Extraction d'une signature cohérente par Empirical Mode Decomposition

Nicolas MAZELLIER & Fabrice FOUCHER

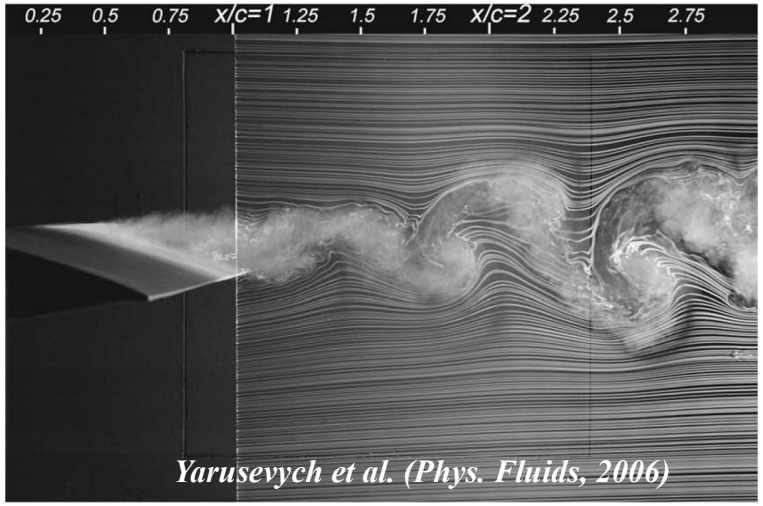
Institut PRISME – Université d'Orléans
Equipe Ecoulements et Systèmes Aérodynamiques

nicolas.mazellier@univ-orleans.fr

Contexte et objectifs : → interactions structures cohérentes / champ turbulent

→ Modélisation turbulente : Effet longue portée (influence des CI / CL)

Hussain (J. Fluid Mech., 1986), Tong and Warhaft (Phys. Fluids 1994), Lavoie et al. (Exp. Fluids, 2005), George (2008)



Yarusevych et al. (Phys. Fluids, 2006)

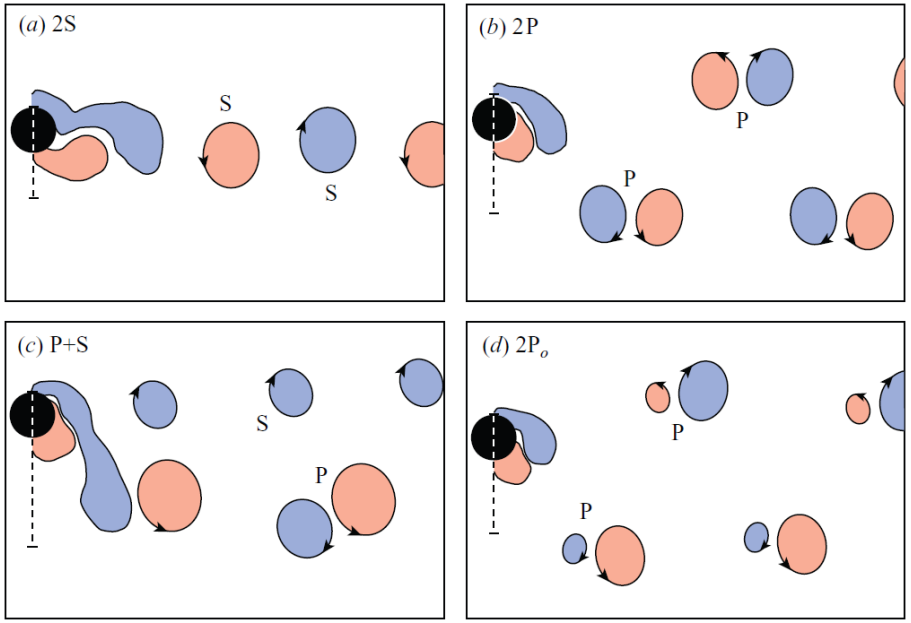
→ Interactions non-linéaires entre la couche de mélange et le sillage proche : aspect contrôle

Bloor (J. Fluid Mech., 1964), Kourta et al. (J. Fluid Mech., 1987), Tian et al. (2006)

→ Instationnarité des performances aérodynamiques : interactions fluide-structure

Bearman and Obasaju (J. Fluid Mech., 1982), Bearman (J. Fluid Mech., 2009)

Morse and Williamson (J. Fluid Mech., 2009)



Empirical Mode Decomposition *Huang et al. (Proc. R. Soc. Lond. A, 1998)*

« détail » (haute fréquence)

$$s(t) = d(t) + r(t)$$

« tendance » (basse fréquence)

avec $$r(t) = \frac{e_{min}(t) + e_{max}(t)}{2}$$

$$\boxed{s(t) = \sum_{k=1}^{N} \mathrm{IMF}_k(t) + r_N(t)}$$

Avantages :

Pas de fonctions prédéfinies (data-driven) $\Rightarrow$ non-linéarité

Pas une approche statistique (local) $\Rightarrow$ instationnarité

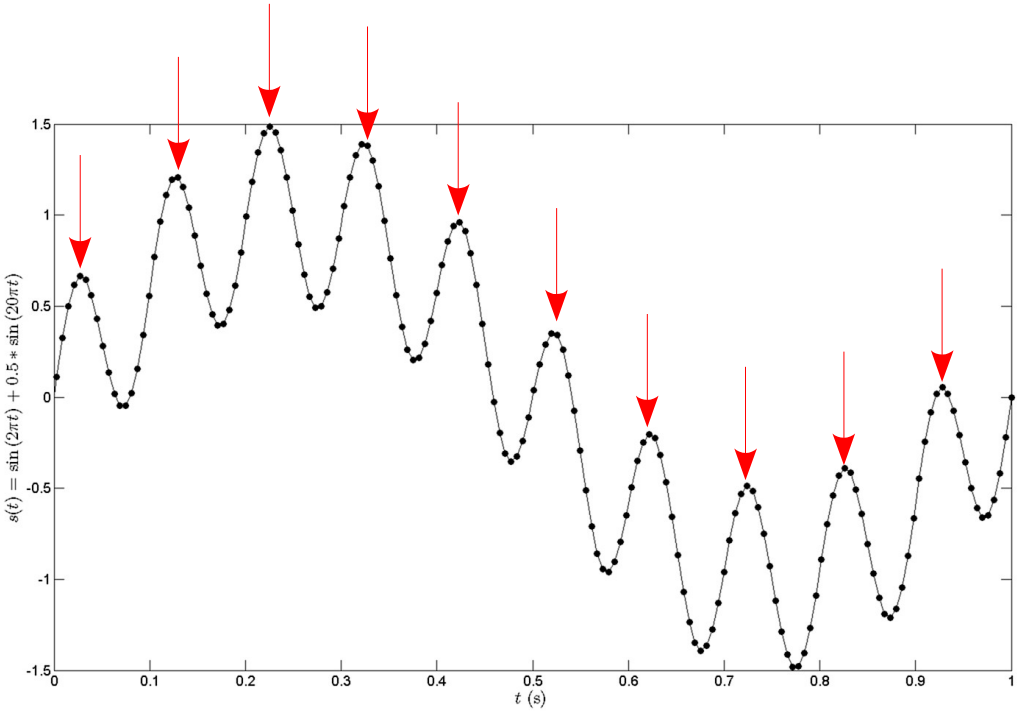
Inconvénients :

