation of mean-flow as the simulation runs!**

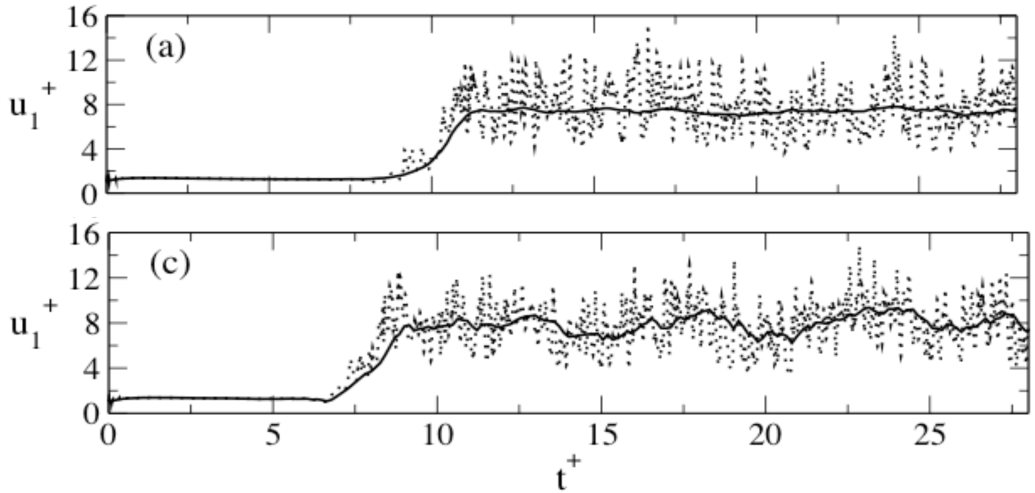
signal processing

Application to complex-geometry unsteady turbulent flows (real-world flows)

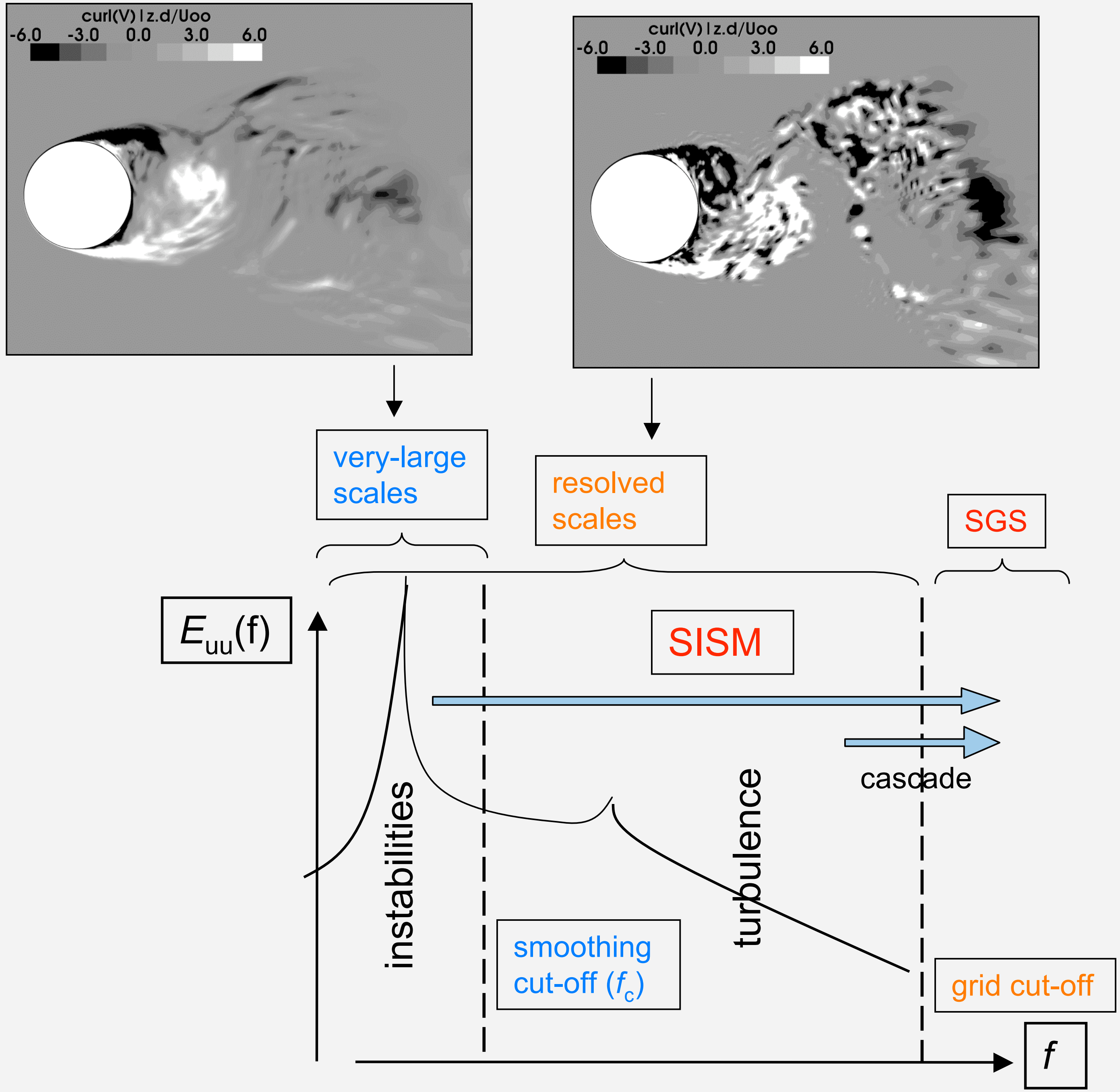
Cahuzac et al., in press Phys. Fluids (2010)

How to extract (or reconstruct) the mean flow as the simulation runs ?

- **physically sound** : smoothing in time
 mean flow = low-frequency component of the flow



-**manageable** from computational viewpoint : recursive filter
-**adaptive** : Kalman filtering


curl(V) | z.d/Uoo
-6.0 -3.0 0.0 3.0 6.0
curl(V) | z.d/Uoo
-6.0 -3.0 0.0 3.0 6.0
very-large scales
resolved scales
SGS
E_{uu}(f)
SISM
cascade
instabilities
turbulence
smoothing cut-off (f_c)
grid cut-off
f

Exponentially-weighted moving average exponential smoothing

$$\langle u \rangle^{(n+1)} = (1 - c_{\exp}) \cdot \langle u \rangle^{(n)} + c_{\exp} \cdot u^{(n+1)}$$ at each grid point

-c_{exp} : smoothing factor
- equivalent tp first-order low pass filter with cut-off frequency f_c :

$$c_{\exp} = \frac{2\pi \, f_c \, \Delta t}{\sqrt{3}} \approx 3.628 \, f_c \, \Delta t$$

PLUS :

→ conceptually simple
→ local in space
→ low memory storage

MINUS :

→ uniform physically-sound cut-off frequency ?
→ unavoidable phase delay : $\Delta t \, / \, c_{\exp}$

