Interaction(s) fluide-structure & modélisation de la turbulence

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GDR Turbulence & GDR IFS

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Fil conducteur

IFS → instationnarité (Cond. Lim.)→ modification écoulement

- Effets sur la turbulence ?
- Conséquence pour la validité des modèles de turbulence ?
- Prévision des efforts exercés sur la structure ?
- Choix pour l’exposé: couche limite turbulente

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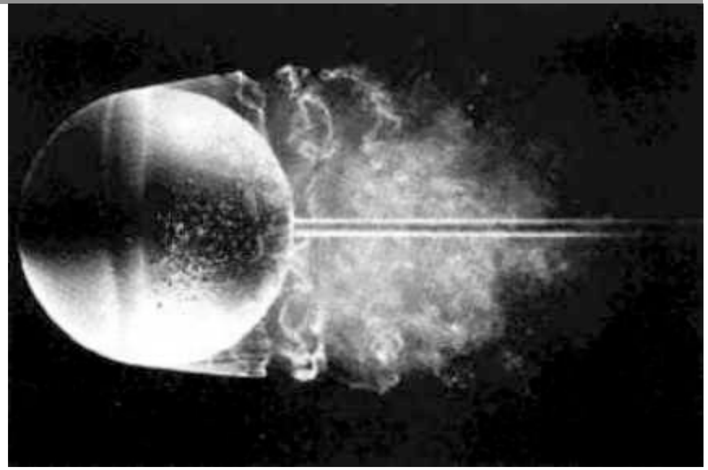
Motivations

- Unsteady simulations are now of common use
  - for theoretical studies
  - for engineering purposes



Courtesy of ONERA, France

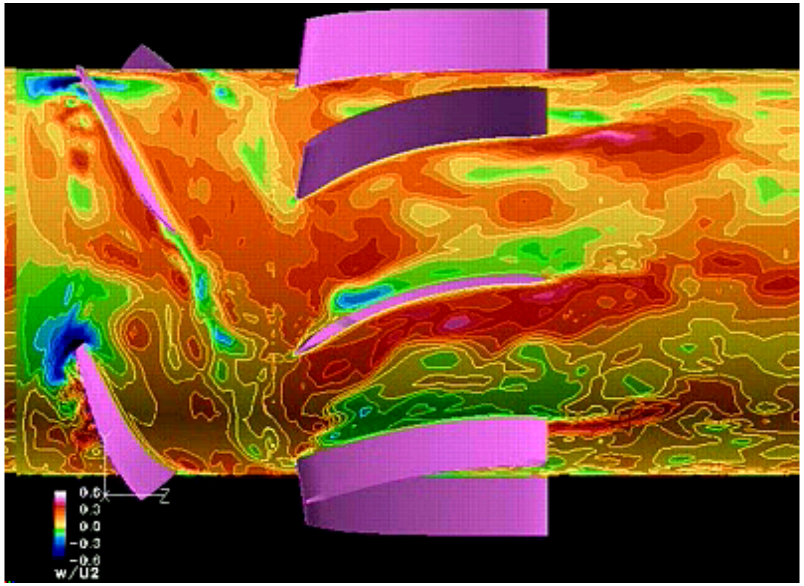
- All scales of turbulent flows cannot be directly captured because of required computing ressources

Number of grid points : $O(Re^{9/4})$

Number of time steps : $O(Re^{1/2})$

⇨ some scales must be modeled

⇨ many available approaches



Courtesy of C. Kato, Tokyo

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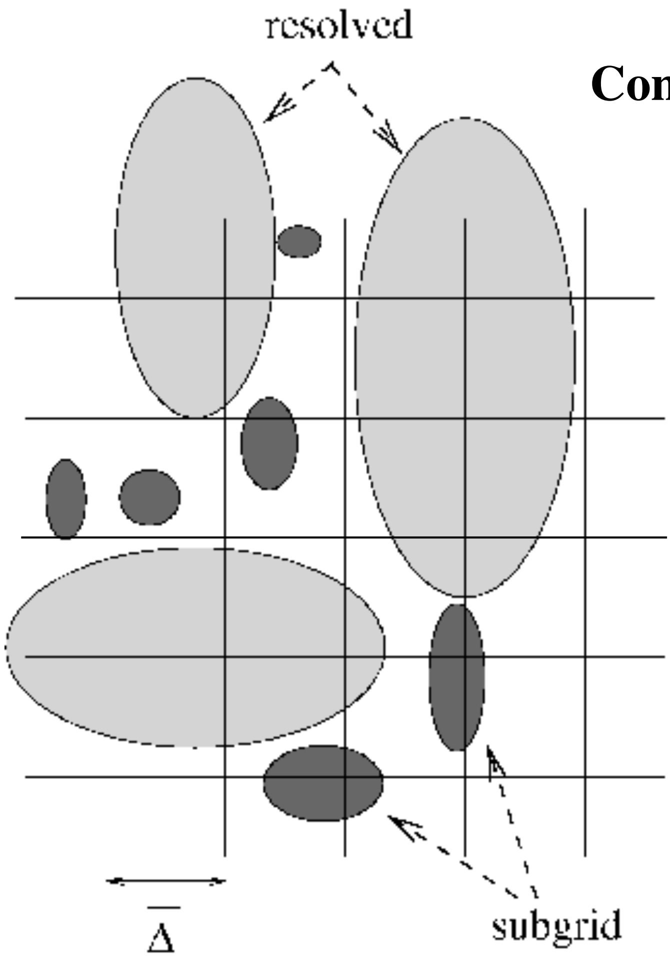
The LES concept (linear filtering)

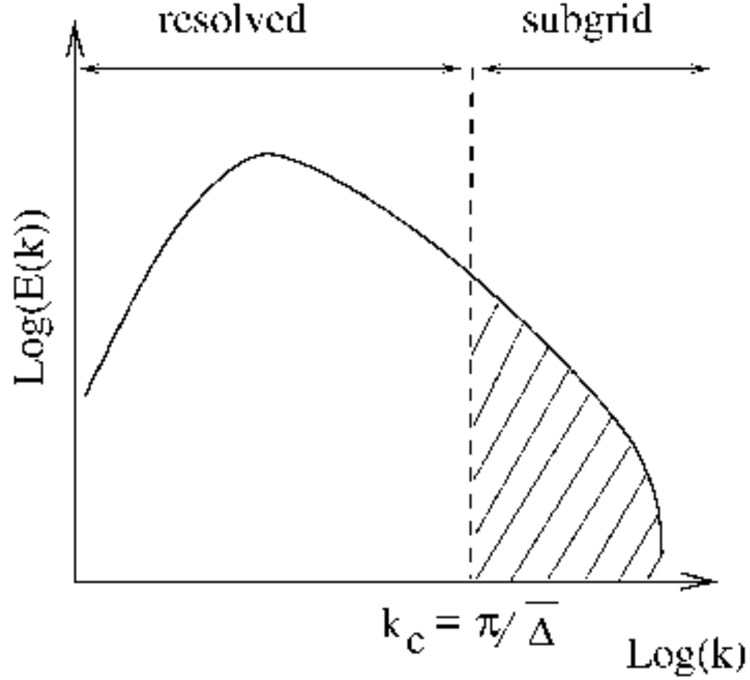
$$\boxed{\bar{u}(x,t) = G \star u(x,t)}$$