Formalisme mathématique

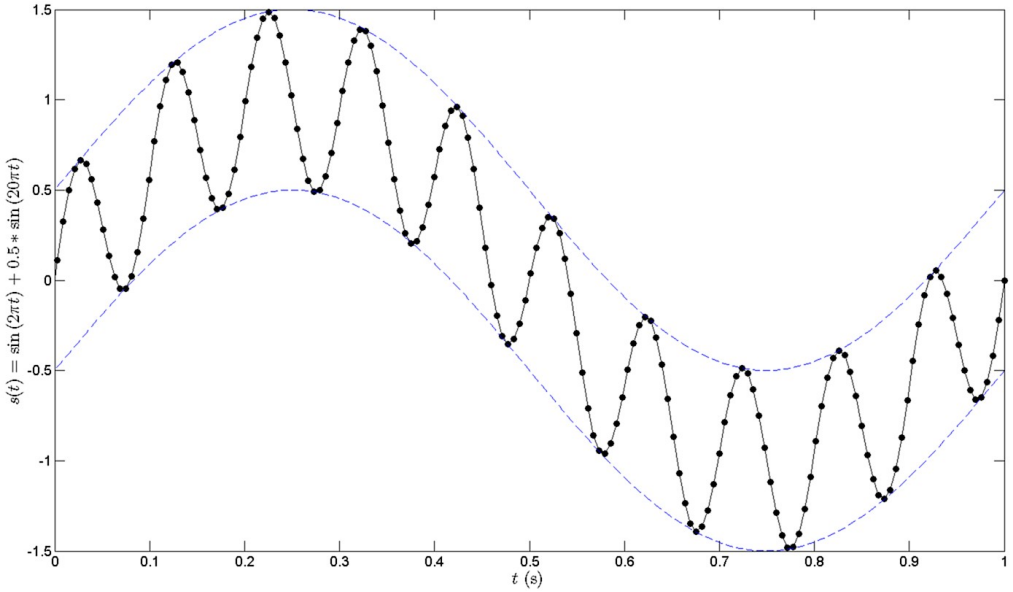
$$s(t) = \sin 2\pi t + \frac{1}{2} \sin 20\pi t$$

Rilling and Flandrin (2008)

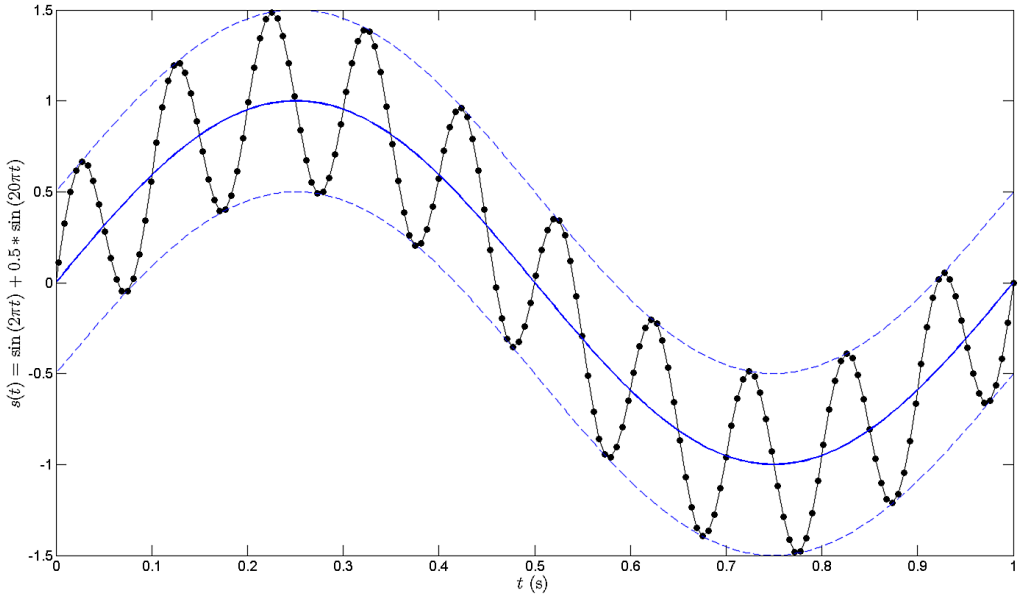
- Détection des mins et max locaux
- Enveloppes
- Moyenne locale *r*(*t*)



- Détection des mins et max locaux
- Enveloppes
- Moyenne locale *r*(*t*)

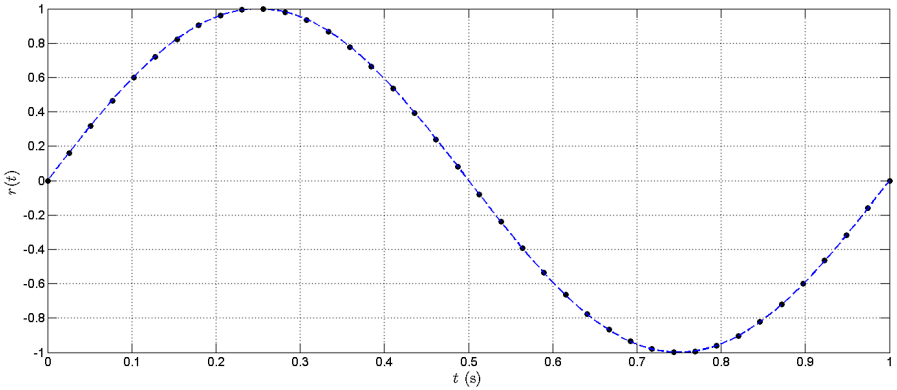


$$s(t) = \sin 2\pi t + \frac{1}{2}\sin 20\pi t$$

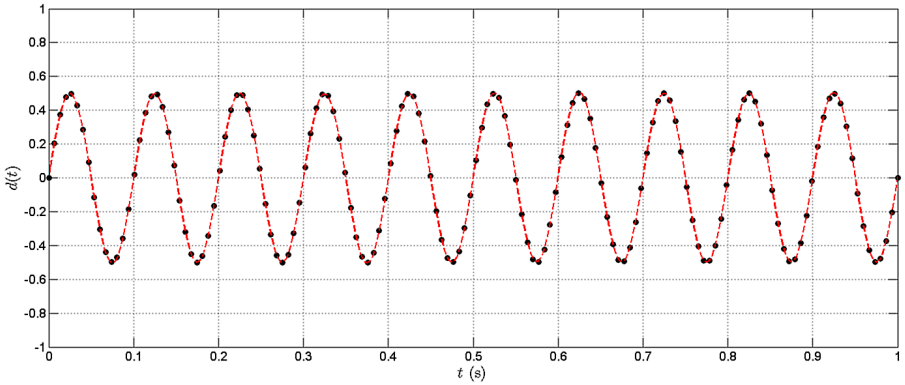


- Détection des mins et max locaux
- Enveloppes
- Moyenne locale *r*(*t*)

Tendance : $\sin 2\pi t$



Détail : $\frac{1}{2}\sin 20\pi t$



Efficacité et intérêt en turbulence ?