Low-pass Kalman filter

adaptive exponential smoothing

- $\langle u \rangle^{(n+1)} = \langle u \rangle^{(n)} + \delta \langle u \rangle^{(n+1)}$: model

control increment = random noise with *fixed variance*

$$\sigma_{\delta\langle u \rangle} = \frac{2\pi \; f_c \; \Delta t}{\sqrt{3}} \cdot u^* \quad \text{why ?}$$

$$\sigma_{\delta\langle u \rangle} / \sigma_{\delta u} = c_{\text{exp.}} \text{ for stationary periods}$$

- $u^{(n+1)} = \langle u \rangle^{(n+1)} + \delta u^{(n+1)}$: observation

deviation from the estimated mean = random noise with variance

$$\sigma_{\delta u}^{(n+1)} = \max \left[u^* \cdot \left| \langle u \rangle^{(n+1)} - u^{(n+1)} \right|, \, 0.1 \cdot u^{*2} \right]$$

predict and update :

$$\langle u \rangle^{(n+1)} = (1 - K^{(n+1)}) \cdot \langle u \rangle^{(n)} + K^{(n+1)} \cdot u^{(n+1)}$$

$K^{(n+1)}$ optimal Kalman gain (given by theory)

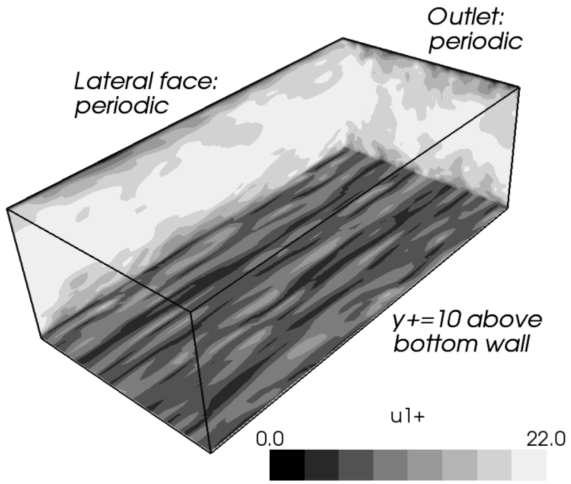
Practical evaluation of the method

Turb'Flow solver (Jérôme Boudet, LMFA):

- Finite volume
- Convective fluxes by centered schemes on four points
- Diffusive fluxes by centered scheme on two points
- SGS model = Shear-Improved Smagorinsky Model (SISM)

Cahuzac et al. Phys. Fluids (2010)

plane-channel Flow at Re_w = 395



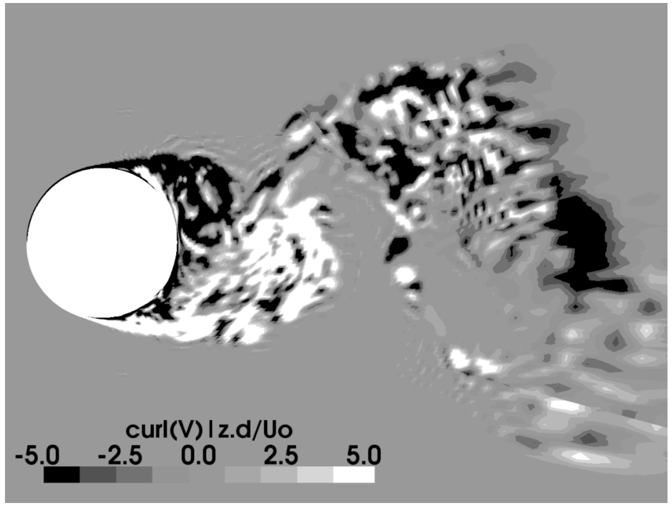
resolution: 49x89x41

Δx+=52, Δz+=31, Δy+=0.5…24

$$u^* = u_{wall}$$ friction velocity

$$f_c = u^*/\delta$$

flow past a circular cylinder at Re_D = 47,000



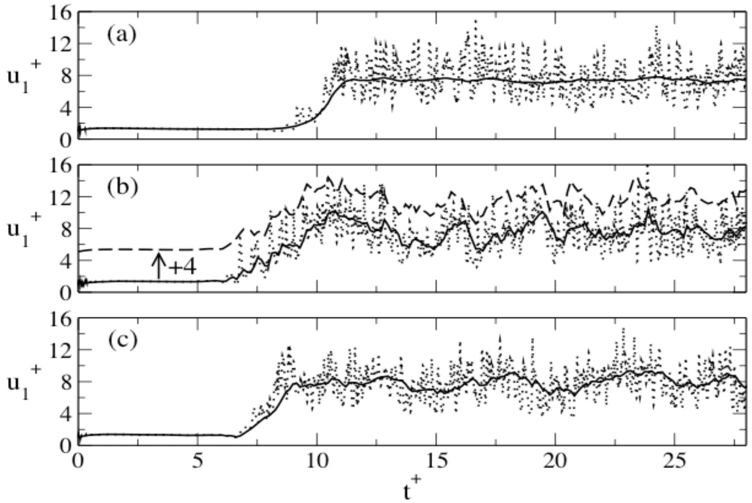
3.10^6 mesh-points

Δr+=1, RΔθ+=20, Δz+=25

$$St = f_s \cdot D / U_\infty$$ Strouhal number

$$f_c = 2 f_s$$

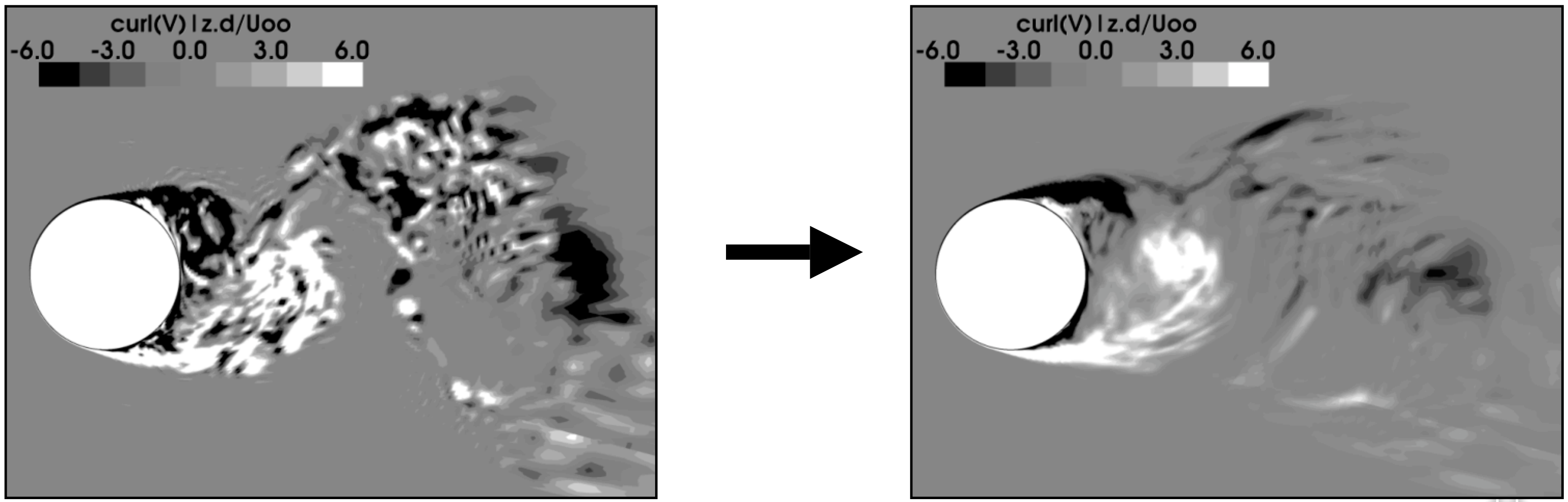
Velocity probes at y^+=10 in the plane-channel flow (Rew=395)