Computed 'filtered' solution

Exact solution

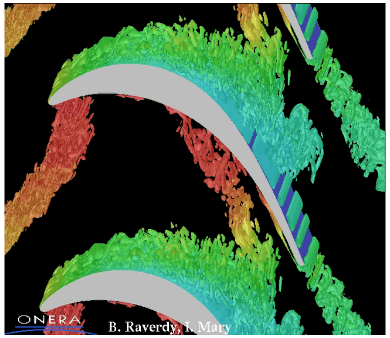
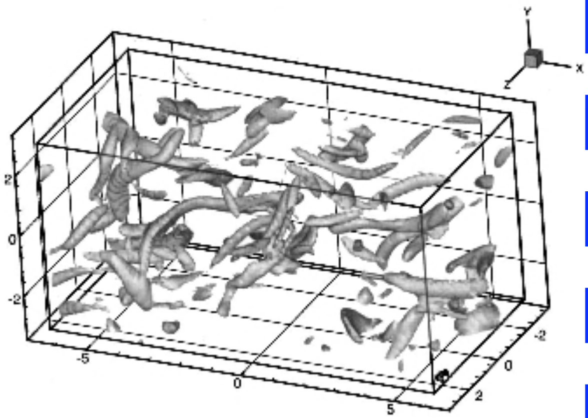
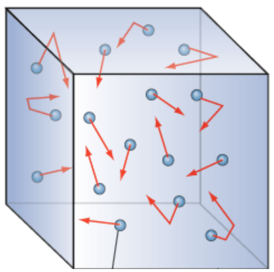


PHYSICAL SPACE



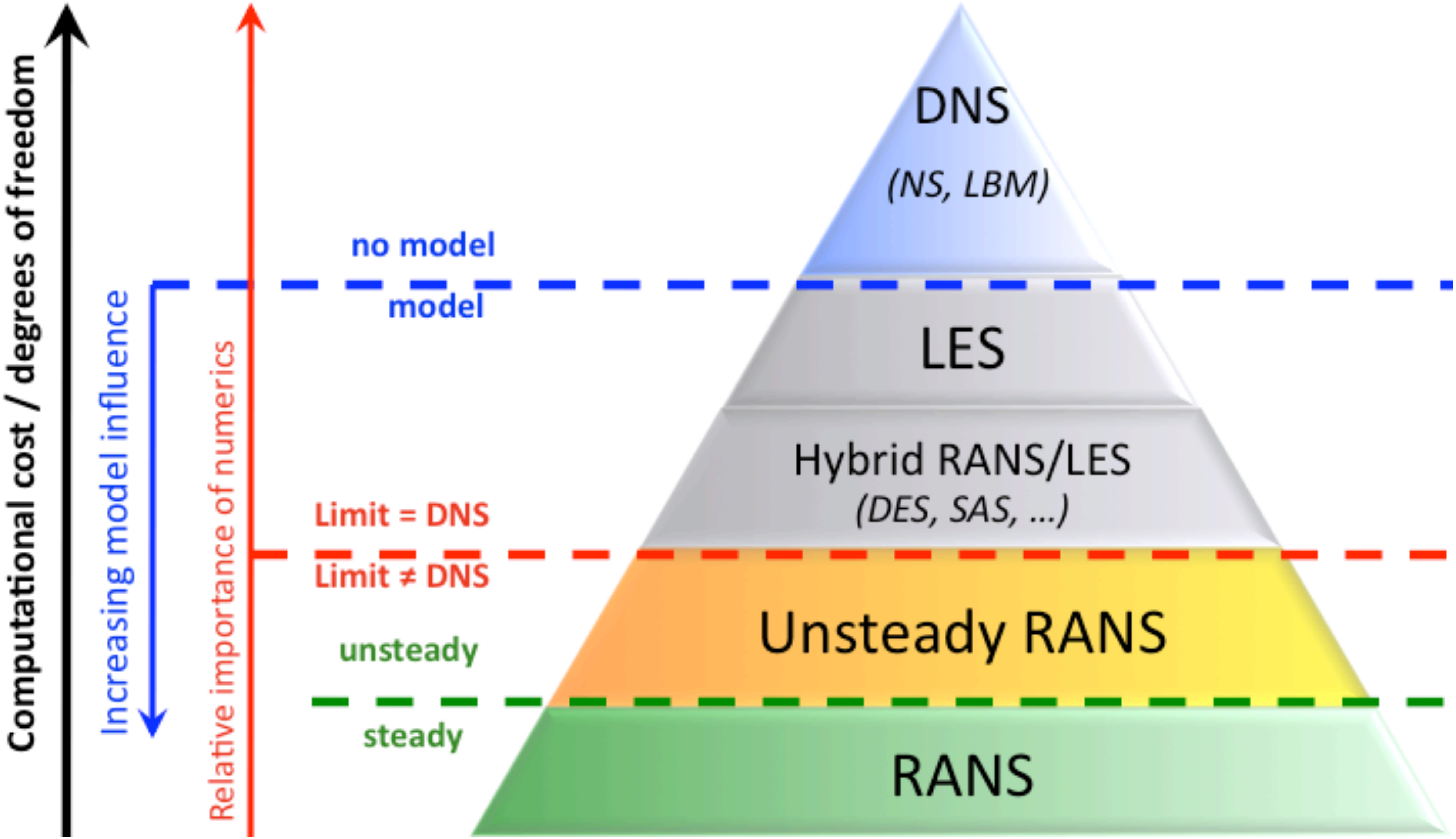
FOURIER SPACE

LBM-LES schematic view

	NS-LES	LBM-LES	Non-linear terms
	Filtered macroscopic $(\bar{u}, \bar{p})$	Filtered microscopic $(\bar{f}, \bar{\rho}, \tilde{u})$	inherited
	macroscopic (u, p)		advection (quadratic)
	microscopic f	microscopic f	collision (exponential form/ Nth-order Hermite)

Hierarchy of CFD methods

« ***Multiscale & Multiresolution approaches for turbulence*** »
Sagaut, Deck & Terracol, Imperial College Press, 2006



RANS mean flow equations

$$\frac{\partial}{\partial t}\bar{u}_i + \frac{\partial}{\partial x_j}(\bar{u}_i\bar{u}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \nu\frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k} + \bar{f}_i - \frac{\partial}{\partial x_j}R_{ij}$$

$$\frac{\partial}{\partial t}\bar{T} + \frac{\partial}{\partial x_j}(\bar{T}\bar{u}_j) = \kappa\frac{\partial^2 \bar{T}}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_j}\overline{T'u'_j}$$

$$\frac{\bar{D}}{\bar{D}t}\phi \equiv \frac{\partial}{\partial t}\phi + \frac{\partial}{\partial x_j}(\phi\bar{u}_j)$$

Material derivative associated with mean velocity field

RANS mean flow kinetic energy

$$\underbrace{\frac{\partial}{\partial t}K + \frac{\partial}{\partial x_j}(K\bar{u}_j)}_{\text{Mean flow advection}} = \underbrace{-\frac{\partial}{\partial x_i}(\bar{p}\bar{u}_i)}_{\text{Pressure diff.}}$$

$$+\underbrace{\nu\frac{\partial^2 K}{\partial x_k \partial x_k}}_{\text{Viscous diff.}} \underbrace{-\nu\frac{\partial \bar{u}_i}{\partial x_k}\frac{\partial \bar{u}_i}{\partial x_k}}_{\text{dissipation}}$$

$$+\underbrace{\bar{u}_i\bar{f}_i}_{\text{external force power}} \underbrace{-\frac{\partial}{\partial x_j}(\bar{u}_i R_{ij})}_{\text{Turbulent diff.}} + \underbrace{R_{ij}\frac{\partial \bar{u}_i}{\partial x_j}}_{\text{transfer}}$$

RANS fluctuating momentum eqs.

$$\frac{\partial u'_i}{\partial x_i} = 0$$

$$\frac{\partial u'_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u'_i \bar{u}_j + \bar{u}_i u'_j + u'_i u'_j - R_{ij}\right) = -\frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_k \partial x_k} + f'_i \quad (i = 1, 3)$$

$$\frac{\partial T'}{\partial t} + \frac{\partial}{\partial x_j}\left(T' \bar{u}_j + \bar{T} u'_j + T' u'_j - \overline{T' u'_j}\right) = \kappa \frac{\partial^2 T'}{\partial x_k \partial x_k}$$