Perturbation numérique

$$u_p(t) = u_i(t) + p_i(t)$$

$$p_i(t) = a_p(t)\sin\left(\phi(t)\right)$$

$$\frac{d\phi}{dt} = 2\pi f_p(t)$$

Signal turbulent expérimental :
Turbulence de grille
Mazellier et al. (J. Turbulence, 2010)

U^a (m/s)	$\langle u_i^2 \rangle^{1/2}$ (m/s)	L_i^b (mm)	λ_i^c (mm)	η_i^d	Re_λ^e
4.1	0.57	9.4	1.6	0.1	59

[a] Streamwise mean velocity,
[b] Integral length-scale computed from Eq. (14),
[c] Taylor microscale computed from Eq. (15),
[d] Kolmogorov scale computed from Eq. (16),
[e] Taylor-based Reynolds number ($\equiv \langle u_i^2 \rangle^{1/2} \lambda_i/\nu$ with ν the kinematic viscosity).

$$\langle u_i^2 \rangle^{1/2} / L_i \approx 60\text{Hz}$$

$$a_p < 4 \text{ m/s}$$

$$f_p < 10 \text{ Hz}$$

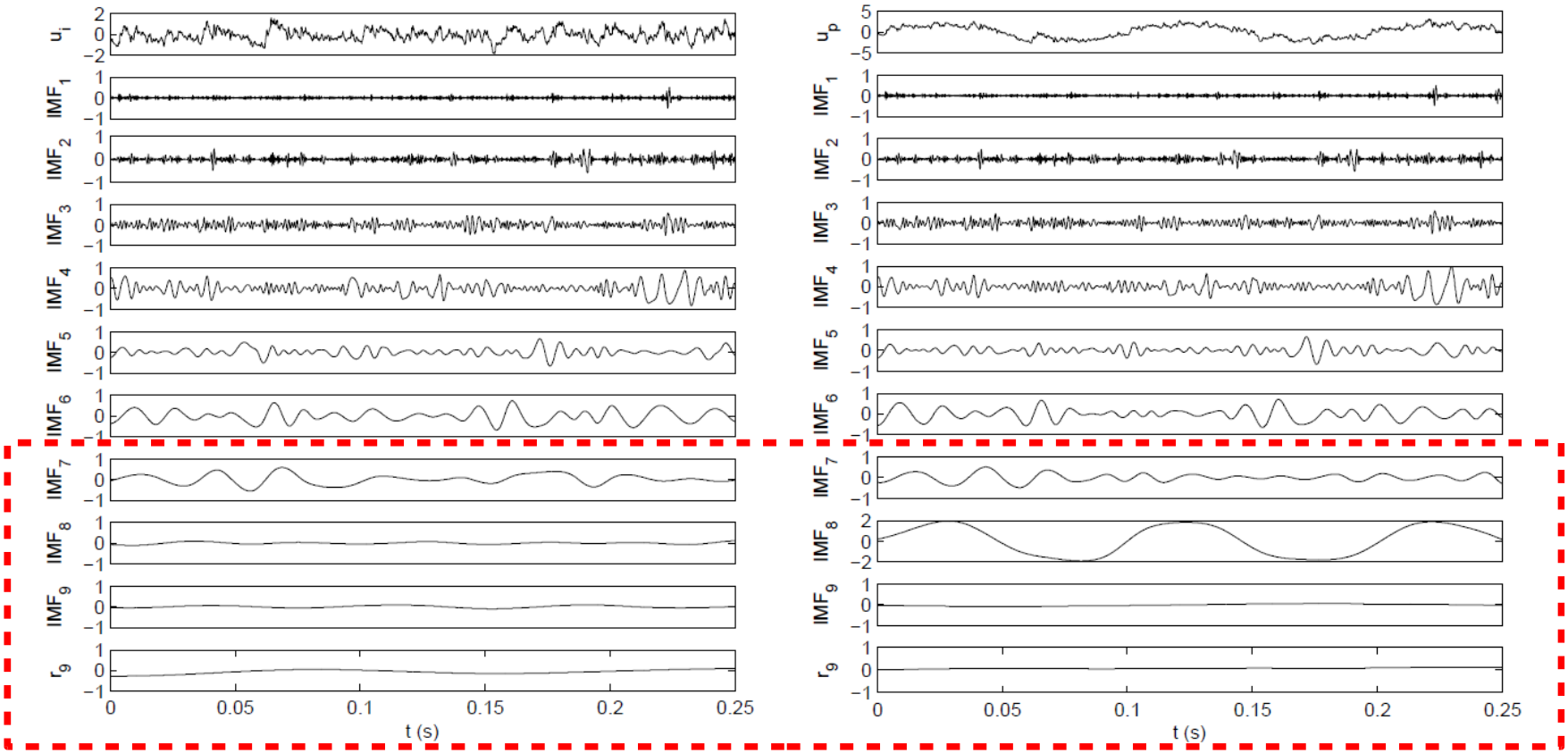
Battement basse-fréquence

mono-fréquence

ou

multi-fréquences

$p_i(t) = 0$ $\qquad$ $p_i(t) = 2\sin(2\pi \times 10)$



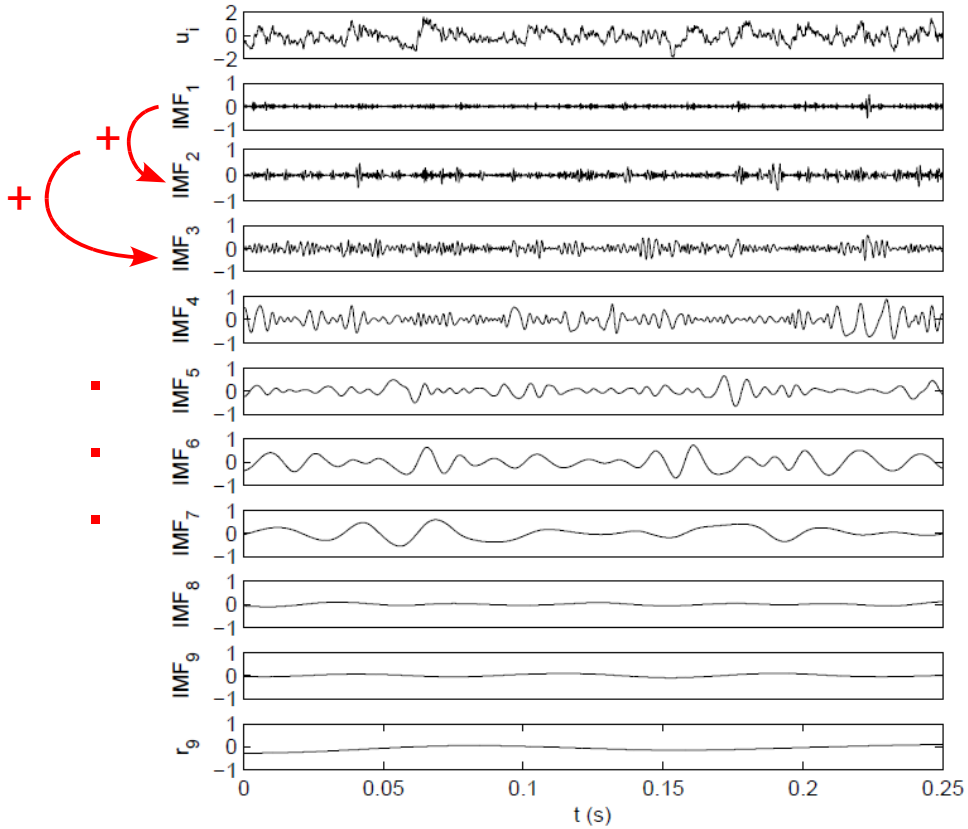
Hautes fréquences contenues dans les premiers modes