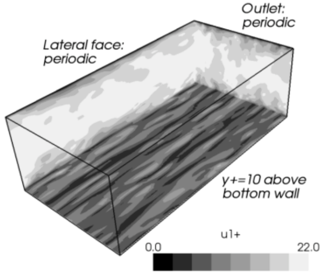


… spatial average (along x-z)

… exponential smoothing compared to spatial average (along x)

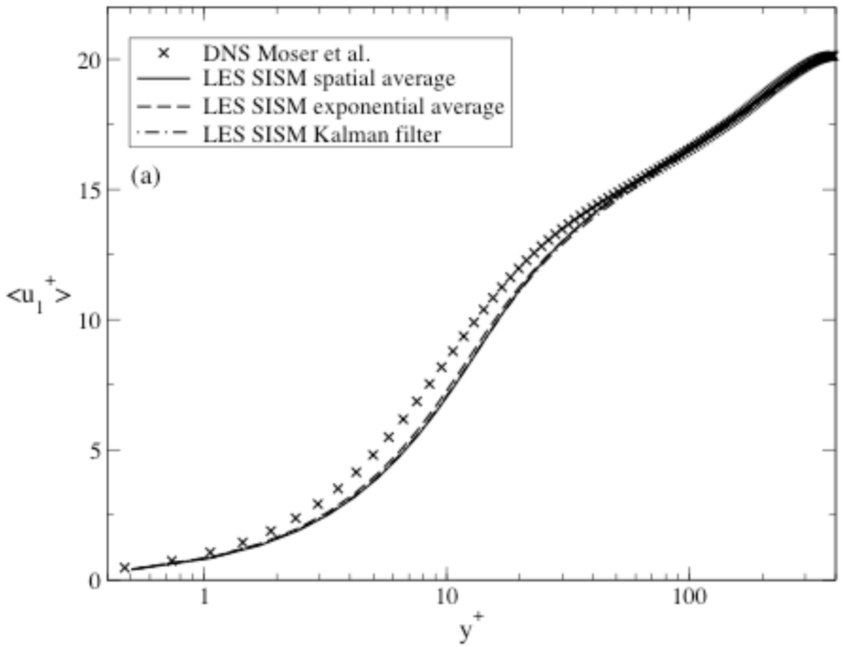
… Kalman filtering



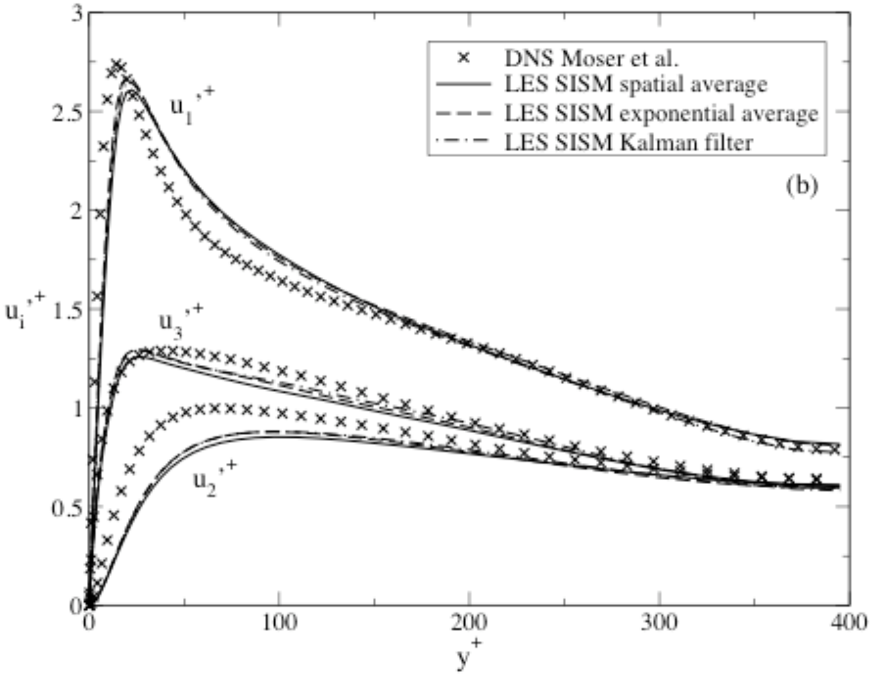


Plane-channel flow

comparisons with DNS data



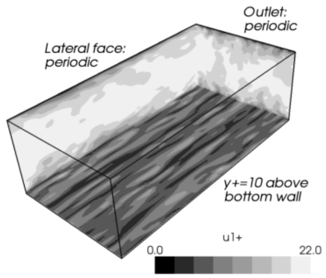
streamwise velocity profile



turbulent intensity profiles

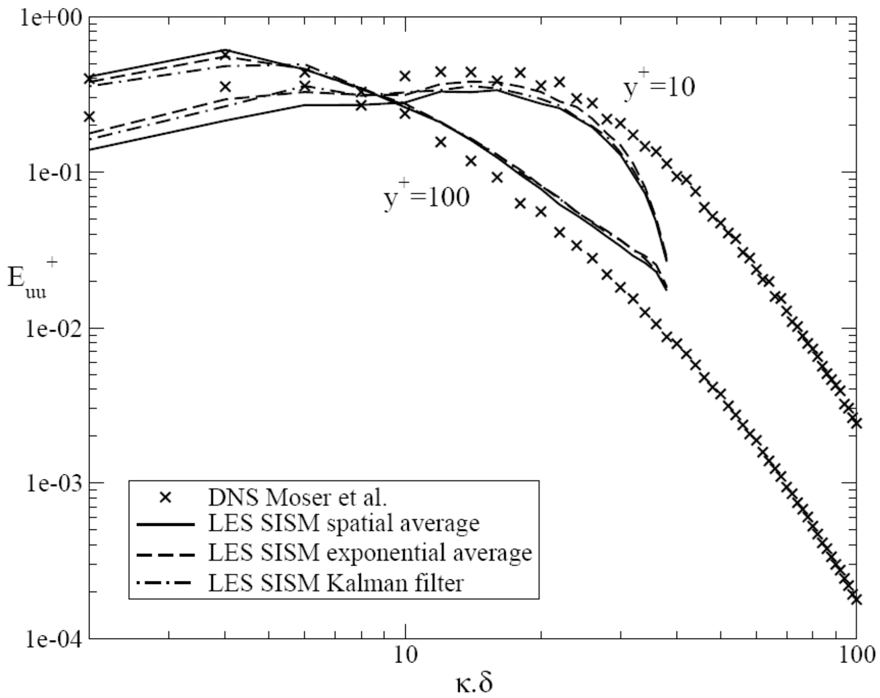
friction velocity: $u_w^{exp.} = 0.54\, m.s^{-1}$