Mean field advection

Reynolds stress

Fluctuating field advection

RANS fluctuating kinetic energy

$$\underbrace{\frac{\partial}{\partial t}\mathcal{K} + \frac{\partial}{\partial x_l}(\bar{u}_l \mathcal{K})}_{\textbf{Mean flow advection}} = \underbrace{-R_{il}\frac{\partial \bar{u}_i}{\partial x_l}}_{\textbf{production}} \underbrace{-\frac{1}{2}\frac{\partial}{\partial x_l}\overline{u'_i u'_i u'_l}}_{\textbf{Turbulent diff.}}$$

$$-\underbrace{\varepsilon}_{\textbf{dissipation}} + \underbrace{\overline{f'_i u'_i}}_{\textbf{External force power}}$$

$$\underbrace{-\frac{\partial}{\partial x_l}\overline{p' u'_l}}_{\textbf{pressure diff.}} + \underbrace{\nu\frac{\partial^2}{\partial x_l \partial x_l}\mathcal{K}}_{\textbf{viscous diff.}}$$

Quasi-steady approximation (RANS)

Quasi-steady approximation:

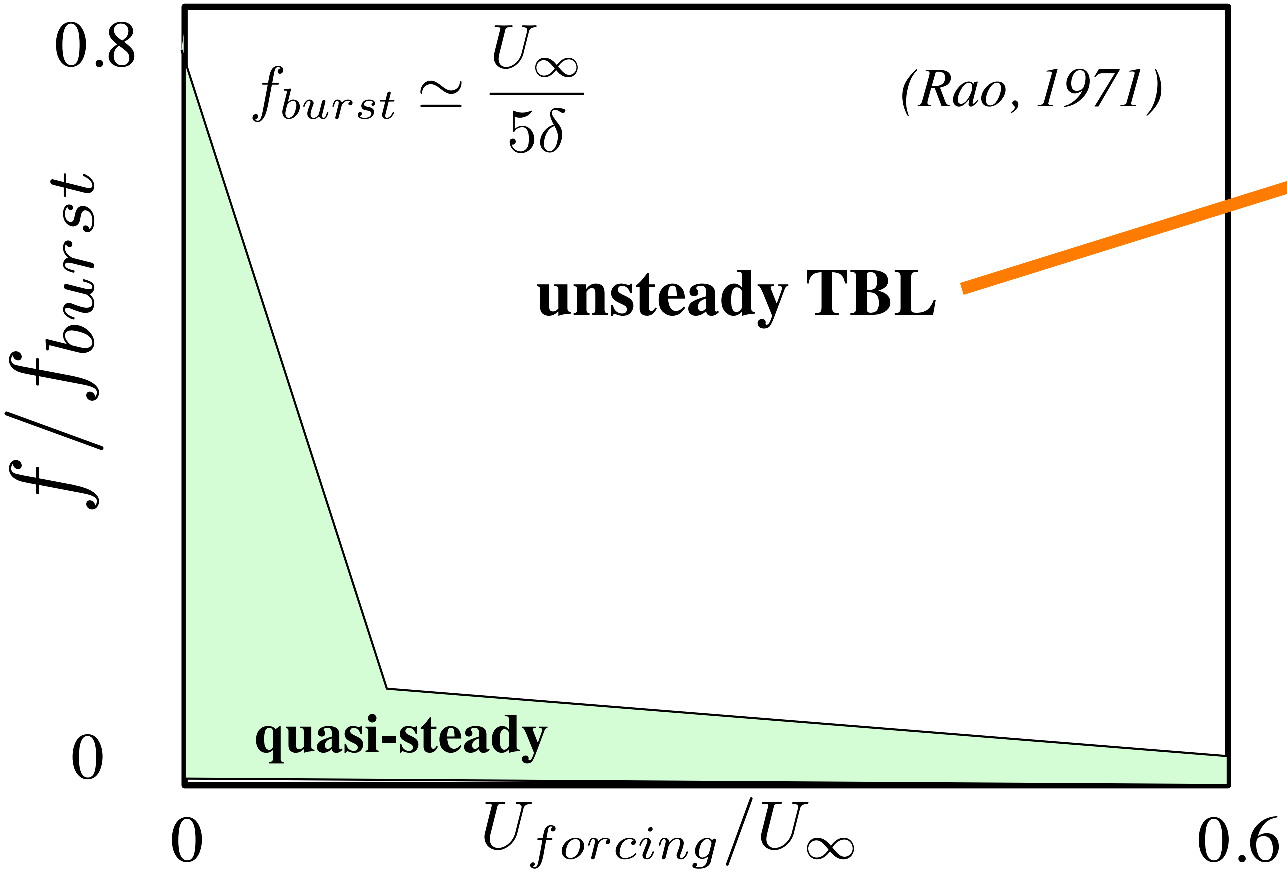
- unsteady forcing of turbulence doesn't induce significant modifications in turbulence dynamics
- usual models can be used
- Flow can be described using sequential steady RANS simulations

$$\frac{\partial}{\partial t}\bar{\phi} \simeq 0$$

Quasi-steady approximation (RANS)

Validity ?

- E.g.: turbulent boundary layer submitted to periodic forcing



Unsteady effects restricted to viscous sublayer if

$$\sqrt{\frac{2\nu}{\omega}}\,\frac{u_\tau}{\nu} < 8$$

(Cousteix, 1985)

RANS eddy-viscosity model sensitivity

$$R_{ij} = \frac{2}{3}\mathcal{K}\delta_{ij} - 2\nu_t \bar{S}_{ij}$$

Jones –Launder (1972) $\nu_t \propto \frac{\mathcal{K}^2}{\varepsilon} = C_\mu \frac{\mathcal{K}^2}{\varepsilon}$

$$\frac{\partial \mathcal{K}}{\partial t} + \bar{u}_j \frac{\partial \mathcal{K}}{\partial x_j} = C_\mu \frac{\mathcal{K}^2}{\varepsilon} \left|\underline{\underline{\bar{S}}}\right|^2 - \varepsilon + \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_t}{\sigma_\mathcal{K}} + \nu\right)\frac{\partial \mathcal{K}}{\partial x_j}\right)$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\varepsilon}{\mathcal{K}}\left(C_{\varepsilon_1}\mathcal{P} - C_{\varepsilon_2}\varepsilon\right) + \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_t}{\sigma_\varepsilon} + \nu\right)\frac{\partial \varepsilon}{\partial x_j}\right)$$

Logarithmic layer recovery

« production = dissipation » equilibrium hypothesis

$$\mathcal{P} = -R_{12}\frac{d\bar{u}}{dy} = -R_{12}\frac{u_*}{\kappa_{\text{VK}}y} = \varepsilon$$

Corresponding eddy-viscosity distribution

$$\mathcal{P} = \varepsilon = \frac{u_*^3}{\kappa_{\text{VK}}y} = \nu_t\left(\frac{d\bar{u}}{dy}\right)^2 \Longrightarrow \nu_t = \kappa_{\text{VK}}u_*y$$

Equalization with model prediction

$$C_\mu\frac{\mathcal{K}^2}{\varepsilon} = \kappa_{\text{VK}}u_*y = C_\mu\frac{\mathcal{K}^2}{(u_*^3/\kappa_{\text{VK}}y)} \Longrightarrow \boxed{C_\mu = \left(\frac{\mathcal{K}}{u_*^2}\right)^{-2} = \text{cste}}$$

Cont'd

Dissipation equation (accounting for the equilibrium condition)

$$0 = \frac{\mathcal{P}\varepsilon}{\mathcal{K}}(C_{\varepsilon_1} - C_{\varepsilon_2}) + \frac{d}{dy}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{d\varepsilon}{dy}\right)$$

From which

$$\boxed{\sigma_\varepsilon = \frac{\kappa^2_{\mathrm{VK}}}{\sqrt{C_\mu}(C_{\varepsilon_2} - C_{\varepsilon_1})}}$$