La perturbation agit essentiellement sur les modes d'ordre élevé

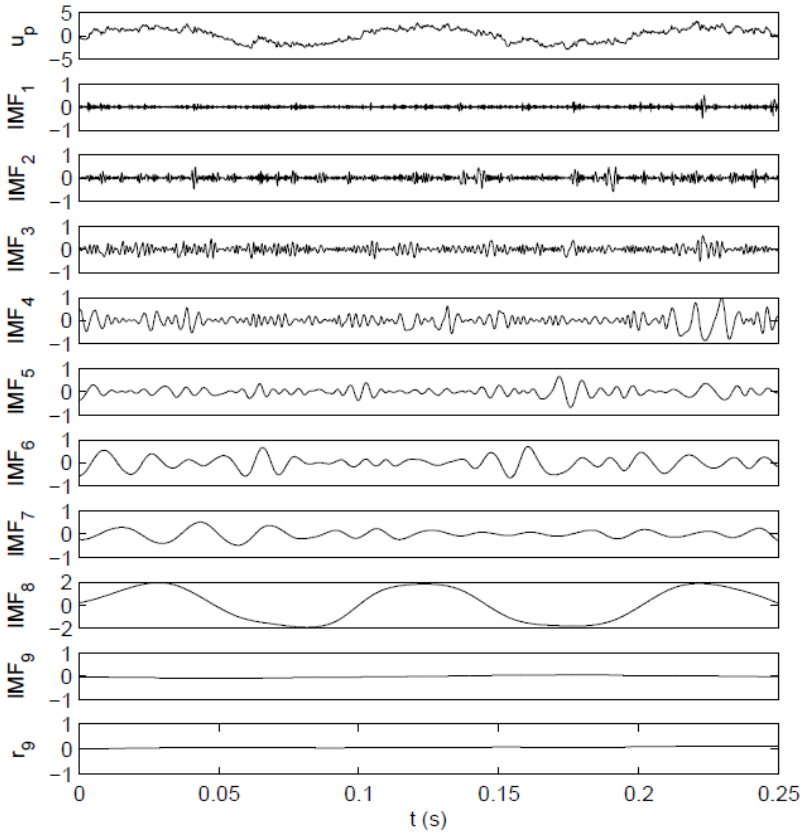
→ Définition d'un critère fiable et robuste pour séparer les modes sains des modes contaminés

Rilling and Flandrin (2008), Foucher and Ravier (Exp. Fluids, 2010)

$p_i(t) = 0$



$p_i(t) = 2\sin(2\pi \times 10)$

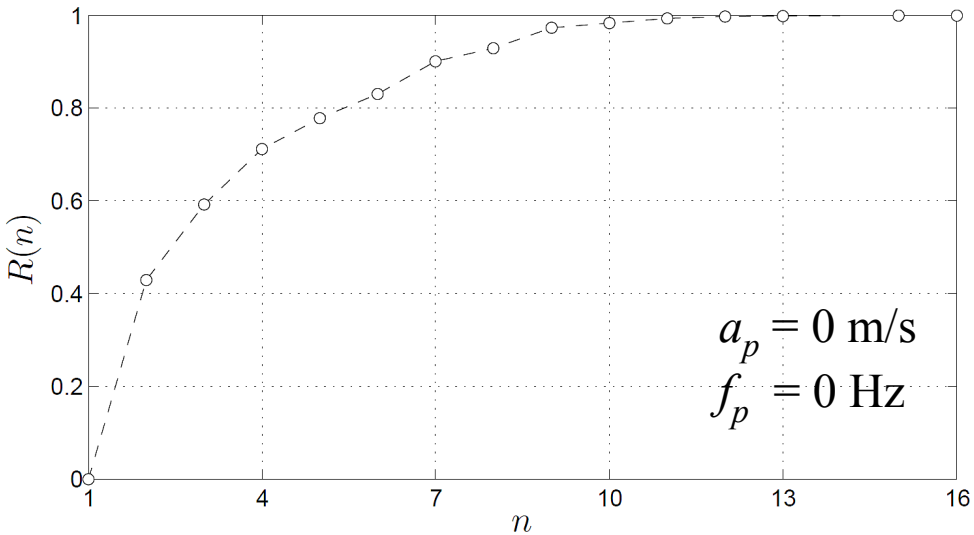


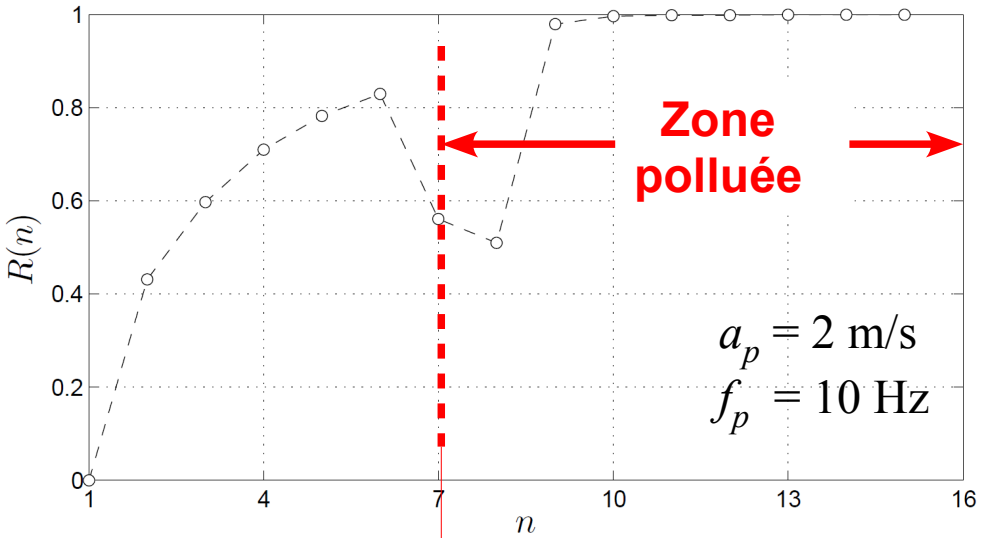
Reconstruction du signal d'origine par ajout successif d'un IMF (ici des hautes vers les basses fréquences)

$$\begin{cases} u_n(t) = \sum_{k=1}^{n} \mathrm{IMF}_k(t) & \text{if } n < N, \\ u_n(t) = \sum_{k=1}^{n} \mathrm{IMF}_k(t) + r_N & \text{if } n = N, \end{cases}$$

Critère de « ressemblance »

$$\begin{cases} R(n) = 0 & \text{if } n = 1, \\ R(n) = \dfrac{\langle u_n(t) u_{n-1}(t) \rangle}{\sqrt{\langle u_n(t)^2 \rangle}\sqrt{\langle u_{n-1}(t)^2 \rangle}} & \text{if } 1 < n \leq N \end{cases}$$