$u_w^{DNS} = 0.59\, m.s^{-1}$ $\rightarrow$ within the 10% level of accuracy reviewed for standard SGS models

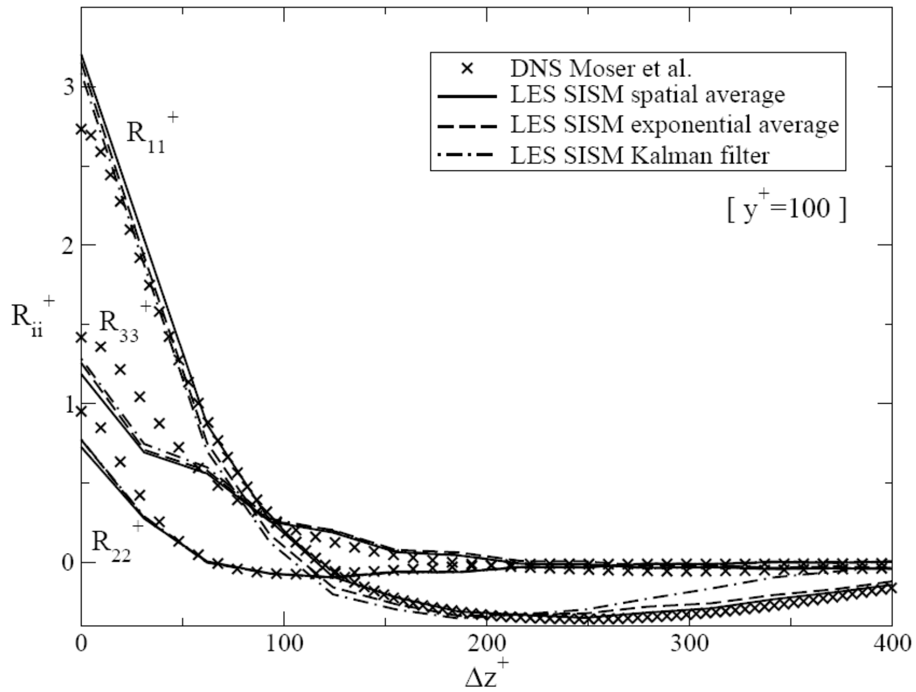


plane-channel flow

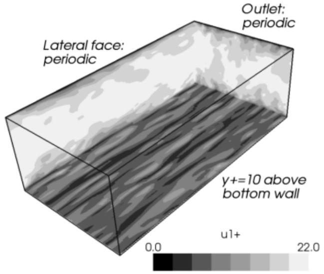
comparisons with DNS data



spanwise spectra
of streamwise velocity

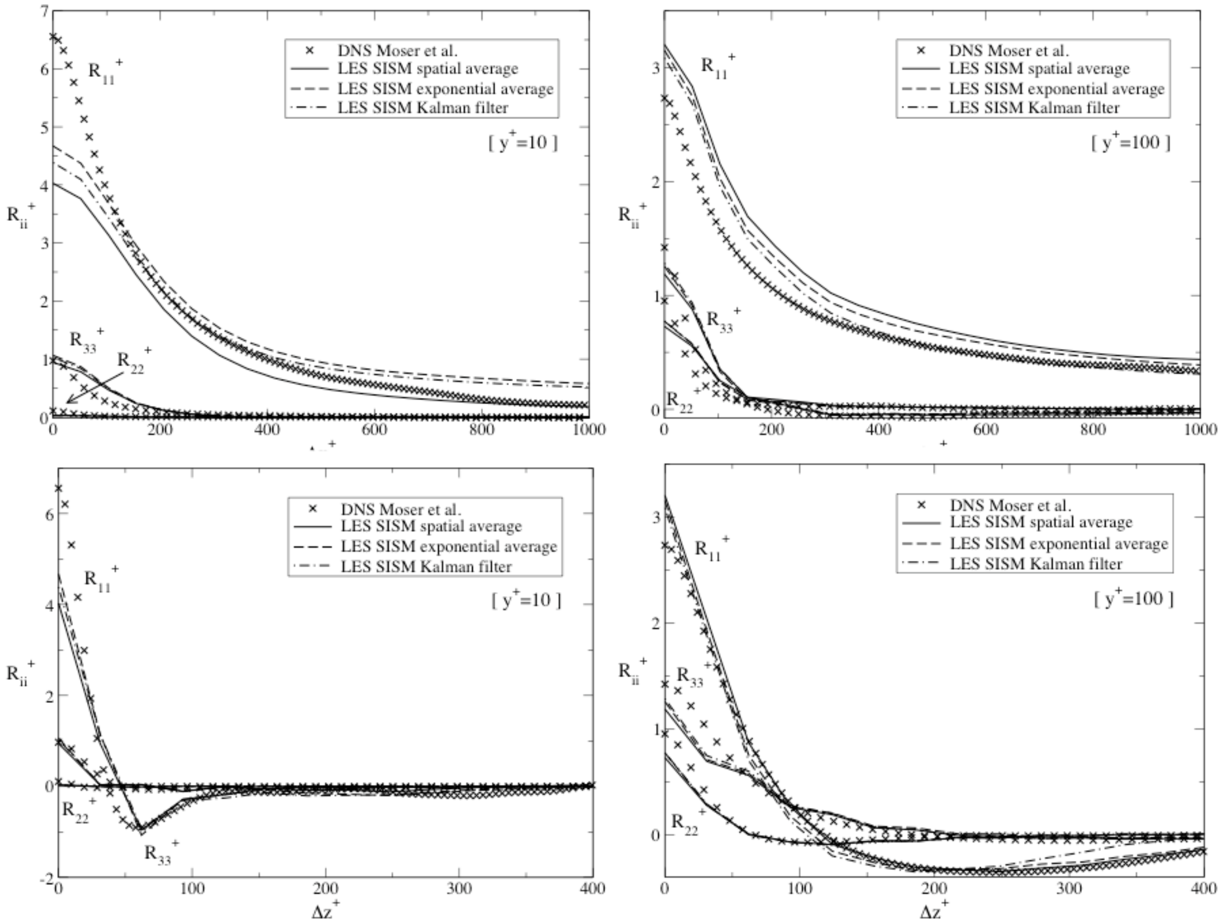


two-point velocity correlation
for spanwise separation



plane-channel flow

comparisons with DNS data

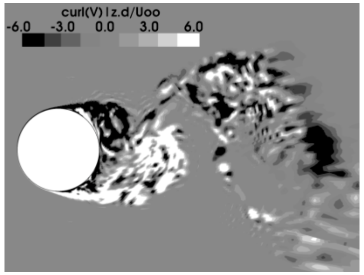


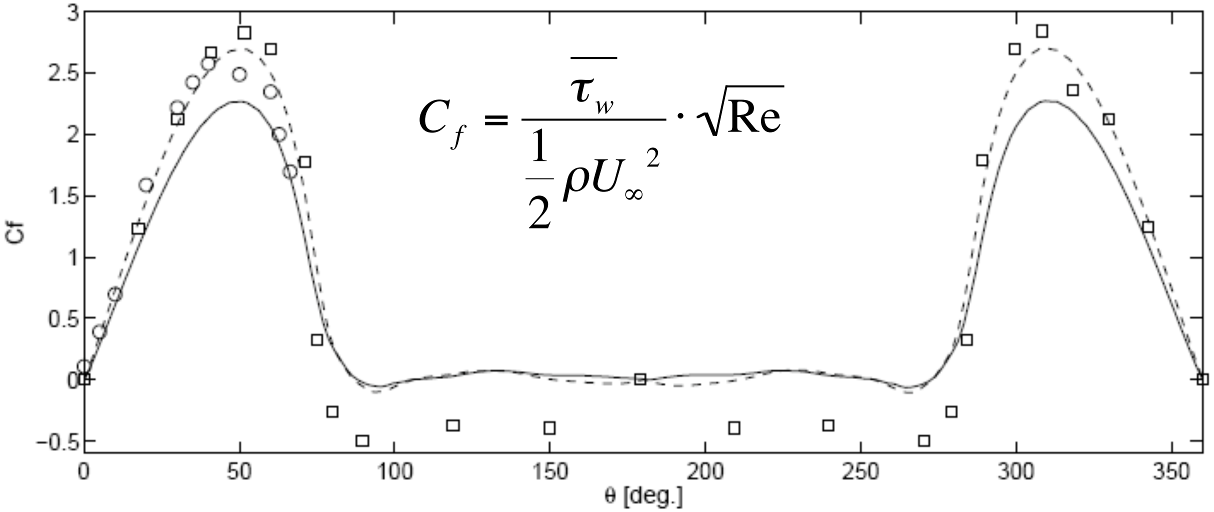
streamwise

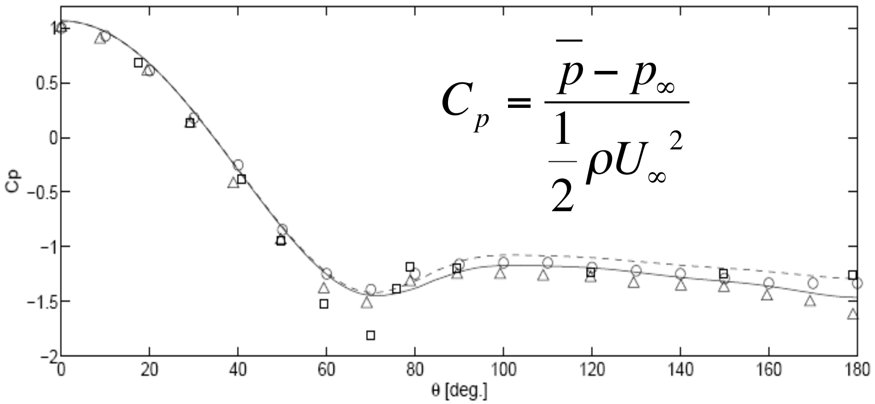
spanwise

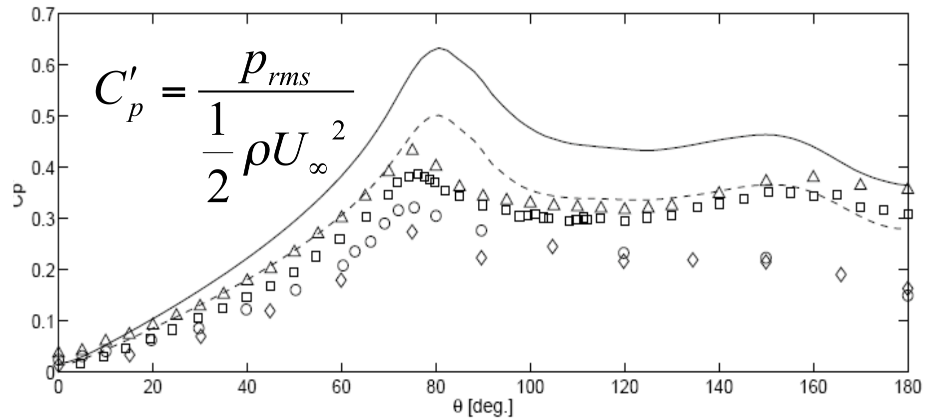
two-point correlation functions… promising for aero-acoustics flow past a circular cylinder