RANS reconstruction of wall-pressure spectrum

Prediction of wall-pressure induced loads on structure

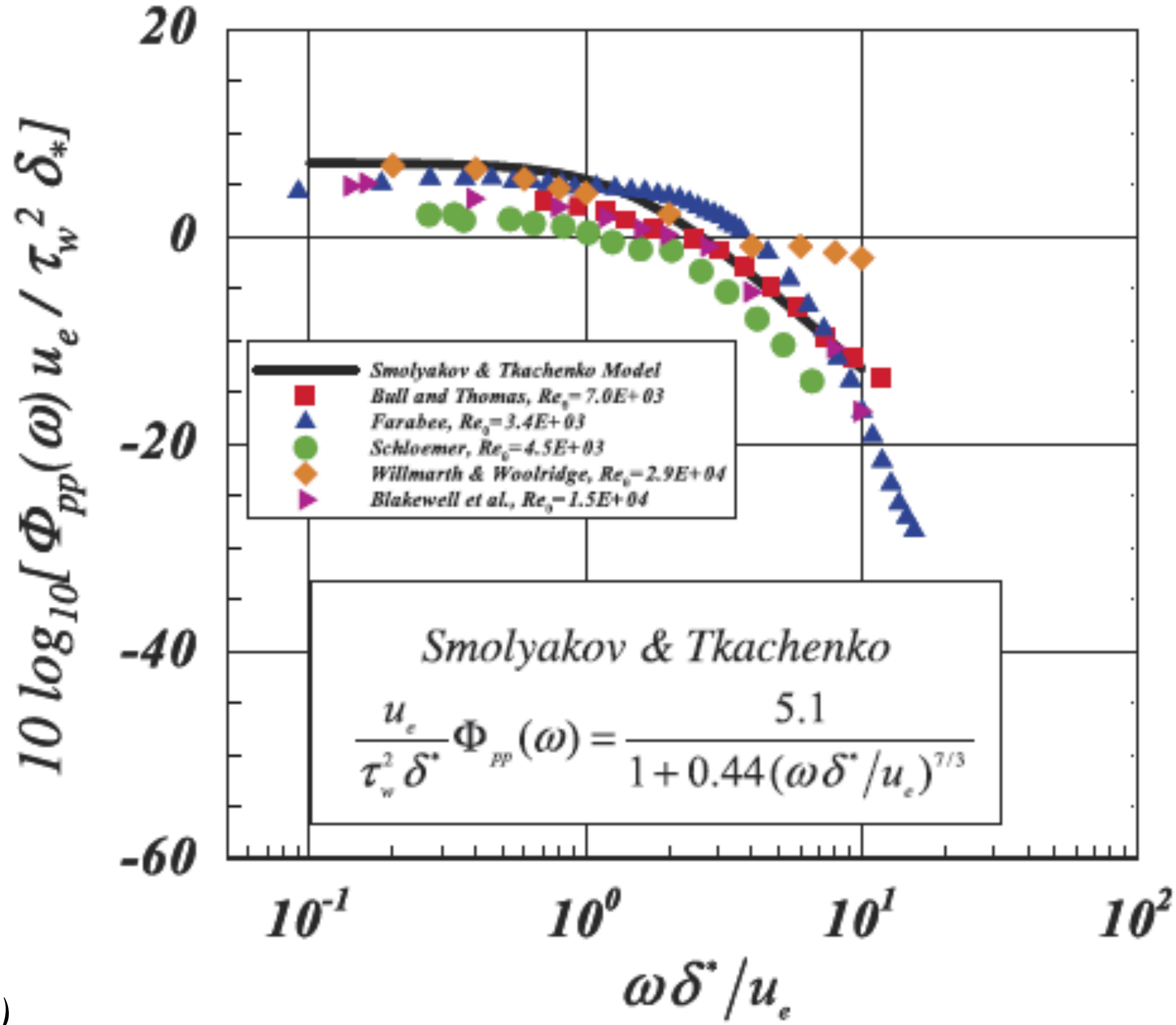
$$\frac{1}{\rho}p(\vec{x}_s,t)=\int_{\vec{x}}-\frac{\partial\left(2U_iu_j+u_iu_j-\overline{u_iu_j}\right)}{\partial x_i\partial x_j}G(\vec{x},\vec{x}_s)\,\mathrm{d}\vec{x}$$

$$\frac{1}{\rho}p(\vec{x}_s,t)=\int_{\vec{x}}-\left[2\left(\frac{\partial U_i}{\partial x_j}u_j\right)\frac{\partial G(\vec{x},\vec{x}_s)}{\partial x_i}+\left(u_iu_j-\overline{u_iu_j}\right)\frac{\partial^2 G(\vec{x},\vec{x}_s)}{\partial x_i\,\partial x_j}\right]\mathrm{d}\vec{x}$$

$$\frac{1}{\rho^2}\overline{p(\vec{x}_s,t)p(\vec{y}_s,\tau)}\equiv\frac{1}{\rho^2}\overline{p(\Delta\vec{x},\Delta t)p(0,0)}\equiv\frac{1}{\rho^2}R_{pp}(\Delta\vec{x},\Delta t)$$

$$\frac{1}{\rho^2}\Phi_{pp}(\vec{\kappa},\omega)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp(-\mathrm{i}(\vec{\kappa}\vec{x}+\omega\Delta t))\frac{1}{\rho^2}R_{pp}(\vec{x},\Delta t)\,\mathrm{d}\vec{x}\,\mathrm{d}\Delta t$$

Semi-empirical formula



(Peltier et al., 2007)

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Full exact expression

$$\frac{1}{\rho^2}\overline{p(\vec{x}_s,t)p(\vec{y}_s,\tau)} = \int_{\vec{y}}\int_{\vec{x}}\left[\begin{array}{l} 4\frac{\partial U_i}{\partial x_j}\frac{\partial V_k}{\partial y_l}\overline{u_j v_l}\frac{\partial G(\vec{x},\vec{x}_s)}{\partial x_i}\frac{\partial G(\vec{y},\vec{y}_s)}{\partial y_k} \\ +2\overline{u_i v_k u_j v_l}\frac{\partial^2 G(\vec{x},\vec{x}_s)}{\partial x_i \partial x_j}\frac{\partial^2 G(\vec{y},\vec{y}_s)}{\partial y_k \partial y_l} \\ +\left\{\left(\overline{u_i u_j v_k v_l} - \overline{u_i u_j}\,\overline{v_k v_l} - \overline{u_i v_k}\,\overline{u_j v_l} - \overline{u_i v_l}\,\overline{u_j v_k}\right)\right. \\ \left.\times\frac{\partial^2 G(\vec{x},\vec{x}_s)}{\partial x_i \partial x_j}\frac{\partial^2 G(\vec{y},\vec{y}_s)}{\partial y_k \partial y_l}\right\} \\ -\left\{2\frac{\partial U_i}{\partial x_j}\overline{u_j v_k v_l}\frac{\partial G(\vec{x},\vec{x}_s)}{\partial x_i}\frac{\partial^2 G(\vec{y},\vec{y}_s)}{\partial y_k \partial y_l}\right\} \\ -\left\{2\frac{\partial V_k}{\partial y_l}\overline{u_i u_j v_l}\frac{\partial G(\vec{y},\vec{y}_s)}{\partial y_k}\frac{\partial^2 G(\vec{x},\vec{x}_s)}{\partial x_i \partial x_j}\right\} \end{array}\right] \mathrm{d}\vec{x}\,\mathrm{d}\vec{y}$$

Cont'd

Quasi-Normal approximation

$$\frac{1}{\rho^2}\overline{p(\vec{x}_s,t)p(\vec{y}_s,\tau)} = \int_{\vec{y}}\int_{\vec{x}} \left[\begin{array}{l} 4\frac{\partial U_i \partial V_k}{\partial x_j \ \partial y_l}\overline{u_j v_l}\frac{\partial G(\vec{x},\vec{x}_s)\partial G(\vec{y},\vec{y}_s)}{\partial x_i \qquad \partial y_k} \\ \quad +2\overline{u_i v_k u_j v_l}\frac{\partial^2 G(\vec{x},\vec{x}_s)\partial^2 G(\vec{y},\vec{y}_s)}{\partial x_i \, \partial x_j \qquad \partial y_k \, \partial y_l} \end{array} \right] \mathrm{d}\vec{x}\,\mathrm{d}\vec{y}$$