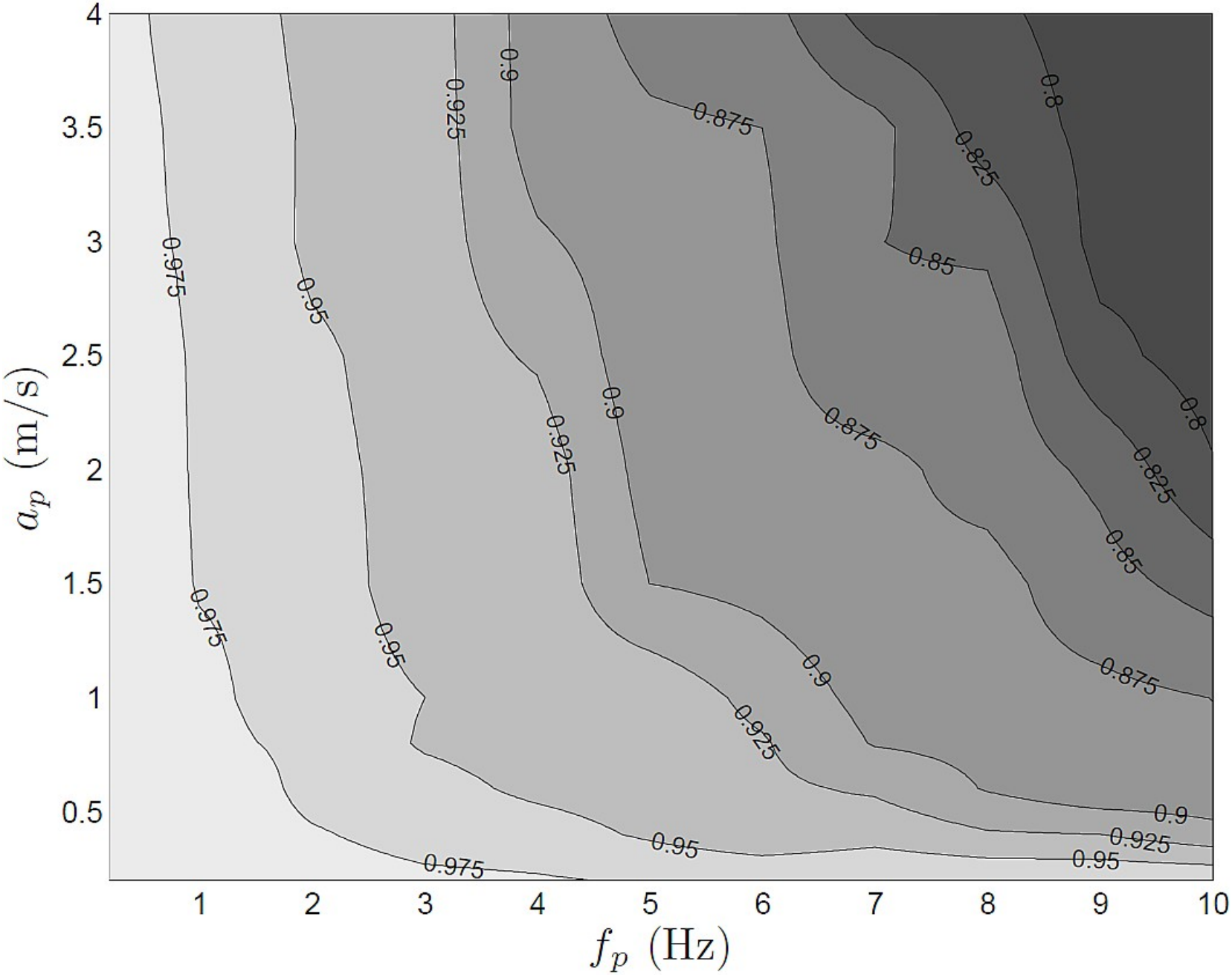


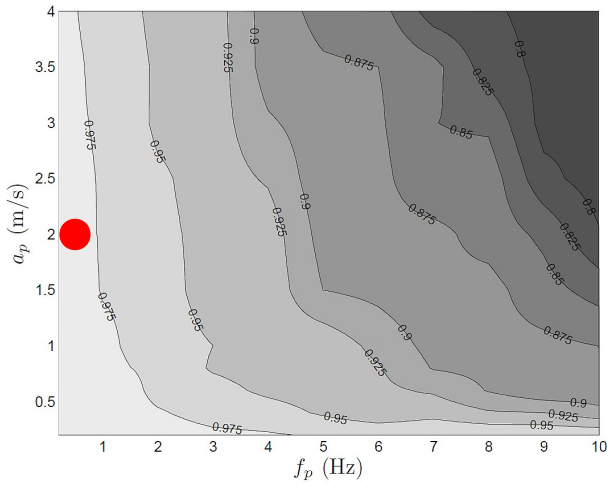
$$\begin{cases} u_r(t) = \displaystyle\sum_{k=1}^{k_c} \mathrm{IMF}_k(t) \longleftrightarrow u_i(t) \\ p_r(t) = u_p(t) - u_r(t) \longleftrightarrow p_i(t) \end{cases}$$

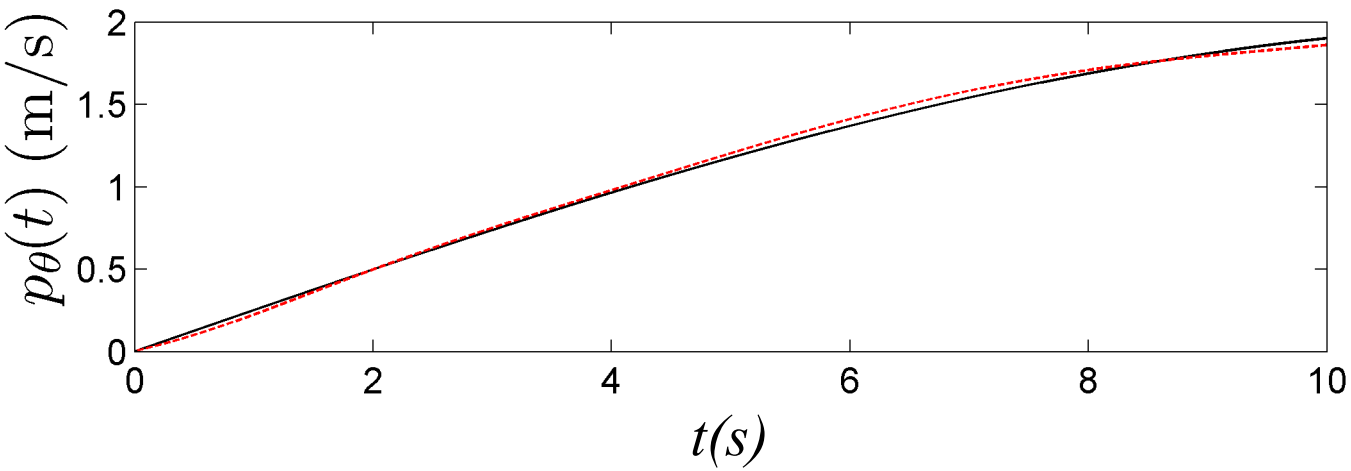
Evaluation de la performance de la reconstruction :

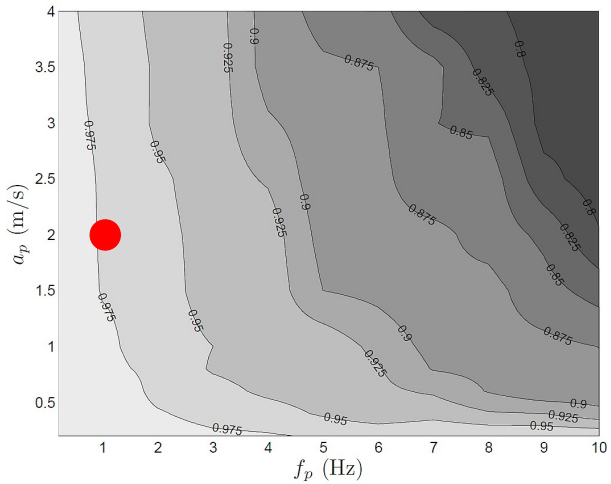
$$C = \frac{\langle u_i(t) u_r(t) \rangle}{\sqrt{\langle u_i^2 \rangle}\sqrt{\langle u_r^2 \rangle}}$$