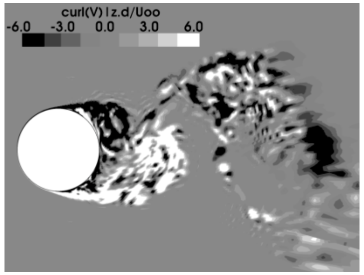
Friction and pressure coefficients





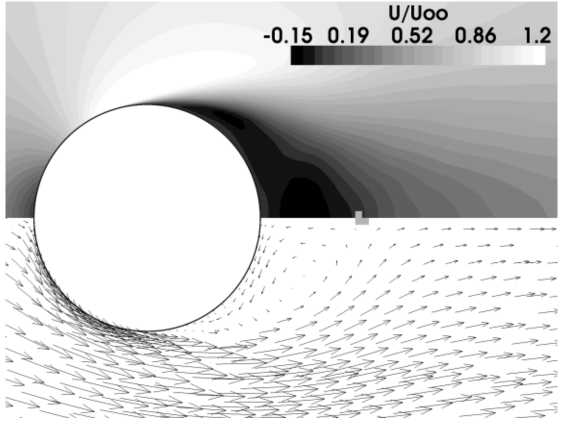


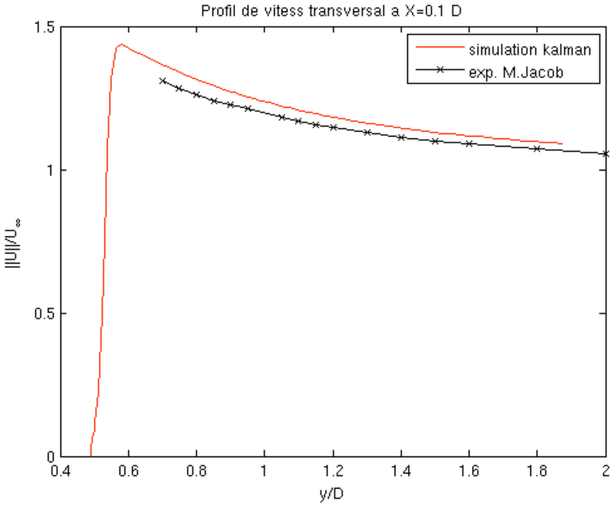


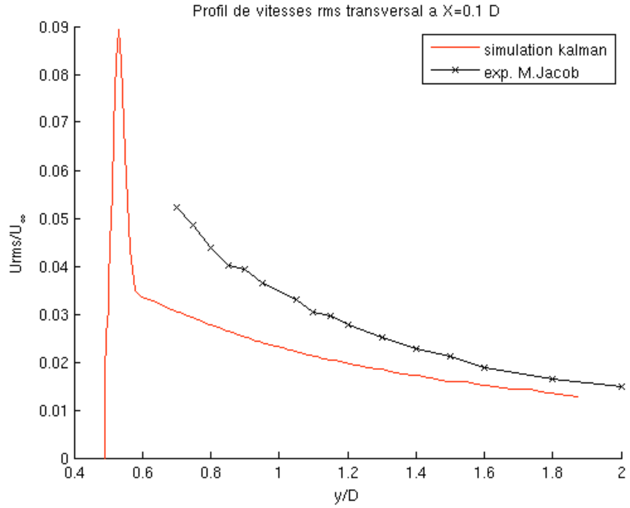


flow past a circular cylinder

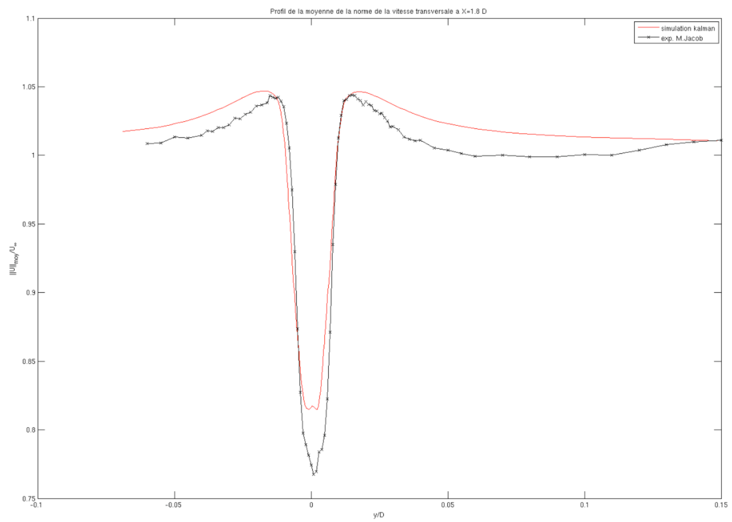
mean flow and turbulence intensities

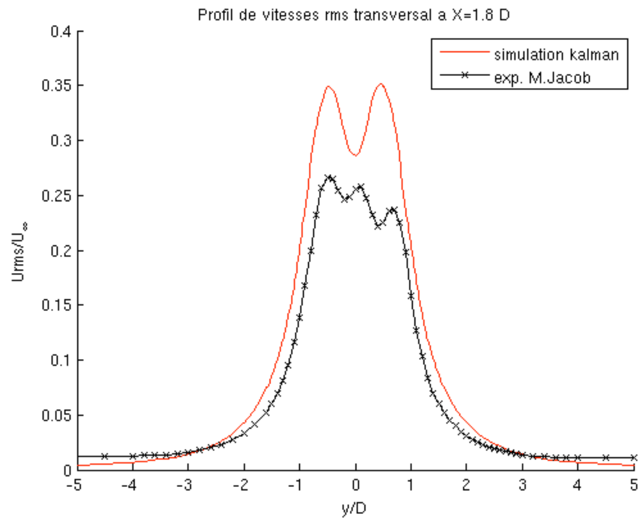


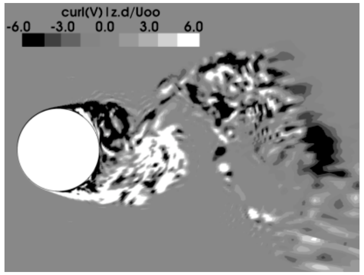




Work in progress...