Assumptions:

$$\overline{u_i v_k} \equiv \overline{u_i u_k}\, C^{ik} \qquad \frac{\partial V_k}{\partial y_l} \approx \frac{\partial U_k}{\partial x_l}$$

$$\frac{1}{\rho^2}\overline{p(\vec{x}_s,t)p(\vec{y}_s,\tau)} = \int_{\vec{x}}\int_{\vec{r}} \left[\begin{array}{l} 4\frac{\partial U_i \partial U_k}{\partial x_j \ \partial x_l}\overline{u_j u_l}\, C^{jl}\frac{\partial G(\vec{x},\vec{x}_s)\partial G(\vec{r},\vec{x},\vec{y}_s)}{\partial x_i \qquad \partial r_k} \\ \quad +2\overline{u_i u_k u_j u_l}\, C^{ik}\, C^{jl}\frac{\partial^2 G(\vec{x},\vec{x}_s)\partial^2 G(\vec{r},\vec{x},\vec{y}_s)}{\partial x_i \, \partial x_j \qquad \partial r_k \, \partial r_l} \end{array} \right] \mathrm{d}\vec{r}\,\mathrm{d}\vec{x}$$

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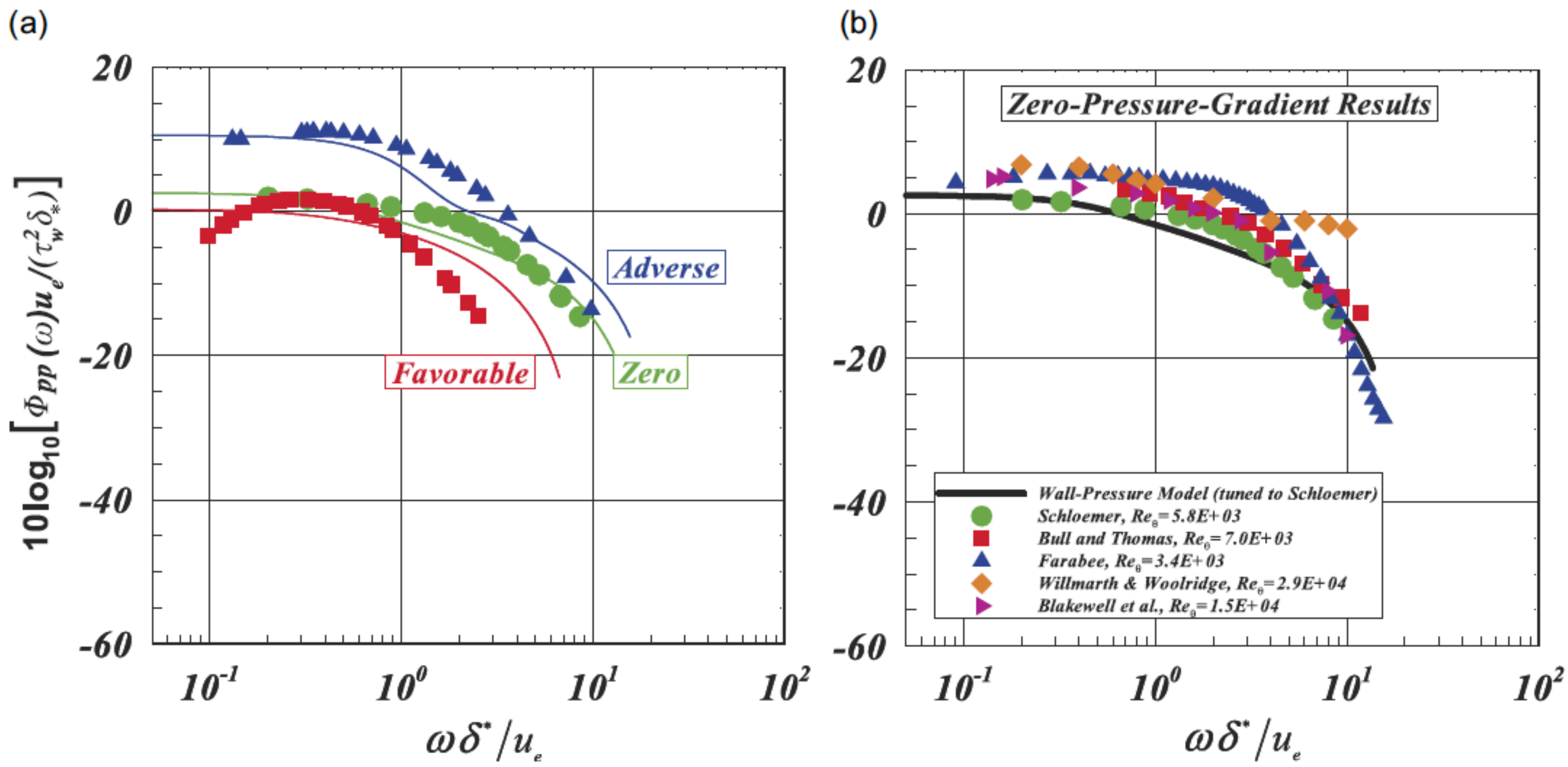

(Peltier et al., 2007)