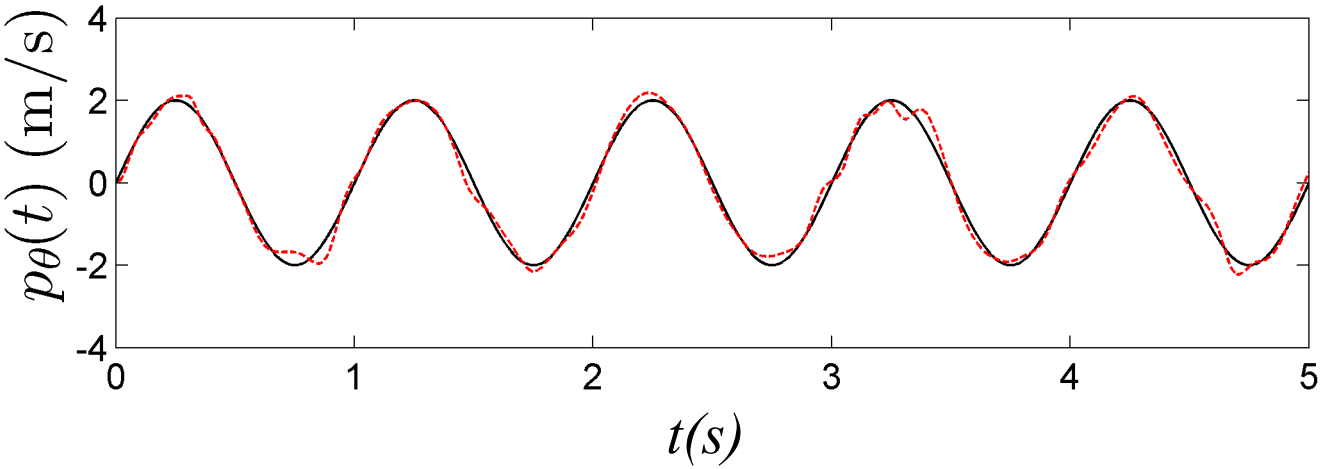
$$C = \frac{\langle u_i(t) u_r(t) \rangle}{\sqrt{\langle u_i^2 \rangle} \sqrt{\langle u_r^2 \rangle}}$$

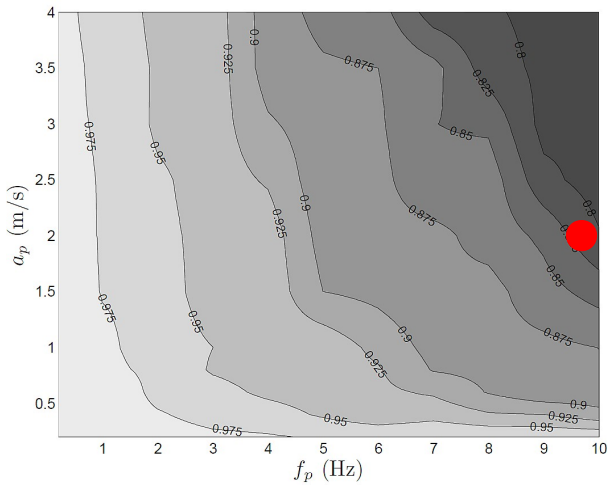


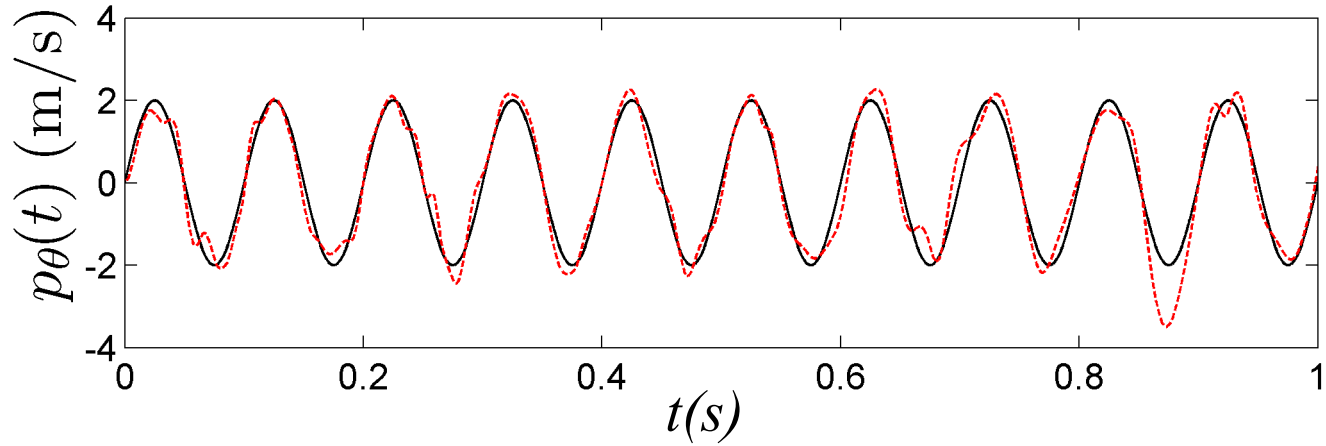

a_p (m/s)
f_p (Hz)