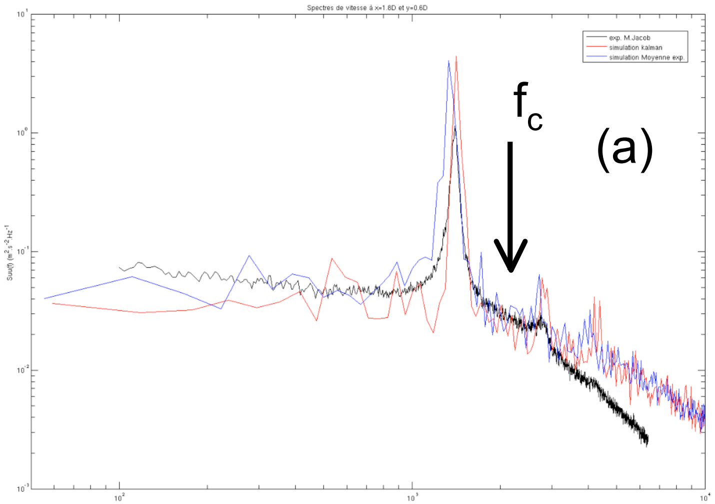


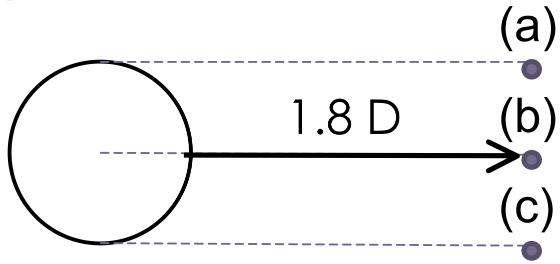


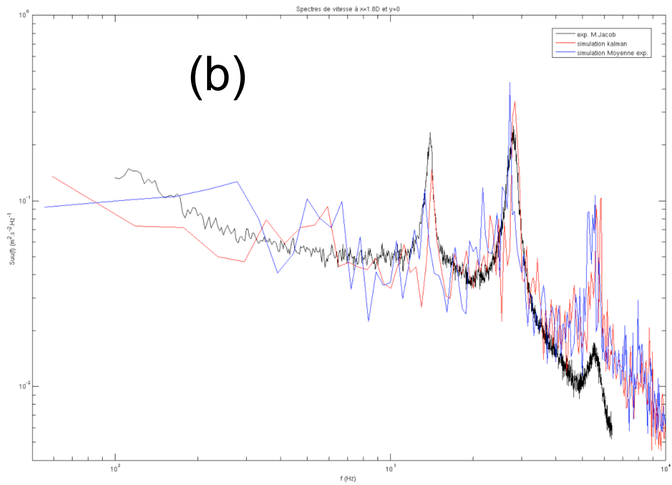


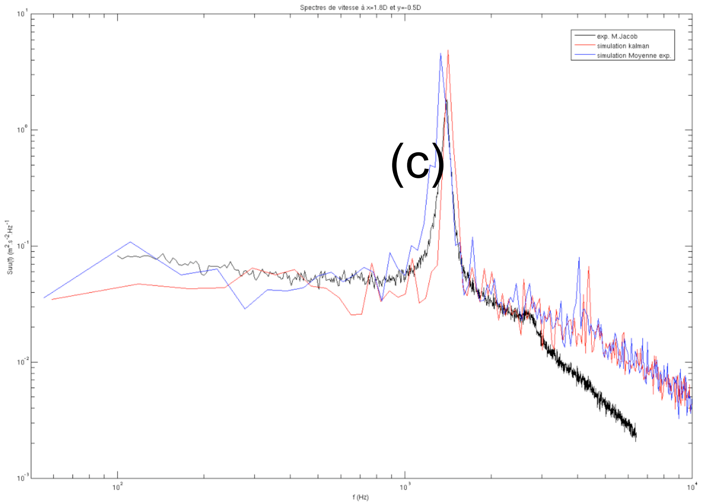
flow past a circular cylinder

comparisons with experimental data spectra in frequency



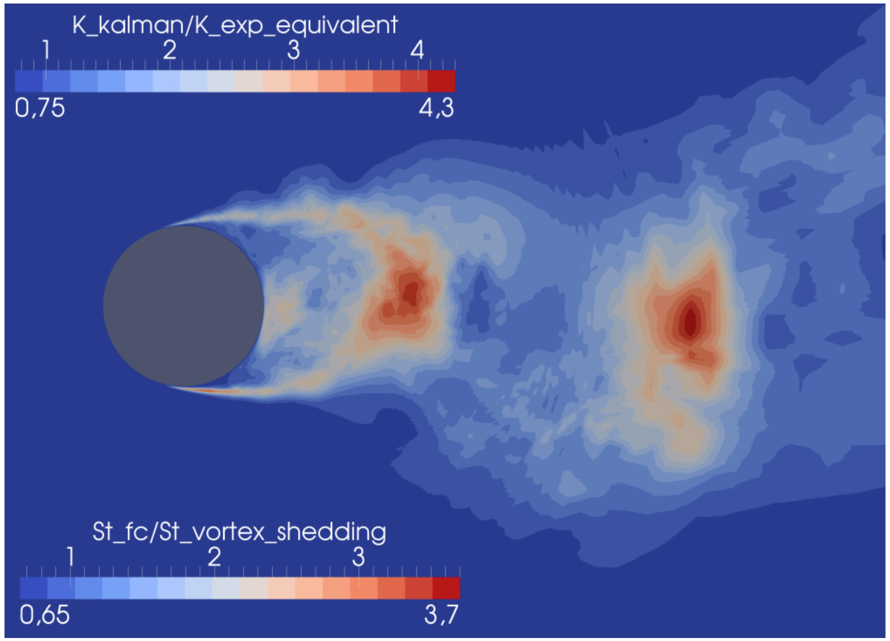


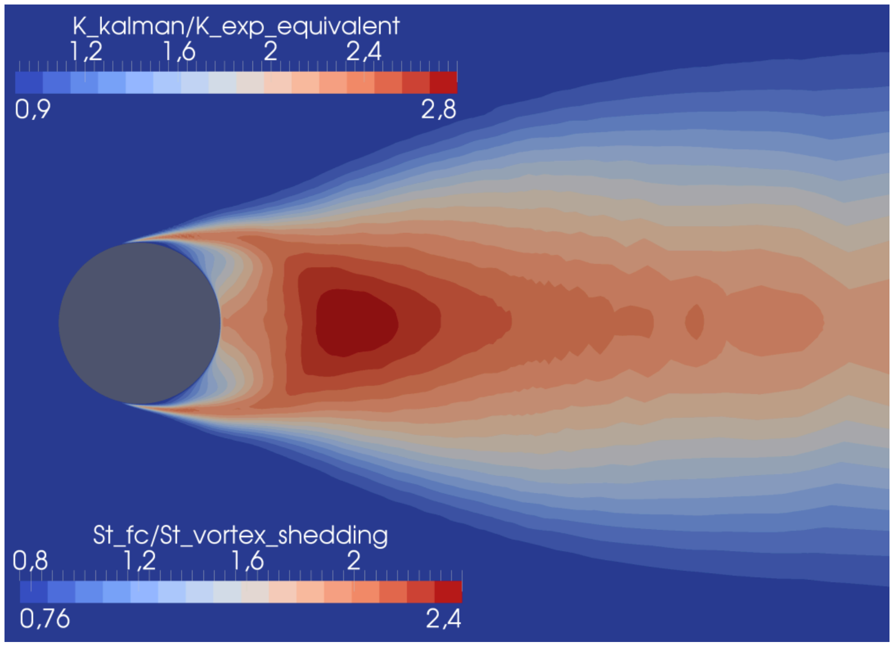


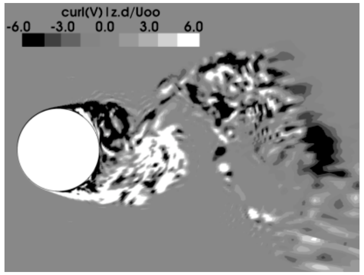