Hierarchy of CFD methods

« ***Multiscale & Multiresolution approaches for turbulence*** »
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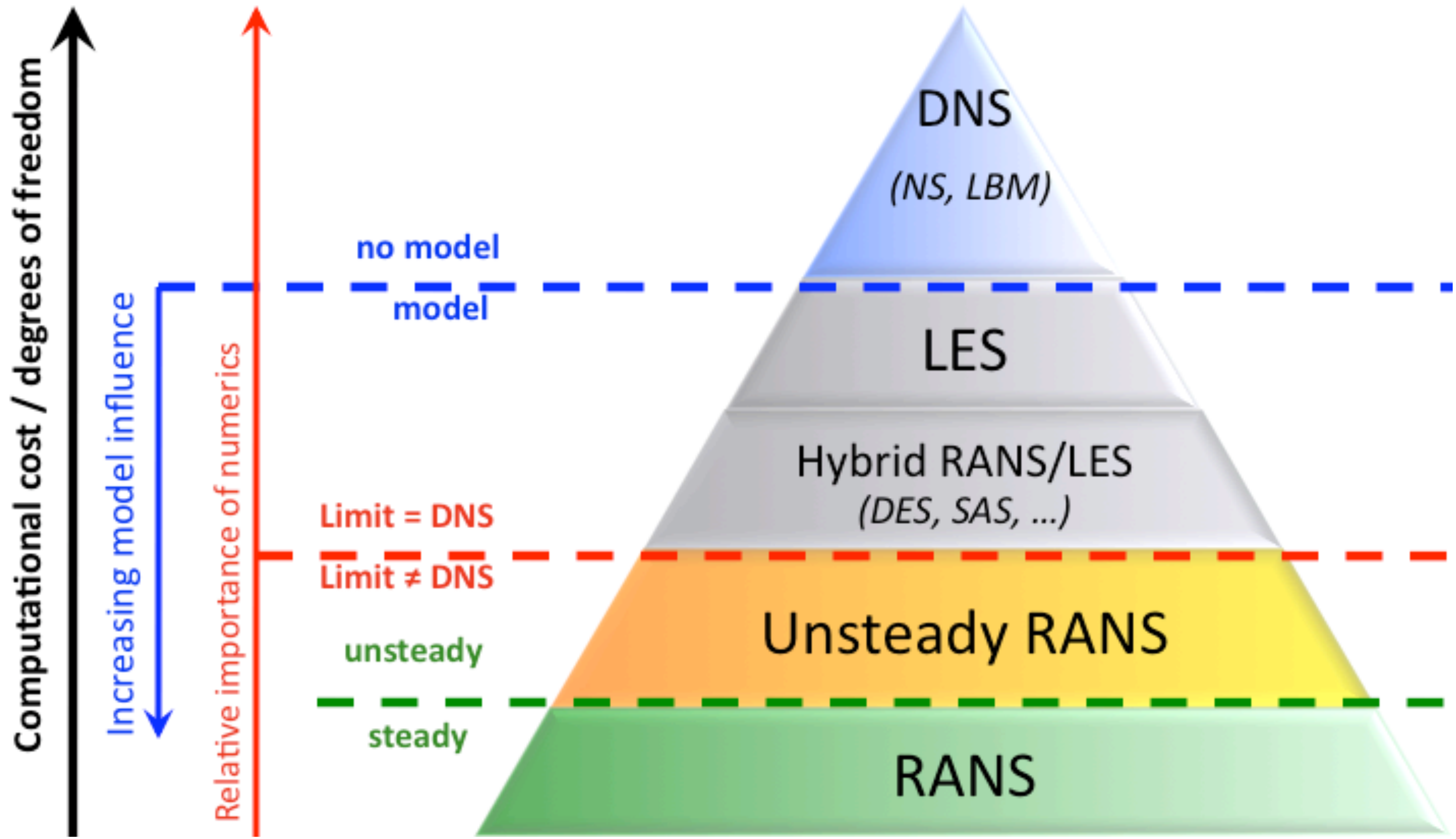
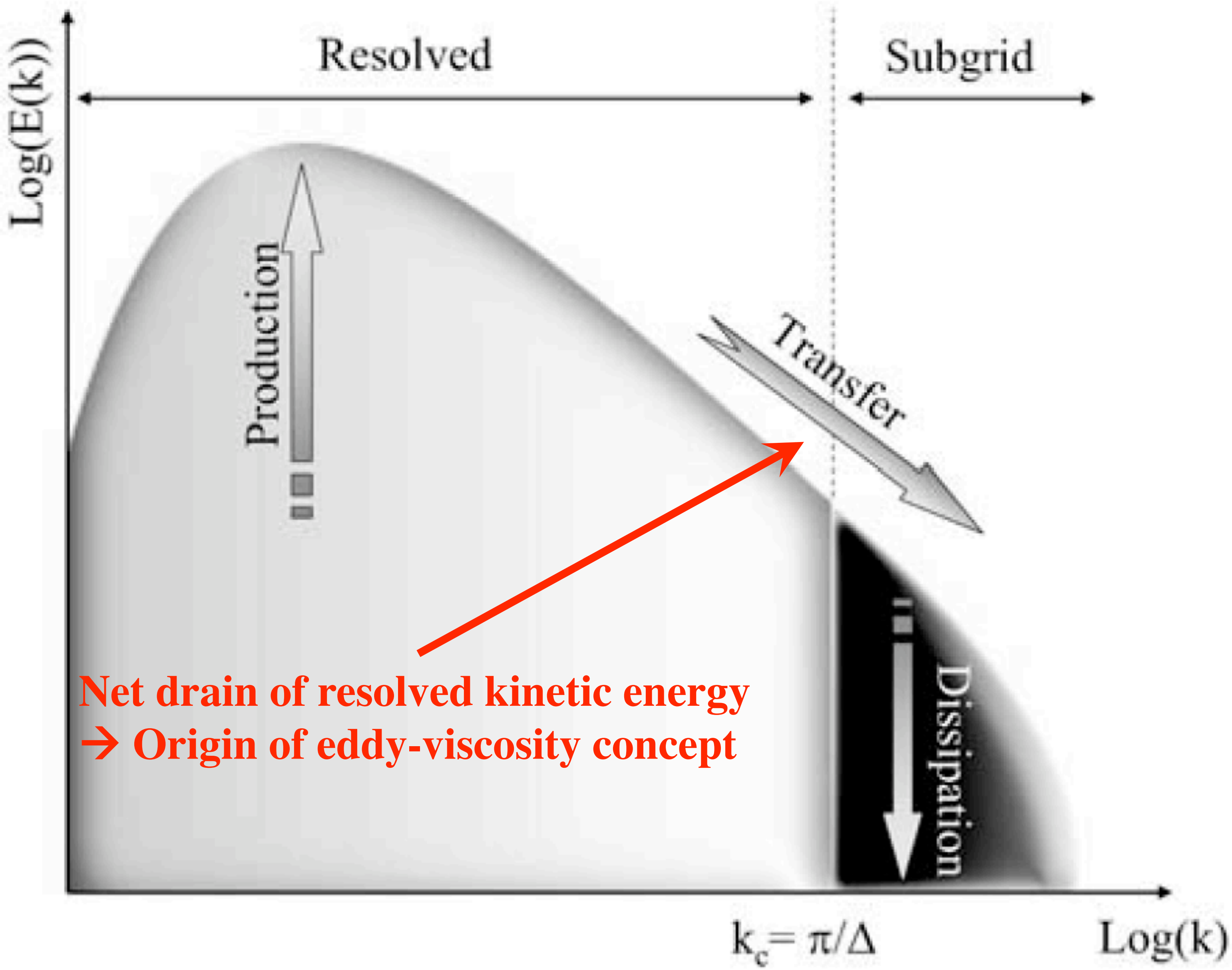


Schematic view at LES dynamics



LES grid resolution

Most subgrid models are eddy-viscosity models

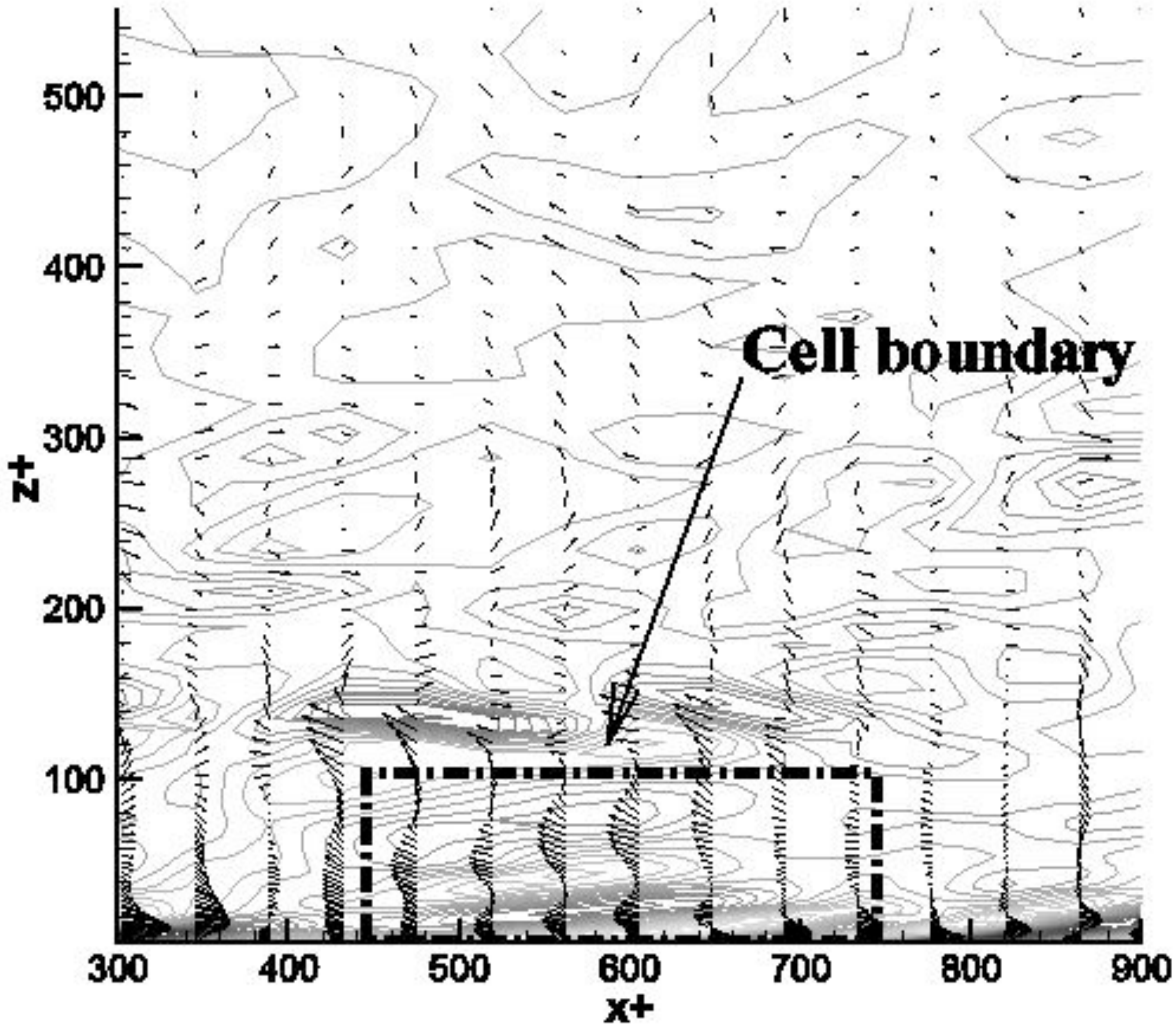
→Turbulence production mechanisms must be directly captured