$p_\theta(t)$ (m/s)
$t(s)$

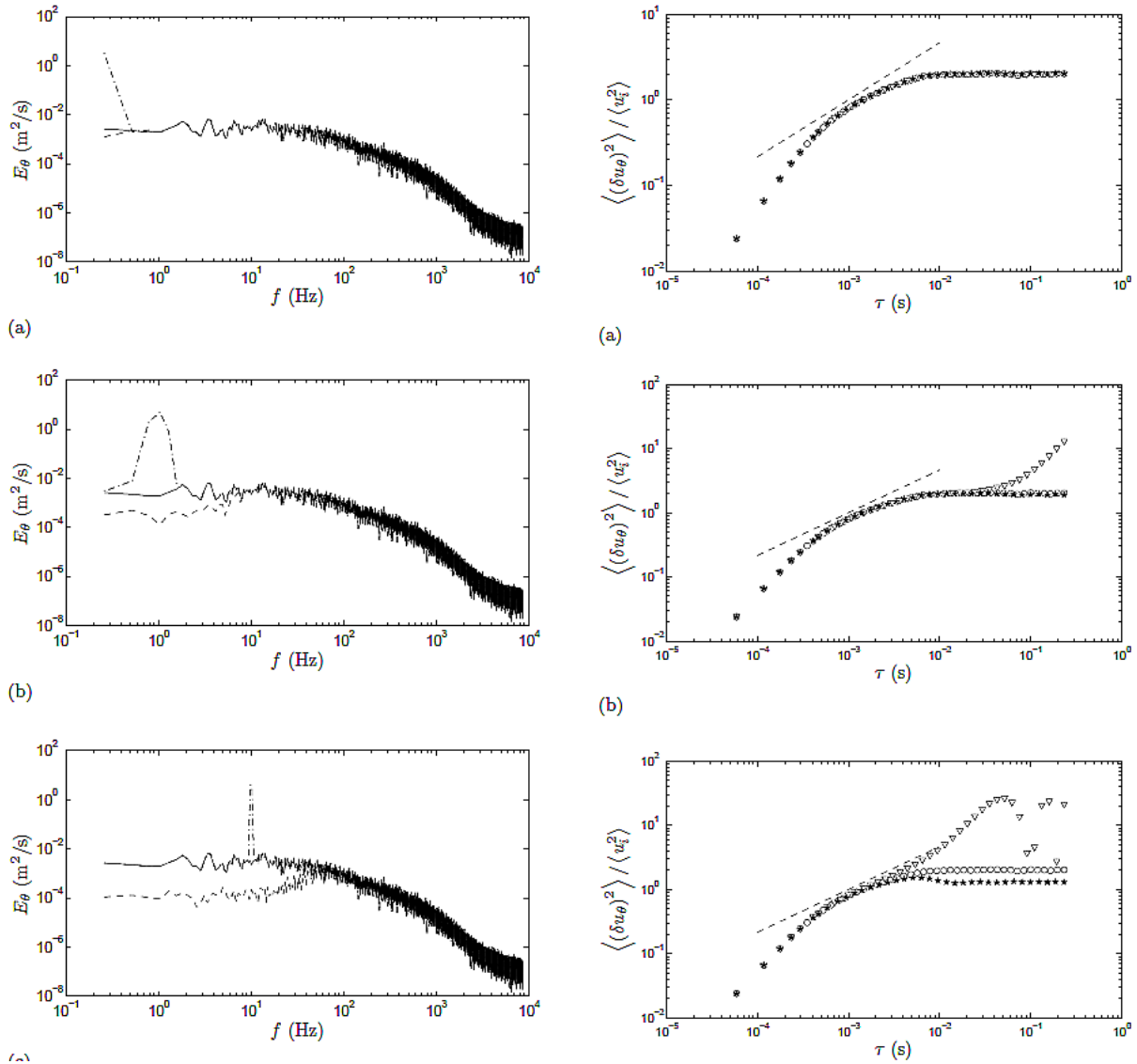

a_p (m/s)
f_p (Hz)


$p_\theta(t)$ (m/s)
$t(s)$


a_p (m/s)
f_p (Hz)


$p_\theta(t)$ (m/s)
$t(s)$

$$\langle u_\theta^2 \rangle = \int_0^\infty E_\theta(f) df \qquad \delta u_\theta(\tau) = u_\theta(t+\tau) - u_\theta(t)$$



Effet de filtrage « passe-haut »

Performances réduites lorsque les fréquences de battement sont proches des fréquences des grandes échelles

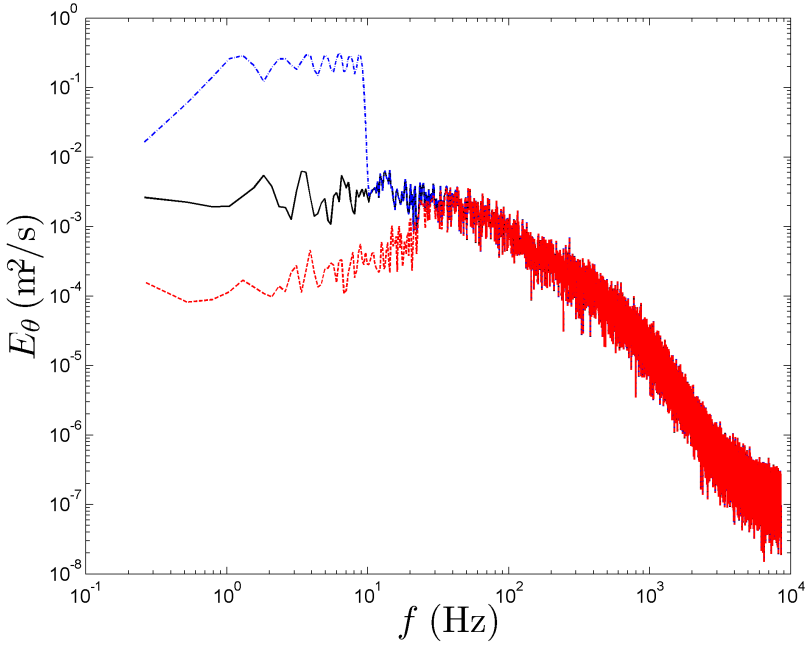
Perturbation multi-fréquences (chirp linéaire)

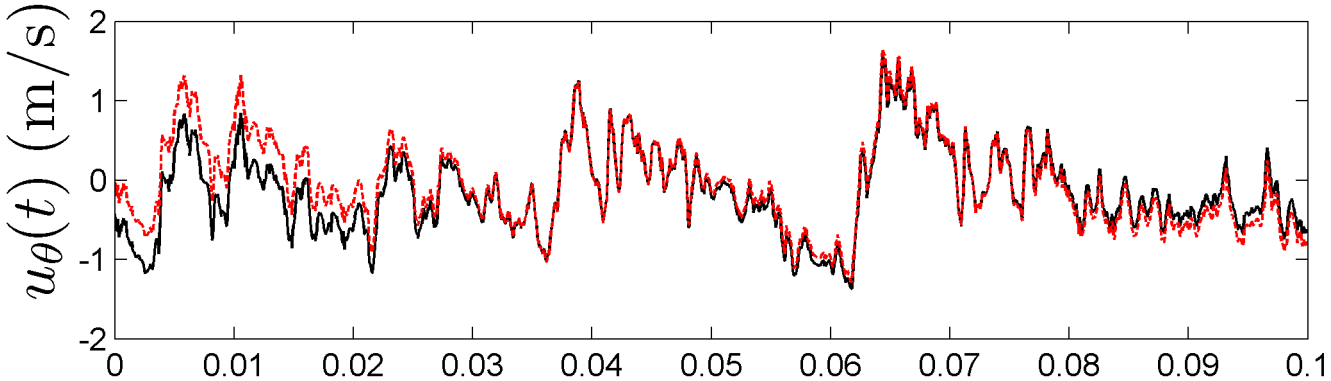
→ $a_p = 2$ m/s

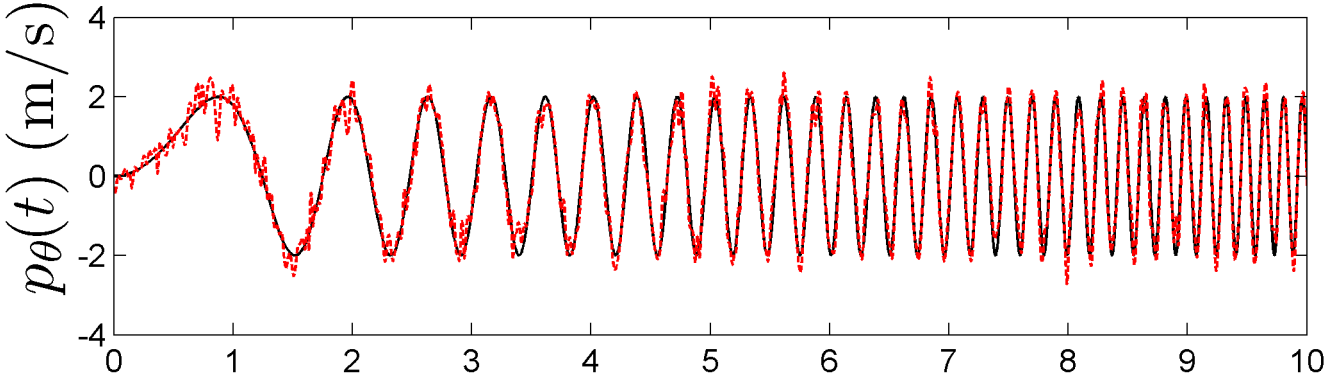
→ $f_p(t) = f_0(t/t_m)$

avec $f_0 = 10$Hz

t_m = durée de l'échantillon (12s ici)







Conclusions

Empirical Mode Decomposition : méthode d'analyse de signaux instationnaires et de dynamique non-linéaire

Signal pleinement turbulent
+
Battement numérique basse-fréquence et énergétique

Critère de « ressemblance » robuste

Résultats très encourageants :