… promising for aero-acoustics

« Adaptivity » of the Kalman filter

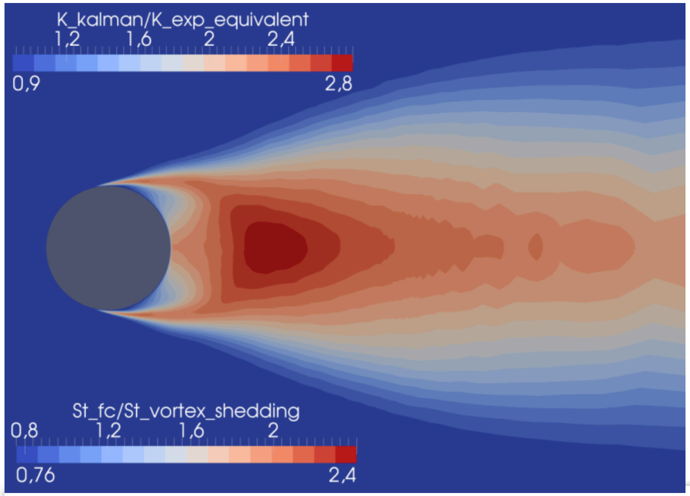






flow past a circular cylinder

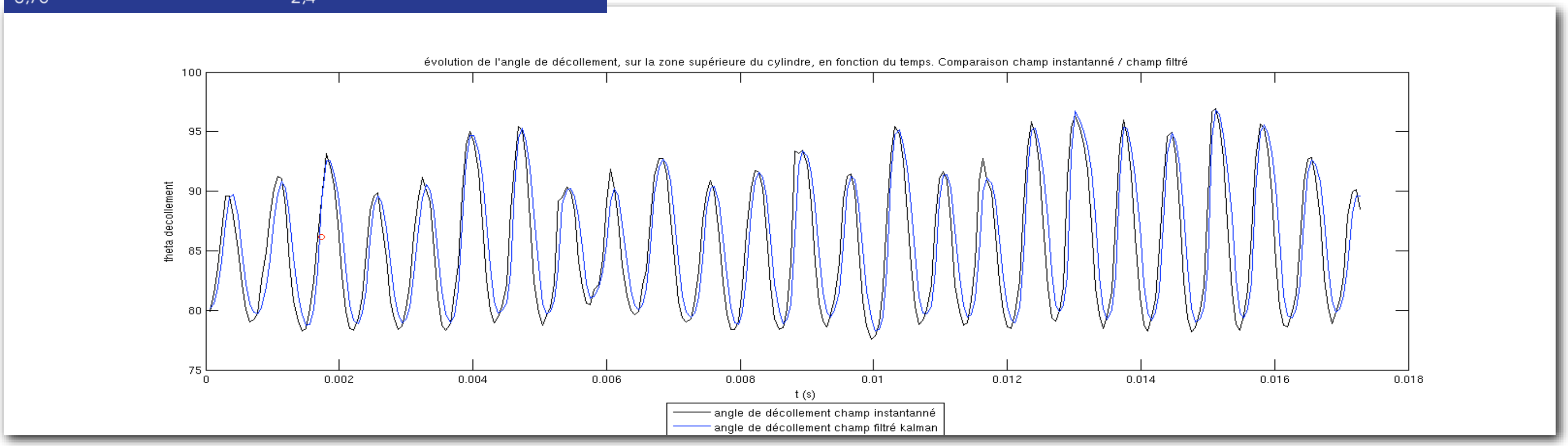
a drawback of the smoothing: phase delay



$$\Delta t / K_{Kalman}$$

$$0 < K_{Kalman} < 1$$

… make K_{Kalman} maximum in separation zone



Conclusion

from a physical viewpoint :