→ very fine grid resolution in TBL

→ most unsteady effects directly captured in fine-grid LES

$$\Delta x^+ \leq 50 - 100, \Delta y^+ \leq 12, \delta z^+ = 1(min)$$

LES wall models



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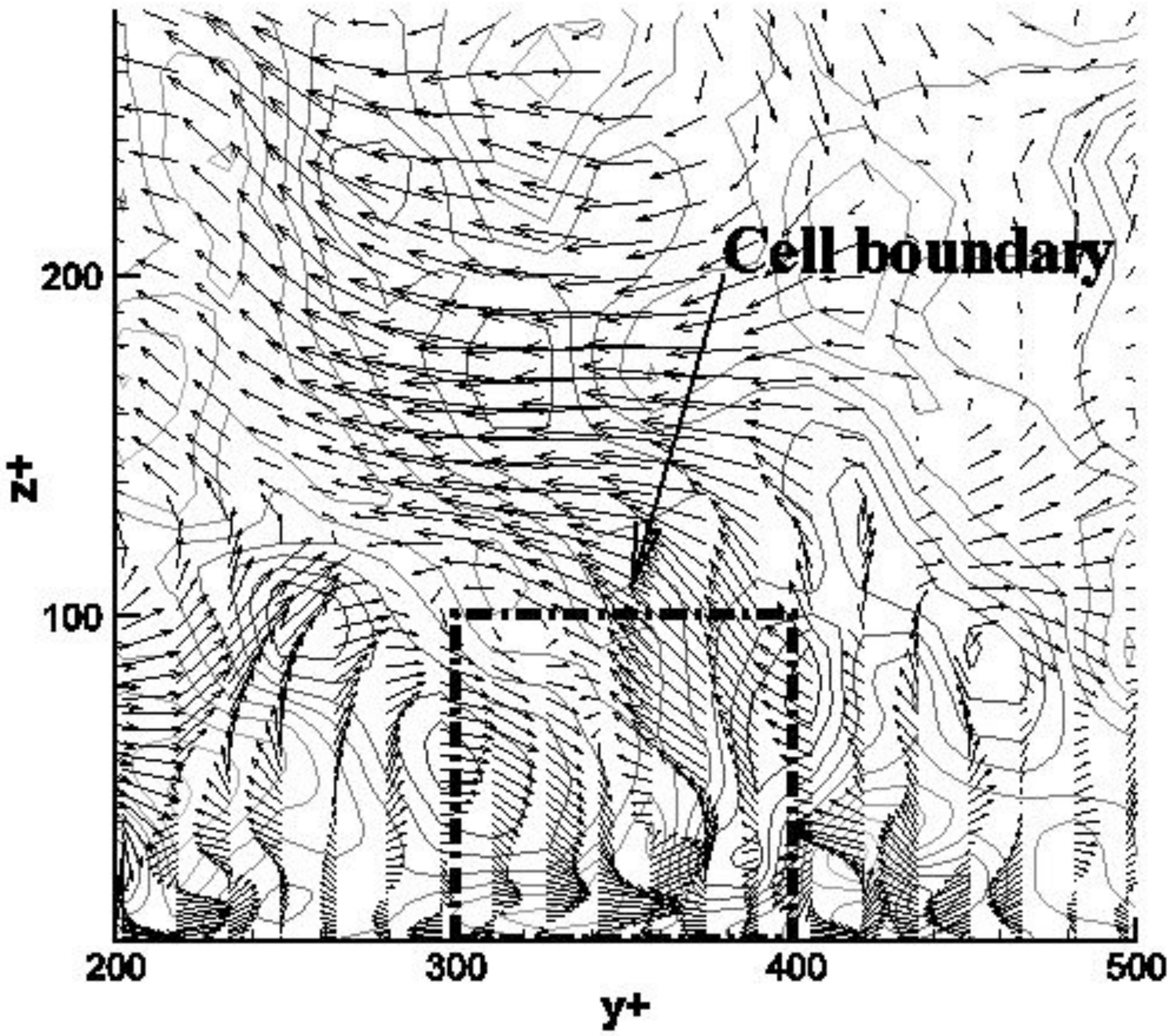
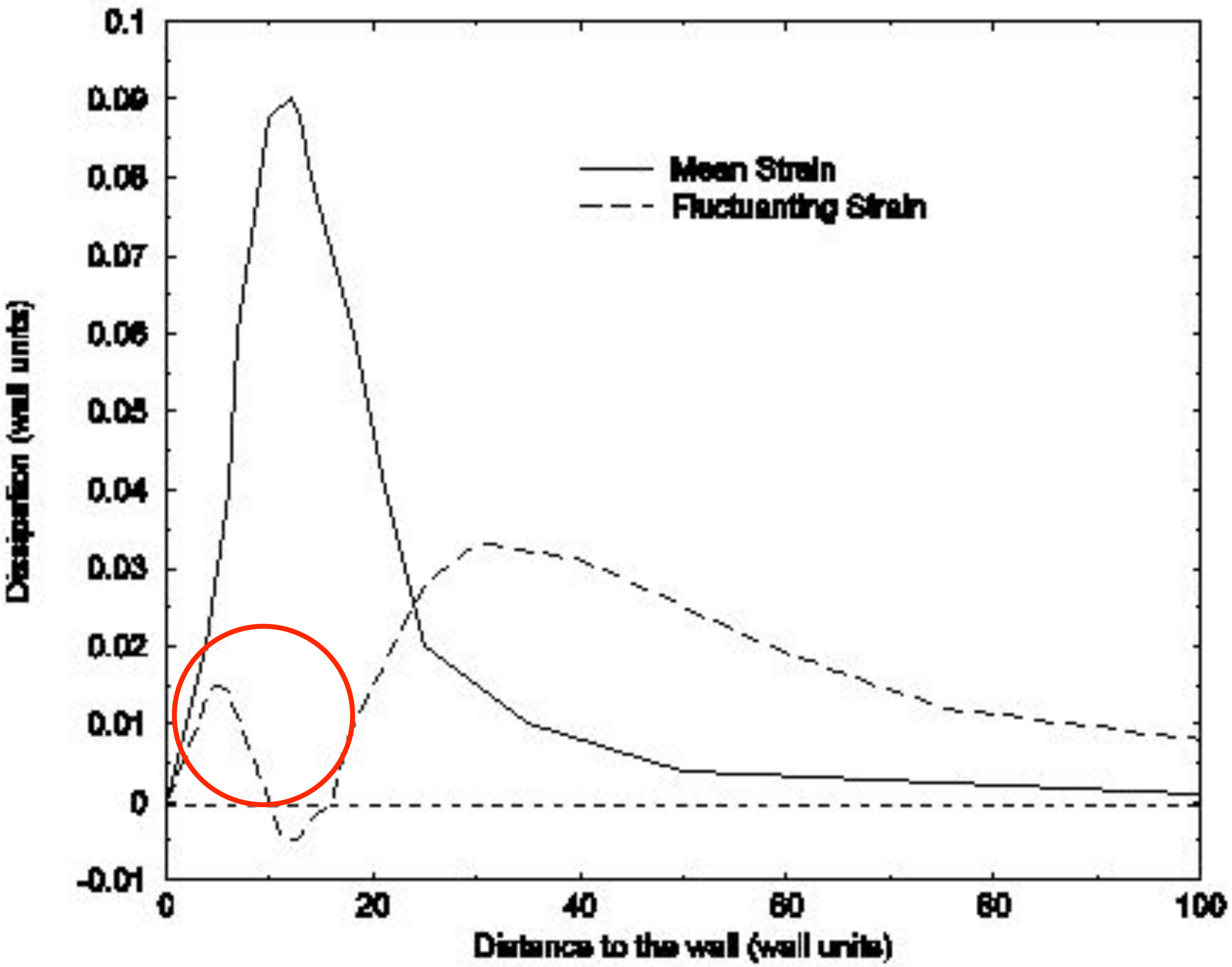