- Retrouve la dynamique haute-fréquence
- Comparaison avec original > 80%

Performances atténuées quand les dynamiques se chevauchent

http://perso.ens-lyon.fr/patrick.flandrin/

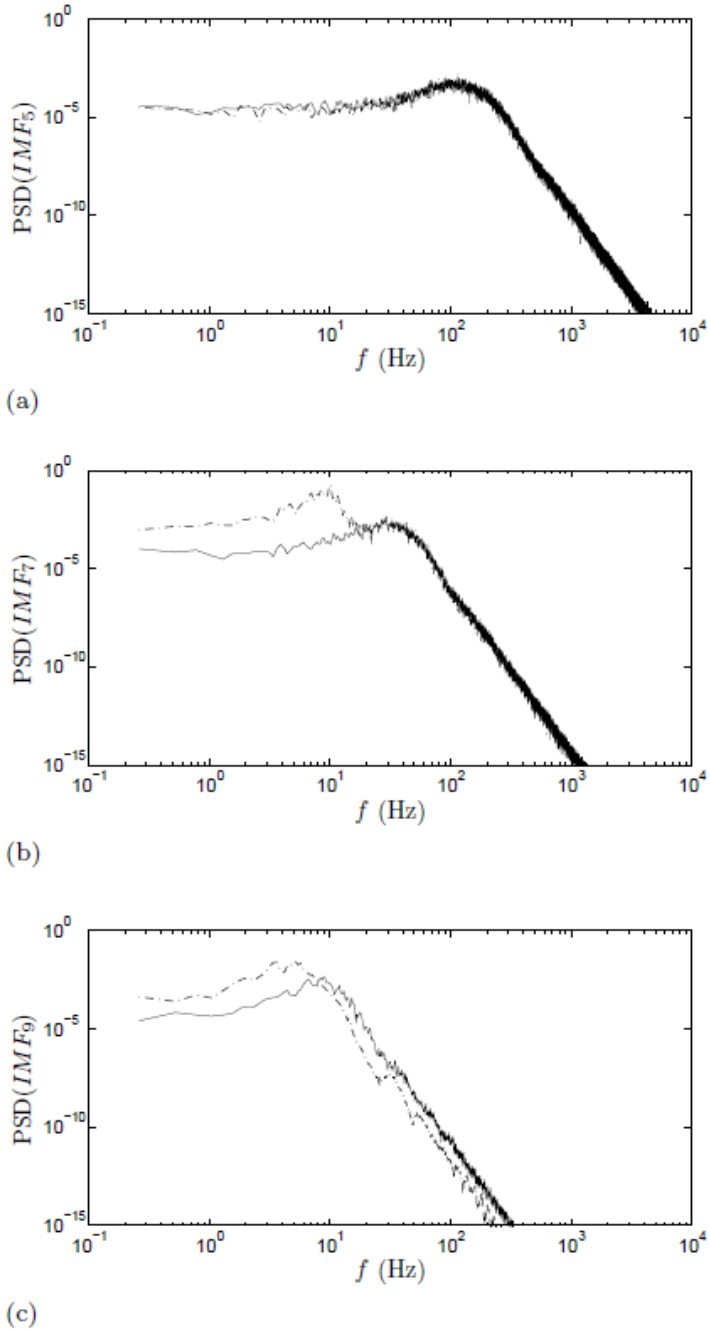
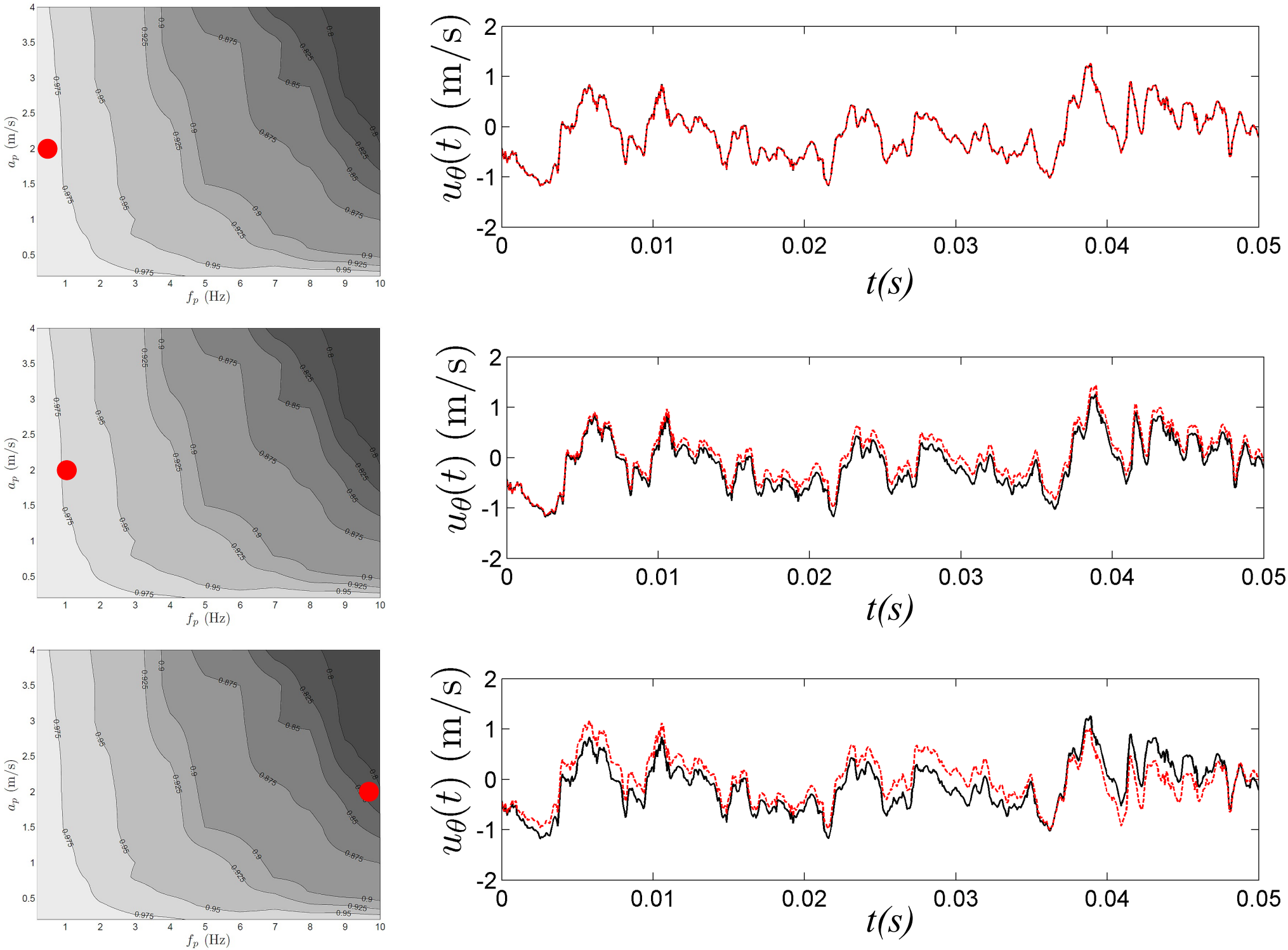


Fig. 3 1D energy spectra of the (a) 5th IMF, (b) 7th IMF and (c) 9th IMF time-series extracted from the reference signal (solid lines) and the perturbed signal (broken lines with dots) with $f_p =$ 10Hz and $a_p = 2$m/s.


a_p (m/s)
f_p (Hz)
$u_\theta(t)$ (m/s)
$t(s)$