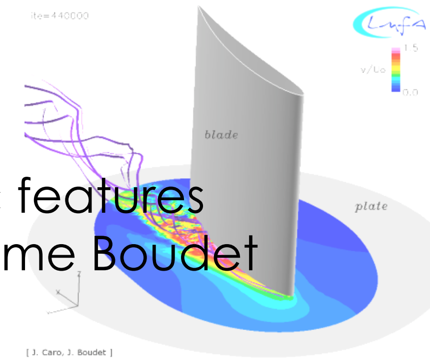
- take into account mean-flow *inhomogeneity* + *unsteadiness :*
  SISM (no ad-hoc parameter) + (adaptive) temporal smoothing

from a numerical viewpoint :

- inconditionnally stable
- low cost algorithms
- easy to parallelize + convenient for boundary conditions

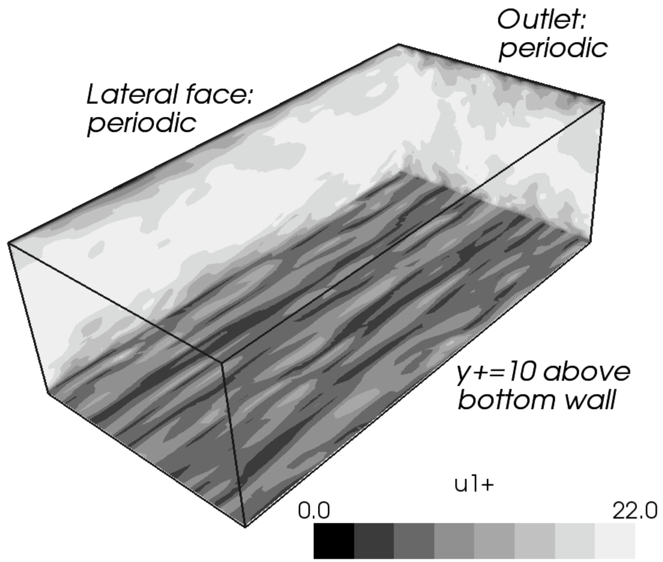
To-Do list:

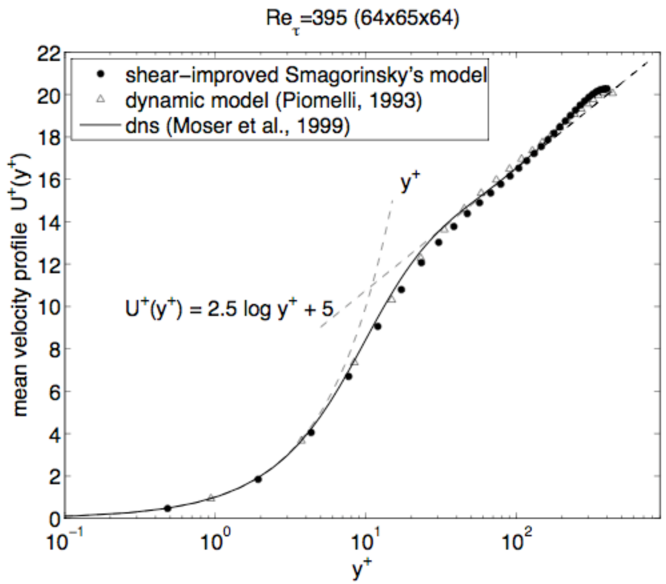
- address realistic turbo-machinery flow / aero-acoustic features
  Flocon project (LMFA) : Adrien Cahuzac + Jérôme Boudet
- reduce phase delay :
  - improve Kalman filter
  - consider Lagrangian filtering (along fluid trajectory)

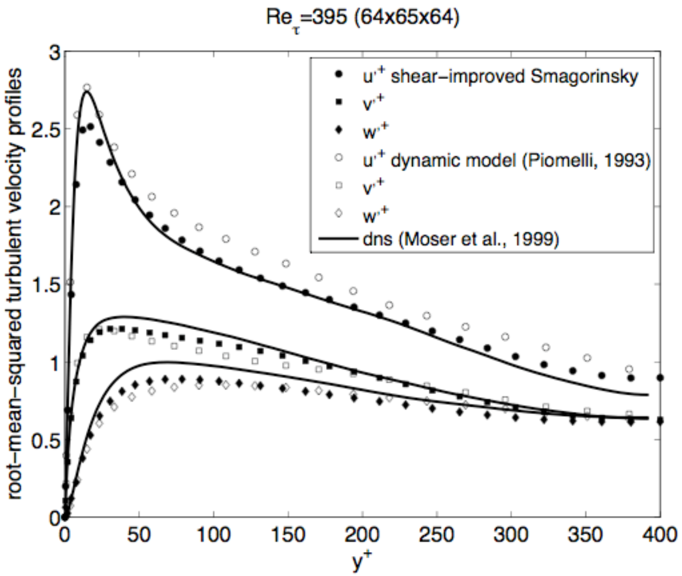


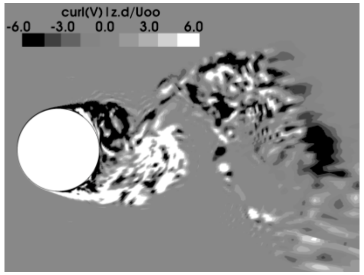
Plane channel flow (academic flow)

Lévêque et al., *J. Fluid Mech.* (2007)
Boudet et al., *J. of Therm. Science.* (2008)
Cahuzac et al. Phys. Fluids (2010)









flow past a circular cylinder

comparisons with experimental data

	SISM-ES	SISM-AKF	data in literature
$\langle C_D \rangle$: mean drag coefficient	1.34	1.23	1.35 ($Re_D = 4.3\ 10^4$), ref.[35] [1.0, 1.35] ($Re_D = 4.8\ 10^4$), ref.[38] [1.0, 1.3] ($Re_D = 4.8\ 10^4$), ref.[39] [1.1, 1.3] ($Re_D \in [10^4, 10^5]$), ref.[37]
C'_D: rms drag coefficient	0.09	0.065	0.16 ($Re_D = 4.3\ 10^4$), ref.[35] [0.08, 0.1] ($Re_D = 4.8\ 10^4$), ref.[40] [0.05, 0.1] ($Re_D \in [10^4, 10^5]$), ref.[37]
C'_L: rms lift coefficient	0.77	0.603	[0.45, 0.55] ($Re_D = 4.3\ 10^4$), ref.[35] [0.4, 0.8] ($Re_D = 4.8\ 10^4$), ref.[40] [0.6, 0.82] ($Re_D \in [10^4, 10^5]$), ref.[37]
St: Strouhal number	0.190	0.204	[0.18, 0.2] ($Re_D = 4.8\ 10^4$), ref.[38] [0.185, 0.195] ($Re_D = 6.1\ 10^4$), ref.[41]

flow characteristics

Mesh resolution