Subgrid dissipation splitting

(Härtel & Kleiser)



Main approaches

- Find an empirical explicit relation between the skin friction and the velocity at the first off-wall point *(Schumann, Grötzbach, Wengle … 1970s)*
  - Algebraic model
  - Based on equilibrium boundary layer mean flow
- Solve a boundary layer equation within the first grid cell *(Balaras et al., CTR group… 1990s)*
  - Gain: pressure assumed to be constant in the wall-normal direction
  - More general

TBL approach

Basis: streamwise momentum TBL equation

$$\frac{\partial}{\partial y}\left((\nu+\nu_t)\frac{\partial u}{\partial y}\right)=F,$$

$$\nu_t=\mathscr{L}_m^2\left|\frac{\partial u}{\partial y}\right| \qquad \mathscr{L}_m=(\kappa y)\left(1-\exp\left(-\frac{y^+}{26}\right)\right)$$

$$(\mathscr{L}_m(y))^2\left|\frac{\partial u}{\partial y}\right|\frac{\partial u}{\partial y}+\nu\frac{\partial u}{\partial y}=Fy+\left.\frac{\partial u}{\partial y}\right|_0$$

Cont'd

$$(\mathscr{L}_m(y^+))^2\left|\frac{\partial u^+}{\partial y^+}\right|\frac{\partial u^+}{\partial y^+}+\frac{\partial u^+}{\partial y^+}=F^+y^++1$$

	$F<0$
$\tau_w<0$	Impossible (due to the constraint that the velocity must be positive at the first cell off-wall)
$\tau_w>0$	$u^+(y^+)=\int_0^{y^*}\frac{2\left(1-\frac{\xi}{y^*}\right)}{1+\sqrt{1-4\mathscr{L}_m^{+2}(\xi)\left(1-\frac{\xi}{y^*}\right)}}d\xi$
	$u^+(y^+)=u^+(y^*)+\int_{y^*}^{y^+}\frac{2\left(1-\frac{\xi}{y^*}\right)}{1+\sqrt{1+4\mathscr{L}_m^{+2}(\xi)\left(1-\frac{\xi}{y^*}\right)}}d\xi$
	$F\geqslant 0$
$\tau_w<0$	$u^+(y^+)=-\int_0^{y^+}\frac{2\left(1+\frac{\xi}{y^*}\right)}{1+\sqrt{1+4\mathscr{L}_m^{+2}(\xi)\left(1+\frac{\xi}{y^*}\right)}}d\xi$
	$u^+(y^+)=u^+(y^*)+\int_{y^*}^{y^+}\frac{2\left(1+\frac{\xi}{y^*}\right)}{1+\sqrt{1-4\mathscr{L}_m^{+2}(\xi)\left(1+\frac{\xi}{y^*}\right)}}d\xi$
$\tau_w>0$	$u^+(y^+)=\int_0^{y^+}\frac{2\left(1-\frac{\xi}{y^*}\right)}{1+\sqrt{1+4\mathscr{L}_m^{+2}(\xi)\left(1-\frac{\xi}{y^*}\right)}}d\xi$

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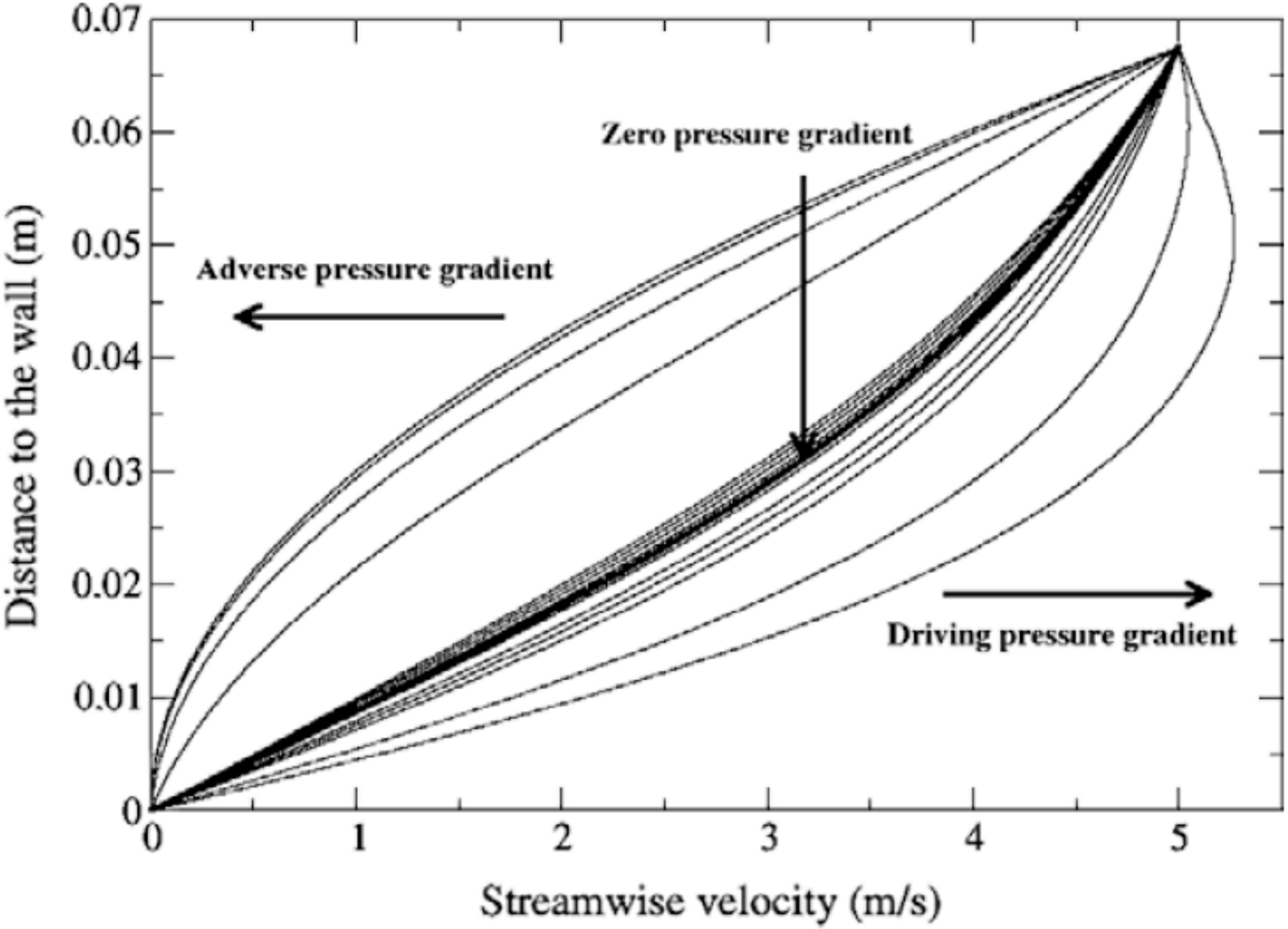

0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Distance to the wall (m)
0
1
2
3
4
5
Streamwise velocity (m/s)
Zero pressure gradient
Adverse pressure gradient
Driving pressure gradient

Thank You